Detecting Non-Power Law Stochastic Gravitational Wave Background

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Abstract

The Stochastic Gravitational Wave Background (SGWB) is the combination of many unknown sources. Unlike known sources lasting a short amount of time, the SGWB is a consistent signal that is always present in the detector data. Current detectors, such as LIGO, have a limited sensitivity hence most SGWB signals that we expect to see look like a power-law spectrum. However, in the future when detector sensitivity increases there will be a problem in how to describe the SGWB because common theories predicted a turnover in the spectrum. Since there is so much we do not know yet of the unknown sources it is pivotal to design a model that will be general and generic so that we can detect a SGWB that does not look like a simple power law. We propose to use current cross-correlation and statistical methods combined with a new method for describing non-power law models to detect SGWB. We propose to use splines and Gaussian processes to define this generic model and test run it with real and/or simulated data.

1 Introduction

Gravitational waves (GW) are ripples in space time that are initiated from extremely energetic sources. Known sources, in increasing order of how difficult they are to detect, include chirps from coalescing binary systems, periodic sources from pulsars, and bursts from supernovae [1]. Sources that are random, with multiple uncorrelated events, are called stochastic gravitational-wave backgrounds (SGWBs). Unlike deterministic sources that last for a certain amount of time, the SGWB are always present. The SGWB can include events directly following the big bang, or more recently-generated signals that we can't necessarily individually detect.

LIGO's first detection in 2015 was a groundbreaking discovery that resulted in a Nobel prize. Events that we can see individually can inform us about the nearby Universe. However, looking at the SGWB from unresolved events and background from the early universe can provide information on a much larger scale. Models are created to predict how certain sources contribute to a SGWB and when an SGWB might be seen. We can do the reverse, and use the data to estimate the values of the parameters associated with each model.

Some models do not have a simple functional form such as a SGWB from a binary coalescences where the result is multiple events adding up together. With current detectors the collection of binary coalescences can be well-described by the simple analytic power law [2]. However, as detectors become more sensitive, we expect to see a "smooth" turnover, which we can not describe analytically. Additionally, we may see compact binary coalescences (CBCs) contribute a similar amount to the SGWB as other sources, like those from the big bang. Although each source might be described by a power law, its sum could not be so easily described analytically. Consequently we need a method that will characterize a SGWB of any "smooth" type.

2 Background

2.1 Current Models

Currently it is assumed that the GWB spectrum is a power law,

$$\Omega_{GW}(f) = \Omega_{GW}(f_{ref}) \left(\frac{f}{f_{ref}} \right)^{\alpha}. \tag{1}$$

Where $\Omega_{GW}(f)$ is the GW energy density, f_{ref} is a reference frequency and α is the spectral index of the signal. Ω_{GW} and α are estimated. Although Ω_{GW} is usually considered a cosmological quantity here it is also used to describe the energy from astrophysical events so that we can compare them to cosmological sources [3].

In figure 1 we show different versions of the SGWB from compact binary coalescences with the assumption of it made up completely of mergers, along with the estimated GWB sensitivity of LIGO detectors. The curves in Fig 1 are well-described power-laws until a certain point were they turn over. The gray sensitivity curves show that for realistic curves (i.e. those with chirp masses below 50 $M_{\odot}$), current generation detectors are not sensitive to this turnover.

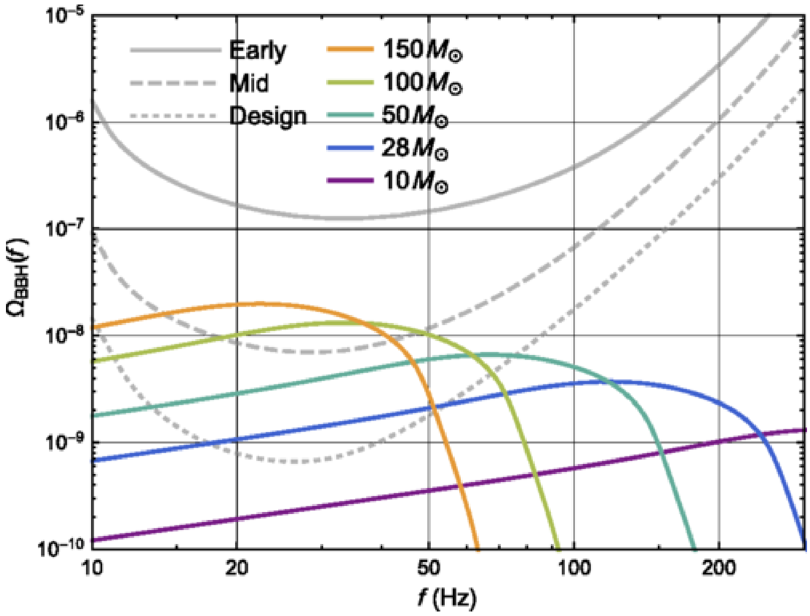

Figure 1: Figure reproduced from [2]. We show the binary black hole's background with various chirp masses with the Fiducial model for stochastic background (colored lines). Power-law integrated curves for one year with Advanced LIGO (grey lines).

This turnover is the astrophysical GWB's non-analytical piece. The turnover depends on the masses of the black holes and becomes more complex as different populations are added to it. Future detectors will be sensitive to these turnovers which can clarify characteristics for CBCs such as: the time it takes for a star to merge in a binary, if properties of the universe contribute to the formation or mass of a black hole, and how the populations of masses and spins of a neutron star and a black hole that enter a binary look like in our spectrum. In addition, there is an unknown cosmological background in CBCs that are not black holes, which motivates another reason to create a new model since we simply do not know what the sources are and how to define them.[3].

2.2 Evolution of Models as Detectors Improve

Once detectors become more sensitive there will be an abundance of individual events. To search for the SGWB, we will subtract the loudest events from the data, which will then change the spectrum of the SGWB. An example of an expected SGWB after subtracting individual events is shown in Fig 2. Evidently the spectrum is no longer a straight line in log space but it does appear "smooth".

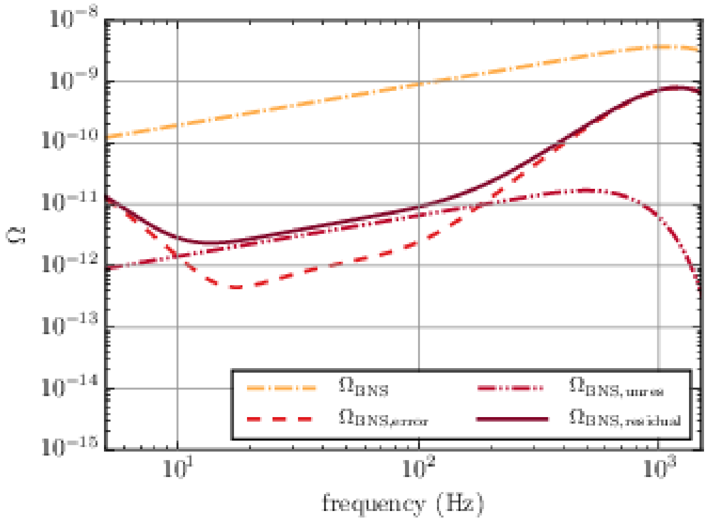

Figure 2: Figure reproduced from [4], the SGWB from all neutron stars (doted orange line) plotted together with background from unresolved (unsubtracted) neutron stars (dotted red line), and the sum of the two (red sold line). With the neutron star summed with unresolved background the line is now longer a power law.

A general and generic model can effectively detect a SGWB of any "smooth" shape at higher sensitivity. The paper [4] predicts most models with a "smooth" look therefore, a new model must capture similar values or change smoothly from one frequency bin to the next. We propose two methods to detect generic models. Both methods are regularly used to fit smooth looking curves, and they are spline fitting and the Gaussian process. As we will discuss in the next section, this new model can also be used to develop a consistency check to verify a detection.

3 Methods

To detect a SGWB we will cross-correlate data between detectors. In the following equations the tilde indicates the use of the Fourier transformation. The detector data ($\tilde{s}_i$) involves both GW signal ($\tilde{h}$) and noise ($\tilde{n}$) with the dependency of frequency,

$$\tilde{s}_1(f) = \tilde{h}_1(f) + \tilde{n}(f). \tag{2}$$

The data of the two detectors is then defined in a cross correlation statistic ($\tilde{C}(f)$) in every frequency bin [5],

$$\tilde{C}(f) = \frac{2}{\tau} \frac{Re[\tilde{s}_1^*(f)\tilde{s}_2](f)}{\gamma_f S_0(f)}, \tag{3}$$

where Re indicates the real part of the cross correlation, τ is the time over which we are analyzing data, and the normalization includes cosmological constants, as well as the overlap reduction function $\gamma(f)$, which we discuss soon. We can substitute Eq 3 into Eq 2 and take an average:

$$\langle \tilde{s}_1^*(f)\tilde{s}_2(f)\rangle = \langle \tilde{h}_1^*(f)\tilde{h}_2(f)\rangle + \langle \tilde{n}_1^*(f)\tilde{h}_2(f)\rangle + \langle \tilde{h}_1^*(f)\tilde{n}_2(f)\rangle + \langle \tilde{n}_1^*(f)\tilde{n}_2(f)\rangle. \quad (4)$$

We then assume that the signal is uncorrelated with detector noise and the noise between the two detectors is uncorrelated. Therefore,

$$\langle \tilde{s}_1^*(f)\tilde{s}_2(f)\rangle = \langle \tilde{h}_1^*(f)\tilde{h}_2(f)\rangle. \quad (5)$$

Next, we note

$$\frac{2}{\tau}\langle \tilde{h}_1^*(f)\tilde{h}_2(f)\rangle = H(f)\gamma(f). \quad (6)$$

Here $H(f)$ is called the gravitational wave power, and the proportionality constant $\gamma(f)$ is called the overlap reduction function. The overlap reduction function is a weight function in frequency that quantifies what fraction of the GW power our detectors are sensitive to. Thus, $\gamma(f) = 1$ means we see all of the GW power in our detectors, but $\gamma(f) = 0.5$ means we see only half of the GW power. Since we know exactly what detectors we are using its value is exactly known [3]. This frequency dependence provides a great insight into the frequencies to which the detectors are most sensitive.
When we substitute Eq. 6 back into Eq. 3, we find that in general,

$$\langle \tilde{C}(f)\rangle = \frac{H(f)}{S_0(f)} = \Omega_{GW}(f) \quad (7)$$

Where the constants $S_0(f)$ are used so that we have the cross-correlation proportional to the energy density.

The shape of the cross-correlation is what we want to model. $\tilde{C}(f)$ is calculated by taking the cross-correlation between our detectors for numerous short time intervals then taking the average of all the runs. We then compare the averaged $\tilde{C}(f)$ to power law spectra to verify if there is any evidence of power law. What we propose to do here, is to instead compare to more generic "smooth" functions like splines or Gaussian processes. Then proceed to test our model with real and/or simulated data.

4 Work Plan

Dates	Events/Tasks
June 14	Program begins
June 15 - July 1	Begin background research on SGWB and learn to use standard SGWB programming tools
June 27 - July 1	Work on and submit Interim report 1
July 5 - 26	Developing new code: working on proposed novel solution to SGWB and translating them into a python pipeline
July 25-29 July 29	Work on and submit Interim report 2 Turn in abstract
July 27- August 12	Testing new code: using new pipeline with real or simulated data
August 15 - 19	Work on and submit final summer report
August 17-19	Final presentation
September 23	Submit Final Report

References

[1] Bruce Allen. The Stochastic gravity wave background: Sources and detection. In *Les Houches School of Physics: Astrophysical Sources of Gravitational Radiation*, pages 373–417, 4 1996.

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[3] Arianna I. Renzini, Boris Goncharov, Alexander C. Jenkins, and Pat M. Meyers. Stochastic Gravitational-Wave Backgrounds: Current Detection Efforts and Future Prospects. *Galaxies*, 10(1):34, 2022.

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