Final Report

Carter Anderson

September 2023

Abstract

Neutron stars are some of the most extreme and relativistic objects in the known universe, with densities well beyond those of typical atomic nuclei. The multi-messenger detection of GW170817, the first measured binary Neutron Star merger gravitational wave event has given new opportunities to investigate these stars and the physics that govern them. However, at the moment, the post-merger signal carrying the greatest impact of the star's nuclear matter hasn't been detected with current gravitational wave detector sensitivities leaving numerical relativity simulations as the only form of investigation. Consequently, we aim to improve this avenue with SpECTRE, the simulation software developed by the collaborative Simulating eXtreme Spacetimes group (SXS), working toward more physical simulations to better understand Neutron Star's, their gravitational wave emission events, and extreme density nuclear physics.

Introduction

In recent years, the detection of binary neutron star merger events has presented an opportunity to learn about the exotic physical properties that are irreplicable in laboratory experiments. These merger events, in which a pair of orbiting neutron stars (NS) collide, are detectable on Earth through the aLIGO gravitational wave detectors.

There are, however, some challenges to detecting binary neutron star mergers; the signals from the gravitational radiation are typically separated into two parts: a *pre* and *post-merger* signal. The pre-merger signal is detectable with well-researched information on the responsible bodies, while the post-merger signal carries the balance of the information necessary to learn about the coalesced object. In the example of GW170817, the first such event to be detected, the pre-merger information was used to measure the radii of the coalescing neutron stars and to place meaningful constraints on both their equations of state (EoS) and their tidal parameters. But no post-merger signal was detected, presumably hidden within detector noise at frequencies beyond the optimized band of the detectors.

Research has identified needed improvements to access post-merger data. For example, [1] proposes that this can be achieved through improving detector sensitivity in the kilohertz range by 2 to 3 times current capabilities through the use of quantum squeezing, which by the Heisenberg uncertainty principle, allows us to better determine a variable at the cost of another. Observing run four, (abbreviated O4) has started and includes some techniques to improve sensitivities (including light squeezing). New signals from O4 could contain post-merger frequencies, providing the ability to improve models and understanding of BNS merger events.

Although we may soon have the technical capabilities to detect merger and post-merger signals, they will be faint and governed by physics we don't yet know everything about. The way we probe the hyper-exotic unknown physics behind NSs is through simulation.

SpECTRE

In the first weeks of my project, we shifted to focus on Simulating eXtreme Spacetime's (SXS) numerical relativity code, SpECTRE. SpECTRE is designed to investigate the universe's most complex astrophysics, including neutron star mergers which is the topic of my project.

My work on the SpECTRE algorithm began with expanding the library of simulatable equations of state (EoS), which describe stellar properties like mass, radius, density, and matter distribution throughout a given star. Our Sun and all other main-sequence stars are polytropic stars, meaning they are governed by a polytropic equation of state. Polytropic equations of state are relatively simple and easily modeled. However, neutron stars hold the densest matter in the universe and are governed by different processes.

Detected binary neutron star merger events have helped eliminate some potential EoS choices for neutron stars, but have not definitively asserted which equation of state best describes neutron stars. However, the LIGO and VIRGO gravitational wave projects are currently observing with unprecedented sensitivity, collecting data searching BNS events. Once detected, comparison with physical models will further inform how neutron stars and ultra-dense matter behaves.

Motivating the creation of more physical models. I began by expanding the possible EoS SpECTRE can simulate, adding two additional EoS's: Enthalpy and Spectral.

I first added the Spectral EoS, a generalized Fourier series presented by [2]. The Spectral, SLyΓ2, equation of state creates a smoother description of extreme-density matter where gravity crushes nuclei into a soup of free protons, neutrons, and electrons.

The second was the Enthalpy EoS presented in [3] which parameterizes stellar structure in terms of specific enthalpy. Specific enthalpy is the internal energy within the star and can be reflected in a stellar EoS as the energy that would be required to inject a unit of rest mass into the star at that given point without breaching the star's equilibrium. This approach proves quite powerful in simulating NSs as it can be implemented as a hybrid EoS, stringing together equations to better customize the model for densities in the stellar interior.

Four mass radius curves are shown below for typical NS equations of state. Shown is typical Polytropic model for a neutron star, a SLyΓ2 (Spectral) model, the DBHF (Enthalpy) EoS followed by a hybrid DBHF_2507 model featuring a phase transition.

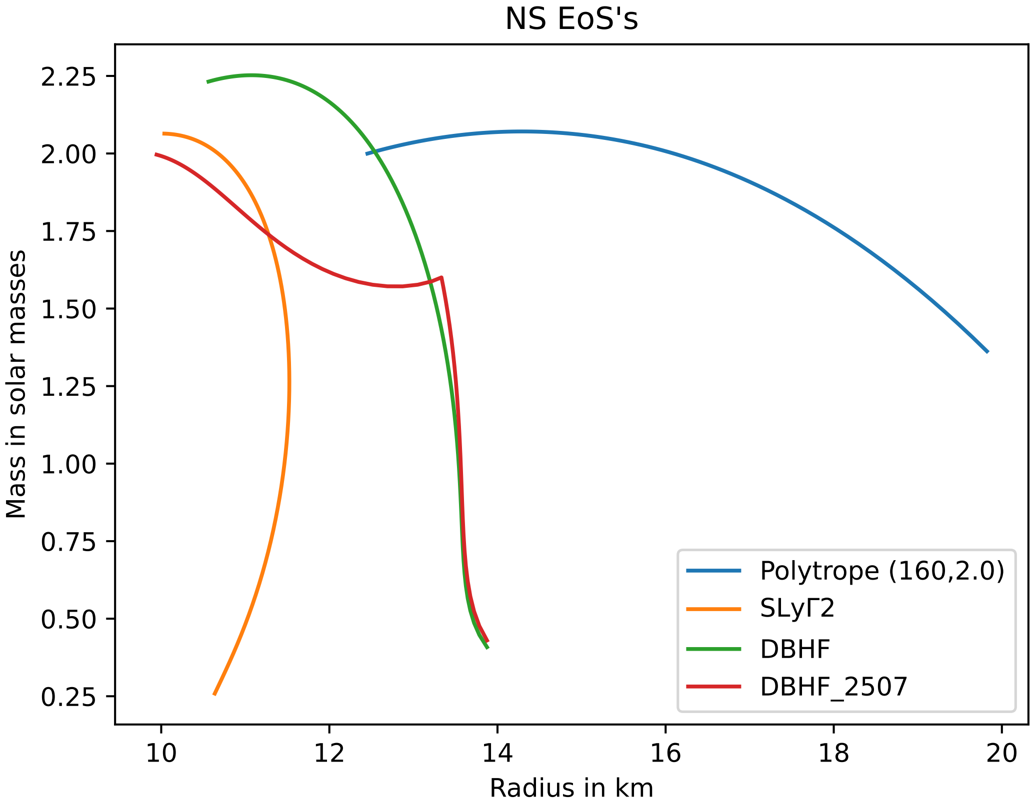

Figure 1: A comparison of various typical neutron star equations of state. SLyΓ2, DBHF, and DBHF_2507 are all now possible to simulate using SpECTRE

With bindings added to allow for simulations with these equations of state, I tried to simulate phase-transitions of the matter at the centers of the colliding stars. Phase transitions generally are non-smooth macroscopic changes in matter, and may occur in a NS. When densities within star grow get high enough new degrees of freedom could be introduced, new particles may form at rapid rates and neutrons could collapse into their constituents. What densities could cause such a transition is itself unknown depending in physics we don't fully understand yet. If however ultra dense regions create a transition in merging neutron stars, we could one day detect the effects in the merger and ring down of BNS collision and perhaps further constrain NS EoS. We managed to create a phase-transition within the merging stars making progress towards more physically accurate simulations. The figure below shows images from the simulation with phase transitions:

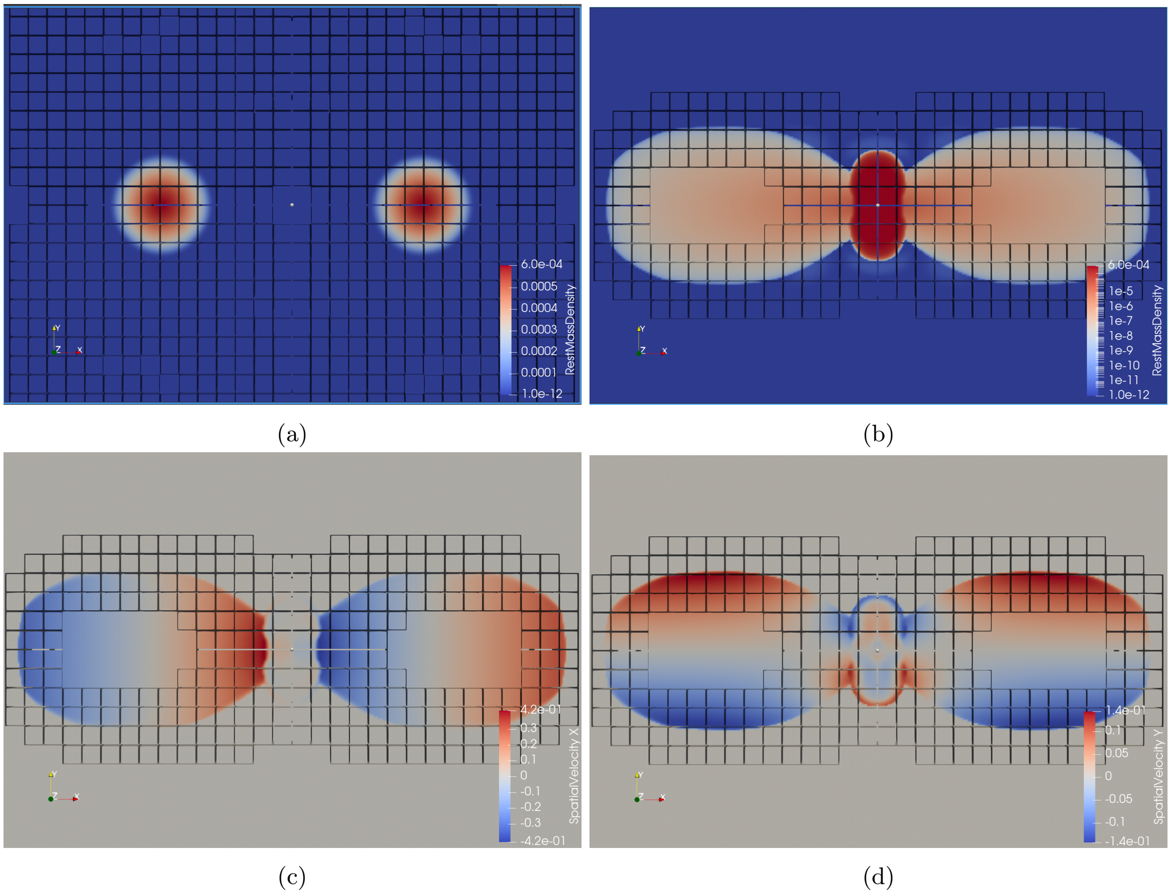

Figure 2: Images from a simulation featuring a phase transition. Image *a* shows the star initial configuration, *b* shows the star post-merger where a highly rigid core ejects matter outwards during the formation of a new remnant, *c* and *d* show velocities of the ejecta, where notably matter is moving at significant percentages of the speed of light.

Analysis

The next goal was to analyze the efficiency of the added EoS, investigating their impact on simulation efficiency. The additional EoS are designed to be more physically accurate, so we were interested in the computational cost of these changes. Shown in figure 3 below is a simulation profile tracking how many finite difference cells are used throughout the simulation and the effect of both using finite difference as well as effect of changing back an forth between FD and DG on the simulation efficiency. At the bottom is the of the maximum simulation density in terms of the initial central density to compare the counts to state of the stars. The two most obvious conclusion from this profiling is that first increasing the finite difference cell count decreases simulation efficiency and that second that a denser (more compact) star uses fewer FD cells and is more efficiently simulated. The added EoS both proved more efficient, performing comparable simulations faster and allowing more matter-containing computational cells to

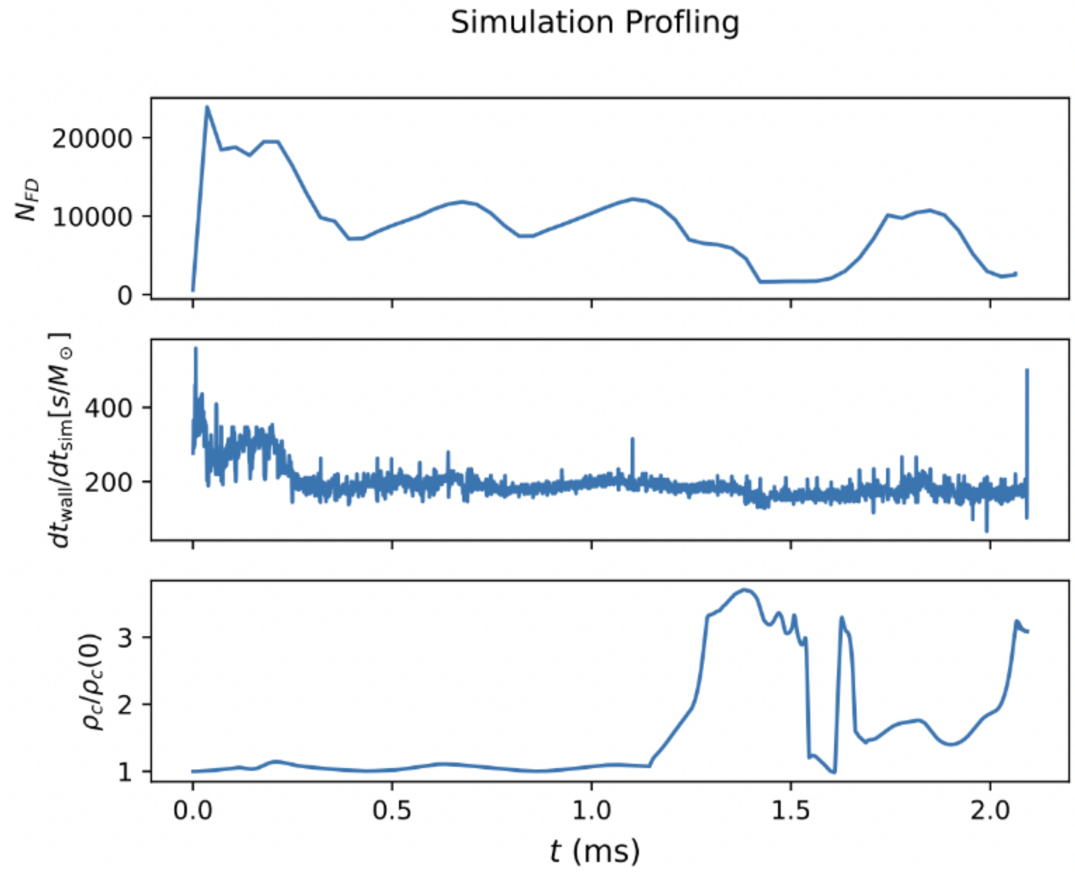

Figure 3: A SpECTRe Simulation profile. *Top:* Number of computational cells using finite difference. *Middle:* Wall time per simulation step. *Bottom:* Maximum density (in terms of initial maximum density).

use the spectral Discontinous-Galerkin mathematical method that the Polytropic EoS had. This we found was due to a few reasons, the first was that the implementation of the new EoSs are smooth themselves, so they perform implicitly better under bases-based numerical method, the second was that the TCI (Trobled Cell Indicator) parameter was both computationally more expensive than originally believed and not set optimally.

Acknowledgments

I would like to thank my mentors Isaac Legred and Katerina Chatziioannou for all their guidance, help and support. I thank the LIGO Laboratory and the Caltech Student-Faculty Programs office for the opportunity to participate in the LIGO SURF program and for their support throughout the summer. This work was funded by the National Science Foundation.

References

[1] A. Torres-Rivas, K. Chatziioannou, A. Bauswein, and J. A. Clark, "Observing the post-merger signal of gw170817-like events with improved gravitational-wave detectors," *Physical Review D*, vol. 99, no. 4, p. 044014, 2019.

[2] L. Lindblom, "Spectral representations of neutron-star equations of state," *Phys. Rev. D*, vol. 82, p. 103011, Nov 2010.

[3] I. Legred, Y. Kim, N. Deppe, K. Chatziioannou, F. Foucart, F. m. c. Hébert, and L. E. Kidder, "Simulating neutron stars with a flexible enthalpy-based equation of state parametrization in spectre," *Phys. Rev. D*, vol. 107, p. 123017, Jun 2023.