Linking the Population of Binary Black Holes with the Stochastic Gravitational-Wave Background

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1 Introduction

Gravitational-waves (GWs) are the product of large scale, highly energetic events that present as perturbations in spacetime. GWs were first observed in 2015 by the Laser Interferometer Gravitational-Wave Observatory (LIGO), located in Livingston, Louisiana and Hanford, Washington, with the detection of GW150914, a binary black hole merger [1]. LIGO is joined by several other GW observatories, including Virgo in Italy, GEO600 in Germany, and KAGRA (Kamioka Gravitational-Wave Detector) in Japan.

LIGO takes the form of a Michelson interferometer, in which an incident laser beam is split into orthogonal reflected and transmitted beam components along the two arms of the detector. The beams are subsequently reflected back toward the beam splitter and recombined. During a GW event, the arms of the detector are compressed and rarefied, causing the two beams to shift out of phase and form a detectable interference pattern.

GW signals are categorized into continuous, compact binary inspiral, burst, and stochastic types. Continuous GWs are produced by large, rotating systems, such as neutron stars, and appear as a sinusoidal pattern of detector strain over long periods of time [6]. Compact binary inspirals arise from mergers of dense objects, such as black hole and neutron star mergers, and are characterized by a chirp signal in time-frequency space [4]. At the present time, LIGO has detected 90 GW events stemming from compact binary inspirals [6]. Burst GW sources include Type II supernovae and are measured on short time scales [2]. Finally, stochastic signals are the sum of numerous unresolved GW sources that form a GW background. LIGO has yet to detect continuous GW, burst, and stochastic signals.

The SGWB is divided into two categories: cosmological and astrophysical. Cosmological sources include events that occurred in the early Universe, such as inflation. In the case of inflation, rapid expansion drove the GWs at the time into a relatively uniform background. Astrophysical sources are comprised of individual events such as mergers and pulsars. Detector resolution limits cause these sources to appear unresolved, the signals of which then overlap to create a SGWB.

The SGWB is particularly important since the involved GWs originate from the very early Universe, not long after the Big Bang. Because the Universe at the time was opaque to photons, the SGWB is one of the only means of studying this era. In addition, correlating the population of binary black holes to the predicted SGWB reveals the significance of source events on the background and the influence of additional factors. Finally, an estimate of the SGWB constrains future searches to a more precise range and reveals information about the expected frequency spread.

2 Background

The sum of individually resolvable GW events predicts a measurable stochastic gravitational-wave background (SGWB). Models of the SGWB are not uniform across all frequencies. Rather, each frequency range exhibits a unique, detector-dependent signal.

The SGWB can be modelled by a power law of the following form:

$$\Omega_{\mathrm{GW}}(f) = \Omega_{GW}(f_{\mathrm{ref}}) \left(\frac{f}{f_{\mathrm{ref}}}\right)^{\alpha}, \tag{1}$$

where $\Omega_{\mathrm{GW}}(f)$ is GW energy density, f is frequency, and α is the spectral index of the signal. The GW energy density can be decomposed as follows:

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_{\mathrm{c}}} \int_0^{\infty} dz \frac{N(z)}{1+z} \left[f_{\mathrm{r}} \frac{dE_{\mathrm{GW}}}{df_{\mathrm{r}}}\right]_{f_{\mathrm{r}}=f(1+z)}, \tag{2}$$

$$\rho_{\mathrm{c}} = \frac{3H_0^2}{8\pi G}, \tag{3}$$

where ρ_c is critical density, $\dot{N}(z)$ is number of GW sources as a function of redshift, z is redshift, dE_{GW}/df_r is spectral energy density, f_r is rest frame frequency, $H_0 = 67.4$ km s^{-1} Mpc^{-1} is the Hubble constant, and $G = 6.67 \cdot 10^{-}11$ m^3 kg^{-1} s^{-2} is the universal gravitational constant. The integral of Equation 2 encompasses the entirety of the Universe's life. The components inside the integral multiply $N(z)$ by spectral energy density weighted by f. At $z = 0$, $f_r = f$, and $\Omega_{\mathrm{GW}}(f) = f(N_0/\rho_{\mathrm{c}})(dE_{\mathrm{GW}}/df)$. As a result, Ω_{GW} is proportional to $N(z)$.

Fractional energy density can be averaged over source parameters θ. In addition, $N(z)$ can be rewritten in terms of event rate, redshift and the Hubble parameter. Therefore, Equation 2 becomes the following after removing f from the integral:

$$\Omega_{\rm GW} = \frac{f}{\rho_{\rm c}} \int_0^{z_{\max}} dz \frac{\dot{N}(z)}{(1+z)H(z)} \left\langle \frac{dE_{\rm GW}}{df_{\rm r}} \Big|_{f_{\rm r}=f(1+z)} \right\rangle, \tag{4}$$

$$\left\langle \frac{dE_{\rm GW}}{df_{\rm r}} \right\rangle = \int d\theta p(\theta) \frac{dE_{\rm GW}(\theta; f_{\rm r})}{df_{\rm r}}, \tag{5}$$

where $H(z)$ is the Hubble parameter as a function of redshift.

Figure 1 depicts the predicted SGWBs across the frequency spectrum, classifying the signal by source:

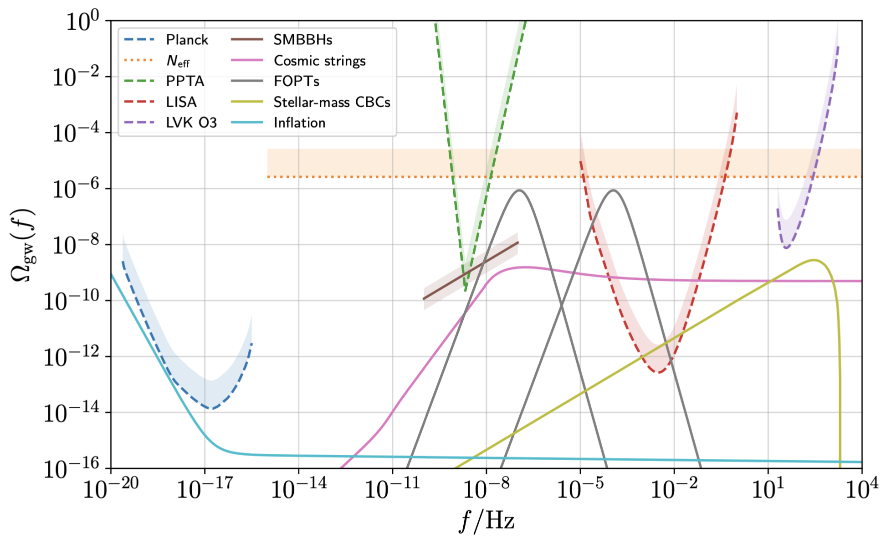

Figure 1: Predicted GW backgrounds from different sources across the frequency spectrum. Figure from [7].

Each color represents a different source of GWs. The project specifically focuses on the frequency sensitivity of LIGO (10 Hz to 10 kHz [5]), which corresponds to the very upper range of Figure 1. Binary black holes (BBH) are incorporated into the figure with the brown line, which represents the predicted background due to supermassive binary black holes (SMBBH). The project also includes stellar mass binary black holes, which are expected to be the majority of the BBH signal in the LIGO frequency range. The predicted SMBBH signal lies in the 10^{-10} Hz to 10^{-7} Hz range, which is outside of LIGO sensitivity, thus suggesting that BBHs between 10 Hz and 10 kHz are not supermassive.

3 Project Plan

The goal of the project is to provide both a theoretical and data-based estimate of the SGWB. The project can be divided into three distinct steps, outlined as follows:

1. Determine representative group of binary black holes to inject into data using source parameters.

   Step 1 involves writing code to generate a list of injections based on parameters such as mass, distance, and spin. Each line of the output represents one BBH and contains the parameter information. This code has already been written.

2. Calculate SGWB both theoretically and empirically.

   Step 2 is split into two parts, the theoretical and the numerical estimate of the SGWB. The theoretical estimate will be made using Equation 4 such that the output incorporates all BBH sources. The empirical estimate will be made using Equation 2 such that the output includes observed BBH sources. The Python library `pygwb` (Python-based library for gravitational-wave background-searches) will be used to inject the signals into the data [8]. Another student will be completing the theoretical estimate, and I will be completing the empirical estimate.

3. Compare theoretical and numerical SGWBs in order to determine the expected frequency range of the SGWB.

4 Appendix A

The Hubble parameter is a measure of the expansion of the universe in km s^{-1} Mpc^{-1}.

$$H(z) = H_0(\Omega_\mathrm{R}(1+z)^4 + \Omega_\mathrm{M}(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda)^{1/2}, \tag{6}$$

$$\Omega_\mathrm{R} = \Omega_\gamma + \Omega_\nu + \Omega_\mathrm{GW} + ..., \tag{7}$$

where $H(z)$ is the Hubble parameter, H_0 is the current Hubble parameter, z is redshift, and Ω is the energy density with R as the radiation component, M as the matter component, k as the curvature, and Λ as the cosmological constant, representative of dark energy. R is composed of photons, neutrinos, and GWs. M is composed of baryons and cold dark matter. The current value of H, H_0, is approximately equal to 67.4 km s^{-1} Mpc^{-1} [3], though the value varies across literature.

The quantity Ω_R is particularly notable at high redshift, which is concurrent with the radiation-dominated era of the cosmological timeline, suggesting that Ω_GW becomes a measurable quantity when probing the early Universe.

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