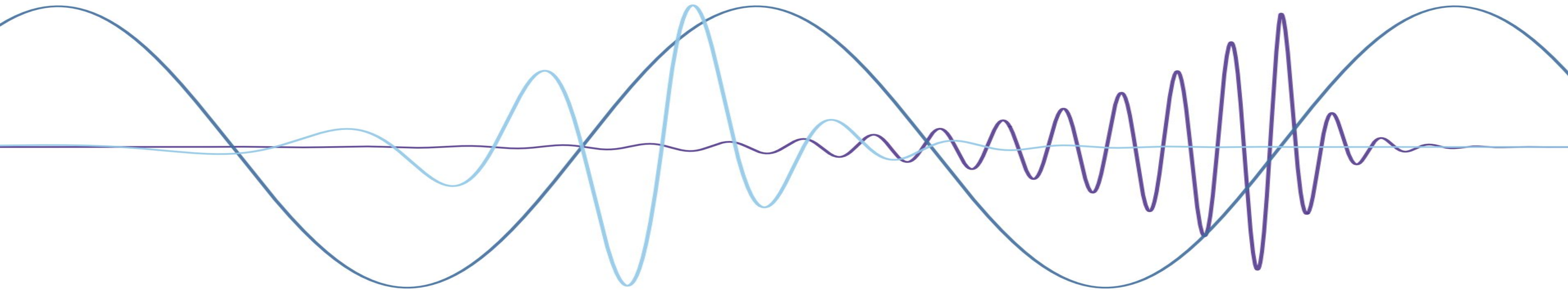
Linking the Population of Binary Black Holes with the Stochastic Gravitational-Wave Background

Olivia Laske
Mentors: Patrick Meyers and Arianna Renzini



Outline

1. PROJECT GOALS
2. BACKGROUND
3. GRIDDED METHOD
4. MONTE CARLO METHOD
5. COMBINED METHOD
6. RESULTS

Project Goals

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  - ➢ Theoretical approach with precomputed mass grids and probability distributions (**Gridded Method**)
  - ➢ Monte Carlo integration with simulated data (**Monte Carlo Method**)

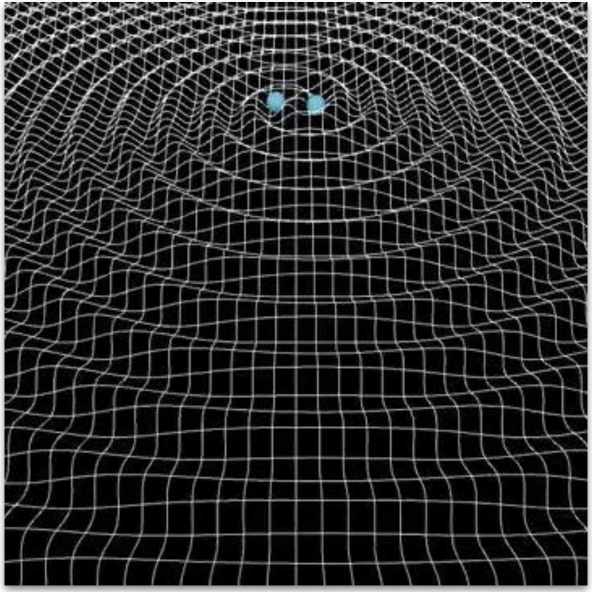
Project Goals

- Understand how different parameters affect the SGWB
  - BBH mass
  - Merger rate

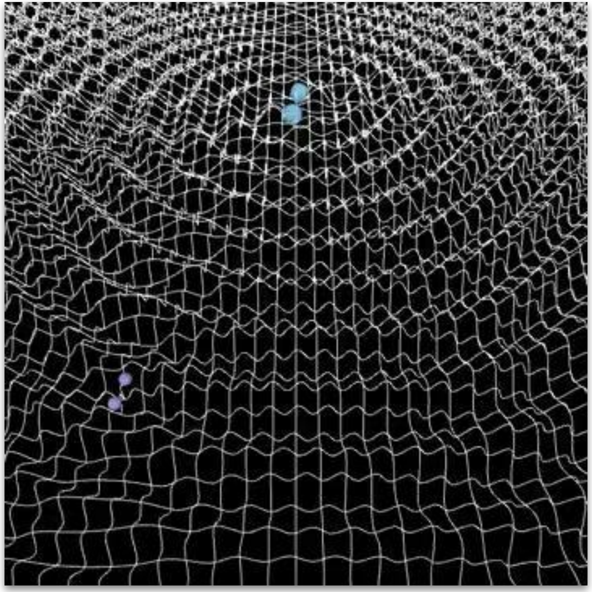
Background

What is the SGWB?

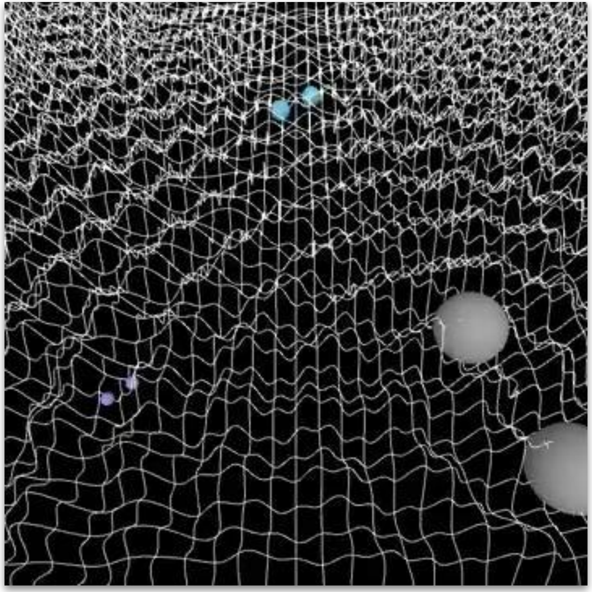
- Composed of overlapping unresolved waveforms



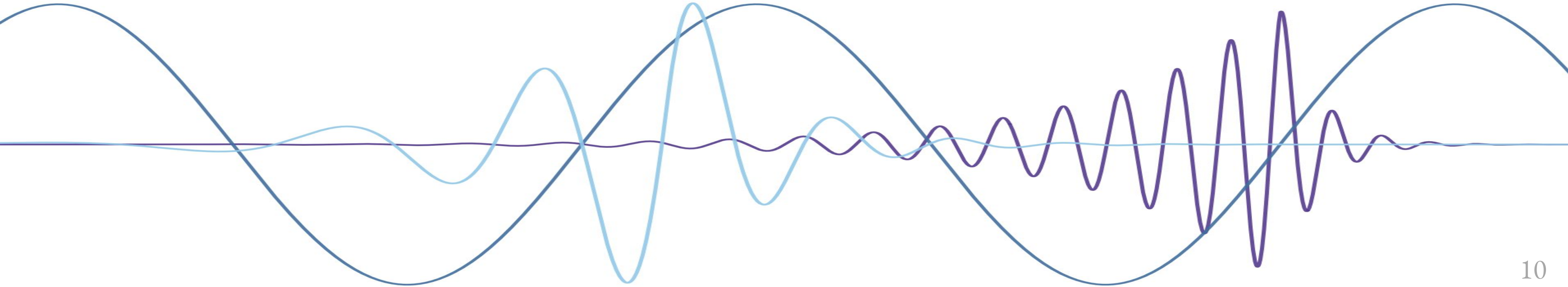
$n = 1$



$n = 2$



$n = 3$



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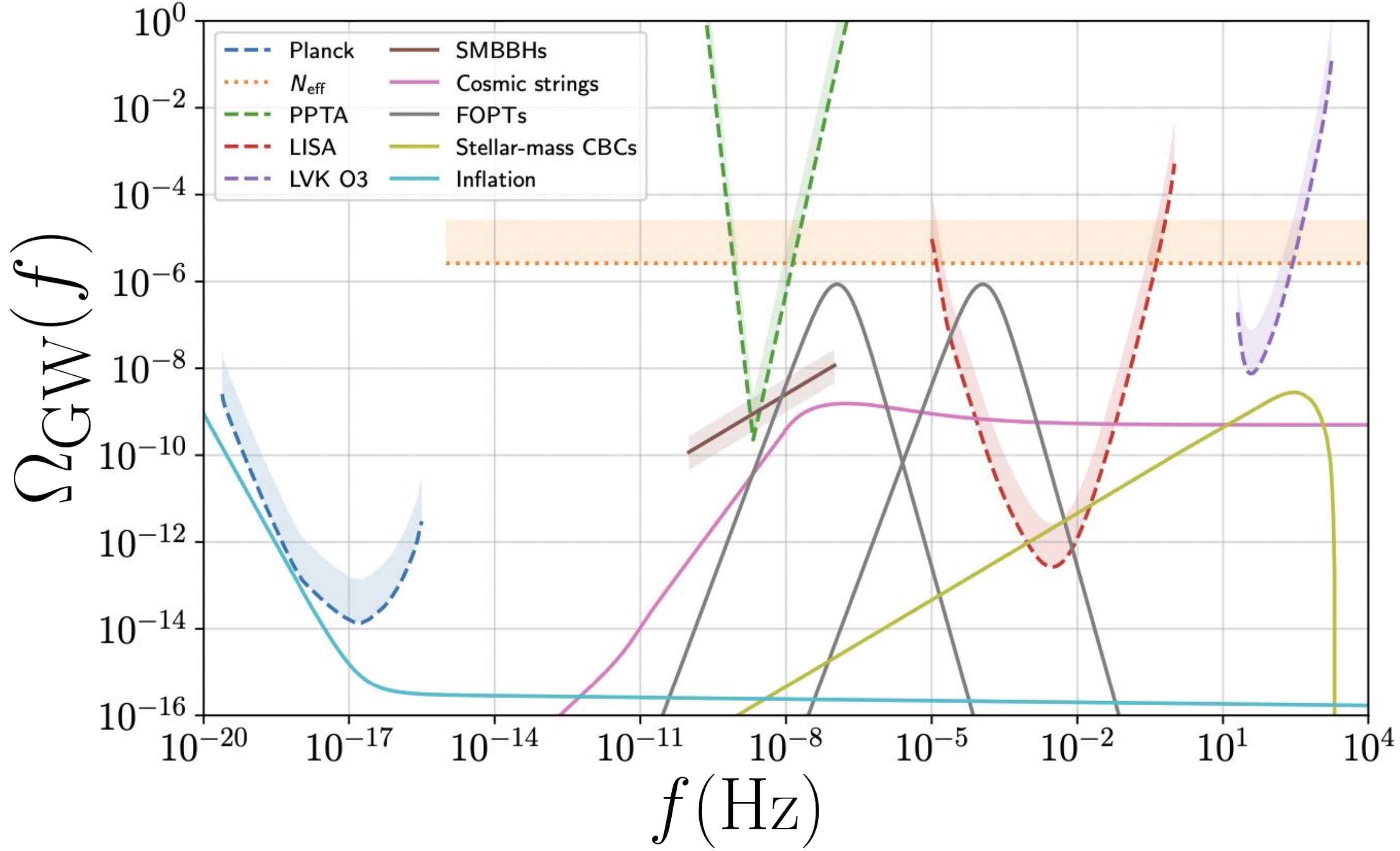
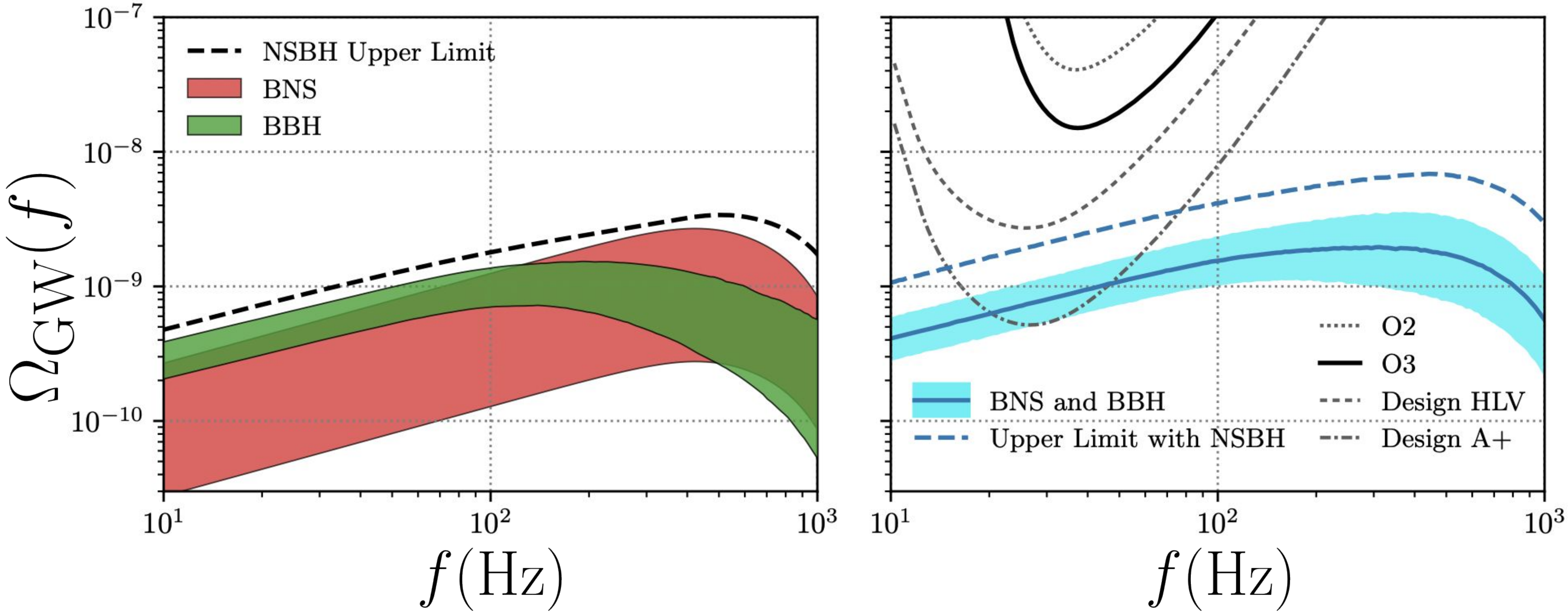


Source: Renzini et al. (2022)

What is the SGWB?

- ❖ Composed of overlapping unresolved waveforms
- ❖ Categorized into cosmological and astrophysical
- ❖ Varies depending on frequency band and source
- ❖ Currently undetectable in the LIGO frequency band



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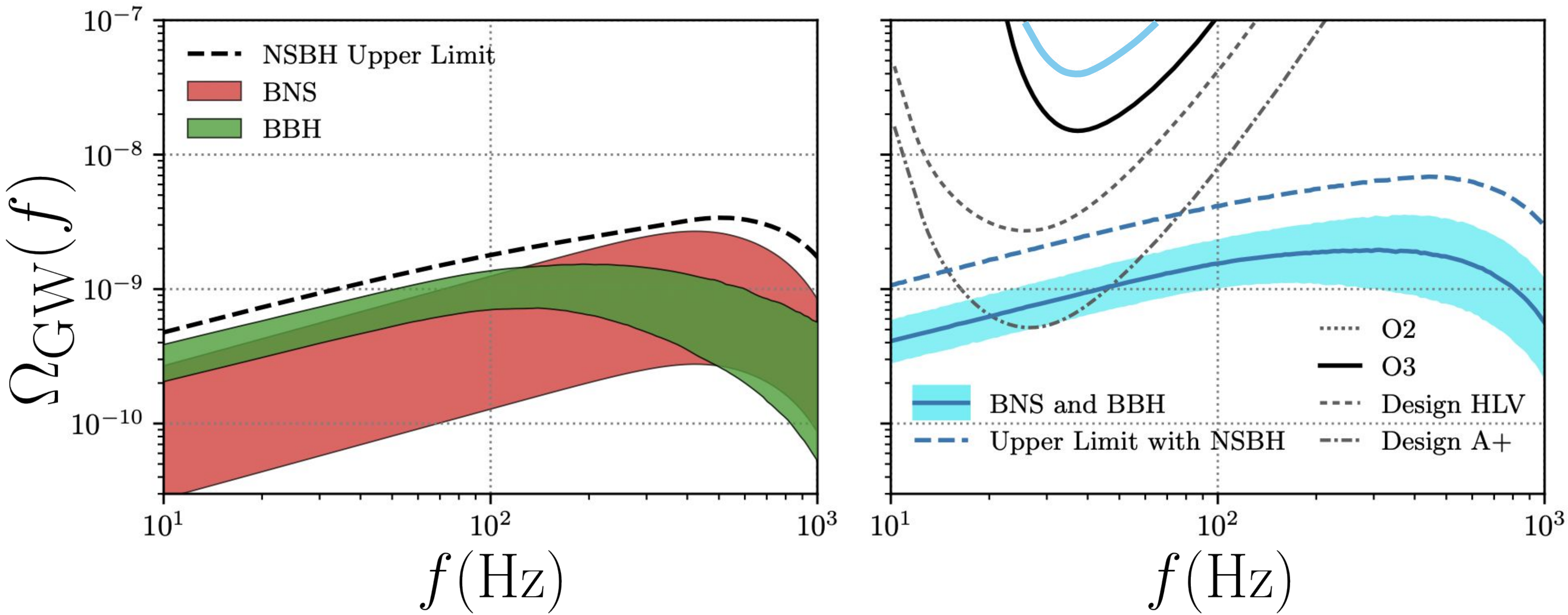

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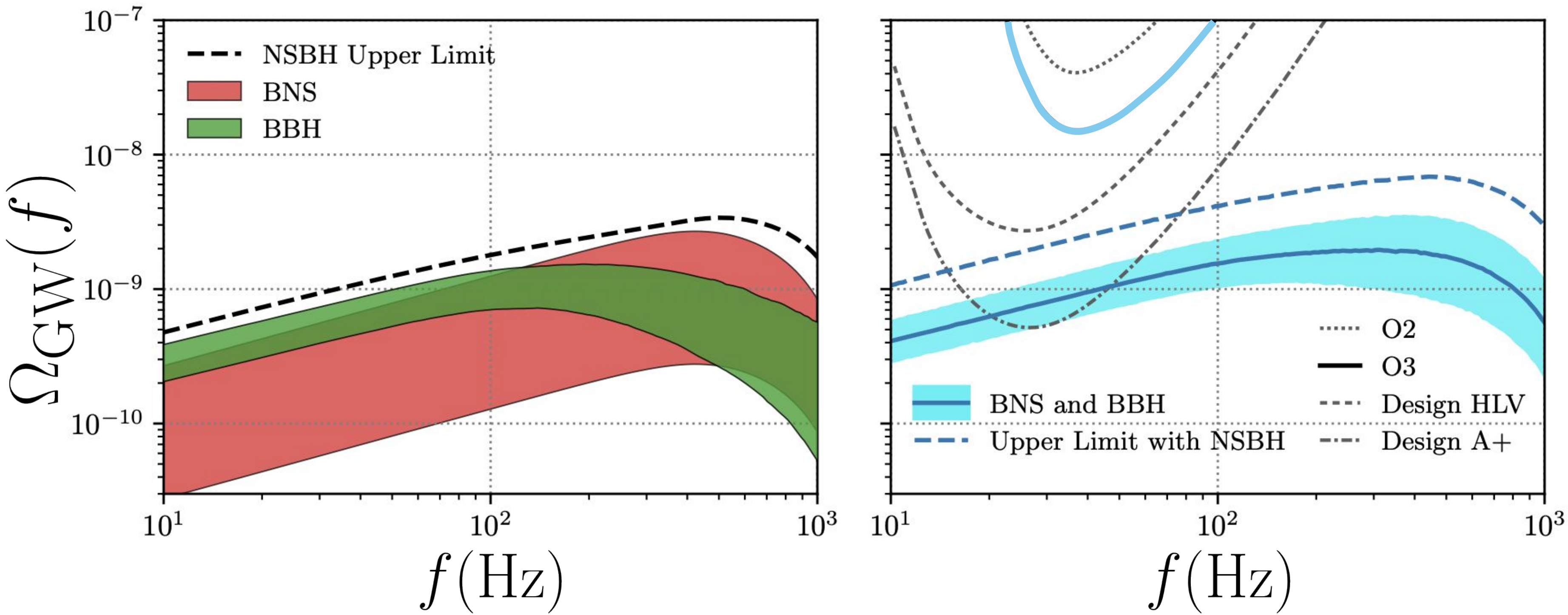

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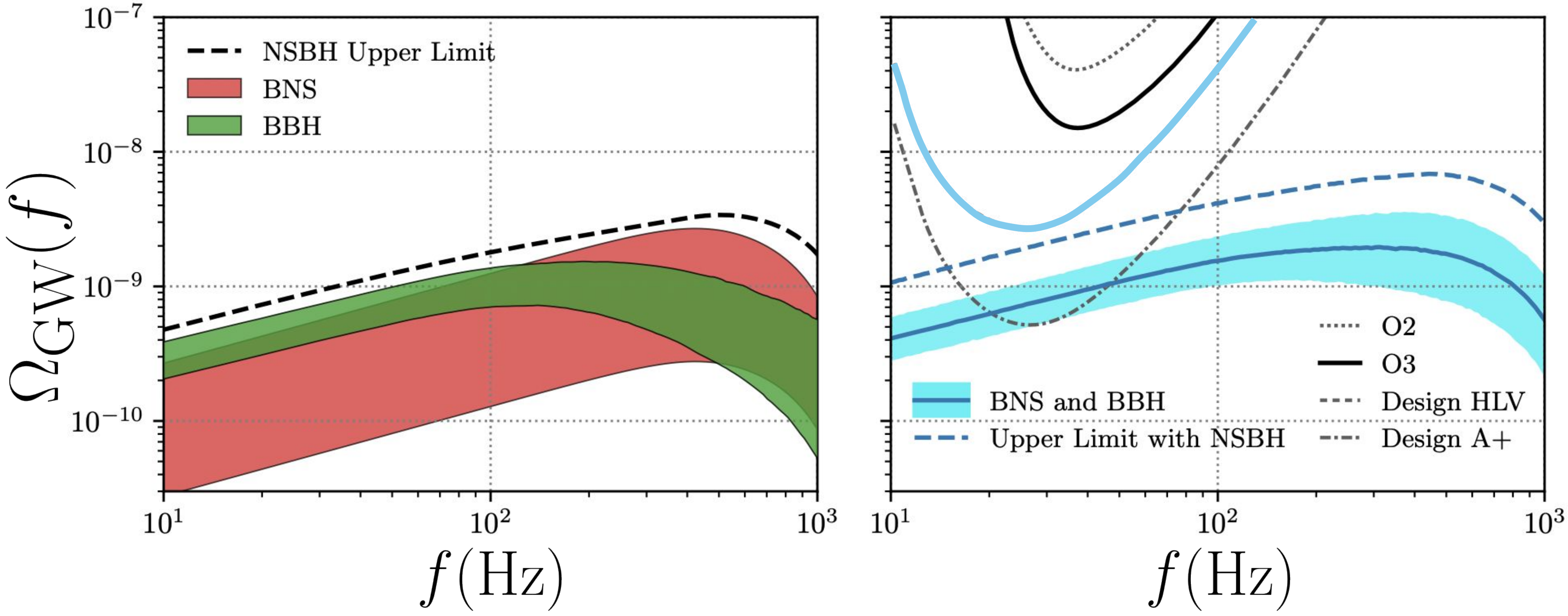

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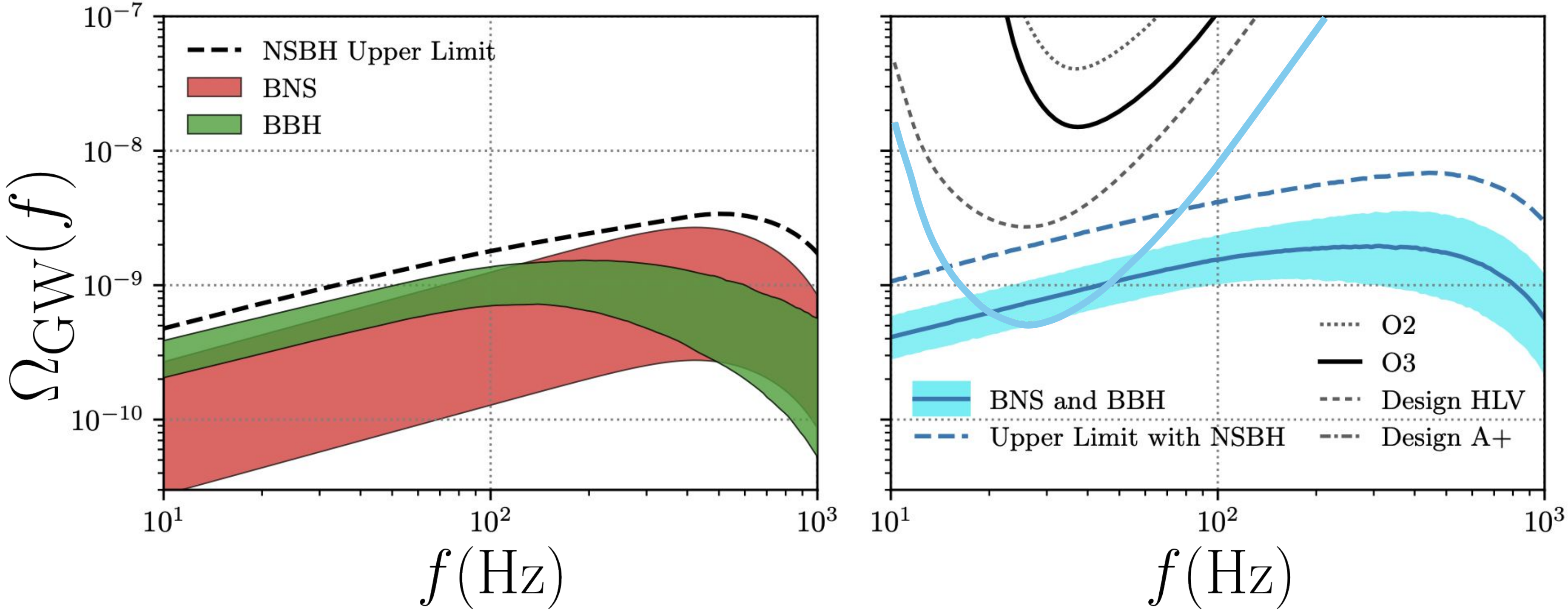

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What is the SGWB?

- Composed of overlapping unresolved waveforms
- Categorized into cosmological and astrophysical
- Varies depending on frequency band and source
- Currently undetectable in the LIGO frequency band
- Recent PTA results suggest SGWB detection at nHz frequencies

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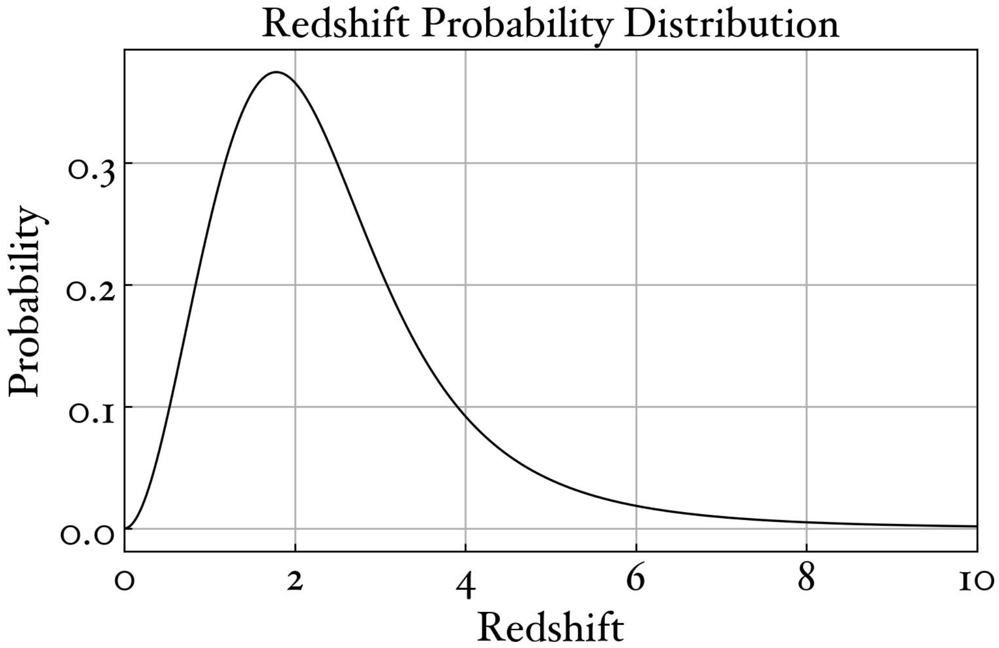
- ❖ Constrain future searches
- ❖ Better understand the evolution of the recent Universe
  - ➢ Star formation, merger history, mass distribution
- ❖ Eventually learn about the early Universe

Methods: Standard Priors

Standard Priors

Prior	Distribution	Parameters
mass_1	Power Law	$\alpha = -2.3$
mass_ratio	Power Law	$\alpha = 1.5$
chi_1	0	0
chi_2	0	0
theta_jn	Uniform	min = 0, max = 2π
geocent_time	Uniform	min = 0, max = T_obs
redshift	$\propto \frac{1}{1+z}\mathcal{R}(z)\frac{dV_c}{dz}$	min = 0, max = 10

Standard Priors: Redshift Distribution



$$p(z) \propto \frac{1}{1+z}\mathcal{R}(z)\frac{dV_{\mathrm{c}}}{dz}$$

Standard Priors: Merger Rate

$$\mathcal{R}(z) = \mathcal{C}(\alpha, \beta, z_{\mathrm{p}}) \frac{\mathcal{R}_0 (1+z)^{\alpha}}{1 + \left(\frac{1+z}{1+z_{\mathrm{p}}}\right)^{\alpha+\beta}}$$

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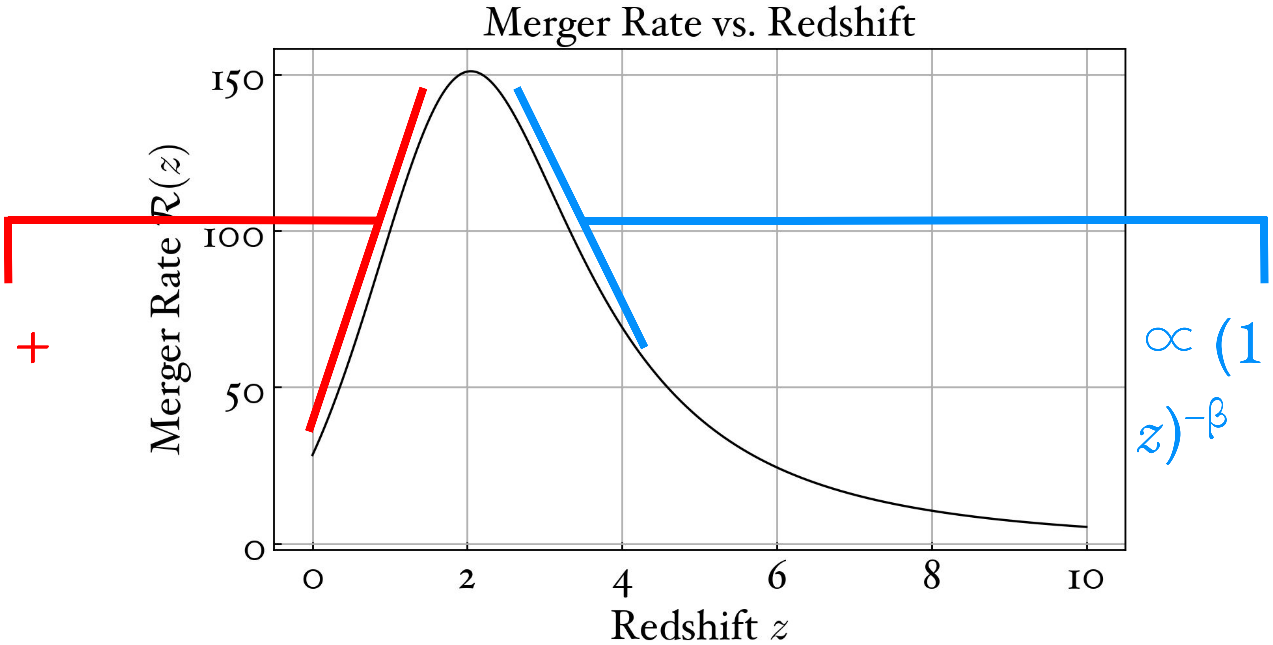
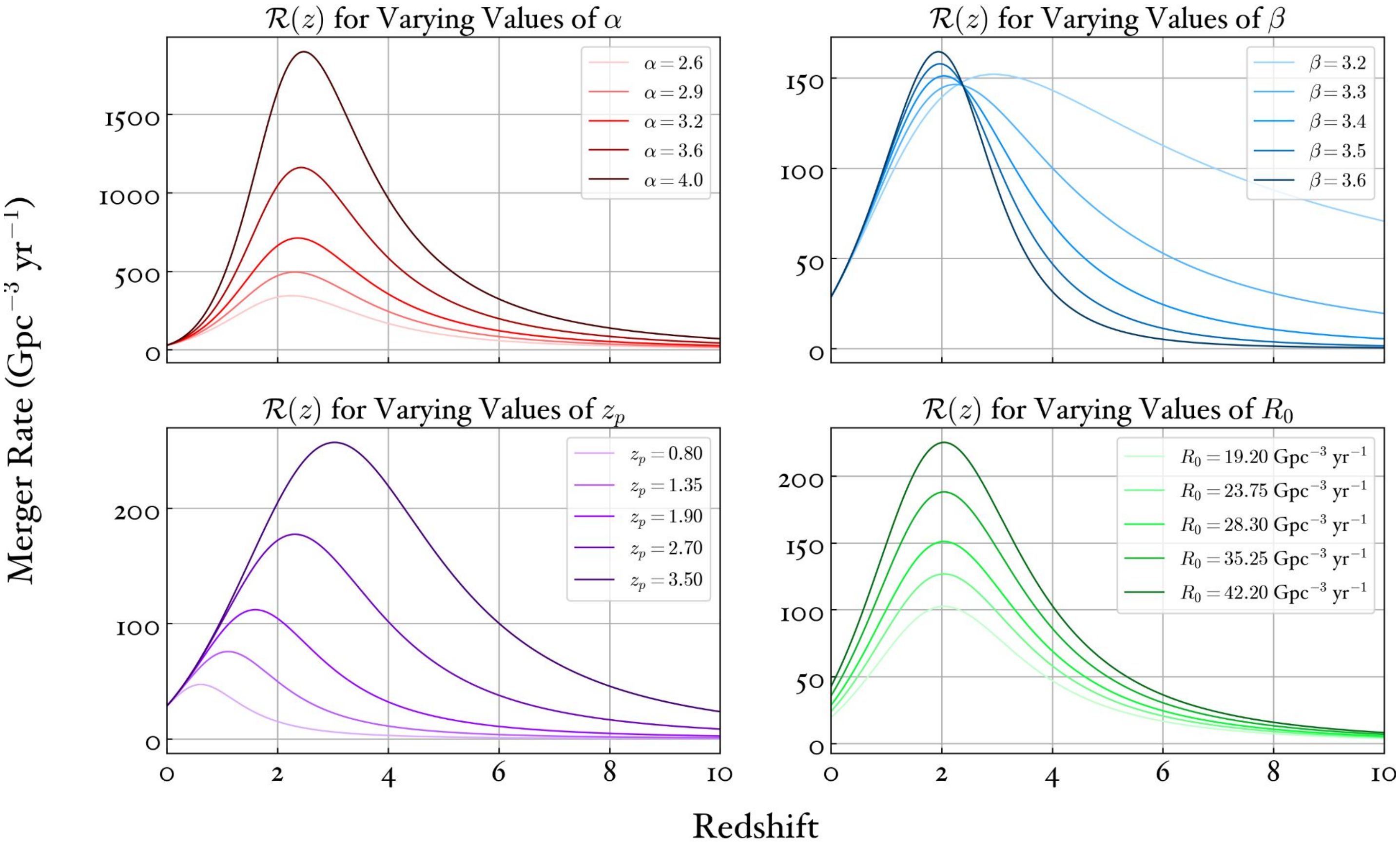


$\propto (1+z)^{\alpha}$

$\propto (1+z)^{-\beta}$



$\mathcal{R}(z)$ for Varying Values of α
$\alpha = 2.6$
$\alpha = 2.9$
$\alpha = 3.2$
$\alpha = 3.6$
$\alpha = 4.0$
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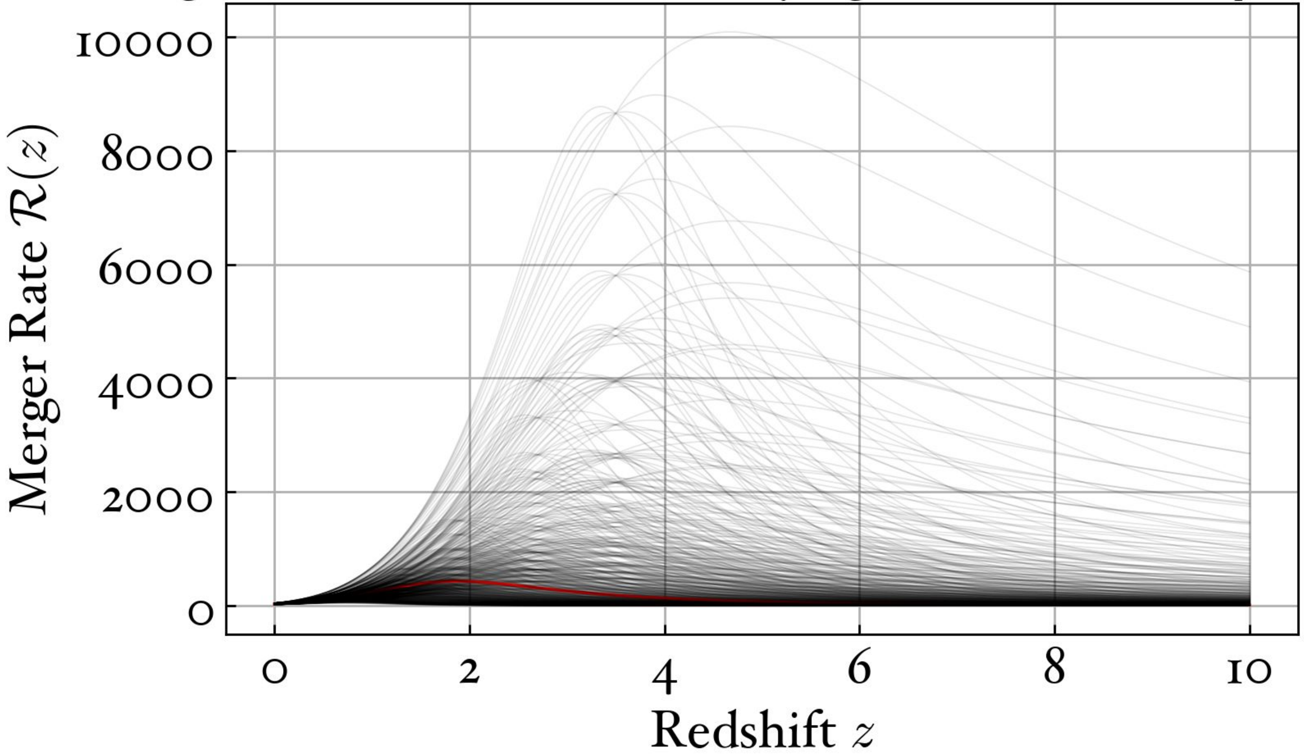
$\beta = 3.2$
$\beta = 3.3$
$\beta = 3.4$
$\beta = 3.5$
$\beta = 3.6$
$\mathcal{R}(z)$ for Varying Values of z_p
$z_p = 0.80$
$z_p = 1.35$
$z_p = 1.90$
$z_p = 2.70$
$z_p = 3.50$
$\mathcal{R}(z)$ for Varying Values of R_0
$R_0 = 19.20$ Gpc^{-3} yr^{-1}
$R_0 = 23.75$ Gpc^{-3} yr^{-1}
$R_0 = 28.30$ Gpc^{-3} yr^{-1}
$R_0 = 35.25$ Gpc^{-3} yr^{-1}
$R_0 = 42.20$ Gpc^{-3} yr^{-1}
Merger Rate (Gpc^{-3} yr^{-1})
Redshift

Merger Rate vs. Redshift at Varying Values of α, β, z_p, and $\mathcal{R}_0$



Methods: Gridded Method

Gridded Method

$$\Omega_{\mathrm{GW}}(f) = \frac{f}{\rho_{\mathrm{c}}} \int_0^{z_{\mathrm{max}}} dz \frac{\mathcal{R}(z)}{(1+z)H(z)} \left\langle \frac{dE_{\mathrm{GW}}}{df_{\mathrm{r}}}\bigg|_{f_{\mathrm{r}}=f(1+z)} \right\rangle$$

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Gridded Method

Merger Rate

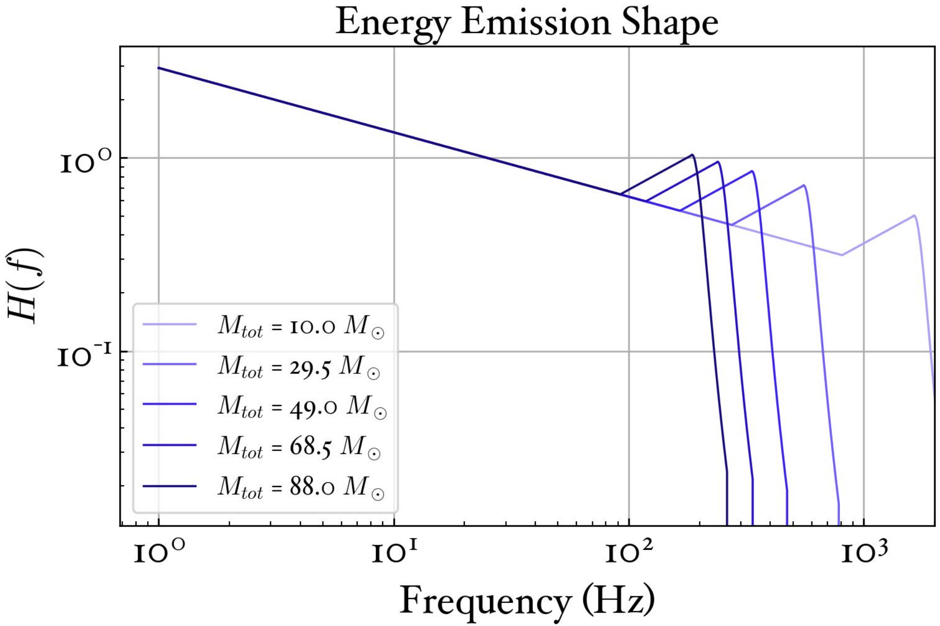
Average Spectral Energy Density

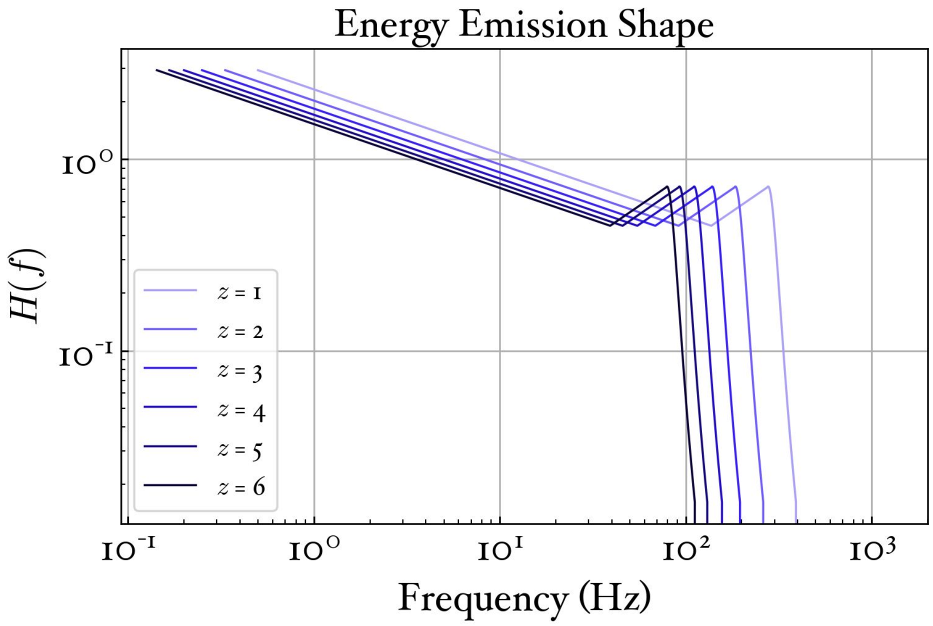
$$\Omega_{\rm GW}(f) = \frac{f}{\rho_{\rm c}} \int_0^{z_{\max}} dz \frac{\boxed{\mathcal{R}(z)}}{(1+z)H(z)} \boxed{\left\langle \frac{dE_{\rm GW}}{df_{\rm r}} \bigg|_{f_{\rm r}=f(1+z)} \right\rangle}$$

Gridded Method:
Average Spectral Energy Density

$$\frac{dE_{\mathrm{GW}}}{df} = \frac{(G\pi)^{2/3}\mathcal{M}^{5/3}}{3} H(f)$$

Gridded Method: H(f)





Gridded Method

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Methods: Monte Carlo Method

Monte Carlo Method

$$\Omega_{\mathrm{GW}}(f) = \frac{2}{T_{\mathrm{obs}}} \sum_{i=0}^{N} \frac{2\pi^2 f^3}{3H_0^2} \left[\frac{dE}{df}\right]_i$$

$$\frac{dE}{df} = |h_+|^2 + |h_\times|^2$$

Methods: Combined Method

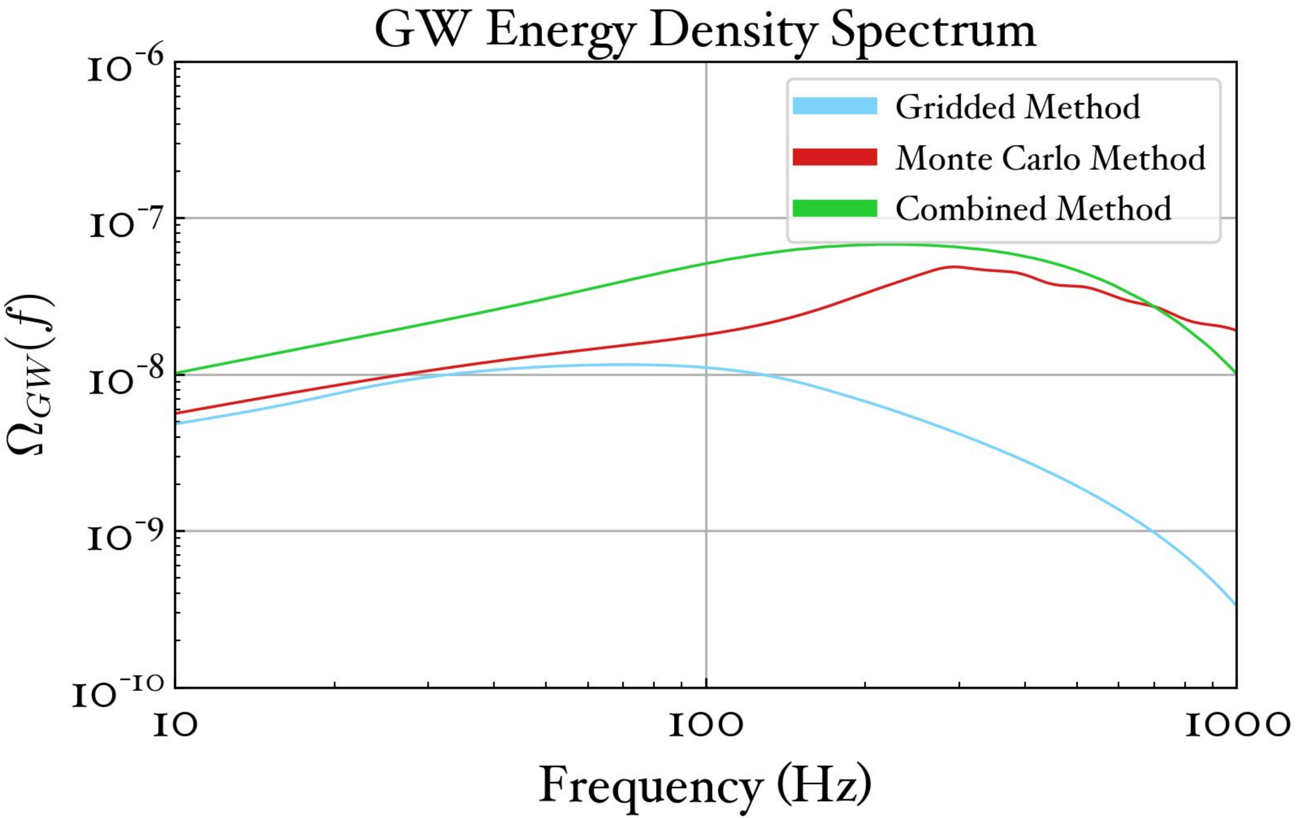
Combined Method

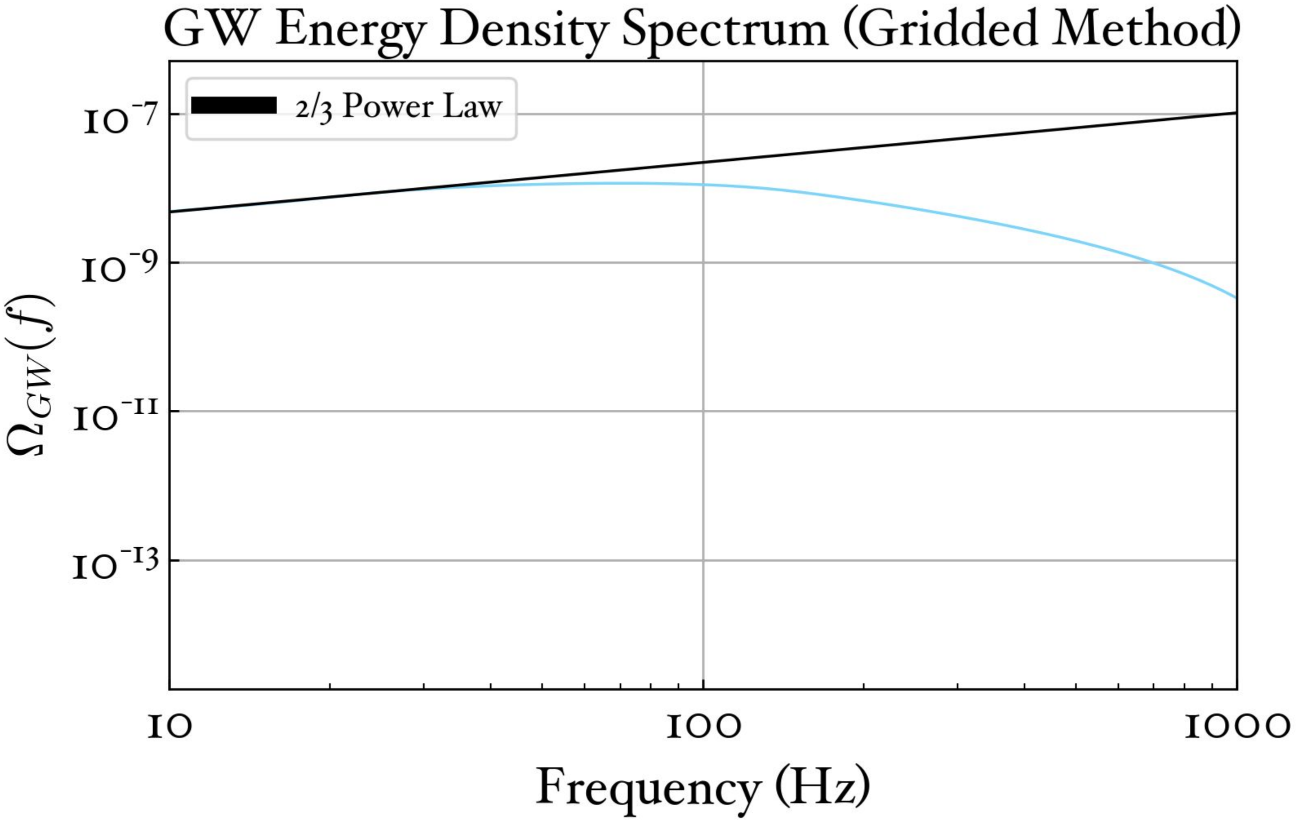
❖ Calculation uses the same equations as the Gridded Method

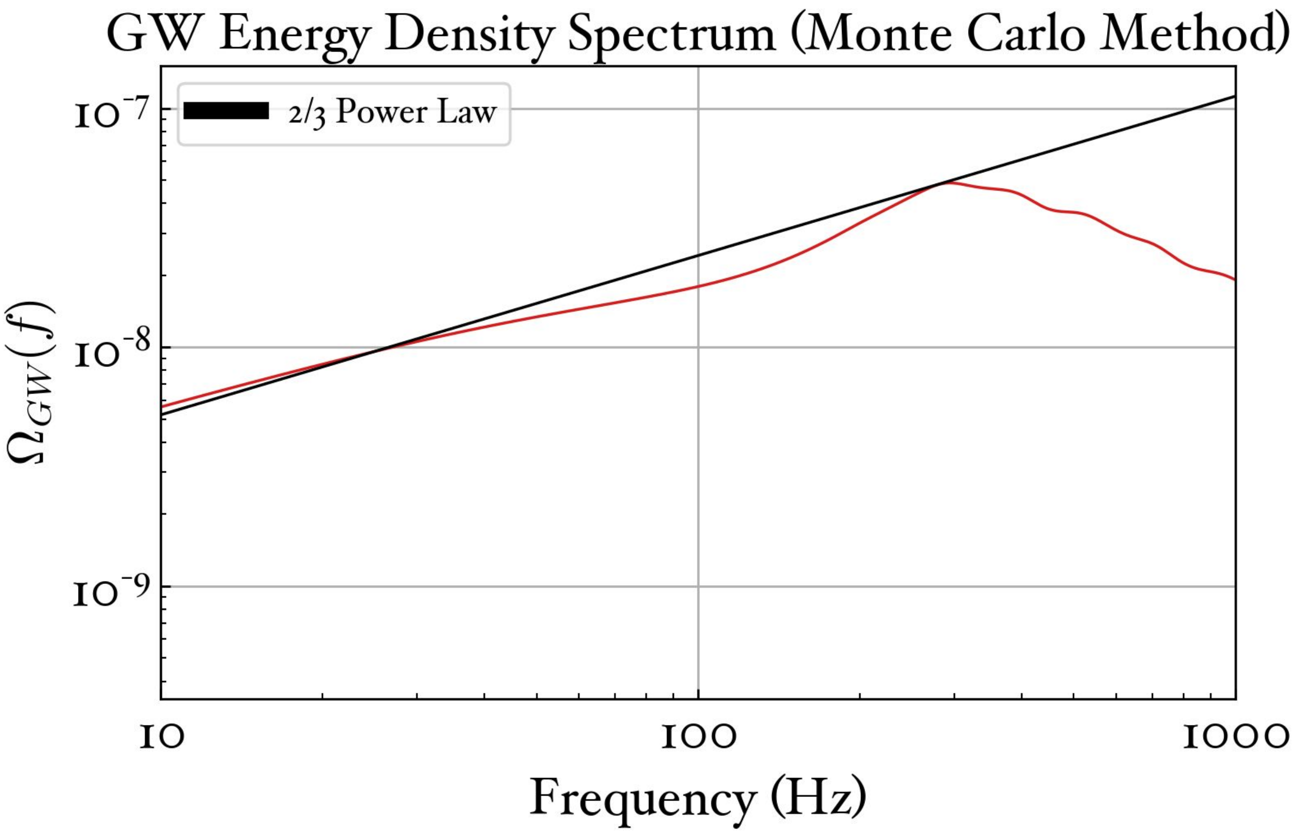
Combined Method

- Calculation uses the same equations as the Gridded Method
- Chirp masses in spectral energy density are sampled from `bilby` priors

Results


GW Energy Density Spectrum
Gridded Method
Monte Carlo Method
Combined Method
$\Omega_{GW}(f)$
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10
100
1000
Frequency (Hz)


GW Energy Density Spectrum (Gridded Method)
2/3 Power Law
10^{-7}
10^{-9}
10^{-11}
10^{-13}
$\Omega_{GW}(f)$
10
100
1000
Frequency (Hz)


GW Energy Density Spectrum (Monte Carlo Method)
2/3 Power Law
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$\Omega_{GW}(f)$
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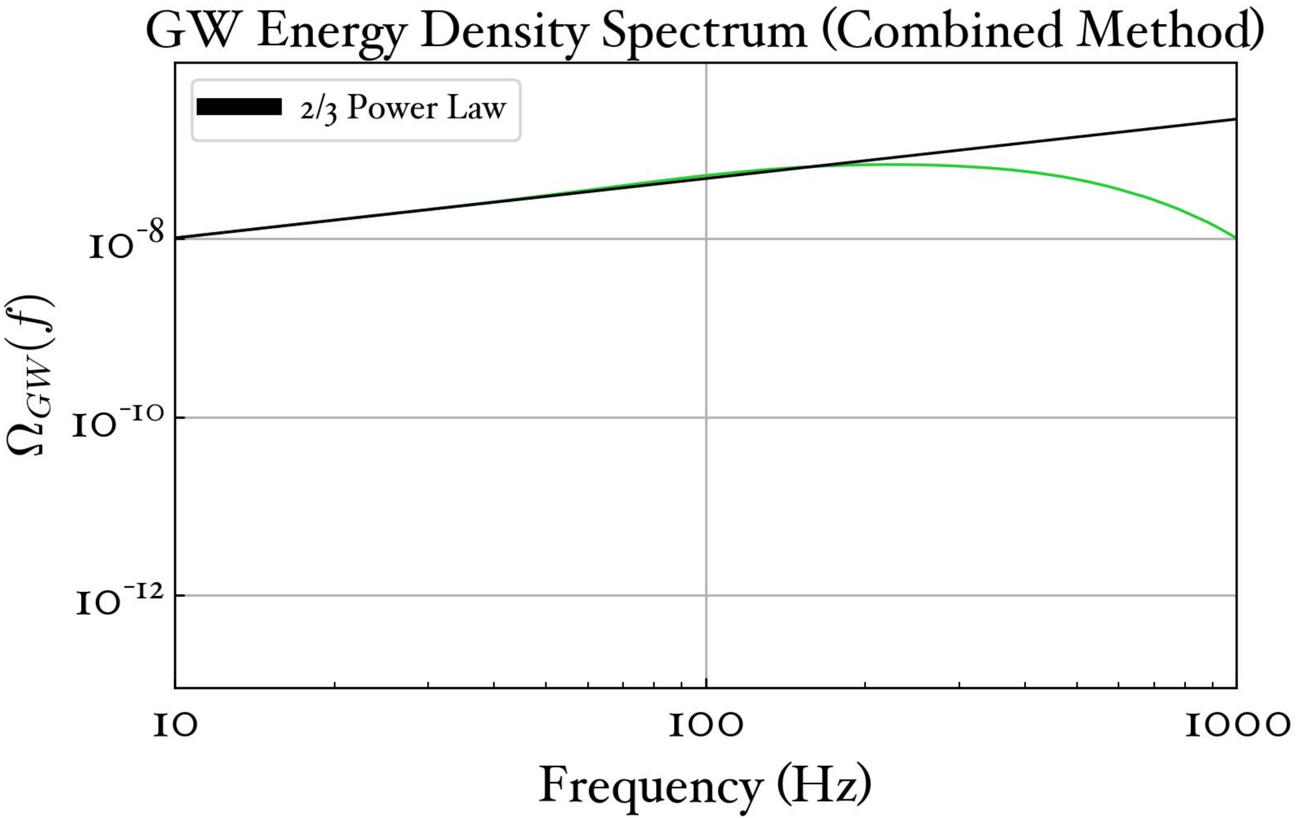
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2/3 Power Law
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10^{-10}
10^{-12}
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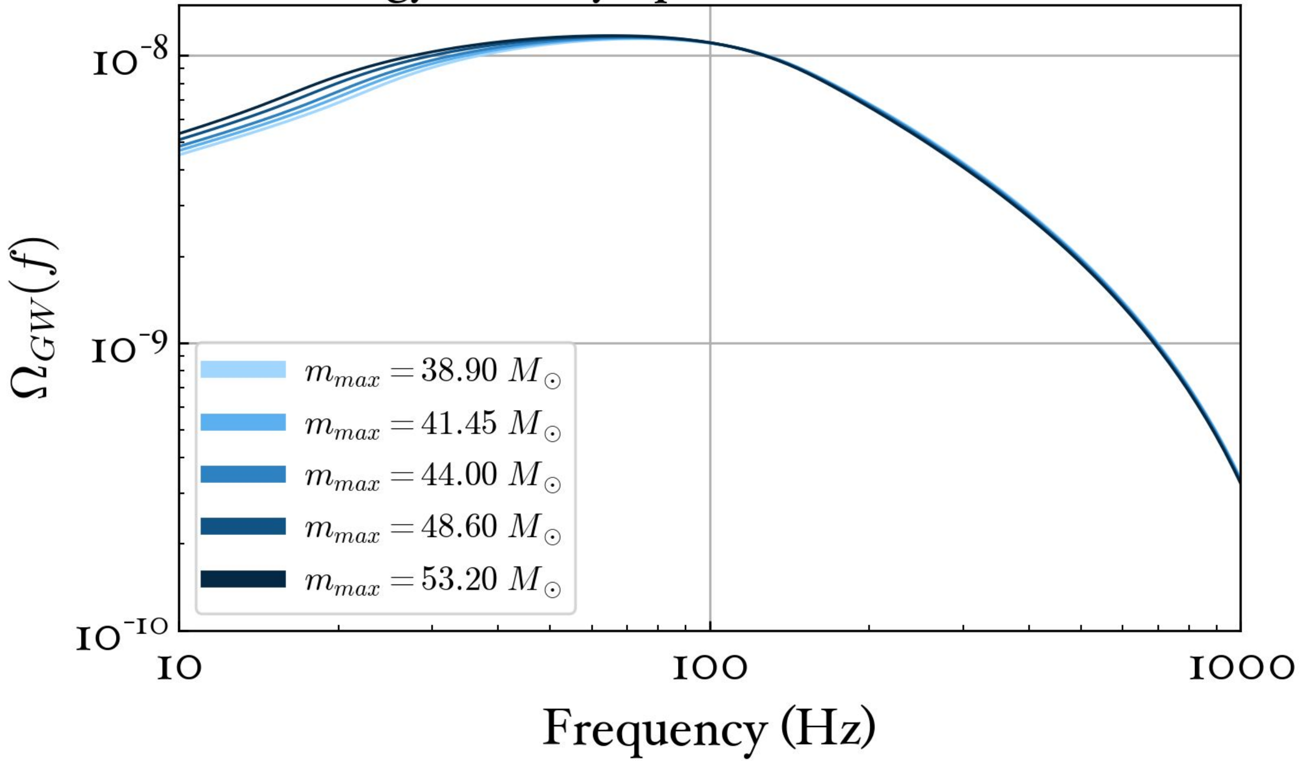
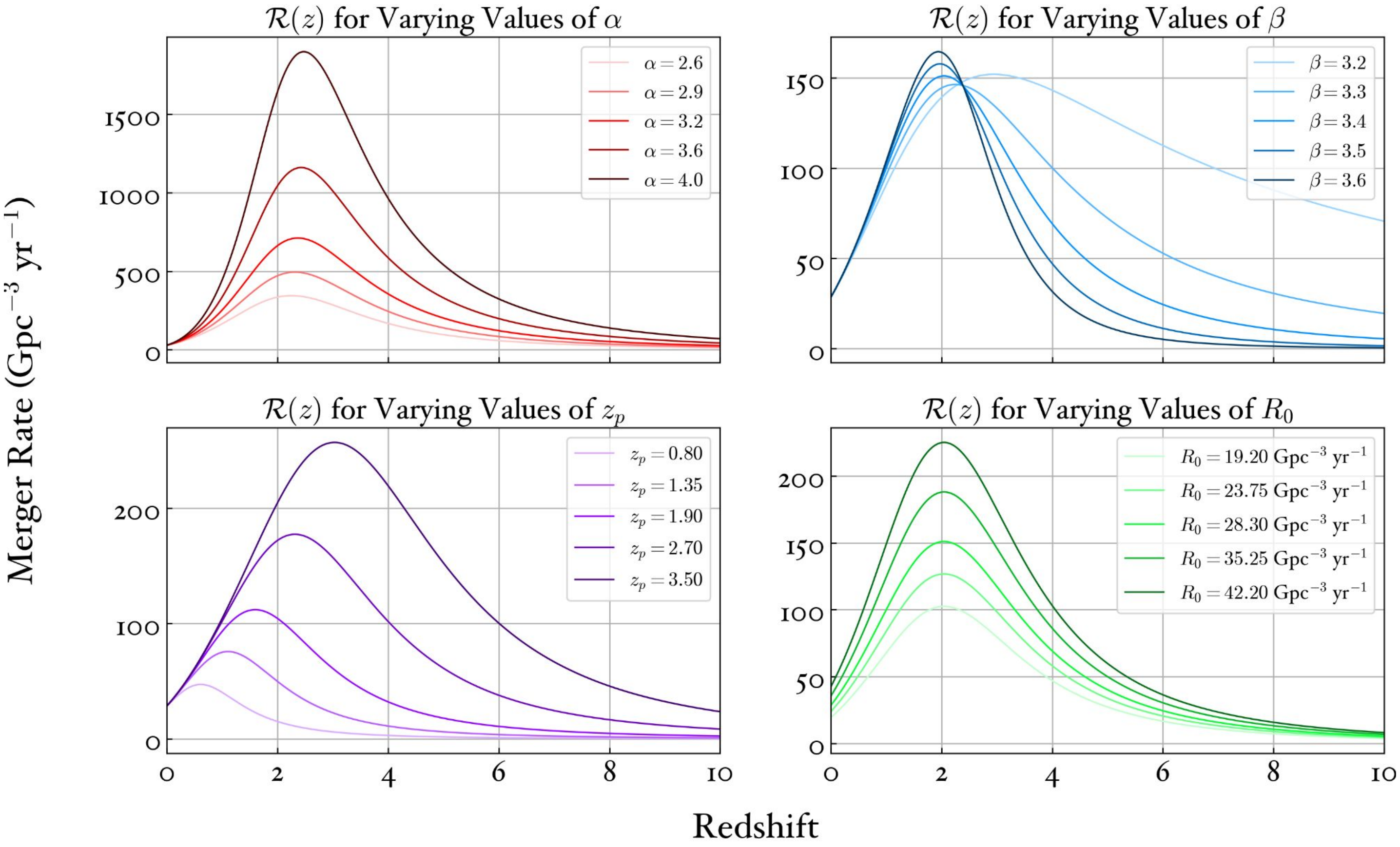
10
100
1000
Frequency (Hz)

GW Energy Density Spectrum (Gridded Method)





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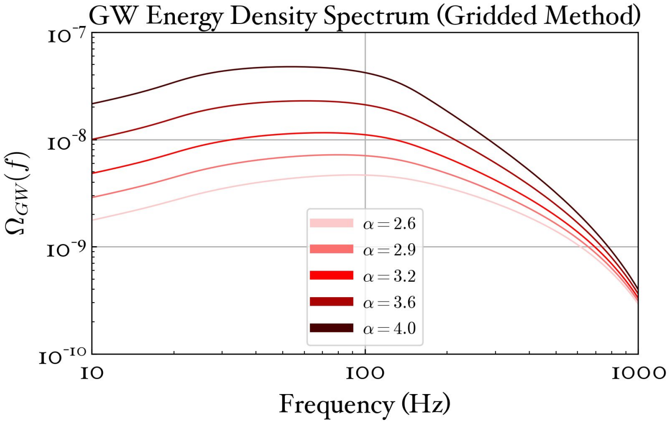
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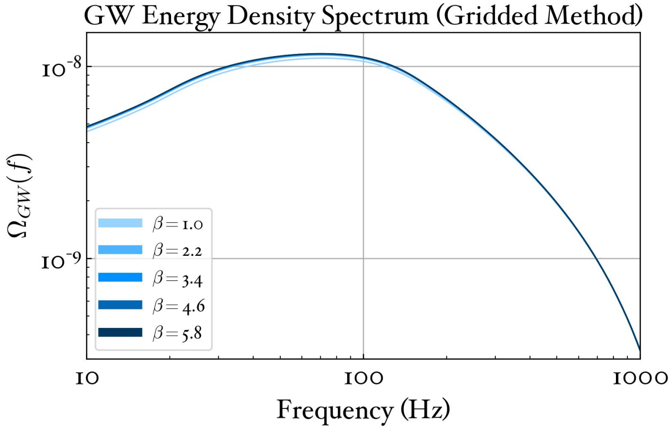
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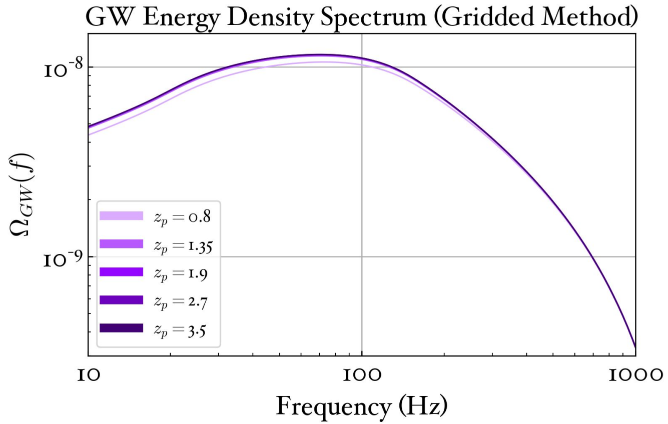
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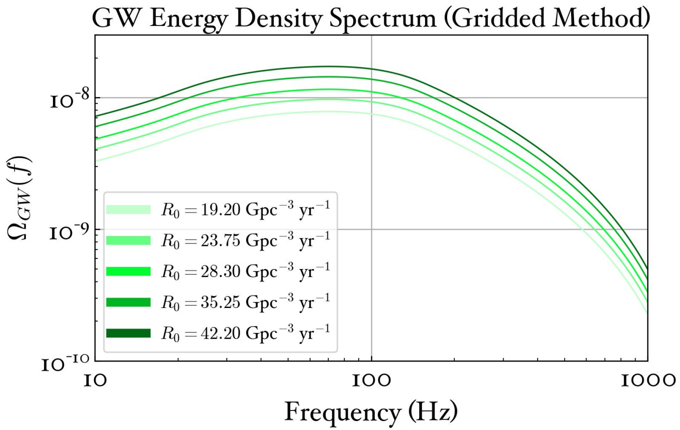
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- ❖ Created a third method that combined the two main methods
- ❖ Compared the spectra generated by each method
- ❖ Explored how BBH mass and merger rate affect $\Omega(f)$

Acknowledgements

- Mentors Patrick Meyers and Arianna Renzini
- Alan Weinstein
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