

# **NIST-PTB bilateral comparison, GW detectors calibration plan**

NIST, PTB, LIGO Hanford  
06/13/2023

# Agenda

- NIST-PTB bilateral study, 2022-2023
  - Calculation of consensus responsivity and bilateral DoE (Matt Spidell's spreadsheet)
  - NEWRAD conference in September 2023
  - Potential publication
- Implementation of the calibration subway map

# Bilateral comparison study

- Matt Spidell prepared a spreadsheet to calculate the consensus responsivity and the Degree of Equivalence
- Modified Matt Spidell's spreadsheet to include
  - Values from the second round measurement at NIST
  - Uncertainty of the pilot lab in calculation of  $u_{rel}(\Delta_{i,j})$  in accordance to the following document by BIPM:  
[https://www.bipm.org/documents/20126/30126060/CCPR-G2+\(rev.+4\)+Guidelines+for+CCPR+key+comparison+report+preparation/ad4d6e1a-bbdb-e272-7d37-73ac80005805](https://www.bipm.org/documents/20126/30126060/CCPR-G2+(rev.+4)+Guidelines+for+CCPR+key+comparison+report+preparation/ad4d6e1a-bbdb-e272-7d37-73ac80005805)

# Modification to the spreadsheet

## From the document

For each NMI  $i$  for each lamp  $j$ , the relative difference  $\Delta_{i,j}$  between NMI measurement (as an average of two rounds) and Pilot measurement is given by,

$$\Delta_{i,j} = \frac{\bar{E}_{i,j}}{E_{i,j}^P} - 1 \quad (4)$$

and its uncertainty by

$$u(\Delta_{i,j}) = \sqrt{u_{\text{rel}}^2(\bar{E}_{i,j}) + u_{\text{rel}}^2(E_{i,j}^{\text{PR}}) + u_{\text{rel,add}}^2(E_{i,j})}. \quad (5)$$

where  $u_{\text{rel,add}}(E_{i,j})$  is an additional uncertainty in the comparison of lamp  $j$  of NMI  $i$ , arising from those components such as changes of the artifact due to transportation (if identified) and different measurement conditions between Pilot and participants that affected comparison results (if applicable) – often related to characteristics of the artifacts.

## From the spreadsheet

$$\Delta_{i,j} = 0; \quad u(\Delta_{i,j}) = 0.1061 \% \text{ for NIST}$$

$$\Delta_{i,j} = 0.00181; \quad u(\Delta_{i,j}) = 0.12476 \% \text{ for PTB (TSA)}$$

$$\Delta_{i,j} = 0.00210; \quad u(\Delta_{i,j}) = 0.12075 \% \text{ for PTB (TSB)}$$

$$\bar{E}_{i,j}^P = -4.35572 \text{ V/W for TSA}; \quad \bar{E}_{i,j}^P = -4.27073 \text{ V/W for TSB}$$

$$\bar{E}_{i,j} = -4.3636 \text{ V/W for TSA}; \quad \bar{E}_{i,j} = -4.27970 \text{ V/W for TSB}$$

$$u_{\text{rel,add}} = 0.001 \%$$

$$u_{\text{rel}}(E_{i,j}^{\text{PR}}) = 0.075 \% \quad \text{This term added for calculating } u(\Delta_{i,j})$$

$$u_{\text{rel}}(\bar{E}_{i,j}) = 0.075 \% \text{ for NIST}$$

$$u_{\text{rel}}(\bar{E}_{i,j}) = 0.09969 \% \text{ for PTB (TSA)}$$

$$u_{\text{rel}}(\bar{E}_{i,j}) = 0.09946 \% \text{ for PTB (TSB)}$$

# $\Delta$ KCRV

## From the document

The KCRV is calculated using weighted mean with cut-off. The cut-off value  $u_{\text{cut-off}}$  is calculated by

$$u_{\text{cut-off}} = \text{average}\{u_{\text{rel}}(\bar{E}_i)\} \text{ for } u_{\text{rel}}(\bar{E}_i) \leq \text{median}\{u_{\text{rel}}(\bar{E}_i)\} \quad (12)$$

$$; i = 0 \text{ to } N$$

The reported uncertainty  $u_{\text{rel}}(\bar{E}_i)$  of each NMI  $i$  is adjusted by the cut-off,

$$u_{\text{rel,adj}}(\bar{E}_i) = u_{\text{rel}}(\bar{E}_i) \text{ for } u_{\text{rel}}(\bar{E}_i) \geq u_{\text{cut-off}} \quad i = 0 \text{ to } N \quad (13)$$

$$u_{\text{rel,adj}}(\bar{E}_i) = u_{\text{cut-off}} \text{ for } u_{\text{rel}}(\bar{E}_i) < u_{\text{cut-off}}$$

The weights  $w_i$  for NMI  $i$  is determined by

$$w_i = u_{\text{adj}}^{-2}(\Delta_i) / \sum_{i=0}^N u_{\text{adj}}^{-2}(\Delta_i) \quad (16)$$

The KCRV,  $\Delta_{\text{KCRV}}$  is determined by

$$\Delta_{\text{KCRV}} = \sum_{i=0}^N w_i \Delta_i \quad (17)$$

The uncertainty of the KCRV (weighted mean with cut-off) is given by

$$u(\Delta_{\text{KCRV}}) = \sqrt{\frac{\sum_{i=0}^N u^2(\Delta_i)}{\sum_{i=0}^N u_{\text{adj}}^4(\Delta_i)}} / \sum_{i=0}^N u_{\text{adj}}^{-2}(\Delta_i) \quad (18)$$

## From the spreadsheet

$$\text{median}\{u_{\text{rel}}(\bar{E}_i)\} = 0.114 \%$$

$$u_{\text{cut-off}} = 0.1061 \%$$

$$u_{\text{rel,adj}}(\bar{E}_i) = 0.1061 \% \text{ for NIST}$$

$$u_{\text{rel,adj}}(\bar{E}_i) = 0.1228 \% \text{ for PTB}$$

$$w_i = 57.3 \% \text{ for NIST}$$

$$w_i = 42.7 \% \text{ for PTB}$$

$$\Delta_{\text{KCRV}} = 0.0836 \%$$

$$u(\Delta_{\text{KCRV}}) = 0.0803 \%$$

# Consensus responsivity and DoE

From the spreadsheet

$$CV_j = (\Delta_{KCRV} + 1) \cdot R_{\text{pilot},j} = -4.35936 \text{ V/W (TSA)}; u(CV_j) = 0.075 \%$$

$$CV_j = (\Delta_{KCRV} + 1) \cdot R_{\text{pilot},j} = -4.27430 \text{ V/W (TSB)}; u(CV_j) = 0.075 \%$$

From the document

The bilateral DoE between NMI  $i$  and NMI  $m$  is given by

$$D_{i,m} = \Delta_i - \Delta_m \quad (24)$$

$$U_{i,m} = k \sqrt{u^2(\Delta_i) + u^2(\Delta_m)} \quad ; k=2 \quad (25)$$

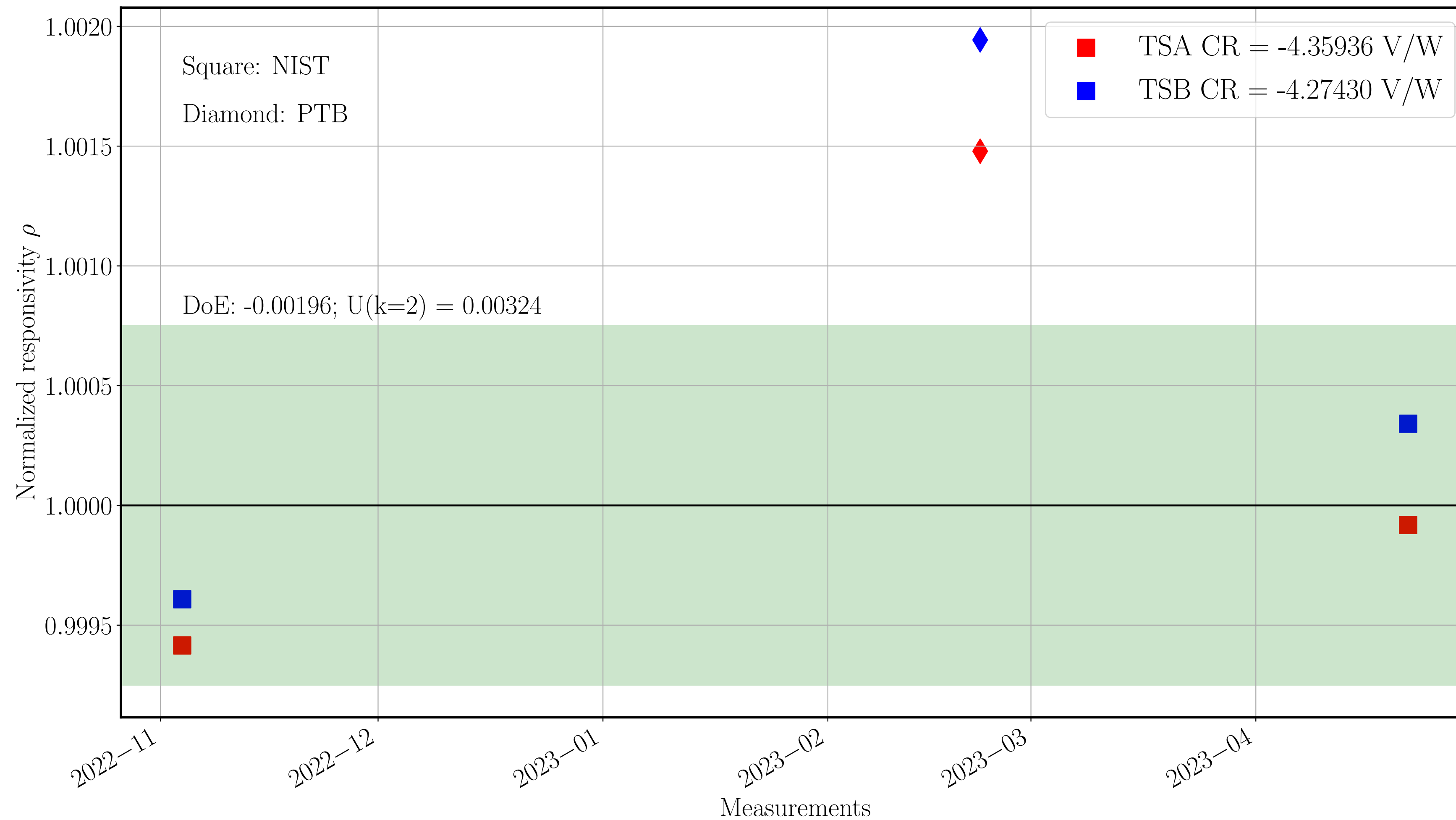
From the spreadsheet

$$\Delta_i = 0 \%; \Delta_m = 0.196 \%$$

$$u(\Delta_i) = 0.1061 \%; u(\Delta_m) = 0.1228 \%$$

$$D_{i,m} = -0.196 \%; U_{i,m} = 0.324 \%$$

# Bilateral data



# NEWRAD, 2023

- Abstract has been accepted for oral presentation

## **Calibrating the global network of gravitational wave observatories via laser power calibration at NIST and PTB.**

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**DCC link: <https://dcc.ligo.org/LIGO-G2300653/public>**



# Calibration subway map

Both transfer standards currently at LIGO Hanford

