NANOGrav: An Overview



NANOGrav

Jerry Sun 6/25/24





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Credit: NRAO

NANOGrav

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The NANOGrav 15 yr Data Set: Search for Signals from New Physics



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The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background

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The NANOGrav 15 yr Data Set: Observations and Timing of 68 Millisecond Pulsars

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The NANOGrav 15 yr Data Set: Bayesian Limits on Gravitational Waves from **Individual Supermassive Black Hole Binaries**

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What is **NANOGRAV**?

- The North America Nanohertz Observatory for Gravitational Waves (NANOGrav) is an international collaboration based in North America
- Our goal is to build a gravitational wave observatory in the nanohertz regime using an array of millisecond pulsars



~86 faculty ~78 graduate students ~36 postdoctoral researchers ~100+ undergraduate students annually via the STARS (Student Teams of Astrophysics Researchers) program

Across over 90 institutions!



An international collaboration dedicated to exploring the low-frequency gravitational wave universe through radio pulsar timing.

nanograv.org

- Millisecond pulsars are fantastic for a few reasons:
 - Incredibly stable rotational frequencies
 - Consistent pulse profile over long timescales
- GWs perturb the distance between the Earth and the pulsar → pulse arrival times change (timing residuals)
- We can detect patterns in these deviations of arrival times to infer gravitational waves



There are also many scientists in NANOGrav focused on understanding millisecond pulsars with excruciating detail and improving the precision of radio astronomy

This work would not be possible without their expertise

Advanced understandings of:

- Pulsar populations
- Interstellar medium effects
- Neutron star physics



Many of these pulsars are found by undergraduates who classify candidates from huge surveys looking for pulsars!

Thank you undergrads

C:







Credit: R. Hurt /Caltech-JPL NASA

Credit: NASA Goddard Flight Center



Credit: NASA Goddard Flight Center

Currently, we're timing 68 pulsars with up to 15 years of data!



• AO 327 MHz	• AO 2100 MHz	• VLA 1400 MHz
• • AO 430 MHz	• GBT 800 MHz	 VLA 3000 MHz
• AO 1400 MHz	••GBT 1400 MHz	

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Gabriella Agazie et al 2023 ApJL 951 L9

Pulsars are sensitive to the GWs expected to originate from supermassive black hole binary systems (SMBHBs)

We can complement other GW detectors by covering this section of GW parameter space



Why are we doing this?

If we look back in time, things are small!

Now things are big.



Why are we doing this?

If we look back in time, things are small!

Now things are big.

They merged! (probably)



Stochastic background (binary ensemble) 100s of ns) **Continuous wave** (single binary) Timing residual (10s Burst (merger event) Timing data span (10 years)



The background helps us learn more about galaxy formation through these mergers







Gabriella Agazie et al 2023 ApJL 951 L8

Continuous GWs create a sinusoidal timing residuals as the shift the Earth and all the pulsars around



We expect to find a few single sources that stick up above the GW background!





100s of ns)

Timing residual (10s

The momentum carried away by the gravitational waves can also be the source for nonlinear gravitational wave **memory**



The momentum carried away by the gravitational waves can also be the source for nonlinear gravitational wave **memory**

In PTA data, as a memory wavefront crosses the Earth, all the pulsars "glitch" at the same time





Credit: Marc Favata

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https://doi.org/10.3847/1538-4357/ad0726



The NANOGrav 12.5 yr Data Set: Search for Gravitational Wave Memory



There's also a lot of other physics we can contribute to as well. Constraints on nanohertz-frequency GWs can provide constraints on:

- Cosmic strings
- Short-duration GW bursts
- Anisotropies in the universe
- Primoridal black holes
- Cosmological phase transitions
- Non-Einsteinian general relativity
- Dark matter
- ... and more!

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The NANOGrav 15 yr Data Set: Search for Signals from New Physics

Abstract

The 15 yr pulsar timing data set collected by the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) shows positive evidence for the presence of a low-frequency gravitational-wave (GW) background. In this paper, we investigate potential cosmological interpretations of this signal, <u>specifically cosmic inflation</u>, <u>scalar-induced GWs</u>, first-order phase transitions, cosmic strings, and domain walls. We find that, with the exception of stable cosmic strings of field theory origin, all these models can reproduce the observed signal. When compared to the standard interpretation in terms of inspiraling supermassive black hole binaries (SMBHBs), many cosmological models seem to provide a better fit resulting in Bayes factors in the range from 10 to 100. However, these results strongly depend on modeling assumptions about the cosmic SMBHB population and, at this stage, should not be regarded as evidence for new physics. Furthermore, we identify excluded parameter regions where the predicted GW signal from cosmological sources significantly exceeds the NANOGrav signal. These parameter constraints are independent of the origin of the NANOGrav signal and illustrate how pulsar timing data provide a new way to constrain the parameter space of these models. Finally, we search for deterministic signals produced by models of ultralight dark matter (ULDM) and dark matter substructures in the Milky Way. We find no evidence for either of these signals and thus report updated constraints on these models. In the case of ULDM, these constraints outperform torsion balance and atomic clock constraints for ULDM coupled to electrons, muons, or gluons.

Looking ahead...

NANOGrav is part of a global effort to time pulsars across the entire sky with the IPTA

Currently, there's a lot of ongoing work to combine several PTA data sets to create the **most sensitive PTA data set** with over **110 pulsars** with up to **18 years** of data









Conclusion

Pulsar timing GW experiments like NANOGrav give us a unique view into large-scale gravitational phenomena

These large-scale phenomena inform our understanding of how the universe became the way it is today

We have been working to test general relativity and contribute to understandings of exotic phenomena in nature!

Conclusion

Pulsar timing GW experiments like NANOGrav give us a unique view into large-scale gravitational phenomena

These large-scale phenomena inform our understanding of how the universe became the way it is

We have been working to test general relativity and contribute to understandings of exotic phenomena in nature!

... Also it leads to a lot of pulsar/neutron star science and radio astronomy which I am criminally under-informed on. The people who study and time pulsars are heroes and I apologize for not discussing their work more here.

This work has been funded by NSF Grants 1430284 and 2020265