Interim Report 1: Incorporating Stepping-Stone Sampling Into BayesWave

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`BayesWave` is used to analyze gravitational wave data that is recieved by LIGO. It uses Bayesian statistics to simulate the signals and compare potential models. This means matching the detected waveform to models we are already familiar with, including glitches and compact binary collisions. `BayesWave` currently uses Thermodynamic Integration (TI) to calculate evidences, which can then be compared to find the best matches for the signal. An alternative method for calculating evidences is called a Stepping-Stone (SS) algorithm. SS methods are as accurate as TI, and are less computationally expensive. I will be testing out SS within `BayesWave` and comparing the results to those acquired from TI, to determine if SS should be used for LIGO's fourth detection run in 2023.

I. INTRODUCTION/BACKGROUND

A. LIGO

Gravitational waves are ripples in space-time caused by high-energy events in outer space, such as supernovae and the collisions of black holes. LIGO [1], which stands for Laser Interferometer Gravitational-wave Observatory, is a large ground-based interferometer used to detect those ripples. It first detected gravitational waves in 2015, and has been carrying out observational runs since. LIGO detects gravitational-waves produced by distant, massive, compact objects by measuring slight fluctuations in its interferometric "arm" length. Once LIGO has collected data from an event, that data needs to be processed and analyzed computationally.

B. Bayesian Statistics

Bayesian statistics is a form of statistics in which earlier probability distributions (priors) can be updated to account for new data to produce new distributions (posteriors). This is done using Bayes' Theorem, given by equation 1.

$$P(A|B) = \frac{\mathcal{L}(B|A)\pi(A)}{P(B)} \tag{1}$$

Some $P(A|B)$ represents the probability of some event A given that B is true, and $\pi(A)$ is the probability of some event A independent of other events. Here, $P(A|B)$ is the posterior, the prior is $\pi(A)$, and the likelihood is $\mathcal{L}(B|A)$. When you integrate such a function over the entire parameter space, the evidence is produced. Evidences can be compared using the Bayes' factor, a ratio of evidences, like so:

$$\frac{P(B_1)}{P(B_2)} \tag{2}$$

Such evidences can be [SH: calculated] with methods such as thermodynamic integration or a stepping-stone algorithm. Being able to compare models is crucial to determining the best match for sources of gravitational waves that LIGO detects. The part of BayesWave which calculates evidences and compares them is the part relevant to this project.

C. `BayesWave`

`BayesWave` [2] is a library of code which analyzes LIGO data using Bayesian statistical methods. is able to account for noise and glitches as it reconstructs observed waveforms. These reconstructed waveforms can then be compared with their Bayes' Factors, to determine which are good fits for the observed data. Signal and glitch models are both constructed as a series of wavelets [2].

When `BayesWave` is used for signal matching, it can execute different types of runs that will match the observed signal with different types of waveforms. `BayesWave` can produce matches that are coherent wavelet models (signal), incoherent wavelet model (glitch), waveform template model (CBC), and many combinations of those. When one of these runs has been completed, `BayesWave` creates and stores model data and diagrams in a directory, useful for analysis. Figure 1 shows a reconstruction of the 150914 signal at the Hanford detector using a coherent wavelet model. Figure 2 shows the reconstructed spectrogram from the same signal.

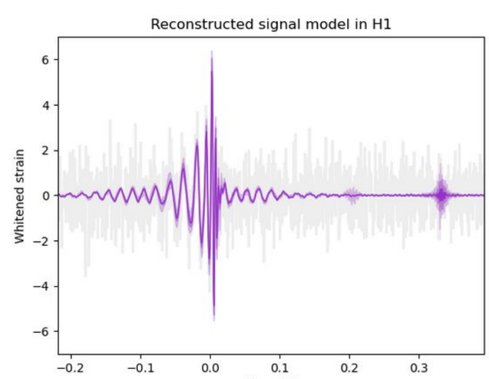

FIG. 1: reconstructed waveform using `BayesWave`

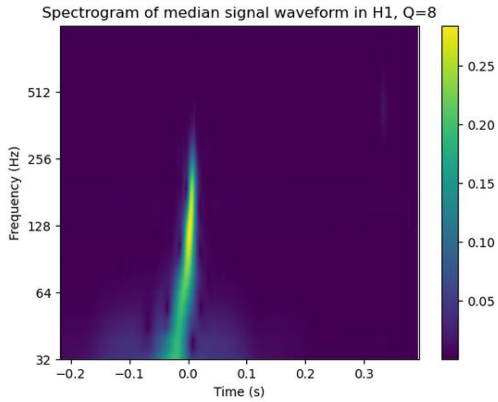

FIG. 2: reconstructed signal spectrogram using `BayesWave`

II. EVIDENCE CALCULATION

To calculate marginalized likelihoods, or evidences, with thermodynamic integration or stepping-stone methods, we start with Bayes' theorem. We rewrite equation 3 into more relevant notation. Here $p_i(\theta)$ is the posterior probability density for some model, $\pi(\theta|M_i)$ is the prior distribution $\mathcal{L}(D|\theta, M_i)$ is the likelihood function of some data, D, given that the prior is true. z_i the evidence (Eq. 4), a normalizing constant also referred to as the marginal likelihood [3].

$$p_i(\theta) = \frac{\mathcal{L}(D|\theta, M_i)\pi(\theta|M_i)}{z_i} \tag{3}$$

$$z_i = p(D|M_i) = \int \mathcal{L}(D|\theta, M_i)\pi(\theta|M_i)d\theta \tag{4}$$

The likelihoods of two models can be compared using a ratio between their evidences known as a Bayes' factor (Eq. 3). this ratio, which we can call μ, is generally scaled logarithmically, as in equation 5.

$$\mu = ln(\frac{z_1}{z_0}) \tag{5}$$

What we want to do is rewrite equation 5 in terms of the symbol β. For this, we need more context about how `BayesWave` solves for evidences. It does so with a Markov chain Monte Carlo (MCMC) process [4]. A chain is a series of points that move through a parameter space to take samples of the probability distribution of the space. It moves in a way determined by the chain's "temperature" until reaching an equilibrium state.

A high temperature corresponds to a chain which, similar to the behavior of a thermodynamic system, has a higher chance of jumping to higher energy, less likely states. This causes the likelihood function to approach the prior distribution (flattening out in parameter space). Likewise, a low temperature corresponds to chains that is more likely to stay in areas of high probability [3], resulting in a peaked probability distribution approaching that of the posterior. Because β is the inverse of the temperature, a β value moving from zero to one corresponds to chains "cooling down", while starting at one and moving to zero corresponds to a chain "heating up".

the result of a series of chains is sample expectations, which can then be used to estimate the integrals we need. An example of this is figure 3, where the changing β value is on the x axis, and the sample expectation values of each chain are on the y axis. The evidence of a model can then be rewritten in terms of β, in equation 6

$$\mu = ln(\frac{z_1}{z_0}) = ln(z_1) - ln(z_0) = \int_0^1 \frac{\partial ln(z_\beta)}{\partial \beta} d\beta \tag{6}$$

III. THERMODYNAMIC INTEGRATION

Currently `BayesWave` uses thermodynamic integration [5] to calculate evidences for potential models. The goal of thermodynamic integration and the stepping-stone algorithm is to do this by estimating the integral form of equation 6. TI is also known as path sampling, because it involves taking samples along a path of temperatures. β travels between zero and one on this path as the temperature changes.

The goal of thermodynamic integration and the stepping-stone algorithm is to estimate the integral form of equation 6. This is done with TI by taking many discrete steps as β moves between 0 and 1 and taking samples at each temperature. The samples can then be used to estimate the integral, as we can see in figure 3 [3]. In figure 4, a curve estimated with TI using 1000 sample points is shown.

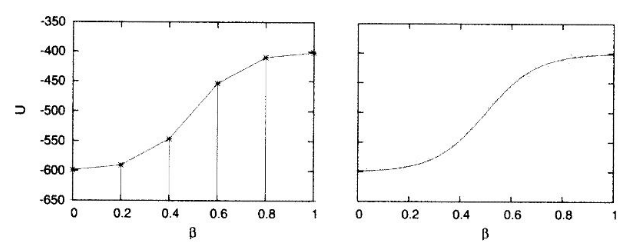

FIG. 3: Using discrete points to estimate an integral

Thermodynamic integration estimates the integral in equation 6 using general path sampling [6]. this involves using the expectation E. This is given in equation 7, read as the expectation of the likelihood, given β. For each chain location along β's path, samples will be taken and a sample average acquired. Then, those values can be used to estimate the integral [6], as per equation 8. This takes some time, as each β chain is run one at a time, rather than in parallel.

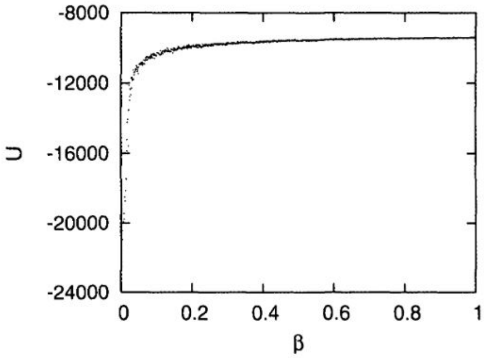

FIG. 4: Estimation using 1000 points

$$E_\beta[log(\mathcal{L}(D|\theta, M))] = \int log(\mathcal{L}(D|\theta, M))\pi(\theta)d\theta \quad (7)$$

$$log(z) = \int_0^1 E_\beta[log(\mathcal{L}(D|\theta, M))]d\beta \quad (8)$$

Thermodynamic integration has proven to be a very reliable method, provided enough samples are taken to minimize bias and produce accurate estimations of the desired curve. TI experiences thermic lag bias [3] as it changes β values and adjusts to each new value. This causes TI to slightly underestimate marginal likelihood values if β begins at zero, and a slight overestimate if it starts at one. It also experiences discretization bias, because of the limits on the accuracy with which a discrete number of values can estimate a continuous integral. Despite these points, and the fact that it is computationally expensive, TI is significantly more accurate than simpler methods, and thus a very helpful tool.

IV. THE STEPPING-STONE METHOD

The stepping-stone algorithm [6] is a method for finding evidences that is similar to TI but is less computationally costly. like TI, SS calculates marginal likelihoods directly, producing similar, and actually slightly more accurate [7] estimates.

the stepping-stone method calculates evidences differently than TI. Rather than calculating the average likelihood at each β value and summing them to estimate the integral, SS compares marginal likelihoods between each discrete β_i value and that of the one before it, in a process called importance sampling. Then the product of those ratios can be used to estimate the evidence [6]. This is shown in equation 9, where K is the number of chains (β values) used.

$$z = \frac{z_1}{z_0} = \prod_{k=1}^{K-1} \frac{z_{\beta_k}}{z_{\beta_{k-1}}} \quad (9)$$

A benefit of this method is that every chain does not need to sample from the posterior distribution, as it refers to the distribution immediately before it instead. This is more accurate than the TI method of averaging samples at each step in comparison to the posterior [7].

LIGO's fourth observational run is expected to detect significantly more events than previous runs, so it is possible that the SS algorithm will make an important addition to the tools `BayesWave` has at its disposal to analyze that data.

V. OBJECTIVES

The main goal of this project is to run `BayesWave` using both TI and SS methods, to compare the speed and accuracy of the two methods. This will help determine if the stepping-stone algorithm would be a worthwhile replacement to TI for LIGO's upcoming run. This will mean doing runs in bulk and comparing both the runtime and evidence convergence.

To determine this, many runs must be done using condor, which automates the run process. The data from these runs must then be organized so that the TI and SS data can be compared. If SS proves to be faster or more accurate, it may be incorporated into the master branch of `BayesWave` for future data analysis.

VI. PROGRESS

So far in weeks 1-3 of the project I have been largely familiarizing myself with the background of this project, as well with how BayesWave is run.

I began in week one by working through the GWOSC open data workshop, in order to understand some of the steps that go into signal matching, noise reduction, and glitch removal. Because I am working in data analysis, and BayesWave is used for signal matching, this was an important basis of understanding for me to have that has made working with `BayesWave` and the data it outputs more intuitive.

To be able to work with `BayesWave`, I learned how to enter the LIGO computing cluster from my computer, and make a branch of `BayesWave` that I can edit and work with, based on the master branch. Additionally, I needed a branch off of Meg Millhouse's version of `BayesWave`, which includes the stepping-stone algorithm that I will be working with. After these were set up and I had learned to navigate them, I did multiple runs using data from signal 150914, including a run using the stepping-stone branch. This was to familiarize myself with the process of doing runs, and with the data that is outputted by `BayesWave` when runs are complete.

The next step was to find where `BayesWave` stored evidence data for runs. After finding the location of the evidences, as well as data for the chains used during the runs. The evidences for TI runs included standard deviations, but the SS evidence files did not. Currently, I am working on a python script which pulls this data into arrays for future use. This will be useful when it comes time to compare TI to SS signal matching.

The next step of the project will be to do many runs of `BayesWave`, and when it is complete, compile the data from the runs using the script. This will be done on simulated data with an injected signal, rather than raw detection data from LIGO. When that has been done, data comparison between TI and SS will be possible. From there, the goal will be to determine how much faster and accurate the stepping-stone method is.

VII. ACKNOWLEDGEMENTS

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