Detecting Non-Power Law Stochastic Gravitational Wave Background

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The Stochastic Gravitational Wave Background (SGWB) is the combination of many unknown sources. Unlike known sources lasting a short amount of time, the SGWB is a consistent signal that is always present. Current detectors, such as LIGO, have a limited sensitivity hence most SGWB signals that we expect to see look like a power-law spectrum. However, in the future when detector sensitivity increases there will be a problem in how to describe the SGWB because common theories predicted a turnover in the spectrum. Since there is so much we do not know yet of the unknown sources it is pivotal to design a model that will be general and generic to detect a SGWB that does not look like a simple power law. We propose to use current cross-correlation and statistical methods combined with a new method for describing non-power law models to detect SGWB. We propose to use splines and Gaussian processes to define this generic model and test with simulated data.

I. INTRODUCTION

Gravitational waves (GW) are ripples in space time that are initiated from extremely energetic sources. Known sources, in increasing order of how difficult they are to detect, include chirps from coalescing binary systems, periodic sources from pulsars, and bursts from supernovae [1]. Sources that are random, with multiple uncorrelated events, are called stochastic gravitational-wave backgrounds (SGWB). Unlike deterministic sources that last for a certain amount of time, the SGWB are always present. The SGWB can include events directly following the big bang, or more recently-generated signals that we can't necessarily individually detect.

LIGO's first detection in 2015 was a groundbreaking discovery that resulted in a Nobel prize. These individual events can inform us about the occurrences of stellar objects in the nearby Universe. However, looking at the SGWB can provide information on a much larger scale with much hope on the early universe. Models are created to predict how certain sources contribute to a SGWB and when an SGWB might be seen. We can do the reverse, and use the data to estimate the values of the parameters associated with each model.

Some models do not have a simple functional form such as a SGWB from a binary coalescences where the signal is generated from many individual events adding together. With current detectors the collection of binary coalescences can be well-described by the simple analytic power law [2]. However, as detectors become more sensitive, we expect to see a "smooth" turnover, which we can not be describe analytically. Additionally, we may see compact binary coalescences (CBCs) contribute a similar amount to the SGWB as other sources, like those from the Big Bang. Although each source might be described by a power law, its sum is not so easily analytically described. Consequently, we need a method that will characterize a SGWB of any "smooth" type.

II. BACKGROUND

A. Current Models

Currently it is assumed that the GWB spectrum is a power law,

$$\Omega_{GW}(f) = \Omega_{GW}(f_{ref}) \left(\frac{f}{f_{ref}}\right)^{\alpha}. \tag{1}$$

Where $\Omega_{GW}(f)$ is the GW energy density, f_{ref} is a reference frequency and α is the spectral index of the signal. Ω_{GW} and α are estimated. Although Ω_{GW} is usually considered a cosmological quantity here it is also used to describe the energy from astrophysical events so that we can compare them to cosmological sources [3].

Current methods to identify the GWB are signal to noise ratio (SNR) and the Bayes Factor. SNR uses the ratio of signal to noise where there is more weight on frequencies that have less uncertainty. This is done by inverse noise weighting, where C is the signal and σ is noise.

$$SNR(f) = \frac{\hat{C}(f)}{\sigma(f)},$$

$$C_{TOT} = \frac{\sum_{i=1}^{N} \hat{C}(f)/\sigma(f)^2}{\sum_{i=1}^{N} \sigma(f)^{-2}}; \tag{2}$$

$$\sigma_{TOT} = (\sum \sigma(f)^{-2})^{-\frac{1}{2}}$$
$$SNR_{TOT} = \frac{C_{TOT}}{\sigma_{TOT}}$$

The SNR tells us how greater the signal is to the detector noise. For GWB searches, when SNR= 3 it is considered as evidence of a GWB and a SNR=5 is a detection of a GWB. Bayes factor is similar to SNR in which the factor is that the data contains a signal divided by the probability the data is consistent with just noise in the detector.

$$Bayes\ factor = \frac{P(\hat{C}|\text{signal})}{P(\hat{C}|\text{noise})} \tag{3}$$

The probability is based on the posterior probability of the algorithm's parameters given the data. θ is used for signal and 0 is for noise,

$$\begin{aligned} P(\vec{\theta}|\hat{C}_i) &= \frac{P(\hat{C}|\vec{\theta})P(\vec{\theta})}{P(\hat{C})} \\ P(0|\hat{C}_i) &= \frac{P(\hat{C}|0)P(0)}{P(\hat{C})} \end{aligned} \tag{4}$$

The evidence is then give by the integral over the numerator. So the integral for signal in eq.4 is,

$$P(\hat{C}|\text{signal}) = \int d\vec{\theta} P(\hat{C}|\vec{\theta})p(\vec{\theta}). \tag{5}$$

In our case for a power-law of SGWB, shown in eq1 the parameters in $\vec{\theta}$ is amplitude A and spectral index α .

Both the SNR and Bayes factors perform well with current detectors and are the base work for a newly proposed method for future detectors. The interpolation model is the method where the height of the control points are interpolated between the parameters of $\vec{\theta}$, this will be further discussed in section III.

In figure 1 we show different versions of the SGWB from compact binary coalescences with the assumption of it made up completely of mergers, along with the estimated GWB sensitivity of LIGO detectors. The curves in Fig 1 are well-described power-laws until a certain point were they turn over. The gray sensitivity curves show that for realistic curves (i.e. those with chirp masses below 50 $M_\odot$), current generation detectors are not sensitive to this turnover.

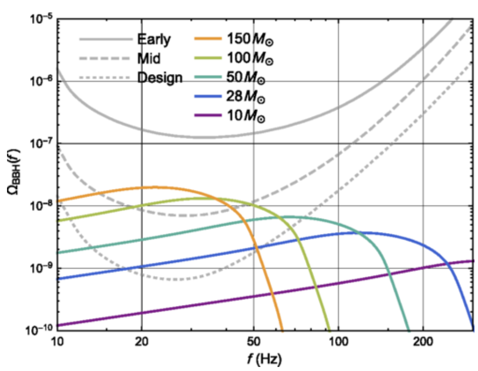

Figure 1. Figure reproduced from [2]. We show the binary black hole's background with various chirp masses with the Fiducial model for SGWB (colored lines). Power-law integrated curves for one year with Advanced LIGO (grey lines).

This turnover is the astrophysical GWB's non-analytical piece. The turnover depends on the masses of the black holes and becomes more complex as different populations are added to it. Future detectors will be sensitive to these turnovers which can clarify characteristics for CBCs such as: the time it takes for a star to merge in a binary, if properties of the universe contribute to the formation and/or mass of a black hole, and how the populations of masses and spins of a neutron star and a black hole that enter a binary look like in our spectrum. In addition, there is an unknown cosmological background in CBCs that are not black holes, which motivates another reason to create a new model since we simply do not know what the sources are and how to define them.[3].

B. Evolution of Models as Detectors Improve

Once detectors become more sensitive there will be an abundance of individual events. To search for the SGWB, we will subtract the loudest events from the data, which will then change the spectrum of the SGWB. An example of an expected SGWB after subtracting individual events is shown in Fig 2. Evidently the spectrum is no longer a straight line in log space but it does appear "smooth".

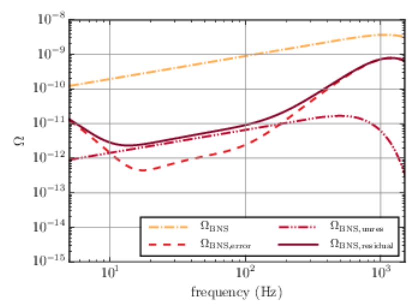

Figure 2. Figure reproduced from [4], the SGWB from all neutron stars (doted orange line) plotted together with background from unresolved (unsubtracted) neutron stars (dotted red line), and the sum of the two (red sold line). With the neutron star summed with unresolved background the line is now longer a power law.

A general and generic model can effectively detect a SGWB of any "smooth" shape at higher sensitivity. The paper [4] predicts most models with a "smooth" look therefore, a new model must capture similar values or change smoothly from one frequency bin to the next. We propose two methods to detect generic models. Both methods are regularly used to fit smooth looking curves, spline fitting and Gaussian process. As we will discuss in the next section, this new model can also be used to develop a consistency check to verify a detection.

III. METHODS

To detect a SGWB we will cross-correlate data between detectors. In the following equations the tilde indicates the use of the Fourier transformation. The detector data ($\tilde{s}_i$) involves both GW signal ($\tilde{h}$) and noise ($\tilde{n}$) with the dependency of frequency,

$$\tilde{s}_1(f) = \tilde{h}_1(f) + \tilde{n}(f). \tag{6}$$

The data of the two detectors is then defined in a cross correlation statistic ($\tilde{C}(f)$) in every frequency bin [5],

$$\tilde{C}(f) = \frac{2}{\tau}\frac{Re[\tilde{s}_1^*(f)\tilde{s}_2](f)}{\gamma_f S_0(f)}, \tag{7}$$

where Re indicates the real part of the cross correlation, τ is the time over which we are analyzing data, and the normalization includes cosmological constants, as well as the overlap reduction function $\gamma(f)$, which we discuss soon. We can substitute Eq 7 into Eq 6 and take an average:

$$\begin{aligned}\langle\tilde{s}_1^*(f)\tilde{s}_2(f)\rangle = \langle\tilde{h}_1^*(f)\tilde{h}_2(f)\rangle + \langle\tilde{n}_1^*(f)\tilde{h}_2(f)\rangle \\ + \langle\tilde{h}_1^*(f)\tilde{n}_2(f)\rangle + \langle\tilde{n}_1^*(f)\tilde{n}_2(f)\rangle.\end{aligned}$$

We then assume that the signal is uncorrelated with detector noise and the noise between the two detectors is uncorrelated. Therefore,

$$\langle\tilde{s}_1^*(f)\tilde{s}_2(f)\rangle = \langle\tilde{h}_1^*(f)\tilde{h}_2(f)\rangle. \tag{8}$$

Next, we note

$$\frac{2}{\tau}\langle\tilde{h}_1^*(f)\tilde{h}_2(f)\rangle = H(f)\gamma(f). \tag{9}$$

Here $H(f)$ is called the gravitational wave power, and the proportionality constant $\gamma(f)$ is called the overlap reduction function. The overlap reduction function is a weight function in frequency that quantifies what fraction of the GW power our detectors are sensitive to. Thus, $\gamma(f) = 1$ means we see all of the GW power in our detectors, but $\gamma(f) = 0.5$ means we see only half of the GW power. Since we know exactly what detectors we are using its value is exactly known [3]. This frequency dependence provides a great insight into the frequencies to which the detectors are most sensitive.

When we substitute Eq. 9 back into Eq. 7, we find that in general,

$$\langle\tilde{C}(f)\rangle = \frac{H(f)}{S_0(f)} = \Omega_{GW}(f) \tag{10}$$

Where the constants $S_0(f)$ are used so that we have the cross-correlation proportional to the energy density.

The shape of the cross-correlation is what we want to model. $\tilde{C}(f)$ is calculated by taking the cross-correlation between our detectors for numerous short time intervals then taking the average of all the runs. We then compare the averaged $\tilde{C}(f)$ to power law spectra to verify if there is any evidence of power law. What we propose to do here, is to instead compare to more generic "smooth" functions like splines or Gaussian processes. Then proceed to test our model with simulated data.

IV. FIRST 3 WEEKS PROGRESS

A. Needed Tools

The LIGO collaboration has several libraries and pipelines that are used specifically for frequency domain spectra. Our work mainly uses the library `bibly` and `pygwb` as well as the pipeline `pygwb_pipe`. Our model is based off of Bayesian statistics which `bibly` is designed for. This library has several function that are needed such as initializing a Power Spectral Density and generating a prior in log uniform and Gaussian distribution. `pygwb_pipe` is the main pipeline to work with SGWB that uses the library `pygwb`. We used this pipeline to simulate data from all three detectors Hanford, Livingston, and VIRGO to then inject a common GW signal with a chosen spectrum into our current data.

Another crucial code is the package `westley`. This code holds a big chunk of what we will be testing. Our model will have control points, referred to as knots, to interpolate between to create a model that we will fit to the data. With this package we can interpolate between the knots using either a piece-wise power law or a cubic spline and run it a number of times to find the probability distribution on where the knots should be placed and turned on to have the best fit. Rather than taking random guesses to find the probability distribution we use the technique Markov chain Monte Carlo (MCMC).

B. Statistics

An MCMC is used to obtain a preferred probability distribution on some set of unknown parameters. For our model, we want to find the probability distribution on the height of each knot and how many are used. With MCMC we are able to make intelligent guesses to where the knots should be placed. A new knot's position is accepted depending on how well the "new" guess will let the model fit the data compared to its current, "old" guess. The likelihood function L tells us how well the model fits the data, and we use a Gaussian likelihood function. The placement of the knot is kept whether,

$$P(accept) = 1, \text{if point has } L_{new} > L_{old}$$

$$P(accept) = \frac{L_{new}}{L_{old}}, \text{if point has } L_{new} \leq L_{old}$$

The proposed distribution function tells the algorithm how we deiced to guess. The way we guess does not interfere with the results, however, it does affect the efficiency of the algorithm time wise.

C. Testing Algorithm

To see if our algorithm works we tested it on known GWB. Our simulated data was reduced to just the signal ($\hat{C}$) and cross correlated with the Hanford and Livingston detector. We set the model to have a max of 20 knots and ran the MCMC for 100000 steps. From the 100000 steps, we discarded the first 50,000 as "burn in" and then recorded the parameters at only every 10^{th} step. Figure IV C below shows a comparison between the injected signal (orange), the data (blue and green), and the recovered signal for 500 MCMC steps (black).

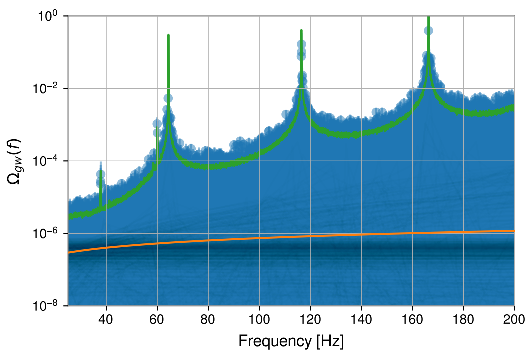

Figure 3. The figure shows the simulated data in blue, noise in green, our proposed injected power law is in orange and the interloped knots in black. The MCMC with 500 steps, relatively fit the injected signal throughout the runs.

The plot tells us the energy density Ω_{GW} of the signal (blue) at a given frequency as well as the noise (green) by the length in Ω_{GW}. Since this is in log space our injection is mostly linear and most of the simulated runs from `westley` fit along the power law. Therefore the code has identified a GW. Figure 4 is a visual representation of which knots where in used in each run of the model.

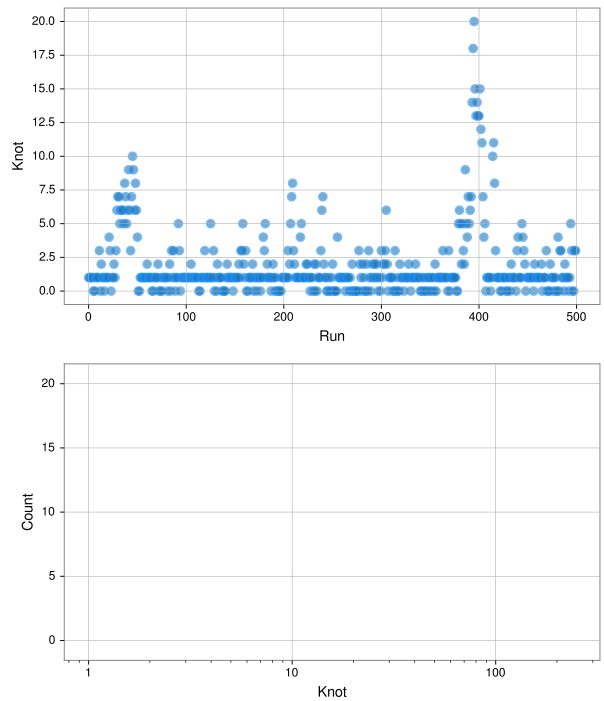

Figure 4. Top, shows when the knot was turned on during a certain run. Bottom, is a histogram to visualize how many times the certain knot was used for all runs.

Currently the model we tested had spaced out the knots evenly throughout the spectra and made its own choice when the knots should be turned on or off. The next model will be incorporating these parameters to modify the way we guess and improve how fast we get to our result.

D. Next Steps

After learning how the pipeline works and getting accustomed to the different packages used for spectra on the frequency domain these passed weeks, I will now be focusing on the new proposed way to look for SGWB. This Method takes concepts from "SNR" and "Bayesian Factor" methods of searching for SGWB. Since the knots are used to fit a signal, they should only be present when a signal is present. Therefor, we should have 1 or more knots when there is a signal present no knots where there is only noise present. For example, figure 4 illustrates the height of each bin which we can use the sum of the heights for all the bins with the y-axis ≥ 1 and divide by the height of the bin at zero to get a log Bayes factor of around 1.3

Recapping from the section above, our model has the questions: how many knots should be in use and where should they be positioned along each axis? We will continue to run our model to enhance how we guess. We will then compare the number of times we weren't given any knots with the times we were and use this calculate the Bayes factor, shown in eq.3. Simultaneously we will also be running the data with the SNR and Bayes factor method. If our new method is working then our factor B will be similar to the calculated other ways and can be compared to the SNR_{TOT}.

The challenges in these upcoming weeks will most likely be similar to those in the past. Learning new code can bring its downfalls for example getting stuck on errors and ending up with the wrong solution but each time I gain more knowledge and it prepares me for the next day.

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[3] Arianna I. Renzini, Boris Goncharov, Alexander C. Jenkins, and Pat M. Meyers. Stochastic Gravitational-Wave Backgrounds: Current Detection Efforts and Future Prospects. *Galaxies*, 10(1):34, 2022.

[4] Surabhi Sachdev, Tania Regimbau, and B. S. Sathyaprakash. Subtracting compact binary foreground sources to reveal primordial gravitational-wave backgrounds. *Phys. Rev. D*, 102(2):024051, 2020.

[5] B. P. Abbott et al. Search for the isotropic stochastic background using data from Advanced LIGO's second observing run. *Phys. Rev. D*, 100(6):061101, 2019.