Measuring a Binary Black Hole Eccentrically Orbiting a Galactic Nucleus with Gravitational Waves

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BACKGROUND AND OBJECTIVES

Since the first detection of gravitational waves (GWs) by LIGO in 2015, GW astronomy has cemented itself as a legitimate and advantageous method for observing binary systems of compact objects, the majority of which are binary black holes (BBHs) [1–3]. Currently, the ground detectors in LIGO/Virgo/KAGRA are most sensitive between 10 Hz to a few kHz, which describes the final moments before merger. In order for a GW detector to be sensitive to frequencies below 1 Hz, it must be built in space, where forming an interferometer with arm lengths much greater than LIGO's is feasible. LISA is a planned space based detector which would specialize in the millihertz band $\sim 10^{-3} - 10^{-1}$ Hz [4].

There are detector concepts which lie in the decihertz range – between LISA and ground-based detectors – such as B-DECIGO [5] and TianGO [6, 7], which may expand the range of astrophysical systems we can observe via GWs. These space-borne GW observatories could monitor inspiraling compact binary systems at times long before their orbital frequencies climb into the frequency bands of active ground-based detectors. Furthermore, since the instantaneous orbital decay timescale due to GW emission varies as $\tau_{GW} \propto f_{orb}^{-8/3}$ [8], a stellar-mass BBH system will remain in the frequency band of these low-frequency observatories much longer than in the frequency band of ground-based experiments. This will allow space-based observatories to monitor properties of a BBH system which may change over time scales on the order of years.

Consider a system consisting of a BBH orbiting around a nearby supermassive black hole (SMBH), which is sketched out in Fig. 1 along with the relative orientation of a hypothetical space-based GW observatory. When a BBH is in such a hierarchical triple system with the SMBH, the orbit of the BBH's center of mass about the SMBH (called the "outer orbit") produces a Doppler shift of the GW frequency [9]. Furthermore, when the semi-major axis of the outer orbit a_o is small enough, the angular momentum of the inner orbit (the BBH), $\boldsymbol{L}_{\rm i}$, will precess about the angular momentum of the outer orbit, $\boldsymbol{L}_{\rm o}$. This coupling between the orbit of the BBH around the SMBH and the inner angular momentum is called de Sitter precession. The inner orbit angular momentum $\boldsymbol{L}_{\rm i}$ evolves under de Sitter precession as

$$\frac{d\hat{\boldsymbol{L}}_{\rm i}}{dt} = \Omega_{\rm dS}\hat{\boldsymbol{L}}_{\rm o} \times \hat{\boldsymbol{L}}_{\rm i} \tag{1}$$

where

$$\Omega_{dS} = \Omega_0 \frac{M_3}{a_0(1-e_o^2)} \tag{2}$$

and $\Omega_0 = \sqrt{M_3/a_o^3}$[1]. For GWs emitted from the BBH with sufficiently low frequency, these perturbations due to the SMBH can be inferred from the waveform. Recently, Hang Yu and Yanbei Chen, with whom I will be working with this summer, studied how observations of the dS precession and Doppler shift in a hierarchical triple can allow inference of the mass of the SMBH, M_3, the semimajor axis of the outer orbit, a_o, and the eccentricity of the outer orbit, e_o, through the modulation of the detected GW waveform [10]. In particular, they find that if the precession period P_{dS} is comparable to the duration of observation by a space-borne GW observatory, then the dS precession of $\boldsymbol{L}_{\rm i}$ can be detected and combined with Doppler shift measurements to generate estimates of the triple system parameters to percent-level precision.

However, the analysis of Ref. [10] assumes that the eccentricity of the outer orbit in the hierarchical triple is $e_o = 0$ - that is, the outer orbit is circular. The general case in which $e_0 > 0$ was not addressed in that study,

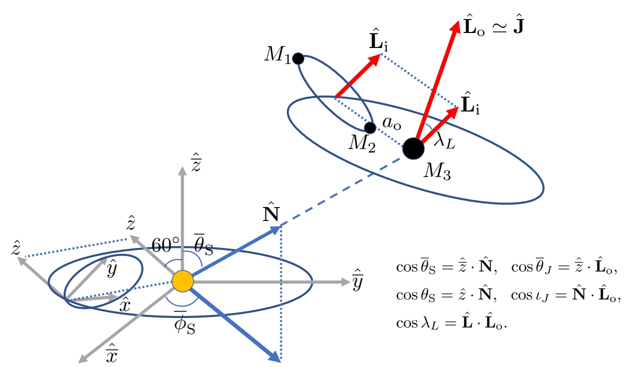

FIG. 1. A cartoon of a BBH+SMBH triple system (top-right) along with the orbit of a space-based GW observatory around the Sun (bottom left). $\boldsymbol{L}_{\rm i}$ is the angular momentum of the two BHs in the binary, and $\boldsymbol{L}_{\rm o}$ is the orbital angular momentum of the BBH around the SMBH, with a_o being the semi-major axis of the outer orbit of the BBH around the SMBH. Figure from [10]. The effect of de Sitter precession is to rotate $\boldsymbol{L}_{\rm i}$ around $\boldsymbol{L}_{\rm o}$ while keeping λ_L constant.

[1] We use geometric units in this paper where $G = c = 1$.

and we expect a significant proportion of BBH+SMBH triples to have non-circular outer orbits. So, in order to develop a more complete understanding of this method to determine SMBH properties from general BBH+SMBH hierarchical triples, we must expand the methods of Ref. [10] to non-zero eccentricities.

The central goal of my work will be to explore how the eccentricity of the outer orbit effects the uncertainty in parameter estimation of the BBH+SMBH triple system. It is expected that, for a given a_o, non-zero eccentricities will enhance the detectability of the de Sitter precession of $\mathbf{L}_i$, as from Eq. 2, the dS precession frequency scales as $(1-e_0)^{-2}$, and a more rapid precession frequency increases the likelihood that the modulation in the waveform due to the precession will be substantial over the duration of a mission. In order to quantify this change, I will ultimately produce figures similar to Figs. 4 and 5 of Ref. [10], which plot how the fractional error that space-borne GW observatories can achieve in estimating BBH+SMBH parameters varies with the true parameters of those systems.

The formation channels of BBH+SMBH triple systems, especially those which are dynamical in nature, are expected to produce a significant population of triples that fail to satisfy $e_o \approx 0$ [10]. Completing this extension of Yu and Chen's work could help with future studies of SMBHs at galactic centers. To determine the mass of an SMBH, astronomers primarily either examine the dynamics of nearby objects, like stars, as they orbit the black hole, or conduct "reverberation mapping" studies of the variation of emission line fluxes in the accretion disk of active galactic nuclei (AGNs) [11]. Yu and Chen claim that the parameter estimation method they outline can achieve precise results for a wider range of black hole masses and for triple systems at farther distances (up to 1 Gpc). Extending their analysis to non-circular outer orbits will be useful to account for more realistic scenarios. Furthermore, in Ref. [12], it is discussed how gravitational lensing of GWs by the SMBH combined with the dS precession of $\boldsymbol{L}_{\rm i}$ can further constrain the parameters of a triple system as estimated by a space-based GW observatory, even in the case of a circular outer orbit. One future extension would be to combine these lensing calculations with the results of my work to characterize the detectability of repeated lensing of GWs in an eccentric BBH+SMBH triple system.

METHODS

This work will proceed in a number of cumulative steps. First, I will learn how to estimate parameter uncertainties with the Fisher matrix formalism [13]. The elements of the Fisher matrix are defined as

$$\Gamma_{ab} \equiv \left(\frac{\partial \tilde{h}(f)}{\partial \theta_a} \Big| \frac{\partial \tilde{h}(f)}{\partial \theta_b} \right) , \tag{3}$$

where

$$\left(\tilde{g} | \tilde{h} \right) = 4 \,\mathrm{Re} \int_0^\infty \frac{\tilde{g}^*(f) \tilde{h}(f)}{S_n(f)} df \,, \tag{4}$$

$\tilde{h}$ is the frequency-domain waveform, $S_n(f)$ is the PSD of the detector noise, and θ_a are the various parameters of the system. For the hierarchical triple systems in question, the relevant parameters θ^a are as follows:

$\boldsymbol{\theta^a}$	**Definition**
$\log \mathcal{M}_z$	Detector Frame Chirp Mass: $\mu^{3/5}(m_1+m_2)^{2/5}$
q	Mass Ratio m_1/m_2
$\log D_L$	Luminosity Distance
t_c	Coalescence Time
ϕ_c	Coalescence Phase
$\overline{\theta}_S$,$\overline{\phi}_S$	Line of Sight of BBH+SMBH Triple
$\overline{\theta}_J$,$\overline{\phi}_J$	Orientation of Total Angular Momentum $\boldsymbol{J}$
λ_L	Angle Between $\boldsymbol{L}_i$ and $\boldsymbol{L}_o$
α_0	Initial Phase of $\boldsymbol{L}_i$ Around $\boldsymbol{L}_o$
$\log M_3$	SMBH Mass
$\log a_o$	Outer Orbit Semimajor Axis
$\phi^{(0)}$	Outer Orbit Initial Phase
e_o	Outer Orbit Eccentricity

TABLE I. Relevant parameters in BBH+SMBH triple system for GW observed by detectors

(The bars over polar angles indicate the Solar System coordinate frame, as in Fig. 1.) The Fisher information matrix is related to the covariance matrix roughly by

$$\Sigma_{ab} = [\Gamma^{-1}]_{ab} + \mathcal{O}(\rho^{-4}) \tag{5}$$

where ρ is the signal-to-noise ratio (SNR). So, in the limit of large SNR, the covariance between two parameters $\Delta\theta_i \Delta\theta^j$ is approximately equal to the corresponding element of the inverse of the Fisher information matrix. If a network of GW detectors were to observe the same system, the Fisher information matrix would scale as the sum of the matrix elements for each detector, or

$$(\Gamma_{ab})^{\rm network} = \sum_{det} \Gamma_{ab}^{det} \,. \tag{6}$$

I will apply the Fisher matrix method to the well-understood case of a simple BBH and confirming my results match standard predictions for parameter uncertainties.

Once I have ensured my understanding of the Fisher matrix method to estimating detectability, I will modify the code that produces the waveform from the BBH in the triple system so that the Doppler phase shift $\Phi_D(t)$ and dS precession effects are generalized to the case of non-zero eccentricity in the outer orbit. I expect that the modification of the code to include elliptical effects will require the most time this summer. After the code is properly modified and tested in well-understood limiting cases (for example, circular orbits), I will determine a "realistic" distribution for outer orbit eccentricities in

Estimated Timeline

Week	Progress Goal
1: June 14-17	Familiarization with Fisher Information Matrix Method
2: June 20-24	Development of Simple Application of Fisher Matrix to BBH
3: June 27-July 1	Testing of Fisher Matrix Code on BBH, Confirmation of Results
4: July 4-8	Outlining Changes Needed to Code Used in [10] For Eccentric Outer Orbit
5: July 11-15	Beginning Implementation of Eccentricity Features into Code
6: July 18-22	Continued Implementation of Eccentricity Features into Code
7: July 25-29	Completion of Modifications to BBH+SMBH Triples Code for Eccentric Outer Orbits
8: August 1-5	Simulation of Population of BBH+SMBH Triples for Statistical Analysis
9: August 8-12	Data Analysis of Simulated Triples to Calculate Parameter Estimation Accuracy
10: August 15-19	Report and Presentation Development

BBH+SMBH triple systems and apply this to simulations of triple systems with randomly-determined parameters. By applying the Fisher matrix formalism utilized in Ref. [10] to the hierarchical triples in the large sample I create, I can use the predicted sensitivities of proposed space-borne GW observatories to calculate the parameter estimation (PE) accuracy for these systems of compact objects.

[1] LIGO Scientific Collaboration and Virgo Collaboration, GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs, Physical Review X **9** (2019), arXiv:1811.12907.

[2] R. Abbott *et al.* (LIGO Scientific, Virgo), GWTC-2: Compact Binary Coalescences Observed by LIGO and Virgo During the First Half of the Third Observing Run, Phys. Rev. X **11**, 021053 (2021), arXiv:2010.14527 [gr-qc].

[3] B. P. Abbott *et al.* (LIGO Scientific, Virgo), Observation of Gravitational Waves from a Binary Black Hole Merger, Phys. Rev. Lett. **116**, 061102 (2016), arXiv:1602.03837 [gr-qc].

[4] P. Amaro-Seoane, H, Audley, S. Babak, J. Baker, E. Barausse, P. Bender, E. Berti, P. Binetruy, et al., Laser Interferometer Space Antenna, arXiv e-prints (2017), arXiv:1702.00786.

[5] S. Kawamura *et al.*, Current status of space gravitational wave antenna DECIGO and BDECIGO (2020), arXiv:2006.13545 [gr-qc].

[6] K. A. Kuns, *Future Networks of Gravitational Wave Detectors: Quantum Noise and Space Detectors*, Ph.D. thesis, University of California, Santa Barbara (2019).

[7] K. A. Kuns, H. Yu, Y. Chen, and R. X. Adhikari, Astrophysics and cosmology with a decihertz gravitational-wave detector: TianGO, Phys. Rev. D **102**, 043001 (2020), arXiv:1908.06004 [gr-qc].

[8] M. Maggiore, *Gravitational Waves. Vol. 1: Theory and Experiments*, Oxford Master Series in Physics (Oxford University Press, 2007).

[9] C. Cutler, Angular resolution of the LISA gravitational wave detector, Phys. Rev. D **57**, 7089 (1998), arXiv:gr-qc/9703068.

[10] H. Yu and Y. Chen, Direct determination of supermassive black hole properties with gravitational-wave radiation from surrounding stellar-mass black hole binaries, Phys. Rev. Lett. **126**, 021101 (2021), arXiv:2009.02579 [gr-qc].

[11] B.M. Peterson, Measuring the Masses of Supermassive Black Holes, Space Sci. Rev. **183** (2014).

[12] H. Yu, Y. Wang, B. Seymour, and Y. Chen, Detecting gravitational lensing in hierarchical triples in galactic nuclei with space-borne gravitational-wave observatories, arXiv e-prints (2021), arXiv:2107.14318.

[13] C. Cutler and E. E. Flanagan, Gravitational waves from merging compact binaries: How accurately can one extract the binary's parameters from the inspiral wave form?, Phys. Rev. D **49**, 2658 (1994), arXiv:gr-qc/9402014.