LIGO SURF project proposal

Estimation of the Stochastic Gravitational Wave Background from binary mergers

By Pritvik Sinhadc
Undergraduate student pursuing Bachelor of Science in Physics
The Division of Physics, Mathematics and Astronomy
California Institute of Technology
psinhadc@caltech.edu
pritvik.sinhadc@LIGO.org
Phone: 1(626)4976425
Caltech UID: 2205710

Mentor and guide:

Dr Alan Weinstein
Professor of Physics
The Division of Physics, Mathematics and Astronomy
California Institute of Technology
ajw@caltech.edu
Phone: 1(626)3952166

Abstract
The ground-based international gravitational wave detector network (IGWN), currently including the Laser Interferometer Gravitational wave Observatory (LIGO) stations at Hanford and Livingston, Virgo and KAGRA [1], has detected gravitational waves (GWs) from Compact Binary Coalescence (CBC) sources [2] in distant galaxies as far away as 8 Gigaparsecs [3], which corresponds to a redshift of slightly greater than 1. More distant sources are too faint to be confidently detected as individual events, but are expected to be so numerous that they can be detectable as a Stochastic Gravitational Wave Background (SGWB) [4]. Whilst stringent upper limits on the strength of the SGWB as a function of frequency in units of the cosmological closure density of the universe, $\Omega_{GW}(f)$ [5], have been made through the IGWN, there has been no observed detection of the SGWB as such. However, early implications for the SGWB from the first observation of Binary Black Hole (BBH) mergers [6] and more recent models from advanced LIGO and VIRGO data [7, 8] have all provided estimates of the CBC merger rate that suggest that we are close to detection of the SGWB. The estimates from [6] come from simple simulations of many individual events, while [7] is based on numerical evaluation on an analytical expression for the SGWB. In this project we will reproduce these estimates, through a thorough analysis and study of the methods used by [6, 7] and study the degree to which they agree with each other, as well as look at the extent to which the results depend on uncertainties in the merger rate as a function of mass and redshift distributions of the sources. Overall, this project aims at investigating the predictions on SGWB parameters and constraining its limits, thereby understanding how the background changes due to uncertainties in several important variables. This incorporation of the latest theoretical models, with a key understanding of the limits and constraints in these frameworks, will aid in the long term goal of refining estimates on the SGWB.

Introduction
The principles of general relativity, specifically the link between the spacetime metric as described by the Einstein field equations and energy-momentum tensor, including matter, momentum, and stress, show that acceleration of hyper-massive objects creates warping or distortions in the fabric of spacetime. This phenomenon of spacetime curvature can propagate through space as a GW in a manner analogous to electromagnetic or even fluid waves spreading out from a source [2].

All GWs that have been detected by the IGWN to date are attributed to CBCs [3], specifically the collision of compact, stellar mass objects [1]. These include events such as the collision of stellar mass objects [3], such as two neutron stars or two black holes [3], or a black hole and a neutron star [1, 3]. During such events, a portion of the rest mass-energy of the colliding objects is converted into GWs, which emanate from the collision site and progressively reduce in amplitude. Analogous to conventional waves, these GWs carry information on the original source via frequency, wavelength, and amplitude [1]. According to general relativity, it is worth noting that GWs also warp space-time as they propagate due to the fundamental interplay between spacetime curvature, matter-energy distribution and momentum.

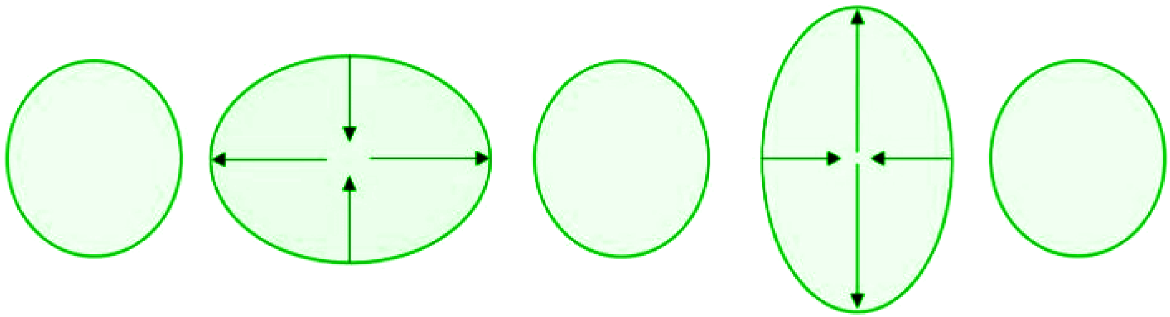

Figure 1: This figure illustrates the deformation of the space-time fabric within an object induced by the passage of a GW, with each image representing a distinct stage in the warping. The object oscillates from maximum longitudinal stretching to maximum latitudinal stretching, with arrows showing the direction of warping of the spacetime fabric. Such a warping is described as linearly polarized. In this case, the effect is exaggerated, since by the time such waves are detected by the IGWN, the warping caused by them results in extremely small changes in distance — less than 1/1000th the diameter of a proton [2].

Source: Image produced by author

This present overview holds paramount significance owing to the fact that the majority of the SGWB is anticipated to emanate from a superposition of CBC events [5]. To elucidate the characteristics or nature of the SGWB, it is imperative to consider the properties of such events as described above [2].

The SGWB is a complex amalgamation of multiple sources of GWs that offer valuable insights into the evolution and history of astrophysical collisions over the universe's timespan [4]. Although numerous theorized sources, including cosmic strings, primordial black holes, etc, have been suggested to contribute to the SGWB, the vast majority of this background is expected to originate from a superposition of deterministic sources, including CBCs, along with less predictable, unmodeled bursts such as core-collapse supernovae [4, 5]. This component of the SGWB is the astrophysical background, and is expected to be made up of the superposition of numerous GW events throughout the universe's history [4, 5]. A much smaller component of the SGWB consists of a cosmological background, including the GWs predicted to be formed immediately after the Big Bang through processes such as the preheating phase at the end of Cosmic Inflation, or even GWs generated during inflation [11, 12, 13]. Other hypothesized sources include baryonic acoustic oscillations, or even further back with contributions from earlier phase transitions [4]. Although this portion of the SGWB is fainter, we note that its frequency lies beyond the detectable range of the IGWN [11, 12, 13] and some of the advanced GW experiments, such as Laser Interferometer Space Antenna (LISA) or even the Pulsar Timing Array (PTA) [9]. Thus, this report focuses solely on the astrophysical component.

The SGWB is expected to be fundamentally stochastic in nature with a source distribution assumed to be isotropic, as well as being randomly distributed across the observable universe [10]. An alternate anisotropy, that of a background centered around galactic filaments, will also be discussed during the research. Figure 2, presented below [10], depicts a prototype of the stochastic signal anticipated to resemble the SGWB.

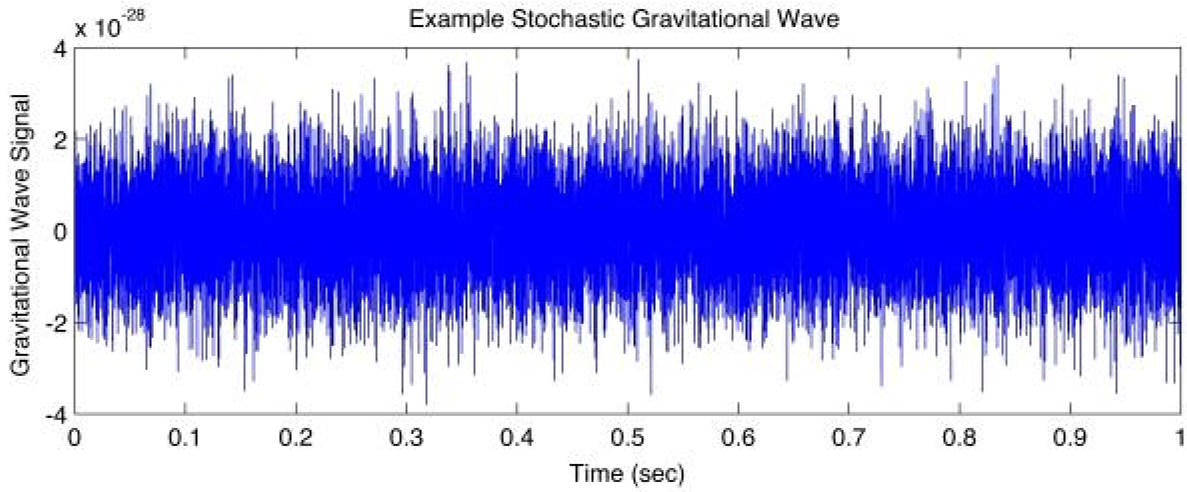

Figure 2: An example signal from an stochastic GW source. The signal is roughly uniform in amplitude and frequency in time, and is very faint [10].

Source: LIGO Scientific Collaboration, https://www.ligo.org/science/GW-Stochastic.php

GWs convey vital information about their sources, and likewise, the SGWB provides valuable insights into the underlying population of astrophysical sources that constitute it, including their mass distribution, the rate of formation of CBCs, and other parameters [5, 6]. Thus, by simulating a SGWB with changing parameters, including amplitude, spectral shape, and angular distribution of sources, a novel window to understand the evolution of CBCs can open, targeting new, in depth insights on how mass distribution of compact binary systems, their isotropy and redshift distribution, impact the SGWB, which will potentially reveal further insights into the astrophysical origins of GWs [14].

This research project aims at investigating the properties of the SGWB resulting from CBCs, with a focus on how different variables such as mass distributions, anisotropies, and redshift distributions impact the background signal. To accomplish this, the simulation techniques employed to model the SGWB will be analyzed in detail, including how such models can be parametrized to account for different variables [6]. The theoretical framework for modeling the SGWB will be developed, including understanding the power spectrum of strain fluctuations generated by the sources, along with a replication of the numerical simulations utilized to generate background signals for different scenarios [6].

More specifically, the simulations will be used to investigate the properties of the SGWB due to different mass distributions of CBCs. The impact of anisotropies in the distribution of CBC sources on the SGWB may also be studied. Additionally, the research project will examine the impact of redshift distributions on the SGWB due to CBCs. This includes investigating the potential for the SGWB to be affected by the evolution of the universe over time.

Overall, the goal of this research project is to gain a deeper understanding of the SGWB due to CBCs and the information it carries about the population of astrophysical sources that compose it. By studying how different variables impact the SGWB, we hope to develop a better theoretical framework for modeling the background signal, which will be crucial for interpreting future observations of the SGWB and will aid in the overarching goal of gaining a better understanding of what to expect when the SGWB is finally detected.

Background

The SGWB is Gaussian (normally distributed), unpolarized compared to an individual source and is expected to be isotropic in nature — or invariant with respect to direction of measurement [11]. This background can be fully characterized by the spectral energy density, and this spectrum can be expressed, as mentioned previously, by the term $\Omega_{GW}(f)$. This term allows for the calculation of the GW energy density within a frequency interval [11]. Specifically, $\Omega_{GW}(f)$ can be described by the equation below [11]:

$$\Omega_{GW}(f) = \frac{f d\rho_{GW}}{\rho_c df} \tag{1}$$

Where $d_{\rho GW}$ is GW energy density, $f \pm df$ the frequency interval, ρ_c the critical energy density needed to have a flat, non curved Universe — calculated as below:

$$\rho_c = \frac{3H_0^2c^2}{8\pi G} \tag{2}$$

Where c is the speed of light, G is Newton's gravitational constant, H_0 is Hubble constant [11].

Equation (1) for $\Omega_{GW}(f)$ derives a relationship between the energy density of the SGWB and the frequency content, thereby allowing us to understand the contribution of GWs for specific frequency intervals [11]. The frequency f that we measure in equation (1) above is of course the frequency measured by a detector. If we take f_{source} as the frequency as observed from source frame [11], we can decompose our equation (1) into another form below:

$$\Omega_{GW}(f) = \frac{f d_{\rho GW}}{\rho_c df} = \frac{f}{\rho_c}\int_0^{10} \frac{R_m(z)dE}{(1+z)H(z)df_{source}} dz \tag{3}$$

In equation (3) [11], we still measure energy density of GWs within the frequency interval for the SGWB, but we now have $\Omega_{GW}(f)$ in terms of new parameters. $R_m(z)$ is the merger rate [11] in $\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$. The term f_{source} is described by the equation $f = \frac{f_{source}}{1+z}$, wherein once again f is frequency in observed frame, and f_{source} is frequency in the source frame [11]. The parameter $H(z)$ is the Hubble expansion rate [11]. Notice that each parameter described (and the integral as a whole) is in terms of z, or the redshift. Typically, we assume that CBCs occur from a redshift of ten (corresponding to the expected time in the universe's history when the first black holes are expected to form) till now [11]. Thus, from equation (3), we have a preliminary link between the energy density of the SGWB, the redshift distribution that we are observing, as well as the mass distribution of CBCs, which the merger rate is dependent upon [11]. Therefore, the overall aim of this research is to recontextualize these equations through simulations. By creating simulations of the SGWB using mathematical models, such as the equation (3) above, we can manually adjust the merger rate through mass distribution, redshift distribution, etc. We can see the impact of variations in parameters to the energy density of the SGWB itself. The term $\Omega_{GW}(f)$ characterizes the spectral energy density of the SGWB [11], and it is a key quantity in the study of the SGWB. It is used to calculate the energy density and SNR of the SGWB, and it provides important insights into the properties of the gravitational wave sources that contribute to the background [11].

Motivation and methods

The primary motivation for our endeavor to compare the differing methods of simulating SGWBs is to further constrain the expected detection of such a background and understand the new insights that can be gathered on the evolution of CBCs over cosmic time. Currently, due to relativistic numerical simulations estimating parameters of the SGWB, as well as new estimates generated by the LIGO, VIRGO, KAGRA (LVK) detectors, we have managed to constrain the limits of the SGWB [6], the expected signal to noise ratio (SNR) needed for detection, and the mean expected energy density of the background. The results can be summarized in the figure below.

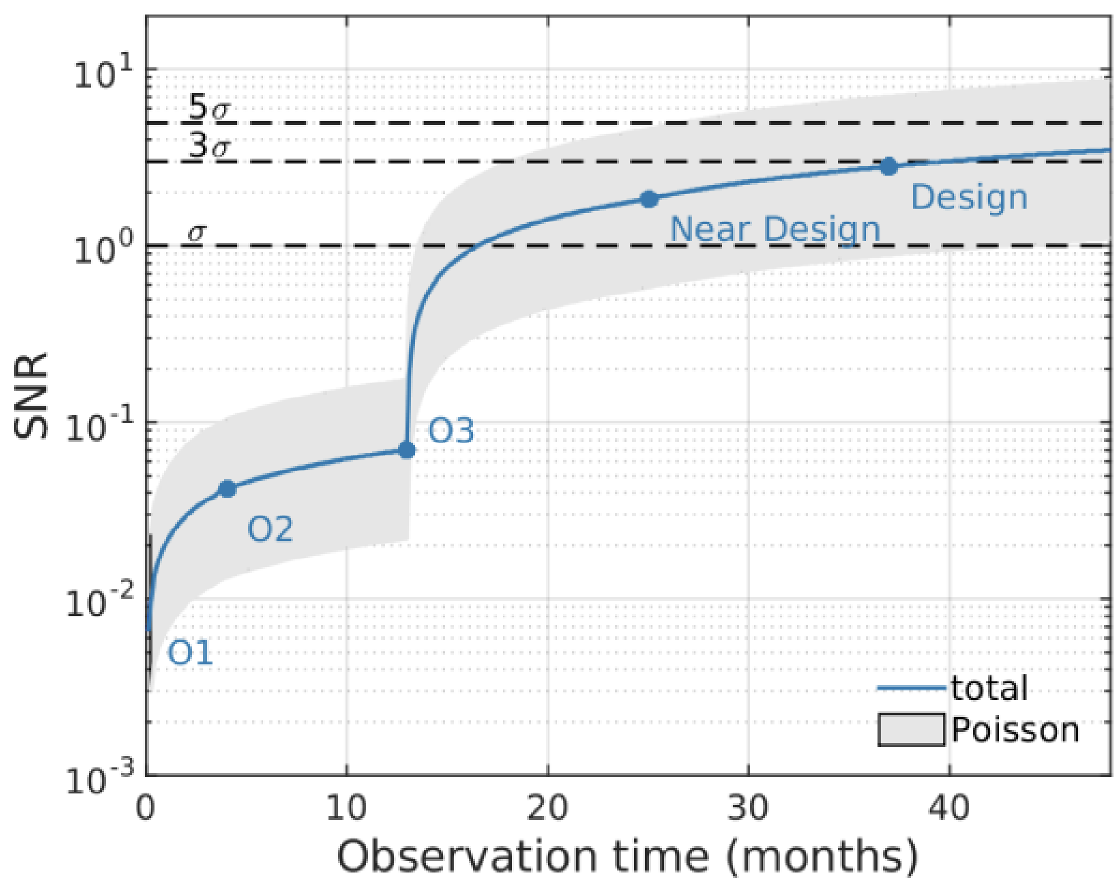

Figure 3: The image above shows the improvements in detector SNR [6]. As SNR will increase, the level of the SGWB will also be reached by the LVK network [6]. Therefore, overall, given the energy densities that can be measurable by upcoming detectors is also taken into account, we can understand that we should be able to detect the SGWB within a few years. The result of this research project will, hence, contribute to further constraining and understanding of the methodologies used to construct predictions of the SGWB, as well as decode the range of possible predictions from simulation [6].

Source: Fig 1 (right), GW170817: *Implications for the Stochastic Gravitational-Wave Background from Compact Binary Coalescences,* B. P. Abbott et al, (LIGO Scientific Collaboration and Virgo Collaboration), *Phys. Rev. Lett,* 120, 091101, Published February 28, 2018, https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.120.091101

This study of the astrophysical SGWB relies on various tools, including numerical integration, specifically of the model used in equation 3, simulations of several gravitational wave events to construct coarse-grain example SGWBs to be generated, and dedicated Python packages, particularly pygwb — the latest released version — for all of the aforementioned gravitational wave science. Numerical integration techniques can be used to better understand the spectral energy density of the SGWB, and generate predictions on sensitivity ranges of various detector and mission operations to observe the presence of an SGWB. Therefore, such techniques remain a critical tool for the final stages for this research. The result of applying these methods on the energy density $\Omega_{GW}(f)$ — as defined by equation 3 — can be seen in the image below [8].

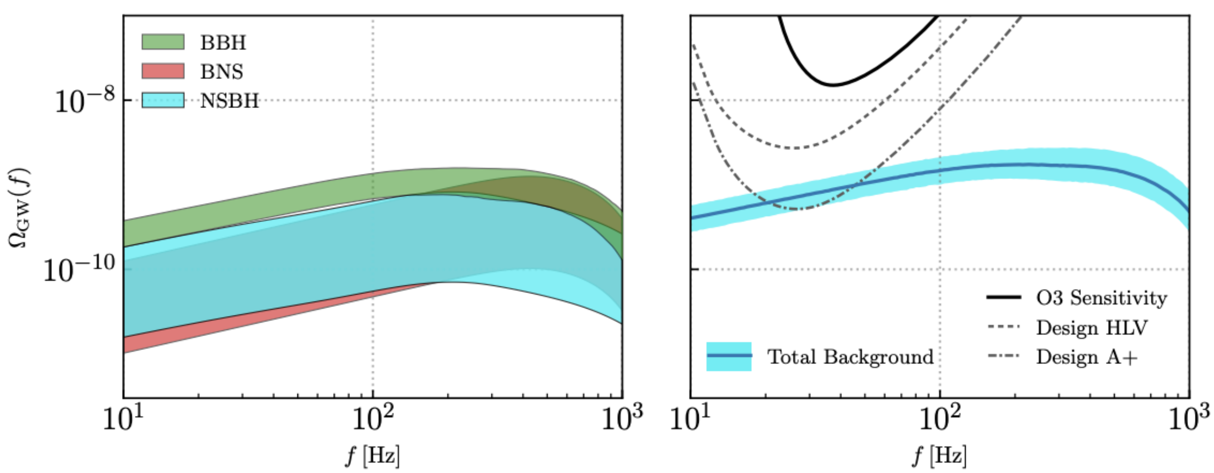

Figure 4 (left) **Figure 4 (right)**

Figure 4: The image above shows the predictions of the SGWB due to CBCs as well as LVK detector sensitivity following Observation Run 3 [8]. Figure 4 (left) shows the expected contributions to the background from various astrophysical sources of gravitational waves, including binary black holes in green, binary neutron stars in red, and neutron star black hole mergers in blue [8]. Figure 4 (right) shows the intersection between detector sensitivity and required parameters needed to reach the SGWB detection [8]. A key part of this research

includes understanding the appropriate uncertainty in merger rate and mass distribution for each source of the CBC SGWB.

Source: *The population of merging compact binaries inferred using gravitational waves through,* GWTC-3, B. P. Abbott et al, (LIGO Scientific Collaboration and Virgo Collaboration), February 23, 2022, https://arxiv.org/abs/2111.03634, section X and Fig 23.

The other key tools that will be used during this research is simulations based on the utility provided by Python packages for gravitational wave science, particularly pygwb [12]. Through simulations and coding, the project aims at utilizing different parameters and approximations for both mass distributions and redshift distributions in my research, apply statistical techniques to prototype SGWBs generated, study SNRs required to probe such backgrounds, etc.

Summary of objectives

The main objectives of this project, to be executed in a final report, presentation, and a possible paper, are summarized below:

1. To reproduce and compare the estimates of the CBC merger rate and the SGWB from [6] and [7], which are based on different approaches, including simple simulations of individual events and numerical evaluation of analytical expressions for the SGWB.
2. To investigate the degree to which these estimates agree with each other and the implications of any discrepancies.
3. To study the dependence of these estimates on uncertainties in the merger rate as a function of mass, redshift distributions of the sources, and potential anisotropies in overall source distribution.
4. To assess the impact of these uncertainties on any potential constraints that could be applied to the SGWB, including the energy density of the SGWB, contributions from different mass ranges of CBCs per frequency band, etc.

Project Timeline

Week 1: My aim will be to continue the Literature Review and my ongoing reading and work on the Background research. This Background research will be solidified during the LIGO SURF's orientation program, and will involve a thorough understanding of the SGWB and its GW potential source contributions, particularly CBCs. I will also aim at consolidating my understanding of the mathematical, computational, and theoretical tools or concepts relevant to the project. In short, a summary of my goals for Week 1 is as follows:

a. Take part in the LIGO SURF orientation program.
b. Research and assess my Literature Review on SGWB and binary mergers.
c. Master the mathematical and computational tools used in SGWB simulations, including a continued review of simulation methods that can be analyzed and compared as the project progresses.

Week 2: My aim will be to understand the theoretical and observational background of the models of interest for simulating an SGWB [6, 7, 8]. This will include decoding the parameters used in simulation to provide preliminary estimates of mass distributions for CBCs, predict the

effect of anisotropies on the SGWB and changes due to variation in red-shift distributions from the gathered Literature Review. In short, a summary of my goals for Week 2 is as follows:
a. Develop a theoretical framework for modeling the SGWB due to binary mergers.
b. Identify relevant parameters, including mass distribution, redshift distribution, and anisotropies.
c. Start coding on the theoretical framework using Python [12], particularly the number of simulated events, with a focus on how to incorporate different masses and redshift distributions with the purpose of making each term parameterizable [12].

Weeks 3-4: My aim will be to simulate and model the SGWB in these two weeks. Starting Week 3, my research will focus on conducting the replication of the first simple methodology used to simulate a SGWB [6]. This will include preliminary simulations of several millions of GW events, constrained by my research in Week 2 to create a superposition of sources, resulting in a simulation of a GWB. In Week 4, I will continue my work with the aim of arriving at a preliminary assessment of how the SGWB changes due to variable parameters. The process of Week 3 will be repeated with a more complex recent methodology for simulating SGWB [7]. The theoretical background from Week 2 will be useful here. In short, a summary of my goals for Weeks 3-4 is as follows:
a. Conduct simple [6] and complex [7, 8] numerical simulations of SGWB due to binary mergers using the theoretical framework developed in Week 2
b. Compare the simulations against existing Literature Review from Weeks 1-2.

Weeks 5-6: My aim will be to comprehend and analyze the degree to which the previously explored estimates agree with each other. My focus will be on decoding the implications of any discrepancies between the models, and understanding any additional complications brought upon by the more complex models [7, 8] as opposed to the simpler one [6]. The extent to which these estimates depend on uncertainties in merger rate mass function as well as the redshift distribution will also be looked at during these two weeks. Overall, this will involve a more sophisticated code detailing how the mass distribution of sources, anisotropy, etc, impacts the SGWB. This will all culminate in a mid-project report. In short, a summary of my goals for Weeks 5-6 are:
a. Investigate the degree to which the previously explored estimates [6, 7, 8] agree with each other and the implications of any discrepancies.
b. Study the dependence of these estimates on uncertainties in the merger rate as a function of mass and redshift distributions of the sources.
c. Assess the impact of these uncertainties on the predictions of the SGWB and use statistical analysis techniques to estimate the uncertainty in the SGWB predictions.
d. Further, parametrize the simulations to account for different variables, such as mass distribution, redshift distribution, and anisotropies.
e. Complete mid-project report review of all facets learned during the Literature Review, model reconstruction and comparison. Ensure to collate all codes for mid-project review.

Weeks 7-8: My aim will be to interpret all results in the frame of attempting to further constrain estimates of the SGWB. This will include understanding and predicting how mass distributions may impact the SGWB, thereby providing a new insight into how the detection of the SGWB can inform us of the evolution of CBCs over time, whether or not the SGWB is isotropic in nature, including the parameters of a anisotropic background, etc. In particular, the results will be

compared with astrophysical predictions on CBC merger rate and mass distributions during events such as Cosmic Noon (peak star formation rate in the universe) [11]. These results and comparisons will be collated for preparation of the final research paper. In short, a summary of my goals for Weeks 7-8 are:
a. Interpret the results from previous weeks in the context of research questions and objectives
b. Utilize results to further constrain SGWB, particularly how a change in parameter space may impact the background itself.
c. Compare results with astrophysical predictions and evolution of the universe's history to determine how mass distribution, red shift, and anisotropy changes may affect the proposed timeline for the evolution of CBCs over cosmic time.
d. Start writing the methodology, results, and conclusions in the form of a research project report, thereby addressing whether or not the estimates from Weeks 5-6 agree with one another, and the new insights that the parameterized models can provide for a greater understanding of the SGWB and its constraints.

Weeks 9-10: My aim will be to complete the final write up and documentation of the project report, including the findings from previous weeks, any potential new insights into how the evolution of CBCs over time may be impacted by the SGWB, the constraints on amplitude, frequency, etc, that can be applied on the SGWB following the research. In short, a summary of my goals for the final two Weeks 9-10 are:
a. Write my research report, ensuring proper documentation of all methodologies, data sources, and analysis.
b. Present my results using figures, tables, data sheets, graphs, codes and any other visual aids.
c. Revise and finalize my project report with the aim of writing a publication-worthy paper under my Mentor and Guide, Dr. Alan Weinstein.
d. Consolidate and prepare a presentation of my research findings for the final SURF Seminar.

Bibliography

[1] *LIGO, VIRGO And KAGRA Observing Run Plans*, IGWN, accessed May 2023, https://observing.docs.ligo.org/plan/

[2] *Gravitational Waves from Compact Binary Mergers seen by LIGO and Virgo*, Alan J Weinstein LIGO Laboratory, Caltech for the LIGO and Virgo Collaborations LIGO-Virgo Open Data Workshop, May 27, 2020, https://dcc.ligo.org/public/0168/G2000797/001/Weinstein_CBCs_GWOSC_ODW_20200527.pdf

[3] *Gravitational-wave Transient Catalog (GWTC)*, LVK Collaboration, https://www.gw-openscience.org/eventapi/html/GWTC/

[4] *Stochastic Gravitational-Wave Backgrounds: Current Detection Efforts and Future Prospects,* Renzini, A.I., Goncharov, B., Jenkins, A.C., Meyers, P.M., *Galaxies 2022,* 14 February 2022, https://www.mdpi.com/2075-4434/10/1/34

[5] *Upper limits on the isotropic gravitational-wave background from Advanced LIGO and Advanced Virgo's third observing run*, R. Abbott et al, (LIGO Scientific Collaboration, Virgo Collaboration, and KAGRA Collaboration), *Phys. Rev.* D 104, 022004, Published July 23, 2021, https://journals.aps.org/prd/abstract/10.1103/PhysRevD.104.022004

[6] GW150914: GW150914: *Implications for the Stochastic Gravitational-Wave Background from Binary Black Holes,* B. P. Abbott et al, (LIGO Scientific Collaboration and Virgo Collaboration), *Phys. Rev. Lett.* 116, 131102, Published March 31, 2016, https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.116.131102;
GW170817: *Implications for the Stochastic Gravitational-Wave Background from Compact Binary Coalescences,* B. P. Abbott et al, (LIGO Scientific Collaboration and Virgo Collaboration), *Phys. Rev. Lett,* 120, 091101, Published February 28, 2018, https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.120.091101

[7] *Upper limits on the isotropic gravitational-wave background from Advanced LIGO and Advanced Virgo's third observing run,* R. Abbott et al, (LIGO Scientific Collaboration, Virgo Collaboration, and KAGRA Collaboration), *Phys. Rev. D,* 104, 022004, Published July 23, 2021, https://journals.aps.org/prd/abstract/10.1103/PhysRevD.104.022004, section V.A

[8] *The population of merging compact binaries inferred using gravitational waves through,* GWTC-3, B. P. Abbott et al, (LIGO Scientific Collaboration and Virgo Collaboration), February 23, 2022, https://arxiv.org/abs/2111.03634, section X and Fig 23.

[9] *The stochastic gravitational-wave background from massive black hole binary systems: implications for observations with Pulsar Timing Arrays*, *Monthly Notices of the Royal Astronomical Society*, A. Sesana, A. Vecchio, C. N. Colacino, Volume 390, Issue 1, October 2008, Pages 192-209, https://doi.org/10.1111/j.1365-2966.2008.13682.x

[10] *Introduction To Ligo & Gravitational Waves, Stochastic Gravitational Waves,* LVK Collaboration, https://www.ligo.org/science/GW-Stochastic.php, accessed May 2023

[11] *Impact of a Midband Gravitational Wave Experiment On Detectability of Cosmological Stochastic Gravitational Wave Backgrounds,* Barry C. Barish, Simeon Bird, and Yanou Cui, 16 June 2021, https://arxiv.org/pdf/2012.07874.pdf

[12] *pygwb documentation*, Copyright 2022, Arianna Renzini, Sylvia Biscoveanu, Shivaraj Khandasamy, Kamiel Janssens, Max Lalleman, Katarina Martinovic, Andrew Matas, Patrick Meyers, Alba Romero, Colm Talbot, Leo Tsukada, Kevin Turbang, https://pygwb.docs.ligo.org/pygwb/

[13] *LISA For Cosmologists: Calculating The Signal-To-Noise Ratio For Stochastic And Deterministic Sources*, Tristan L. Smith, Swarthmore College, tsmith2@swarthmore.edu, R. R. Caldwell, 11-15-2019, https://works.swarthmore.edu/cgi/viewcontent.cgi?article=1378&context=fac-physics

[14] *Stochastic gravitational wave background: methods and Implications*, Nick van Remortel, Kamiel Janssens, Kevin Turbang, 3 Oct 2022, https://arxiv.org/abs/2210.00761