Gravitational Wave Open Data Workshop

Gravitational Wave Searches for Compact Binary Mergers

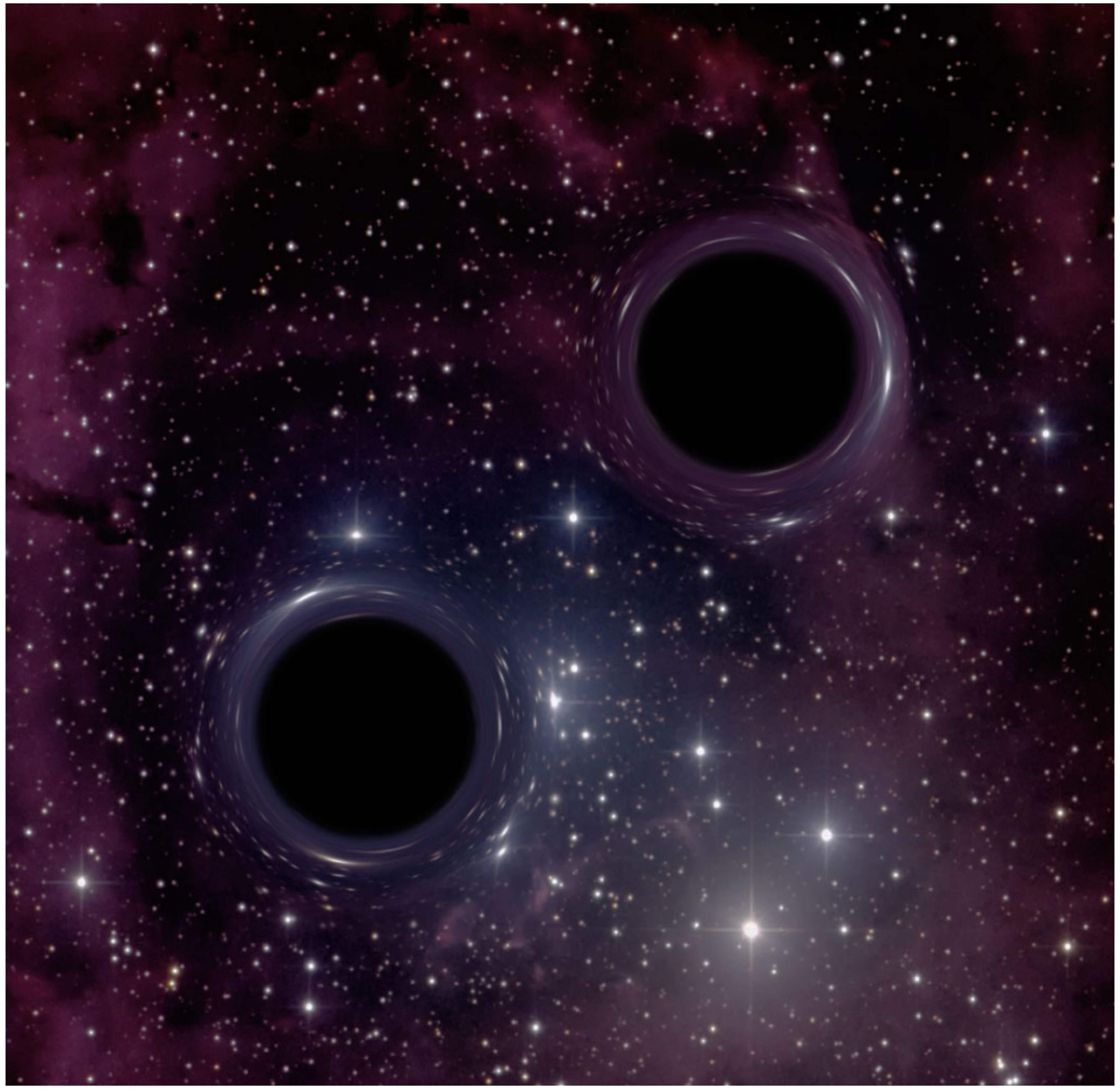
Koustav Chandra

16th May 2023

IIT BOMBAY

Talk Layout

- Introduction
- What does the signal look like?
- What does the data look like?
- How do we find the signal?
- Limitations and how to overcome them



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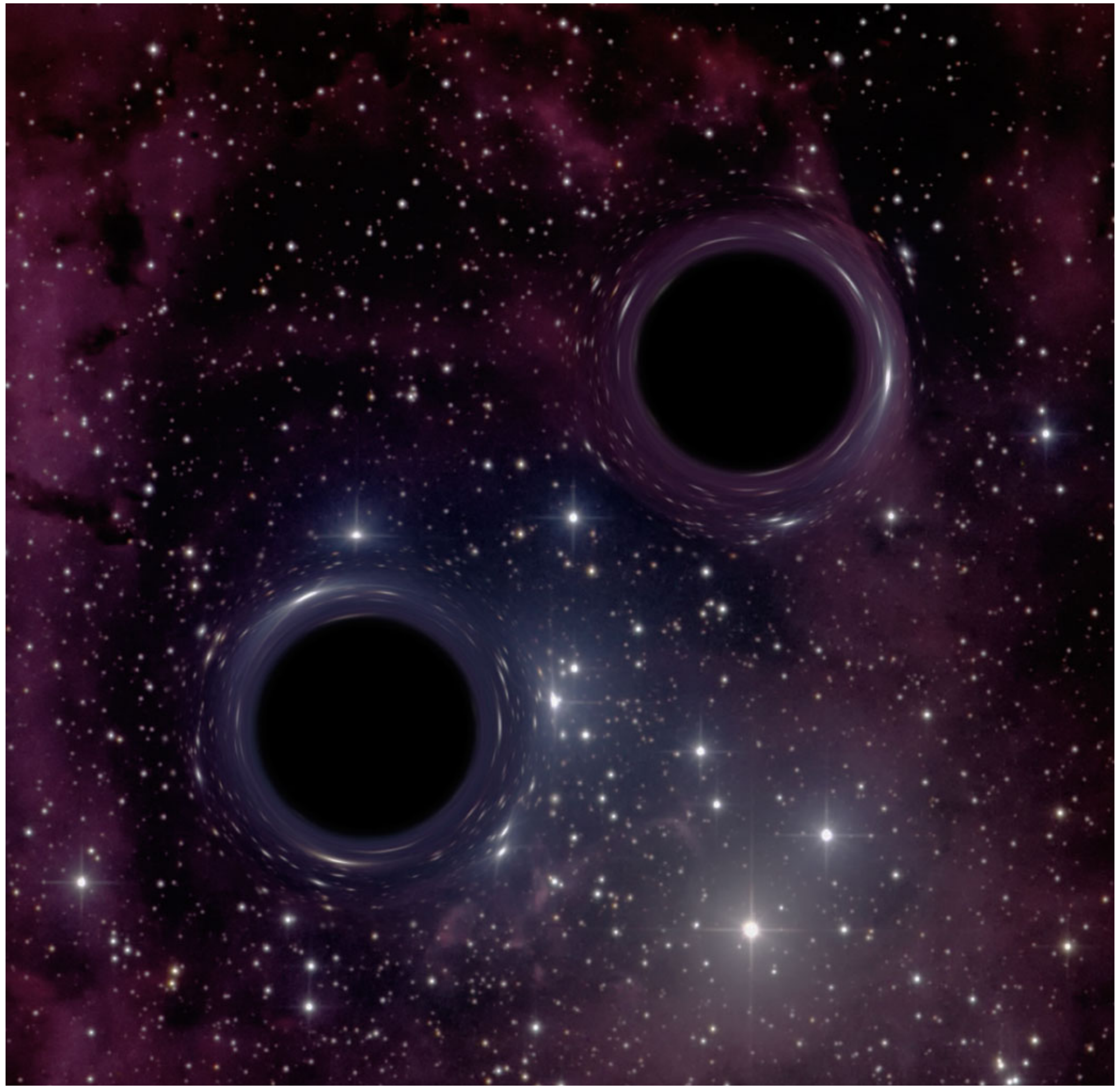
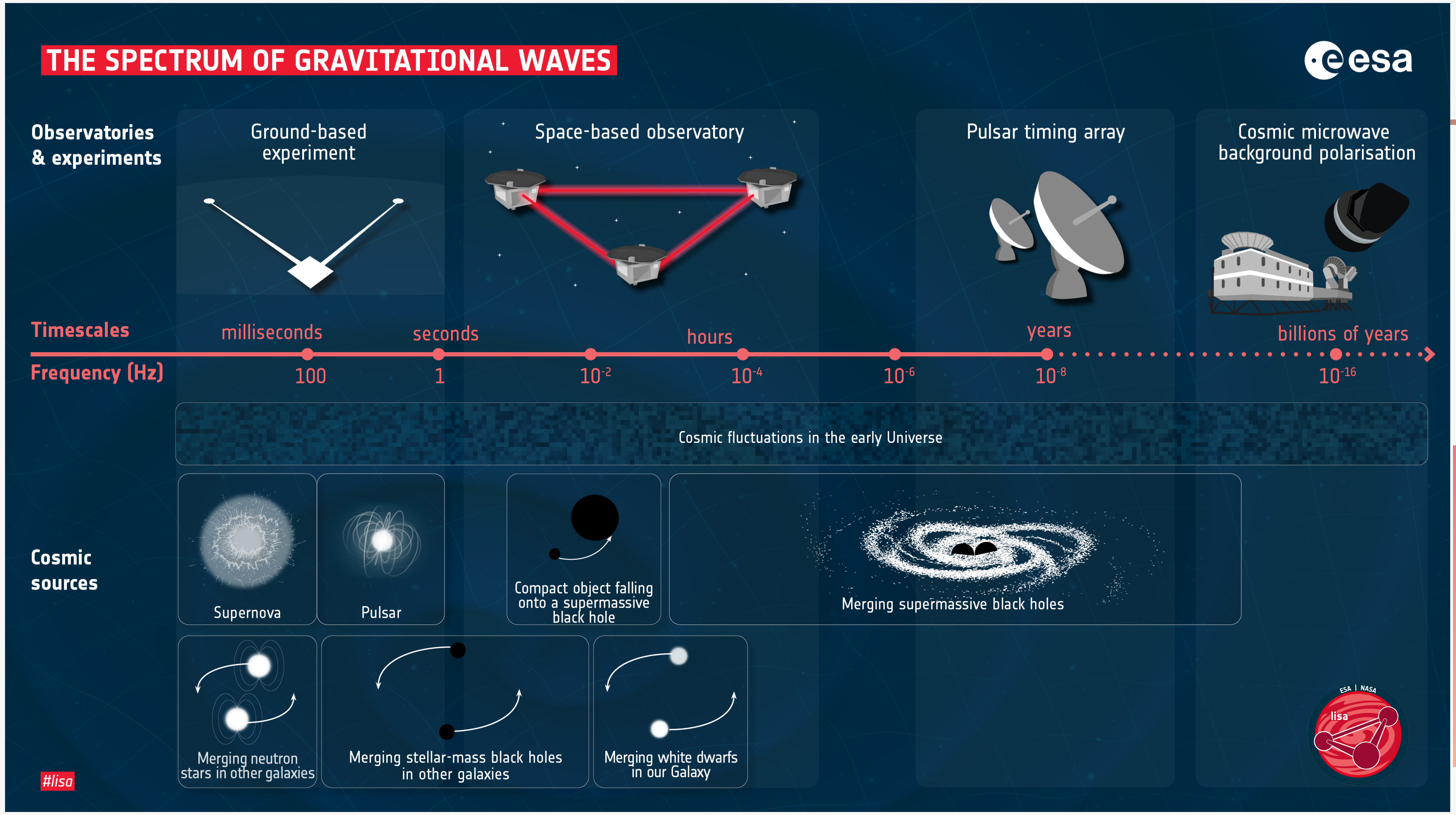


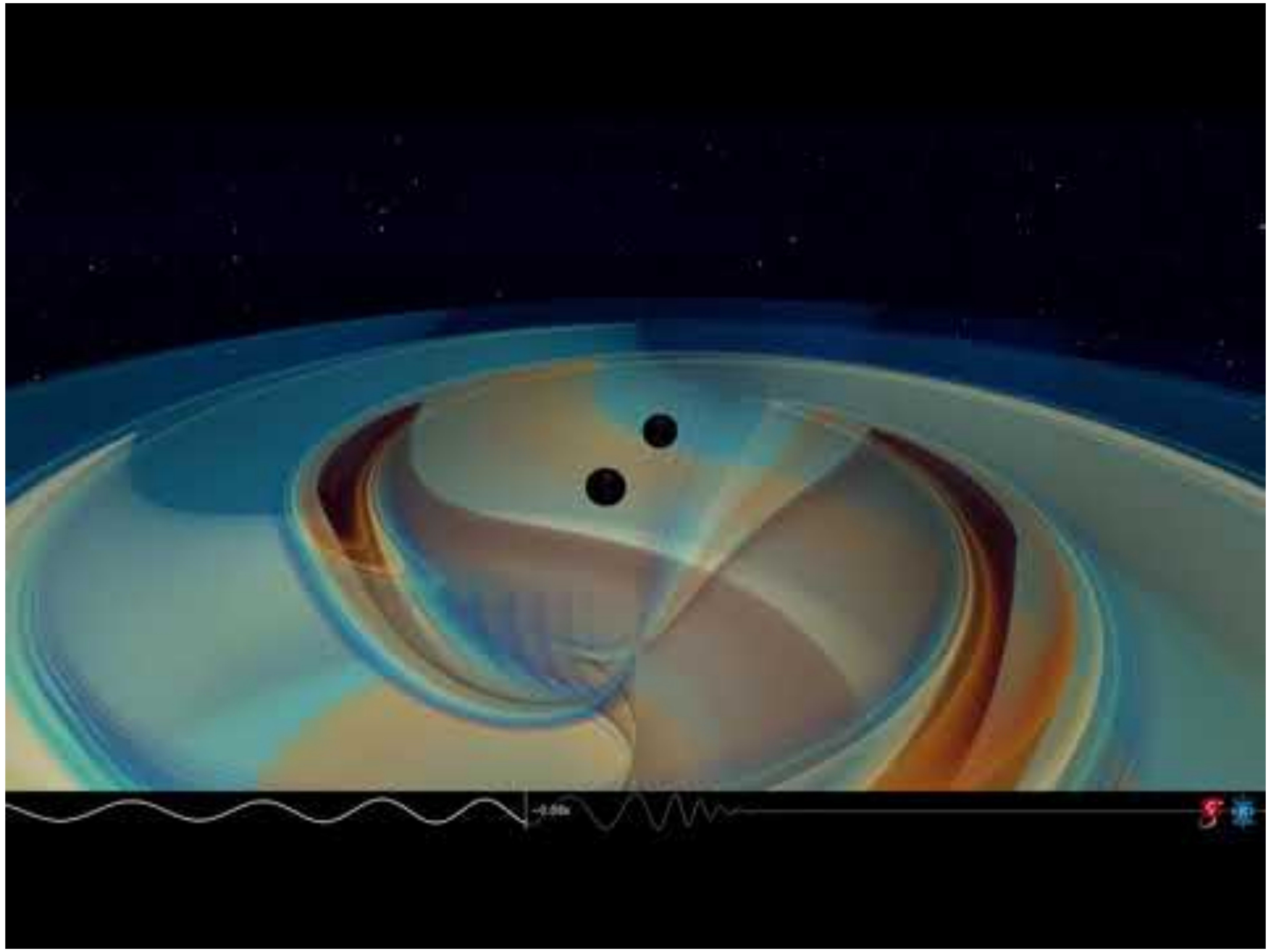
Gravitational Wave Sources



- One of General Relativity's bold predictions — Gravitational Waves (GWs) — ripples in spacetime
- Any time-varying non-axisymmetric mass distribution can produce gravitational waves
- Current ground-based detectors can observe high-frequency gravitational wave sources ($\sim 10\text{Hz}$ to a few 1000Hz)
  - Compact Binary Coalescences (CBCs), Supernova Explosion, Rotating Neutron Stars, etc..
- Focus here: CBCs

Compact Binary Mergers in LIGO/Virgo bandwidth

- Compact Binaries refers to binaries consisting of a pair compact objects — $\text{Radius} \propto \text{Mass}$
- Compact objects include white dwarfs, neutron stars, and black holes.
- LIGO/Virgo detectors observes binary neutron star [BNS], binary black hole [BBH] and neutron star-black hole [NSBH] mergers.
- Strain $= s = \dfrac{\Delta L}{L} \sim 10^{-21} \rightarrow \Delta L \sim 10^{-18}\text{m}$ [Given $L \sim \mathcal{O}(1 \text{ km})$]
- Check Viola's slides to know why observe compact binary mergers



[Link to video]

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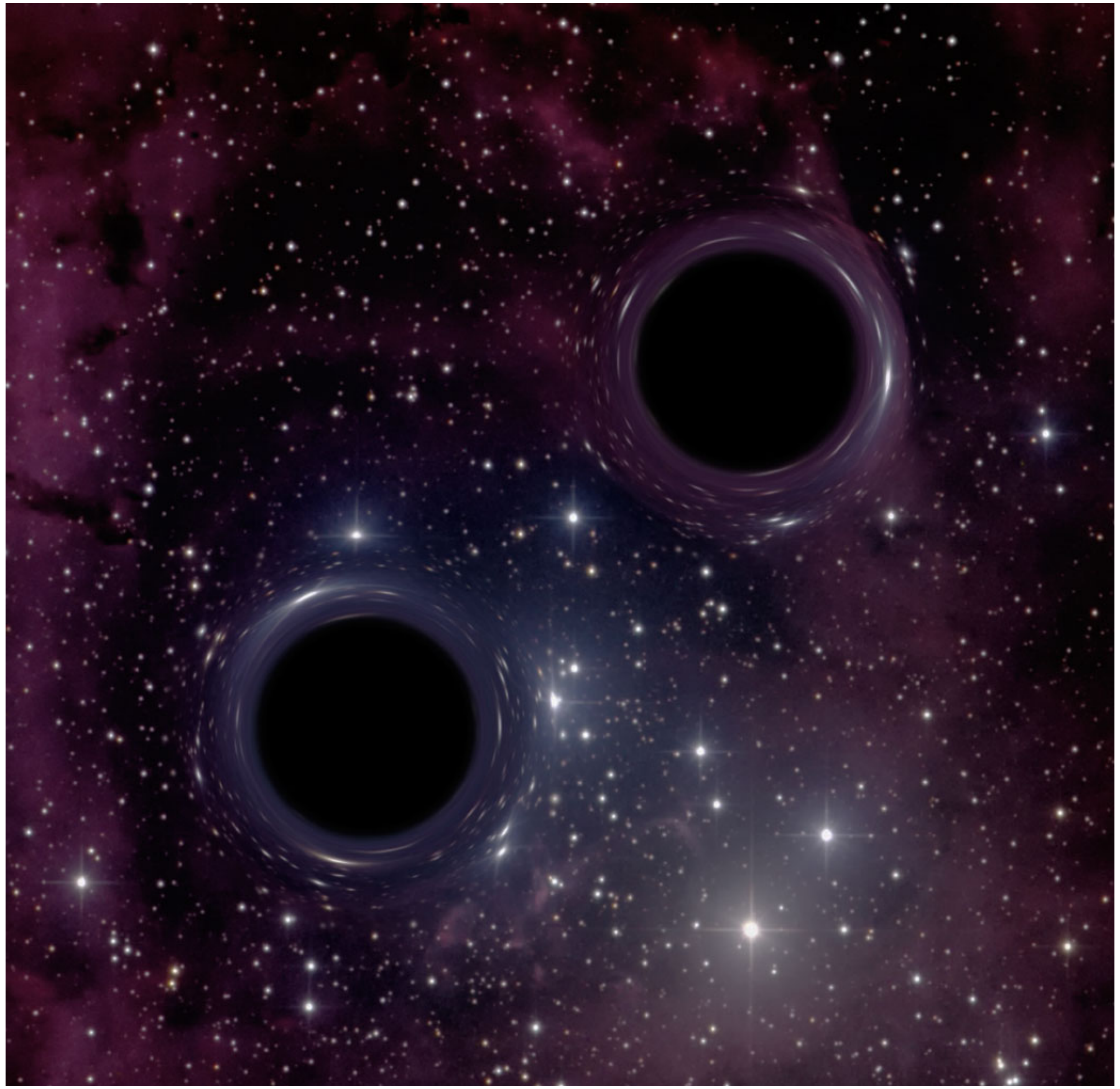


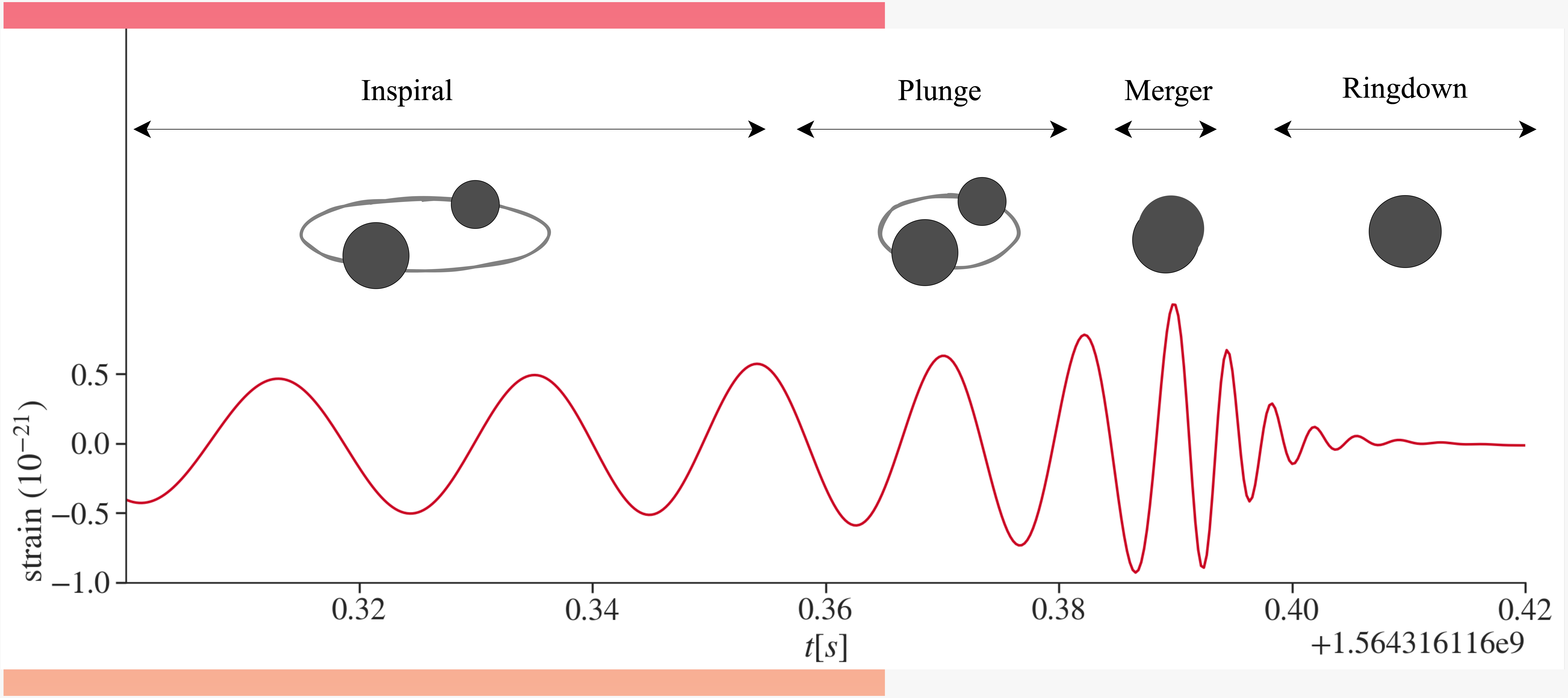
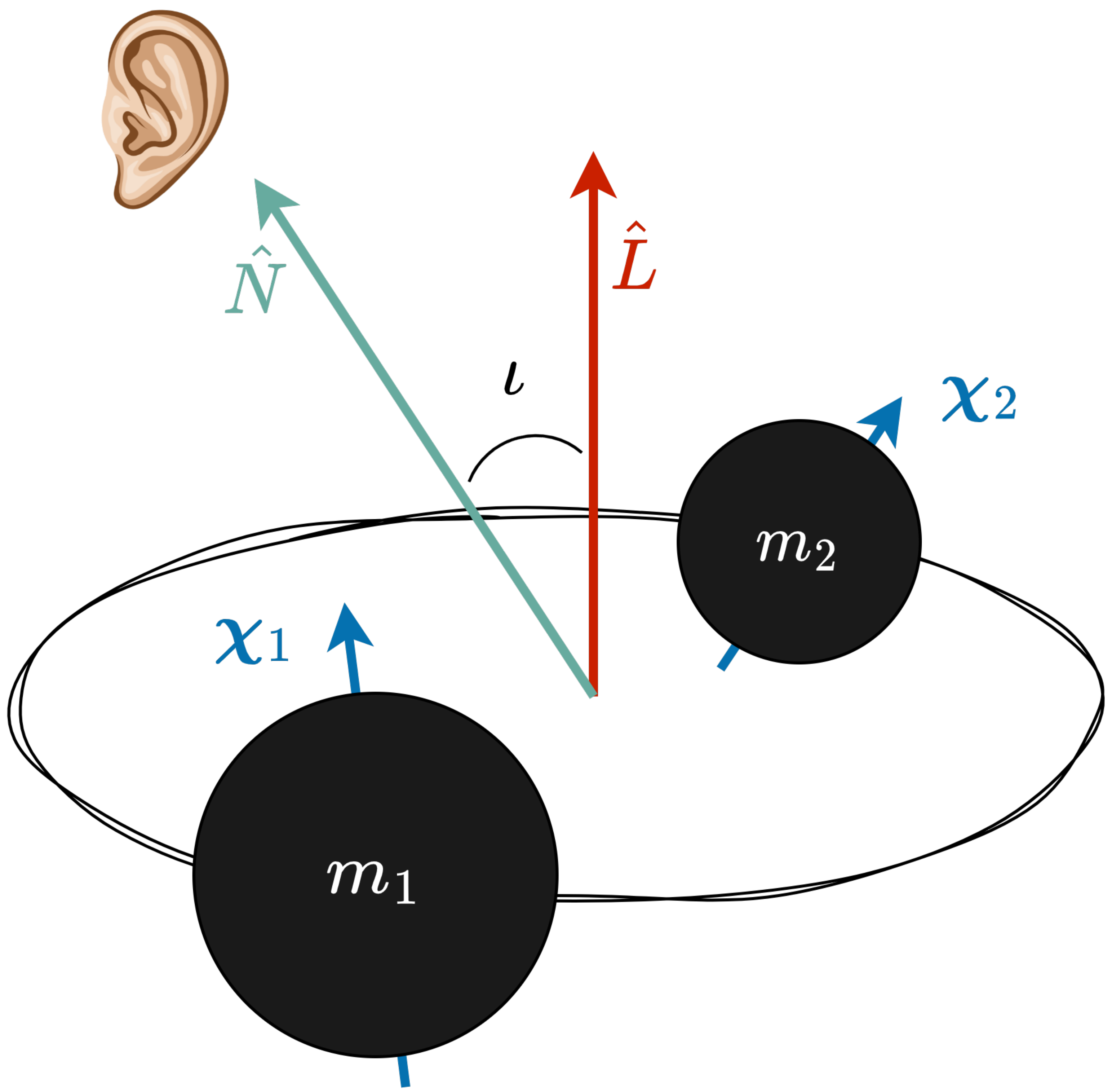


Fig: Gravitational waveform of a non-spinning black hole binary

Compact Binary Parameters

- In General Relativity, *quasi-spherical black hole binaries* are described by $\boldsymbol{\theta}$ which consists of 15 parameters.

- Intrinsic:
  - Two component masses: $m_1, \ m_2$
  - Six spin Components: χ_1, χ_2



- Extrinsic:
  - Sky Location: $(\alpha, \ \delta)$
  - Luminosity distance: D_L (Or equivalently the redshift z)
  - Binary orientation parameters: $(\iota, \ \varphi)$
  - Polarisation angle: ψ
  - Merger time: t_c

- More parameters required if matter or new physics is included

$\hat{L} \rightarrow$ orbital angular momentum direction

$\hat{N} \rightarrow$ Line of sight

P. Schmidt FSPAS (2020)

Phenomenology of Black hole binaries

Effect of total mass

Detector Frame Total Mass = Redshifted source frame mass → Gravitational waves are redshifted due to spacetime expansion

$$s(t \mid \boldsymbol{\theta}) \propto \left(M_T(1+z)\right)^{5/6} \sqrt{\frac{q}{1+q^2}}$$

Leading order

$$q = \frac{m_1}{m_2} = \frac{\text{Heavier black hole}}{\text{Lighter black hole}} \geq 1$$

Heavier binary → Larger amplitude

Phenomenology of Black hole binaries

Effect of mass ratio

$$s(t \mid \boldsymbol{\theta}) \propto \left(M_T(1+z)\right)^{5/6} \sqrt{\frac{q}{1+q^2}}$$

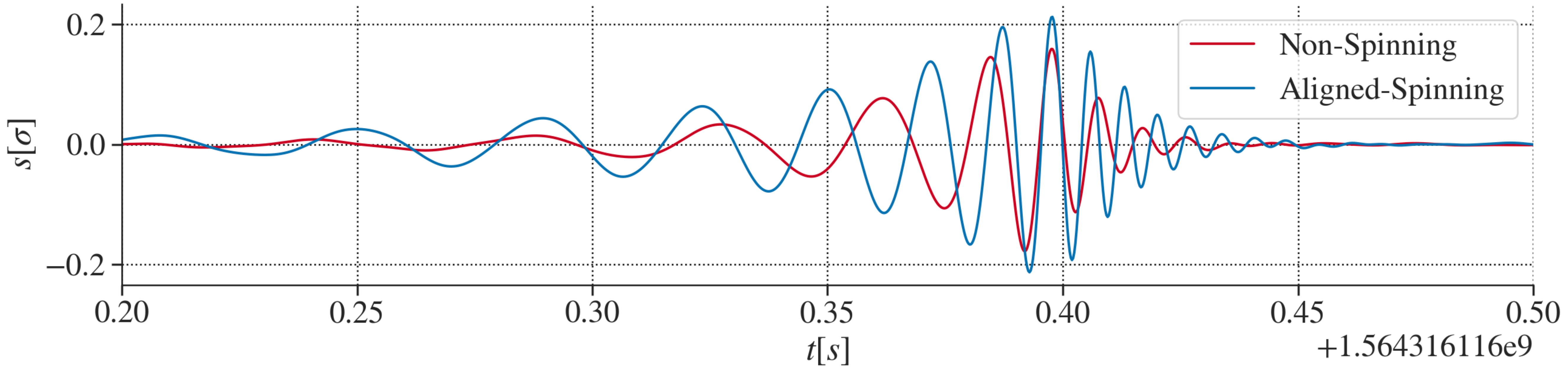
Leading order

$$q = \frac{m_1}{m_2} = \frac{\text{Heavier black hole}}{\text{Lighter black hole}} \geq 1$$

More Symmetric → Larger amplitude

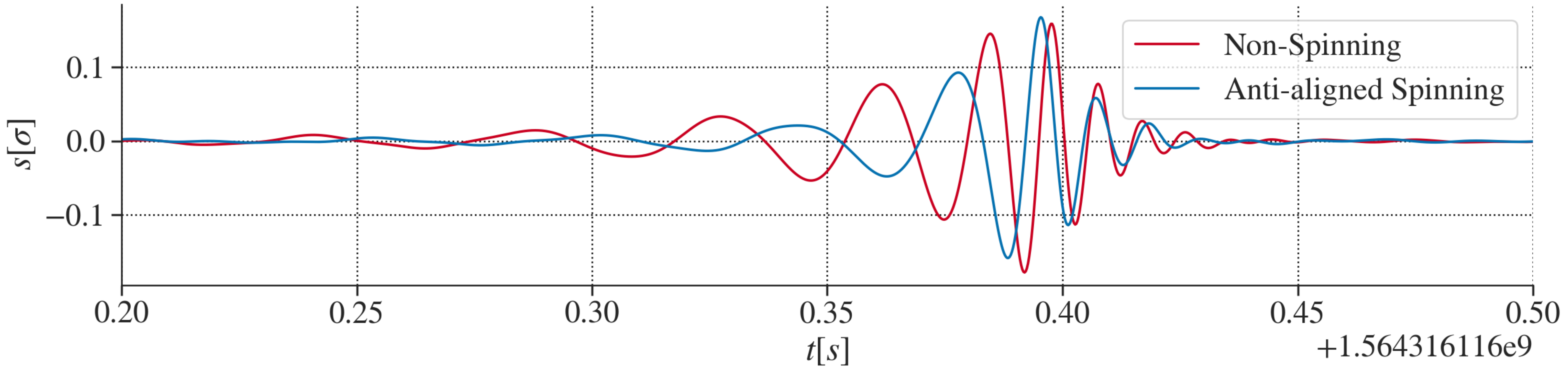
Phenomenology of Black hole binaries

Effect of spins



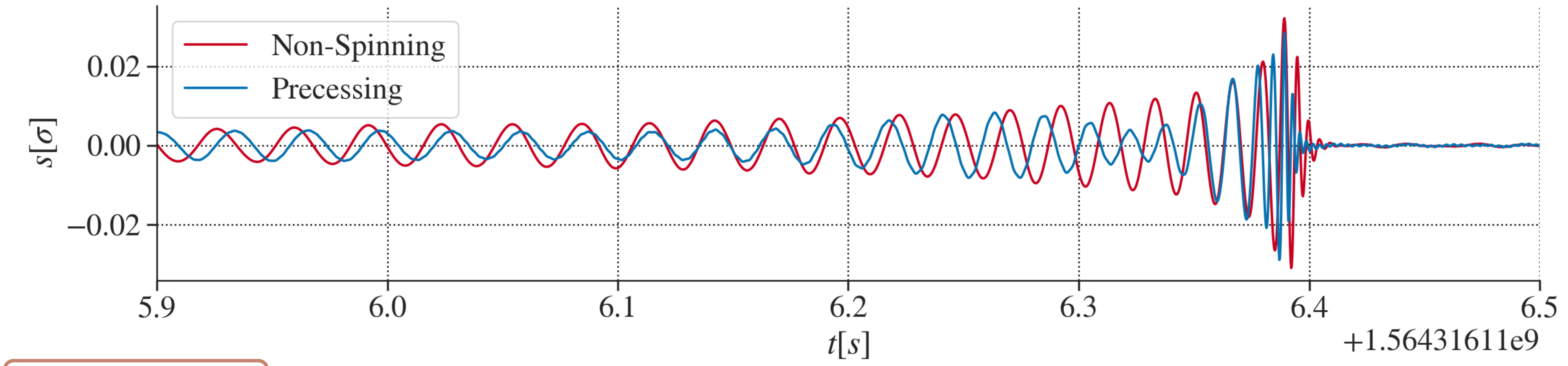
Black holes with aligned spins

χ_1, χ_2 are aligned with $\hat{L}$ → black holes inspiral to closer separation → longer, stronger GWs



Black holes with anti-aligned spins

χ_1, χ_2 are aligned opposite to $\hat{L}$ → black holes can't inspiral to closer separation → shorter, weaker GWs



Black holes with misaligned spins

If χ_1, χ_2 are misaligned with $\hat{L}$ → orbital precession → GW signal with modulating amplitude and phase

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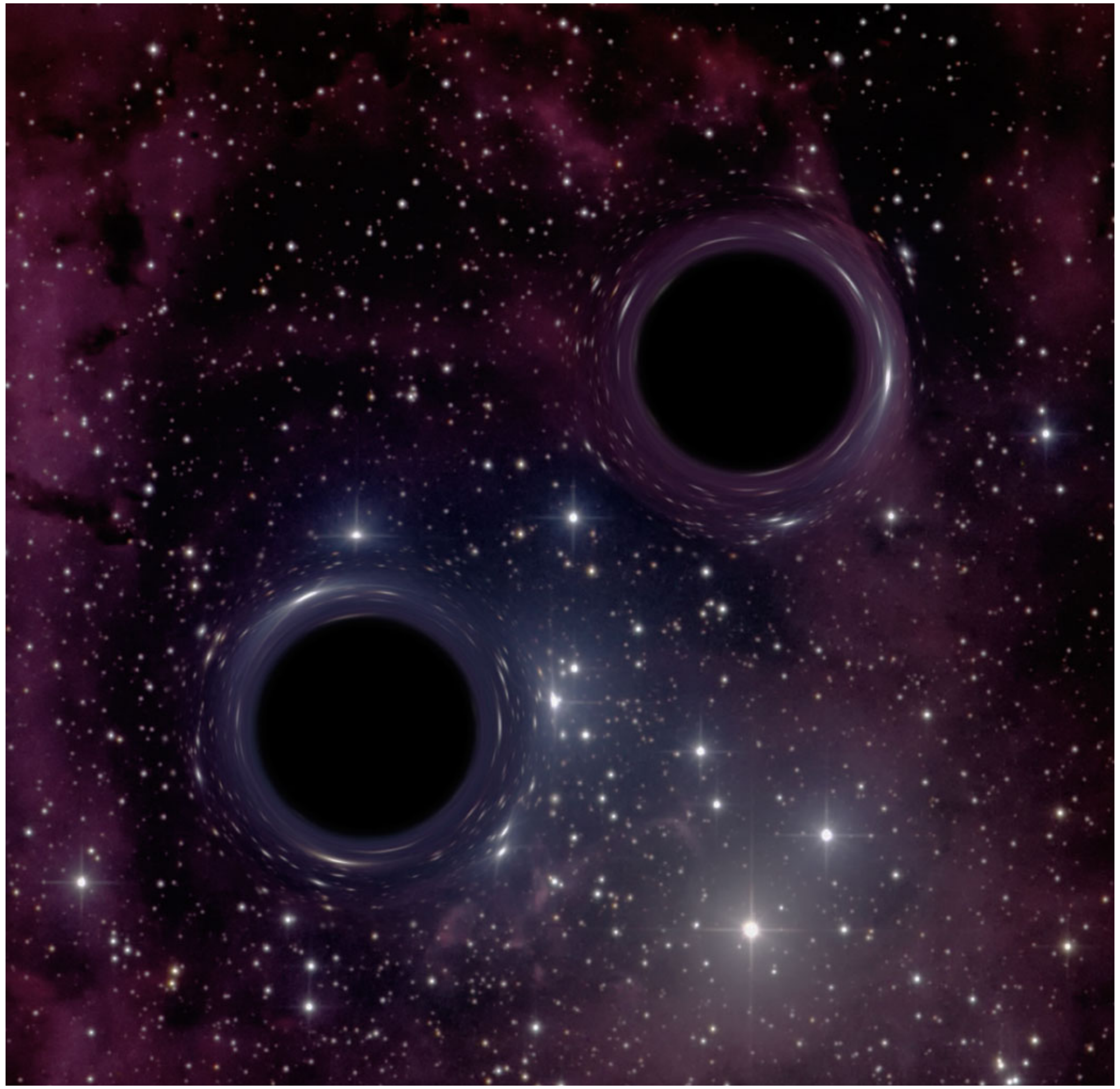


Gravitational Wave Detector Data

- GW interferometers record data as a discretely sampled time series $\boldsymbol{d} = \left\{ d\left(t_1\right), \ldots, d\left(t_N\right) \right\}$ at sampling frequency $f_s = 16\text{kHz} \rightarrow N_{\text{samples}} = T \times f_s$ where T = data segment duration

Assuming linear detector response,

$$\boldsymbol{d} = \boldsymbol{s}\left(\boldsymbol{\theta}\right) + \boldsymbol{n}$$

- $\boldsymbol{d}$: data
- $\boldsymbol{s}(\boldsymbol{\theta})$: signal strain: Deterministic for CBC
- $\boldsymbol{n}$: noise: Stochastic
  - Contains contributions from myriad of noise sources
  - $|\boldsymbol{n}| \gg \left|\boldsymbol{s}\left(\boldsymbol{\theta}\right)\right| \rightarrow$ Needle in a haystack problem!

Refer to Victoria's and Ronaldas's talk for more details

Goal: To find a template or model waveform $\boldsymbol{h}\left(\boldsymbol{\theta}'\right) \sim \boldsymbol{s}\left(\boldsymbol{\theta}\right)$ such that $\boldsymbol{r} = \boldsymbol{d} - \boldsymbol{h}(\boldsymbol{\theta}') \sim \boldsymbol{n}$

Noise Model

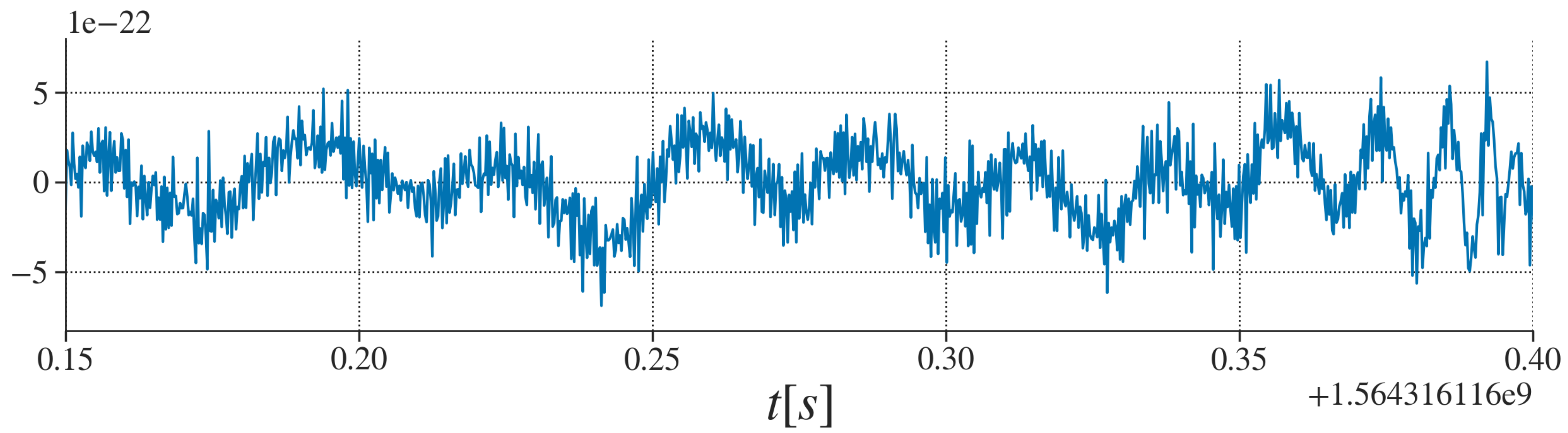
- Assumption: Noise in each detector follows a zero-mean wide-sense stationary multivariate Gaussian distribution (Noise's DC component can always be subtracted out).

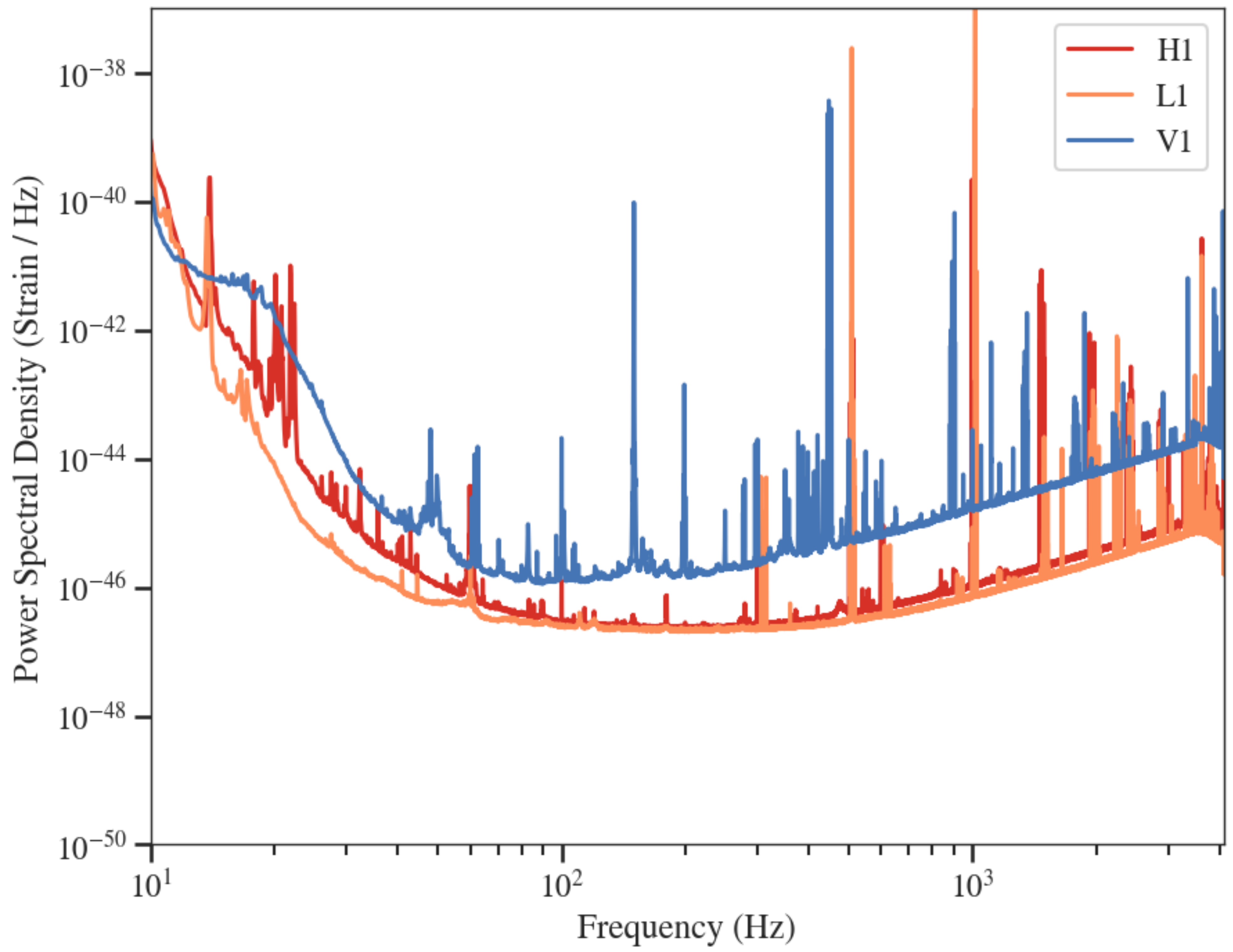
- **Wide-sense stationary**: Elements of noise correlation matrix $C\left(\left|t_i - t_k\right|\right) = \langle n(t_i)\; n(t_k)\rangle \rightarrow$ depends on time-lag between samples.

- In Fourier domain, the noise correlation matrix is diagonal, $\left\langle \left|\tilde{n}(f_k)\right|^2 \right\rangle = \frac{T}{2} S_n(f_k),\; S_n(\boldsymbol{f})$ = Noise power spectrum → calculate using Welch method.

- **Zero-mean multivariate Gaussian**:
$$\mathfrak{L}\left(\boldsymbol{d} \mid \text{noise}, S_n\right) \propto \prod_i \exp\left[-\frac{2\left|\tilde{d}\left(f_i\right)\right|^2}{T S_n\left(f_i\right)}\right],$$

- Presence of a signal adjusts the mean value





Refer to Ronaldas's talk for more details

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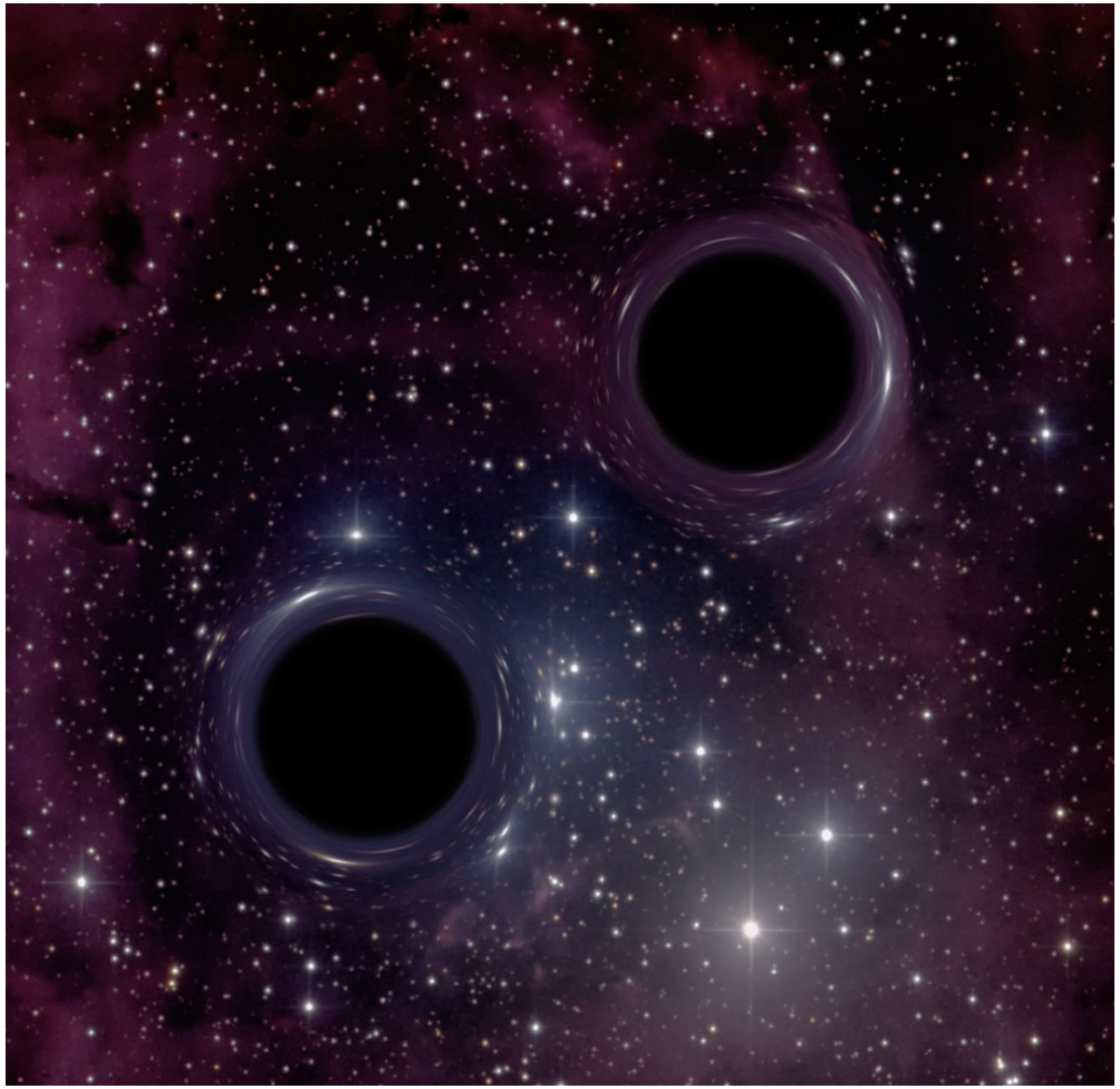
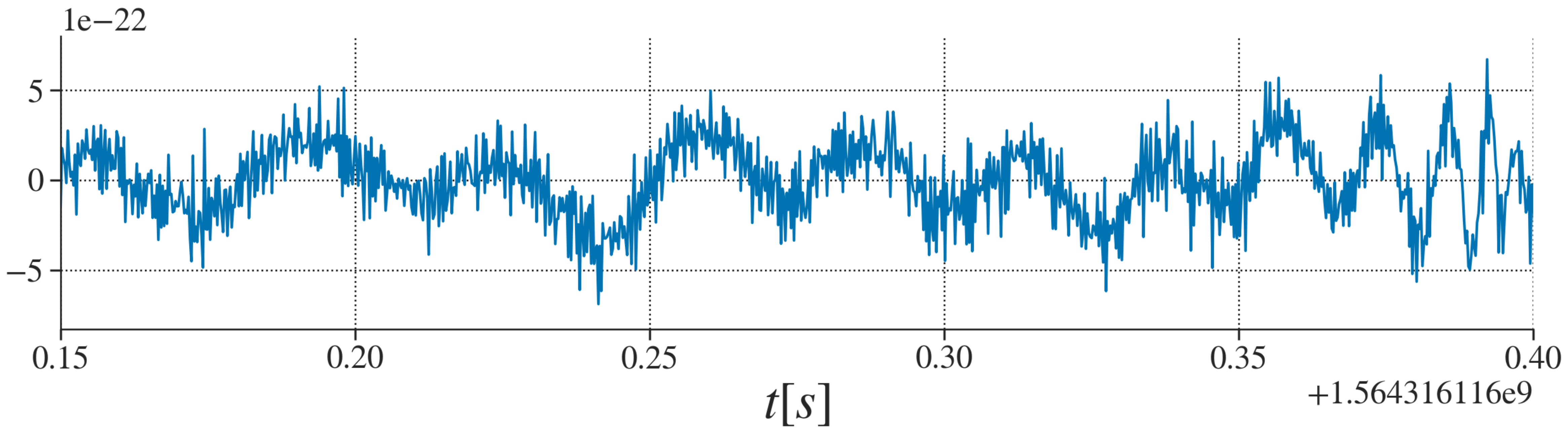
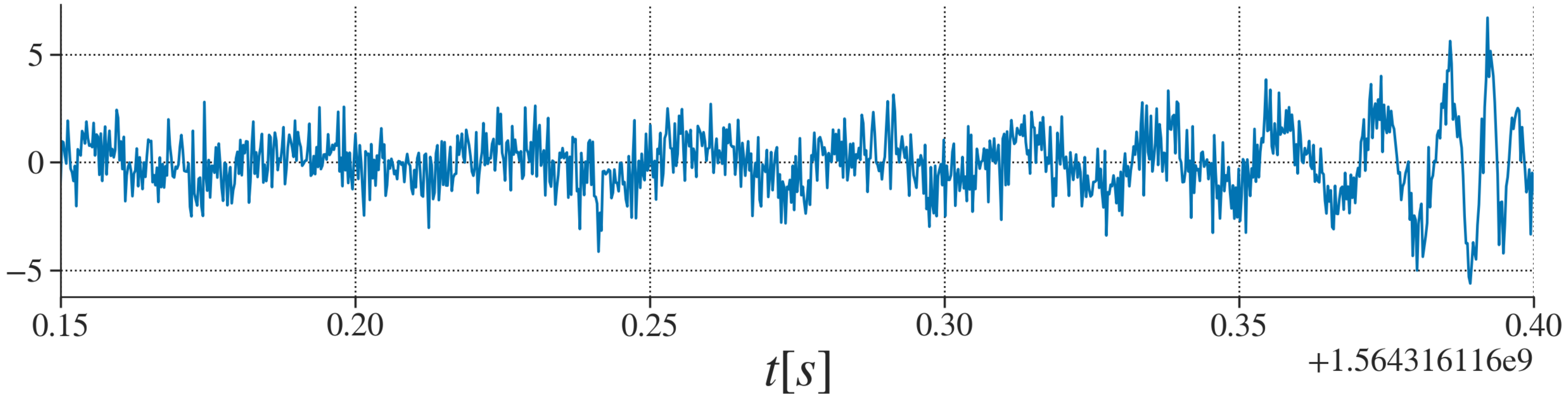


Matched Filtering

One way to find a GW signal is matched filtering.

- Step-1: Whiten the detector data: $\boldsymbol{d} \rightarrow \dfrac{\tilde{d}(f_i)}{\sqrt{S_n(f_i)}}$

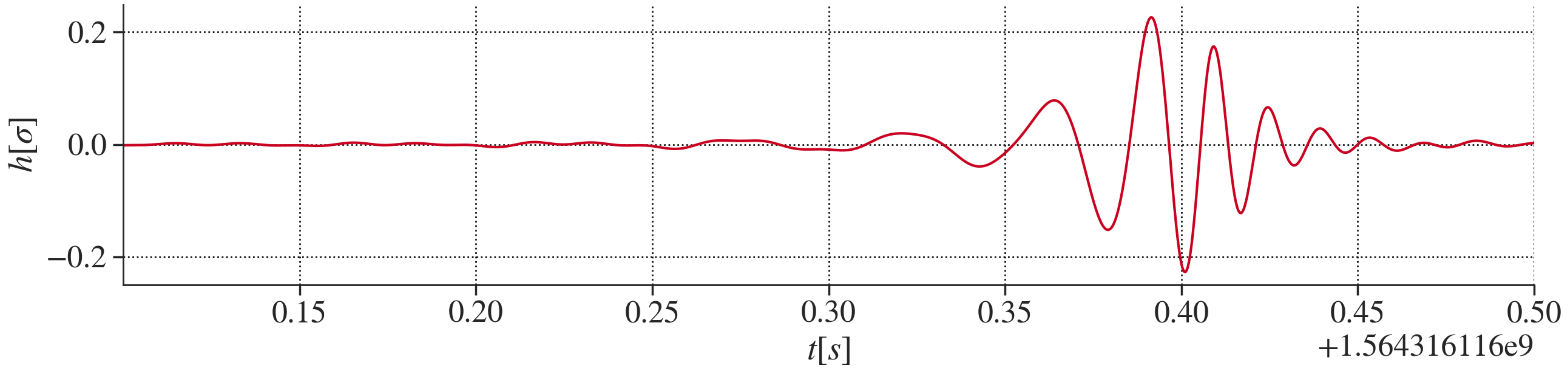




Whitening normalises the power at all frequencies so that any excess power at any frequency becomes obvious.

- Step-2: Whiten the template: $\boldsymbol{h} \rightarrow \dfrac{\tilde{h}(f_i \mid \boldsymbol{\theta}')}{\sqrt{S_n(f_i)}}$

Adjust the template's amplitude at each frequency to account for the detector's noise level



- Step-3: Calculate the optimal signal-to-noise ratio (SNR) of the template

$$\rho^2_{\text{opt}} = \left(h \mid h\right) = 4\Re \sum_{f_i} \frac{\tilde{h}^*(f_i \mid \boldsymbol{\theta})\tilde{h}\left(f_i \mid \boldsymbol{\theta}'\right)}{S_n(f_i)} \Delta f,$$

Δf = frequency resolution

- Step-4: Cross correlate the whitened data and whitened normalised template

$$\rho = \frac{\left(d \mid h\right)}{\sqrt{\left(h \mid h\right)}} \rightarrow \text{matched-filter SNR}$$

Sathyaprakash + Dhurandhar, PRD 44, 3819 (1991)

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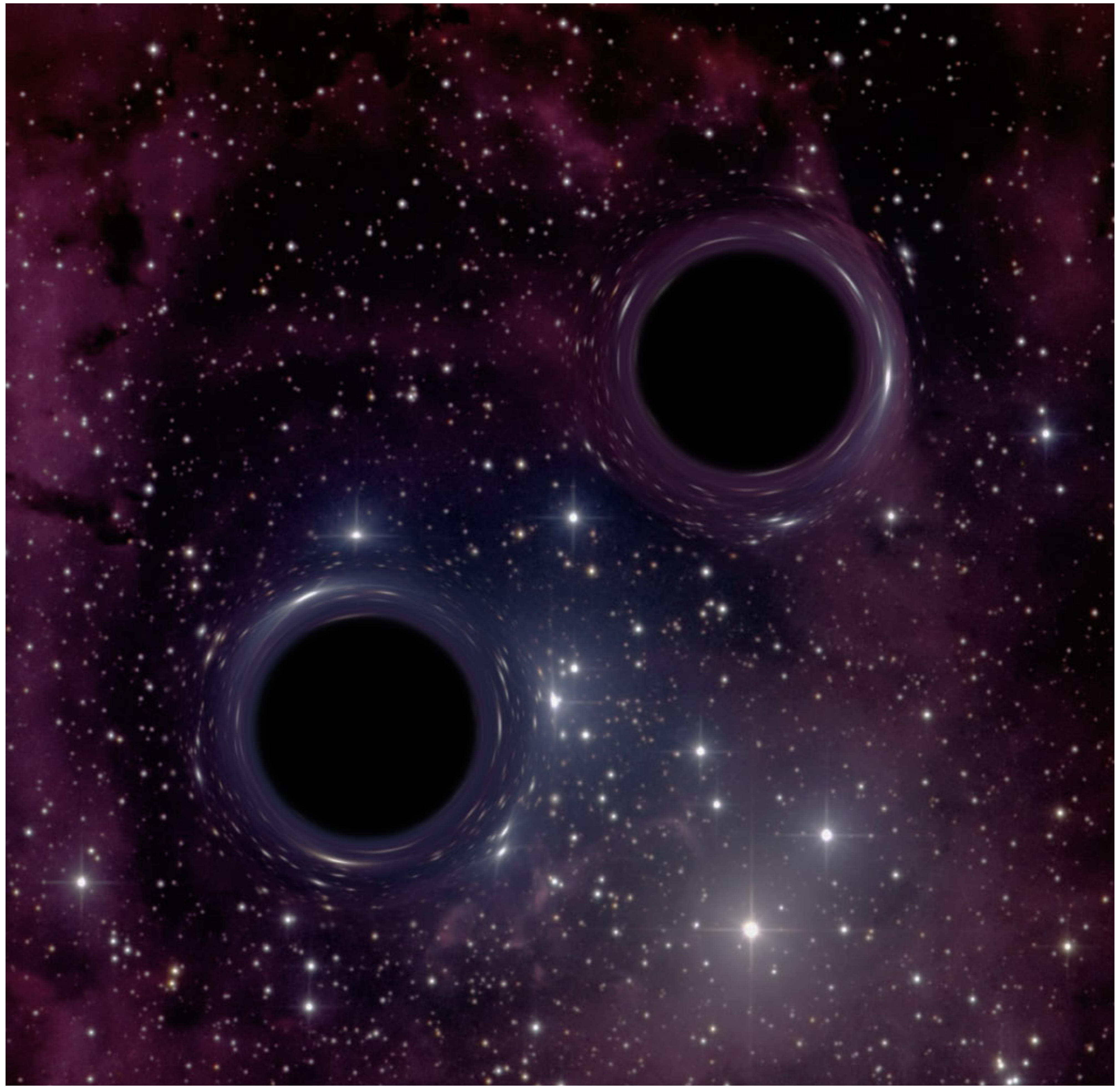


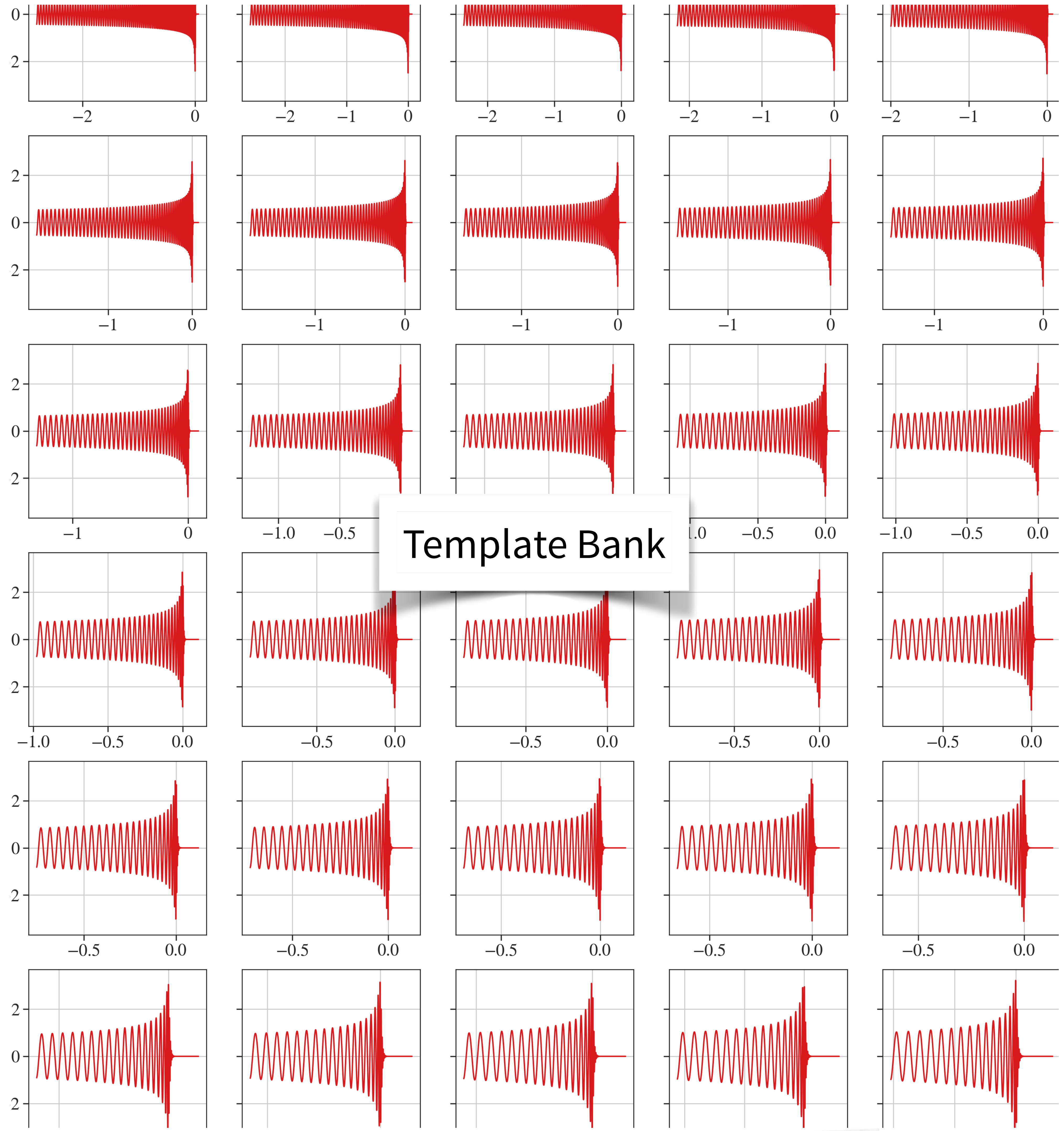
Matched Filtering with unknown binary parameters

- We don't know the signal's merger time t_c → matched filter as a function of time and find the peak of $\rho(t)$.

- Matched filtering is very sensitive to signal's phase evolution + we don't know the binary parameters a priori → numerically maximise $\rho(t)$ using a template bank →Template with highest $\rho(t)$ is the best-matched template.

- Computationally infeasible to search for every possible binary parameter combination → assume signal is adequately represented by quasi-circular (non-precessing) quadrupole modes → search using a template bank parameterised by (m_1, m_2) and $\left(\chi_1 \cdot \hat{L}, \chi_2 \cdot \hat{L}\right)$.

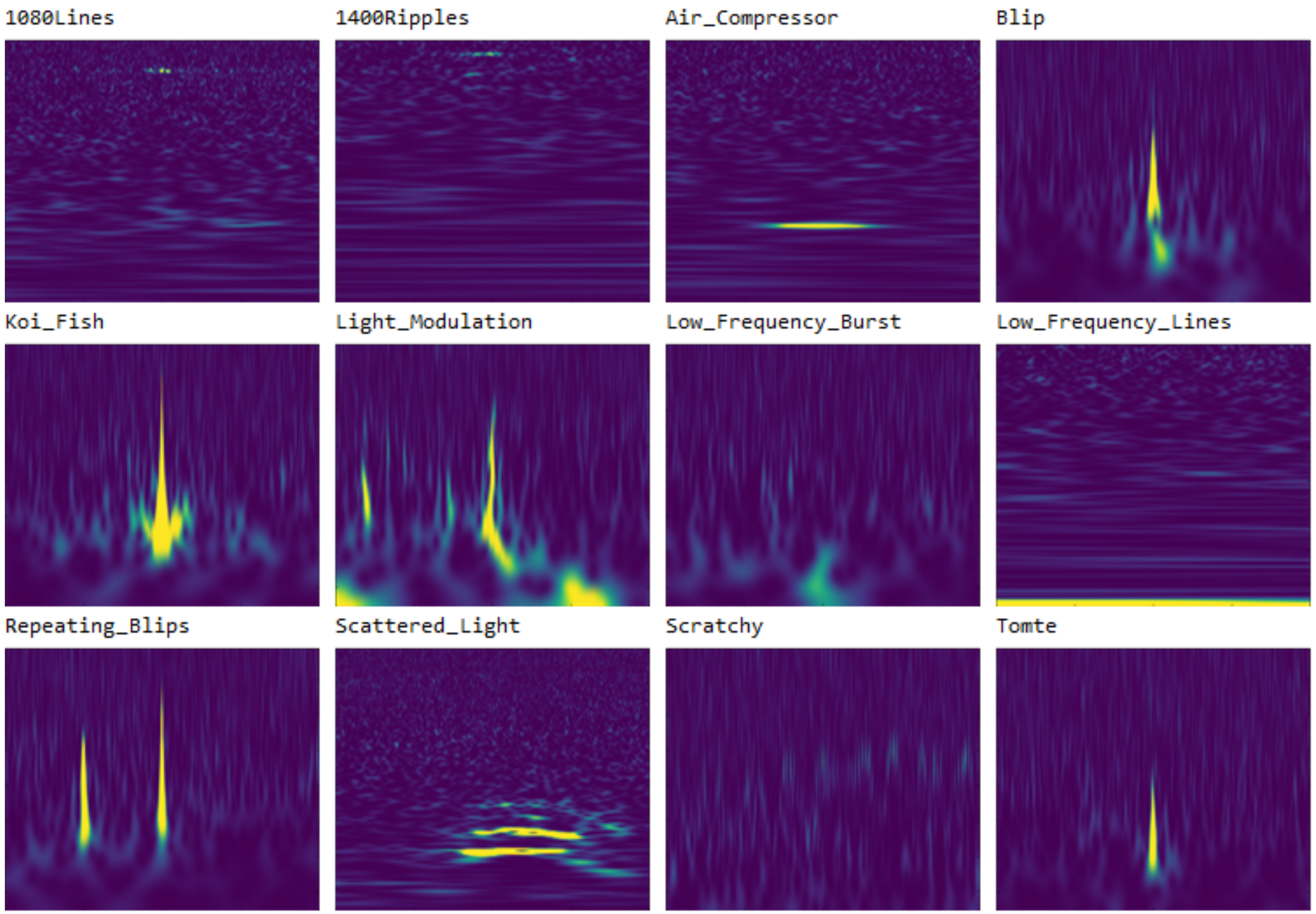
- Note: Neighbouring templates in the bank are not *too* dissimilar.



Sathyaprakash + Dhurandhar, PRD 44, 3819 (1991)

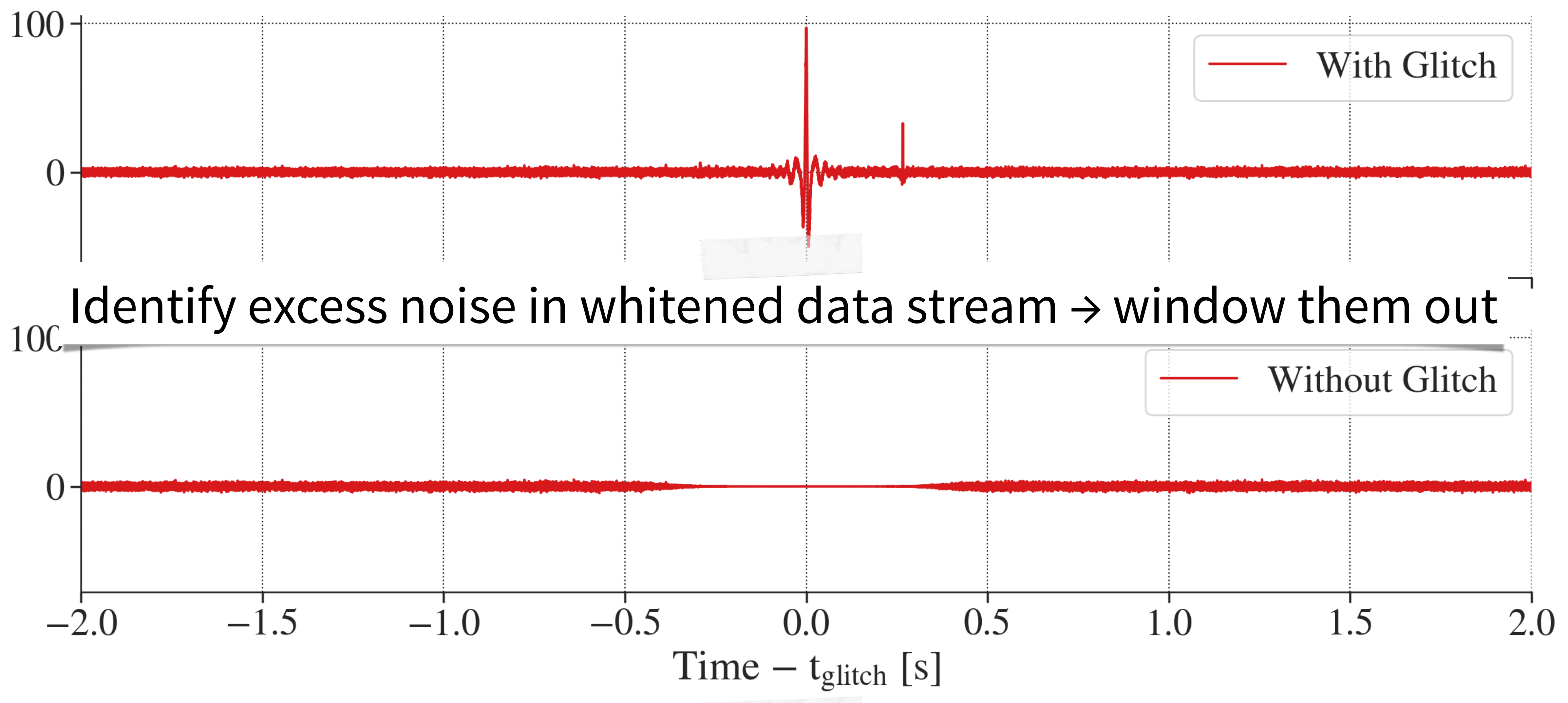
Headaches!

- Matched filtering is optimal if detector data is Gaussian.
- ~~Gravitational wave data can be modelled to be wide-sense stationary Gaussian process.~~
- GW data is plagued with intermittent non-Gaussian transients or glitches → raises false alarms & reduce search performance
- Solution: Use a combination of vetoes, gating, coincidence tests and signal-noise discriminators to penalise/remove noisy glitches.
- The four templated searches namely PyCBC, GstLAL, MBTA and SPIIR implements slightly different methods to handle the non-ideal noise properties.



vetoes → Refer Ronaldas talk

Gating



Identify excess noise in whitened data stream → window them out

Usman et al. CQG 33 (2016) 21, 215004

integrated **D**ata **Q**uality

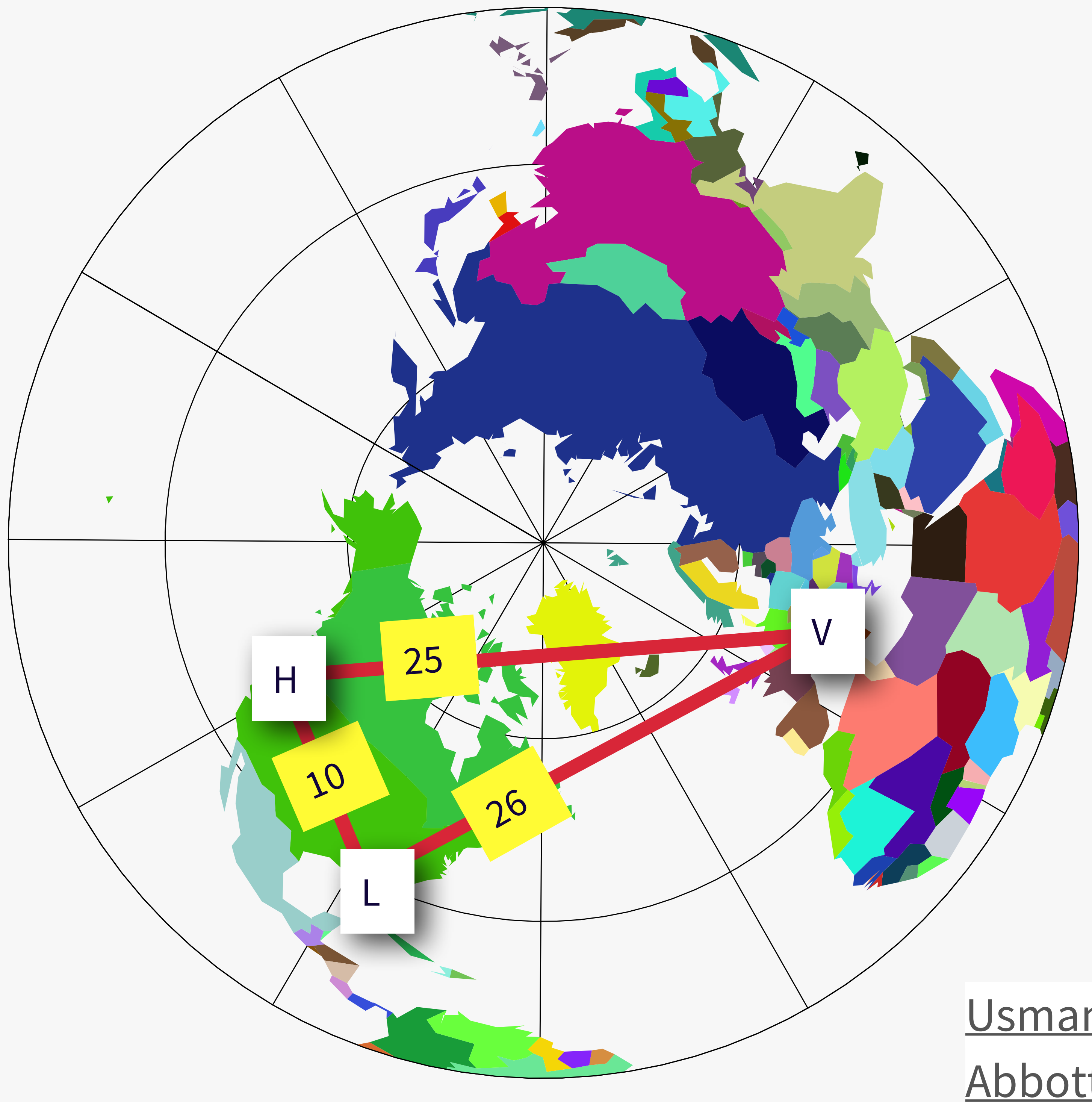
- Use machine learning and data from auxiliary channels to predict the likelihood of a glitch being present in the strain data.
- Clean data → improves statistical significance

Essick et al. (arXiv:2005.12761)

Coincidence test

Demand: if the trigger is of astrophysical origin then:

- must be observed within physically allowed time-delays across the detector network.
- must share the same best-matched template

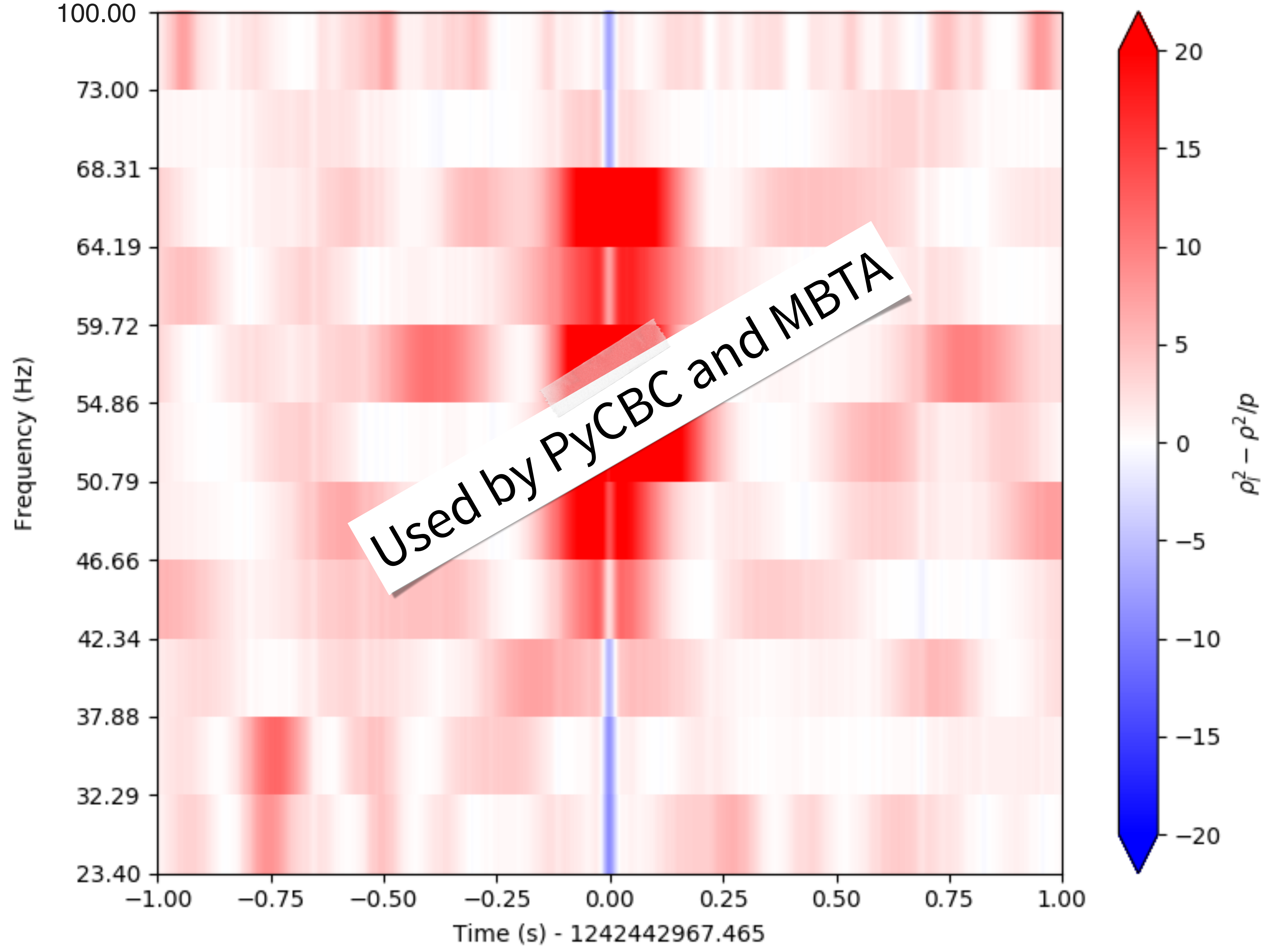


Courtesy: Sathya

Instrumental noises are of local origin!

Usman et al. CQG 33 (2016) 21, 215004
Abbott et al. PRL 116, 061102

χ_r^2- test



- Step-1: Divide the template into p frequency bands of equal expected power.

- Step-2: Calculate $\chi_r^2 = \frac{p}{2p-2} \sum_{l=1}^{p} \left(\rho_l^2 - \frac{\rho^2}{p} \right)$

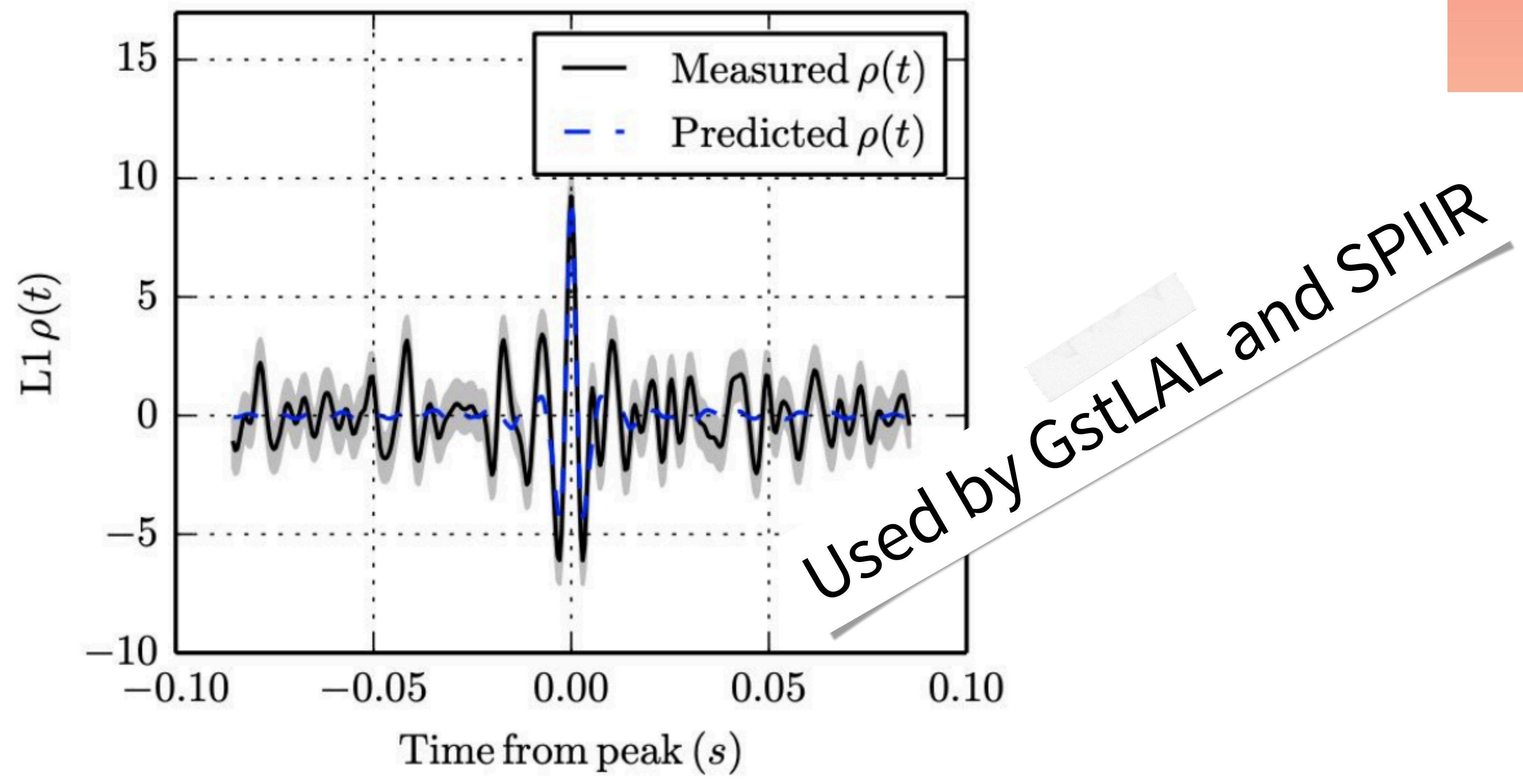
- Trigger consistent with template $\chi_r^2 \to 1$.

- Use χ_r^2-output to calculate $\varrho = f(\rho, \chi_r^2) \to$ amended ρ

Allen PRD 71 (2005) 062001
Usman et al. CQG 33 (2016) 21, 215004

Auto-correlation test

- Matched filtering doesn't produce just an SNR peak, but a time-series of SNR data.

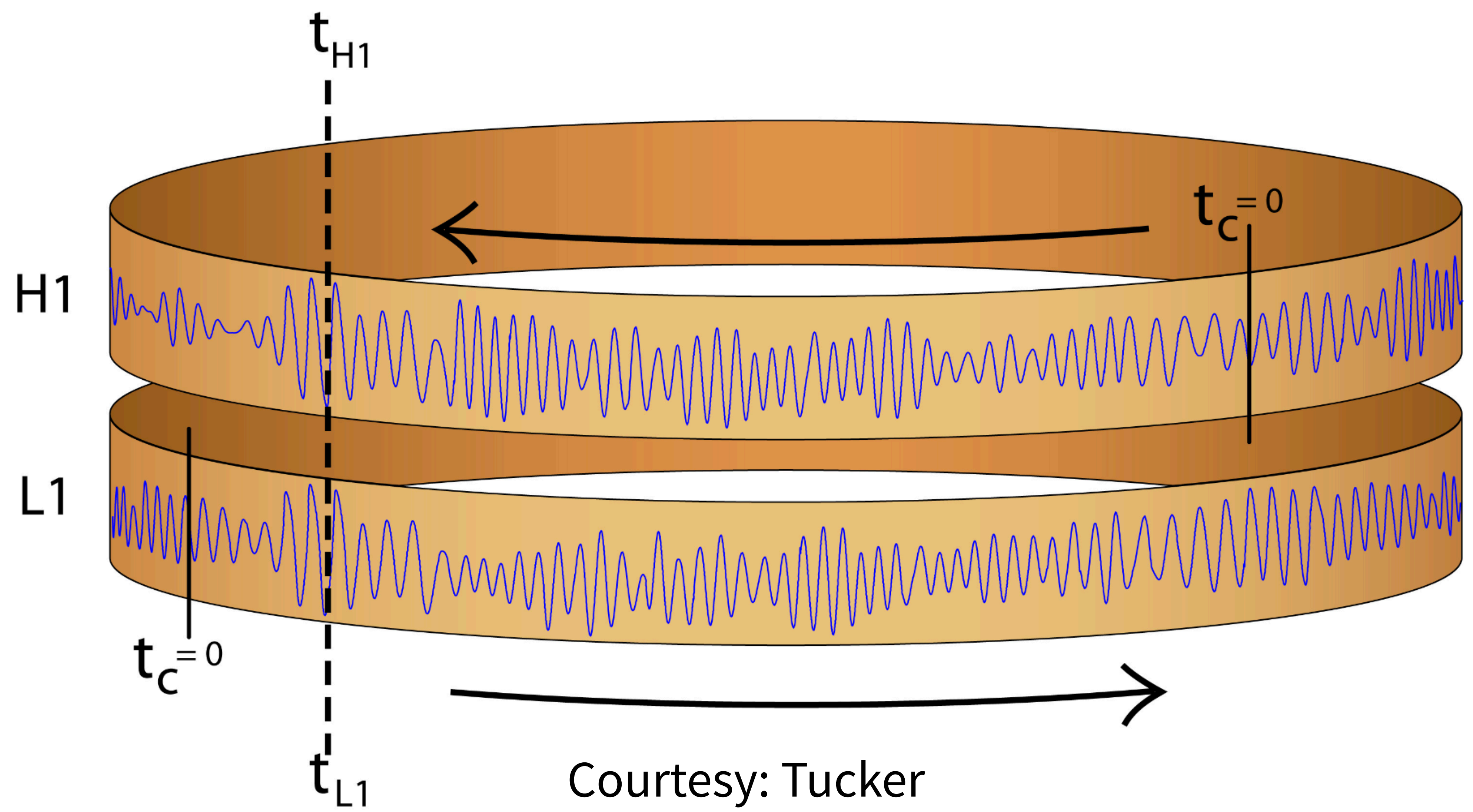


- Compare the SNR time-series shape to the predicted shape for a template waveform.

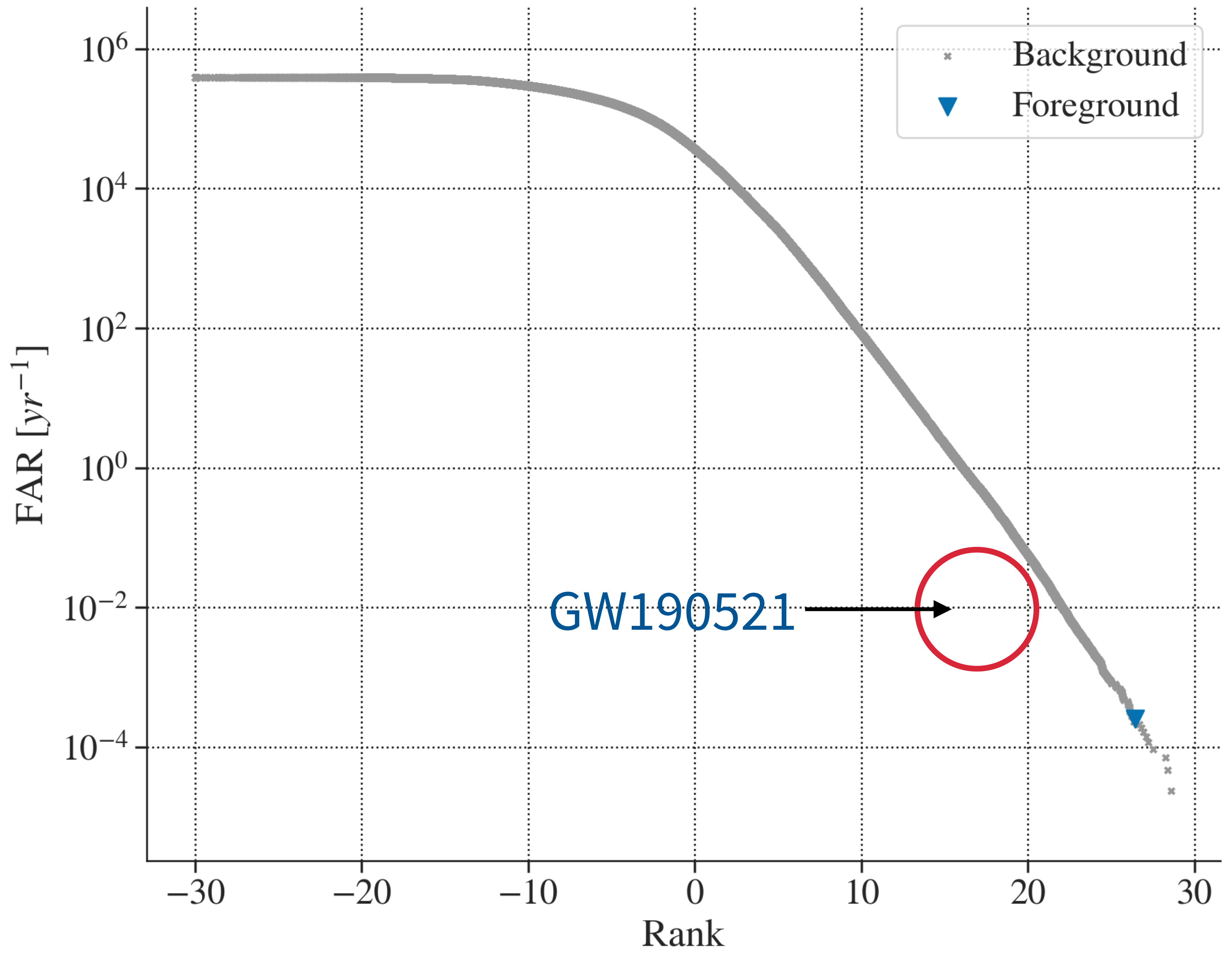
Messick et al PRD 95, 042001 (2017)

Statistical Significance

Step-1: Rank coincident candidates → PyCBC calculates $R = \sqrt{\sum_{k=1}^{N_{\rm ifo}} \varrho_I^2}$, where $N_{\rm ifo}$= # of detectors in the network. Other pipeline do this differently



- Step-2: Generate background triggers by time-slides method → shift one detector's data with respect to other(s) and look for *accidental* coincidences. (GstLAL doesn't use this method.)



- Step-3: Calculate false alarm rate or $\mathrm{FAR} := \frac{1 + n_b(R_b > R)}{T_b}$, n_b = # of background triggers with rank $R_b > R$ in time T_b

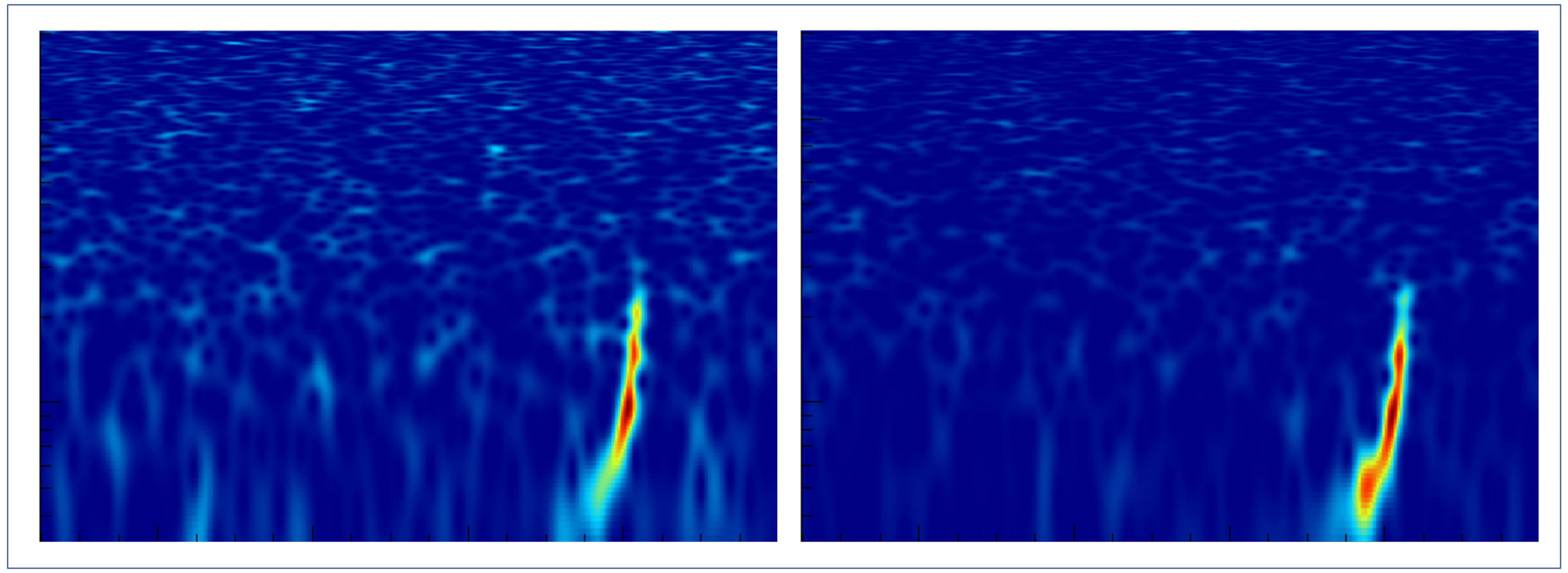
- Related to false alarm probability $p = 1 - e^{-T/\mathrm{FAR}}$

Was et al. CQG 27 (2010) 015005

Non-templated searches as an alternative

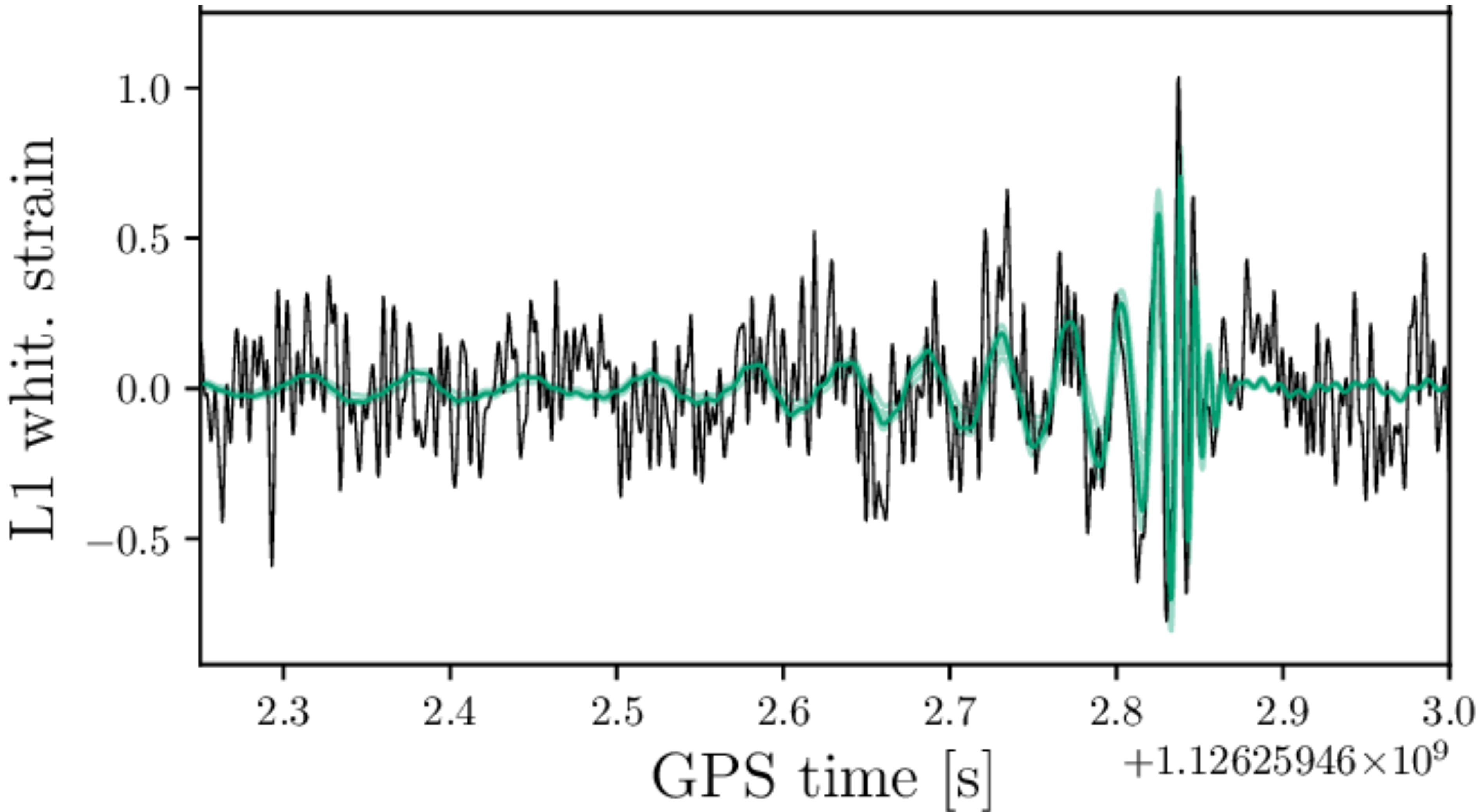
Templated searches assumes that the putative signal is well-modelled by the template waveforms → Need not be the case → Search is less flexible

Alternative-1: use a non-templated search such as coherent WaveBurst or oLIB.



- Astrophysical transients emit short-lived gravitational waveforms.
- This waveforms create localised excess in energy in the time-frequency plane.
- Identifying such excess in energy coherently across the detector network is a strong indication of an event.

Alternative-2: use a search that models GW signals in a morphology-independent through a sum of sine-Gaussian waveforms (Morlet-Gabor wavelets). Eg: BayesWave



Klimenko et al. CQG 33 (2016) 21, 215004
Lynch et al. PRD 95, 104046 (2017)
Cornish et al. CQG 32 (2015) 13, 135012

Summary

- GW signals from compact binary mergers are pretty well-modelled.
- Matched filter searches use these waveforms to find the signals.
- Matched filtering is extremely sensitive to signal's phase evolution and is optimal only when detector noise is Gaussian → not the case.
- Therefore templated searches use different techniques to account for non-Gaussianities .
- Use non-templated searches to catch the unexpected.
- Need to improve our analysis as detectors continue to improve.