LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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Optical contact bonding for mitigating clamping losses in silicon resonators

Ojo Akinwale. Mentors: Aaron Markowitz, Christopher Wipf, Rana Adhikari

California Institute of Technology
LIGO Project, MS 18-34
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project, Room NW22-295
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
Route 10, Mile Marker 2
Richland, WA 99352
Phone (509) 372-8106
Fax (509) 372-8137
E-mail: info@ligo.caltech.edu

LIGO Livingston Observatory
19100 LIGO Lane
Livingston, LA 70754
Phone (225) 686-3100
Fax (225) 686-7189
E-mail: info@ligo.caltech.edu

http://www.ligo.caltech.edu/

Abstract

This experiment aims to improve the quality factor of silicon resonators in order to mitigate the thermal noise present in the mirror coatings and suspension systems in the LIGO Michelson Interferometer. The oscillation of the cantilever is analyzed and used to calculate the mechanical quality factor through the exponential decay of the oscillations with the ringdown method. The clamping losses among other extrinsic sources of loss are studied to understand the dissipation mechanisms affecting the quality factor. The design of the clamp was made to use optically contacted layers of silicon in order to reduce the acoustic waves radiating away from the resonator into the substrate, in order to increase the quality factor of the cantilever. The quality factor of the silicon cantilever with the optically contacted layers averages around $4 * 10^3$ which did not show improvement over the cantilever alone. The oscillation data that was collected had a weakened signal which skewed the calculations of the last ringdown data collection. It is possible that a better oscilloscope could improve the accuracy of the ringdown experiments and show that optical contacted layers of silicon decrease clamping losses. Further experimentation may show that thermoelastic losses is the dominant source of loss within the cantilever.

1 Introduction

Gravitational-wave astrophysics makes use of propagating disturbances in the spacetime medium to detect sources of gravitational waves. Gravitational waves thus open up the possibility of discoveries that can't be observed using traditional observational techniques. Accelerating masses in a relativistic field theory of gravity produces gravitational waves. An accelerating object can produce fluctuations in the curvature of spacetime that propagate at the speed of light in the form of gravitational waves. LIGO uses laser interferometry for gravitational wave detection where the mirror test masses are displaced when a gravitational wave passes. The effect from even the most extreme gravitational waves is only detectable with a very sophisticated detector and is limited by a number of noise sources.

One such noise source is caused by thermally induced vibrations which show up as noise in the detectors. An improvement of LIGO's observational capabilities will be necessary to maximize the potential of the astrophysics obtainable through gravitational waves. A large part of the thermal noise comes from the internal friction of the mirror and suspension elements which hold the mirrors. LIGO's cavity design features a quadruple pendulum suspension system used to support the test mass mirror vertically against the force of gravity while allowing free motion in the arms of the interferometer. As next-gen laser interferometers plan to operate at cryogenic temperatures, reducing the cryogenic Brownian thermal noise of the mirror coating is increasingly becoming essential with advances in the reduction of quantum noise [1]. Silicon is a material with extremely high mechanical quality. At cryogenic temperatures silicon is beneficial since unlike fused silica, its mechanical loss decreases with temperature. The expansion coefficient vanishes around 120 K which eliminates the thermoelastic component of the thermal noise [2]. Thermal noise leads to mechanical dissipation according to the fluctuation dissipation theorem, driving the motivation for studying the quality factor of silicon cantilevers. We can investigate the loss of the surface of the mirrors with a small cantilever blade. The cantilevers are thin strips of the bulk material from which we can extrapolate how the noise sources would be transferred to a LIGO test mass mirror.

Thermal losses can also be explained as friction converting mechanical energy to thermal energy. The fluctuation–dissipation theorem says that when there is a process that dissipates energy, turning it into heat (e.g., friction), there is a reverse process related to thermal fluctuations [3]. No system is truly as stable as on the quantum level there is constant Brownian motion due to the inherent random motion of particles in a medium. The mirrors and the suspension always contain Brownian noise since the reverse process to the dissipative mechanism of kinetic energy is always in effect where the thermal energy of the particles is transformed into kinetic energy. The silicon macro-mechanical cantilever allows direct measurement of thermal noise at cryogenic temperatures. The mechanical loss angles of the silicon coating materials of the mirror and suspensions are measured using the cantilever ring down method. The ring down measurements will be used to determine the effect losses associated with the clamps on the mechanical quality factor.

2 Thermal motion and Brownian Noise

The fluctuation-dissipation theorem is derived from statistical physics proving that thermodynamic fluctuations in a physical variable predict the response quantified by the impedance of the same physical variable [3]. The theorem says that when there is a process that dissipates energy, turning it into heat, there is a reverse process related to thermal fluctuations. The amount of thermal motion within a system at a given temperature depends on the internal material properties of that system [9]. Systems are never truly at rest as on the molecular level there exists microscopic motion of its particles. This phenomenon referring to the random motion of particles within a medium is known as Brownian motion. Brownian motion is an antecedent to the general theorem of the fluctuation-dissipation relation.

The same random forces that cause the erratic motion of a particle in Brownian motion are also responsible for drag. Drag dissipates kinetic energy, turning it into heat. The corresponding fluctuation is Brownian motion. An object in a fluid is not really still but moves around with a small and rapidly changing velocity, as molecules in the fluid bump into it. Brownian motion converts heat energy into kinetic energy. The same applies for resistance and johnson noise. Resistance dissipates electrical energy, turning it into heat; but an electric current within a wire with resistance is never exactly zero. The corresponding fluctuation is Johnson noise. A small and rapidly-fluctuating current caused by thermal fluctuations of the electrons and atoms in the resistor [10]. Johnson noise converts heat energy into electrical energy, the reverse of resistance.

The underlying physical laws that govern the motion of collections of particles are extremely complex. The quantum system that encompasses the particle motion has a wave function that holds a large amount of information making analytical calculations difficult. In quantum mechanics, the many-body system is in a superposition of combinations of single particle states [11]. The dynamics of more than three quantum-mechanical particles is infeasible for many physical systems, thus a many-body theoretical model of thermal noise would require a computationally intensive approach. The greater implication of the fluctuation-dissipation relation is that to study thermal noise models of the particle motion itself is unnecessary, since the noise of a system is related to dissipation within it. The fluctuation-dissipation theorem can be written as:

$$S_f(\omega) = 4k_bTRe\{Z(\omega)\}$$

[9]

where $S_F(\omega)$ is the force noise spectral density, and $Z(\omega) \equiv \frac{F(\omega)}{v(\omega)}$ is the mechanical impedance of the system.

3 Mechanical Impedance and Silicon Cantilevers

The fluctuation-dissipation theorem provides a straightforward way to study the thermal noise of the mirror coatings and suspension system in LIGO, thus driving the motivation for studying the quality factor of silicon cantilevers. Any sort of dissipation guarantees fluctuating forces when the system is at rest which, in the case of LIGO's mirrors and mirror suspensions, masks the gravitational wave signals. However, it also implies that one does not need to make a detailed microscopic model to predict the dissipative mechanisms associated with thermal noise. The noise spectral density of thermal fluctuations scales inversely with the quality factor. We can investigate the loss of the surface of the mirrors with a small cantilever blade. The cantilevers are thin strips of the bulk material from which we can extrapolate how the noise sources would be transferred to a LIGO test mass mirror.

4 Quality Factor of Silicon Resonators

The quality factor describes how much loss of energy there is for an oscillator or resonator in a given cycle of oscillation. The magnitude of the quality factor dictates the rate of energy loss where the oscillations deteriorate slower with a high quality factor. Thus, high quality factor translates to more sensitive gravitational wave detection in the interferometer. Losses can be sourced to air damping, radiation, dielectric material deterioration, phonon-interaction or of the metallic resistance. Silicon was chosen for the test masses for its natural resistance to mechanical and thermal loss.

The dissipation mechanisms can be divided into intrinsic dissipation sourcing qualities of the resonator itself, and extrinsic dissipation related to the resonator's interactions with its environment [12]. The total quality factor is a sum of the intrinsic and extrinsic losses

$$\frac{1}{Q} = \frac{1}{Q_{int}} + \frac{1}{Q_{ext}}$$

The intrinsic mechanisms include damping from surface effects, thermoelastic damping, and phonon-phonon loss. The extrinsic sources include gas damping and clamping losses. Surface loss originates from cracks and contaminations in the surface of the silicon. As the surface-to-volume ratio of a sample increases, the relative contribution of these surface effects decreases [9]. The material type and quality thus greatly affect the internal friction of the resonator. As its atoms move when the resonator oscillates the friction between the atoms creates damping. Phonon-phonon noise comes from the redistribution of phonons within a solid due to an external oscillation with a longer wavelength than that of the phonon disturbing its equilibrium. The redistribution of phonons generates entropy which causes loss. However, it is only a contributing factor at low temperatures [13]

Clamping loss arises when the resonator oscillates and pulls on the clamping points where the resonator is attached to the substrate causing acoustic waves to radiate outward into the substrate which siphons the energy of the resonator. The other main extrinsic source of loss, gas damping, is caused by collisions between the gas molecules and the resonator. The sum of these loss sources determines the quality factor of the resonator

$$Q^{-1} = \sum_i Q_i^{-1} = Q_{gas}^{-1} + Q_{phonon}^{-1} + Q_{TED}^{-1} + Q_{surface}^{-1} + Q_{clamping}^{-1} + Q_{other}^{-1}$$

[5].

The current method to measure the quality factor requires a resonator to be driven at the resonance frequency then to remove the drive signal. When the drive signal is switched off for the resonator, the free-decaying resonance amplitude follows the exponential law $A(t) = A_0 e^{-t/\tau}$. As the resonator oscillates it experiences a decay in its amplitude proportional to a time constant τ, measuring how long it takes for the amplitude of motion to decrease to 1/e of its original value. From this an expression for the factor can be resolved for how much the amplitude decreases during a single period. The Q factor equals 2pi times the exponential decay time of the stored energy times the optical frequency.

$$Q = \pi f_0 \tau$$

The fit equation used has various parameters encompassing an exponentially decaying cosine wave. When the time constant and frequency of the cantilever's oscillation is extracted from the fit it is plugged in to the Q factor equation

$$A(t) = Acos(2\pi\omega t + \phi)e^{-t/\tau} + D$$

.

5 Clamping Losses and Optical Contacting

The main sources of loss for the cantilever will be surface losses, gas damping, and clamping losses. Surface losses are more easily altered by increasing the quality of the resonator's material, thus will not be the focus of the experiment. The cantilever will be tested within and outside of a vacuum chamber up to 10^-6 torr to measure the effect it has on the quality factor. The primary source of interest in the experiment is clamping losses. Clamping losses can be described by a number of terms including anchor losses, clamping losses, radiation losses, mounting losses, and phonon tunneling losses.One solution to mitigate clamping losses can be to use optical contacted silicon stacked at the clamp. Because optical contacting has no adhesives, it produces highly reflected structures. If you add material to alter the geometry near the clamp of the resonator you might introduce new sources of dissipation as in some designs such as double clamping face these issues.

Optical contacting silicon stacked near the clamping point would work to increase the mechanical quality factor by reducing the amount of acoustic waves radiating the energy of the resonator away into the substrate. Phonons exhibit tunneling properties given that heat can flow across a gap through the phonons that act as a tunnel between two materials. With optically contacted silicon layers will contact the cantilever at the nodal points in its vibrations which will increase the nullification of the acoustic radiation loss. The size and mass difference between the resonator and the clamp creates an impedance mismatch at the connecting points [12]. The impedance mismatch causes a partial reflection of acoustic waves from the resonator. As acoustic waves travel through the clamping points the resonator loses energy. An analytical approximation of the clamping loss can be derived from finding the density of states overlap between the resonator and substrate modes. Maximizing the impedance mismatch between the cantilever and the clamp should decrease the clamping loss. This has been shown in trampoline resonators, where increasing the thickness of the surrounding substrate results in a larger Q [15].

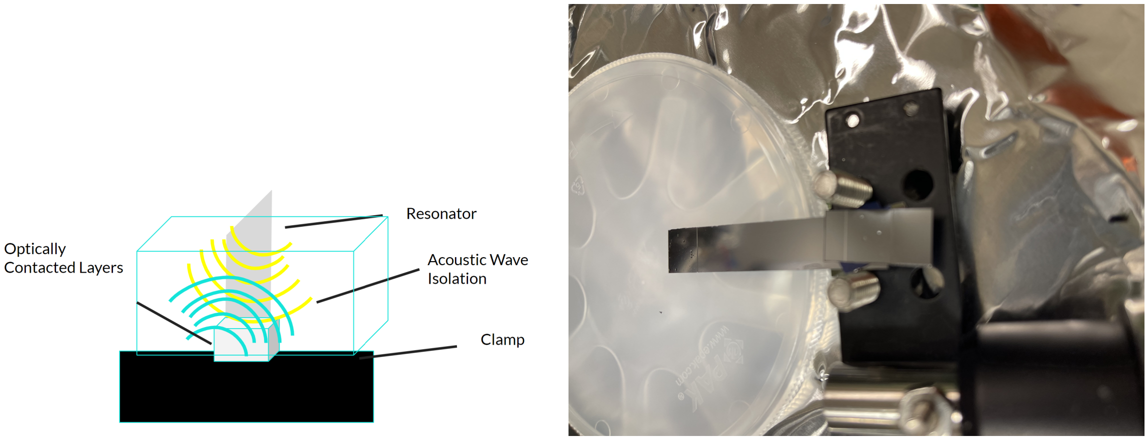

Figure 1: Optical contacting conceptual schematic (left). Applied layers of silicon stacked on cantilever (right).

Nested structures can emulate analog filters to isolate vibration. Nested low-pass filters have been shown to reduce the contribution of clamping loss [12]. The optically contacted layers of silicon emulate the nested resonator structure where one resonator is nested within another ancillary resonator, or a series of them. The various resonators have slightly different frequencies with different geometries, being abstract pieces of silicon. The optical contacted layers have different frequencies such that the overall coupling between them is suppressed. The central nested resonator, the cantilever, can be acoustically isolated from its surroundings, reducing the amount of energy that is dissipated from clamping loss. The masses surround the nested resonator act as an acoustic filter that can be modeled using the power transfer function

$$T(\omega_m) = \frac{\omega_4^4}{(\omega_a^2 - \omega_m^2)^2 + \Gamma_a^2 \omega_m^2}$$

where ω_m is the angular frequency of the isolated resonator, and ω_a & Γ_a are the angular frequency and damping rate of the ancillary or surrounding resonator respectively. Stacking n resonators results in mechanical suppression scaling of $(\omega_a/\omega_m)^{2n}$ [16].

6 Project Timeline

The goal of this project is to explore clamping losses and determine what effect optically contacted silicon stacked at the clamp will have. This study expands upon this area with the aim of minimizing dissipative sources that diminish the sensitivity of the optical cavity mechanisms on LIGO detectors.

The start of the project in the first 3 weeks included setting up an optical lever to measure small displacements of the resonator's oscillation, the ringdown method. The silicon cantilever was clamped down to a breadboard placed inside of a vacuum chamber to isolate external vibrations. The next 2 weeks were spent measuring the Q factor of various cantilevers and refining the calculation method. The Q factor calculation was done in matlab with the built in fit function to extract the time constant. The rest of the project focused on measuring the Q factor at low pressures and reducing clamping losses by stacking layers of silicon at the clamping point.

7 Optical Setup and Clamp design

The clamp design features two steel rectangular blocks screwed together with the base steel beam on the end like a flag so that the cantilever can be attached separately from its mount.

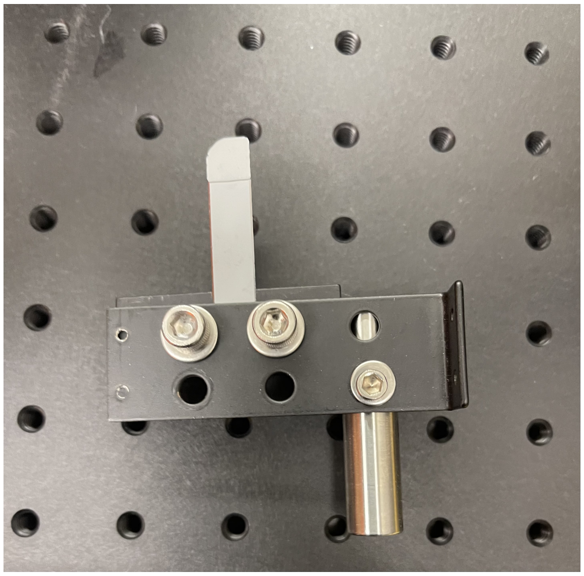

Figure 2: Silicon Cantilever with steel base

The optical setup uses a helium-neon laser aimed at a mirror which reflects onto the cantilever in the cryostat chamber, and then onto a quadrant photodiode. As the cantilever oscillates the motion of the laser is recorded as a changing amplitude that decays over time. The current setup has an extra mirror added to the optics to lengthen the laser beam and eliminate a shadowed laser spot that could affect the measurements on the QPD. The cantilever isn't excited through any technical means but is done by banging the table.

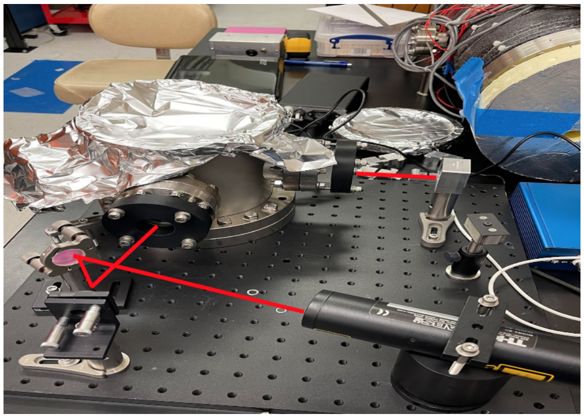

Figure 3: Optical lever setup for cantilever quality factor measurement with its projected laser path to the Quadrant Photodiode.

8 Oscillation Data Collection

The ringdown method fits the oscillation data to a fit of an exponentially decreasing cosine wave. The previous Q factor calculations were derived from a general exponential decay fit for comparison. The fit equation with more parameters more accurately matches the cantilever ringdown but the input values need to be tightly chosen for fit to work well.

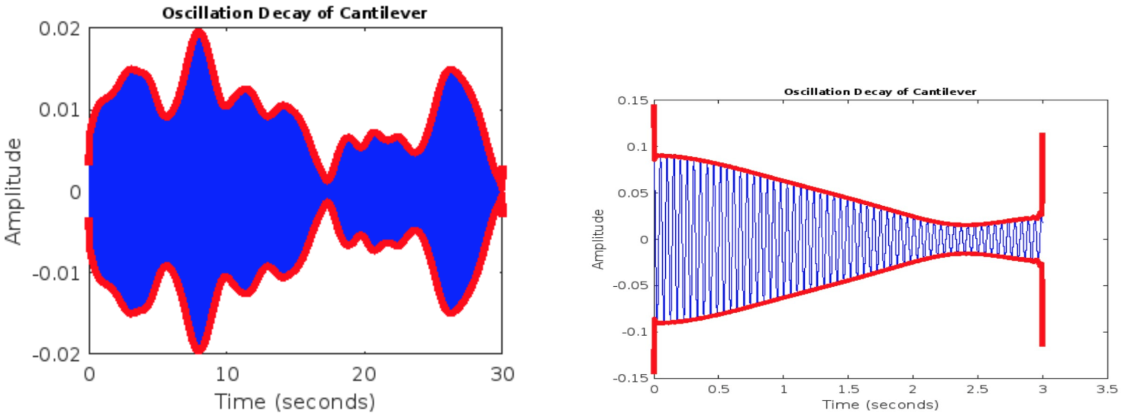

Figure 4: Sample of oscillation data of silicon cantilever in early experiments. The waveform contains lots of noise making it hard to resolve the ringdown sections.

The oscillation data that was captured in previous analysis did not look accurate to resonator ringdown outputs as it is hard to see the exponential decay. The signal also was extremely deformed when the signal from the QPD was ported serially to a board and transmitted. The resonator data obtained directly from the oscilloscope looked more like a natural ringdown sequence for which analysis can more confidently be conducted.

The cantilever previously recorded a Q value of around 500. The cantilever was replaced

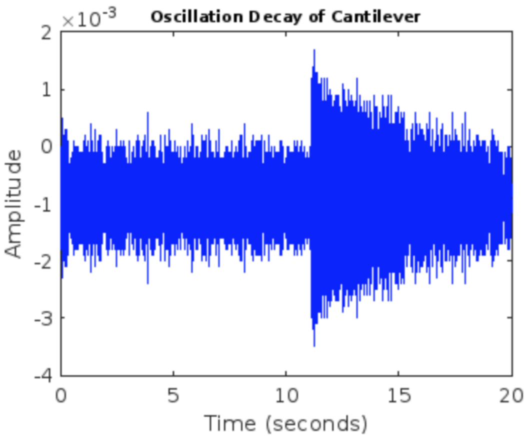

Figure 5: Cantilever ringdown fitted to an exponentially decaying sine wave.

due to structural damage and 10 ringdown samples were collected. The mean quality factor from the ringdowns ranged between $1 - 4 * 10^3$. The final cantilever measurements were conducted in a vacuum chamber with the optically contacted layers of silicon surrounding the cantilever. Silicon naturally has a high mechanical quality factor, so surface losses and intrinsic Q factor reductive processes are negligible compared to extrinsic sources such as gas damping and clamping losses, however in these experiments the cantilever has lots of surface defects which limits the Q factor. Another major source that limits the Q factor is thermoelastic loss which arises from the heat gradient created by elastic motion. Thermoelastic damping limits the Q factor to 10^5. Gas damping within a vacuum limits the Q factor to 10^7.

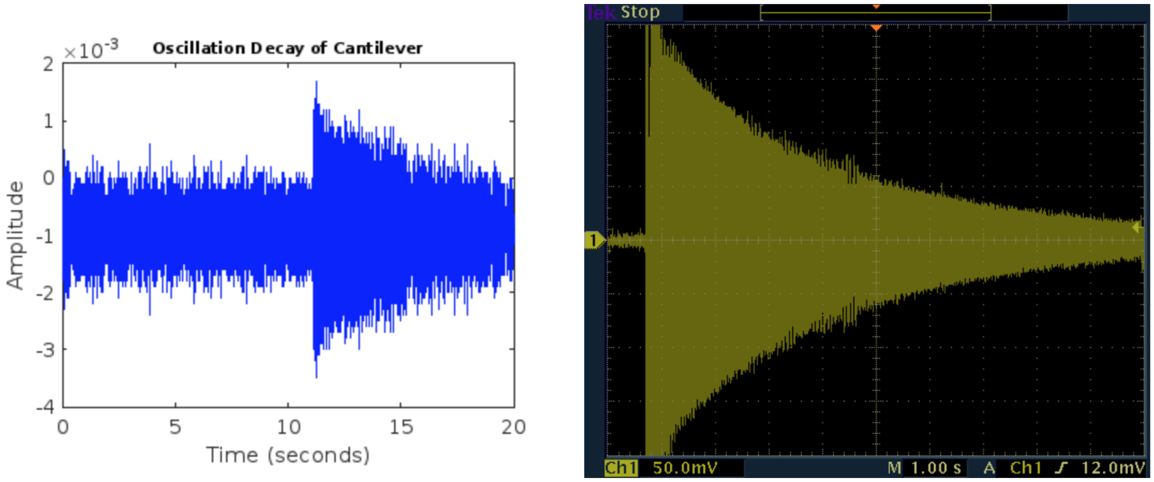

Figure 6: Current ringdown data collected from oscilloscope (left) compared with a ringdown obtained from [8].

9 Ringdown Q Factor Calculations

1st Cantilever mean Q Value- 500

2nd Cantilever mean Q Value- 4594

Optical Contacted Cantilever mean Q value- 4623

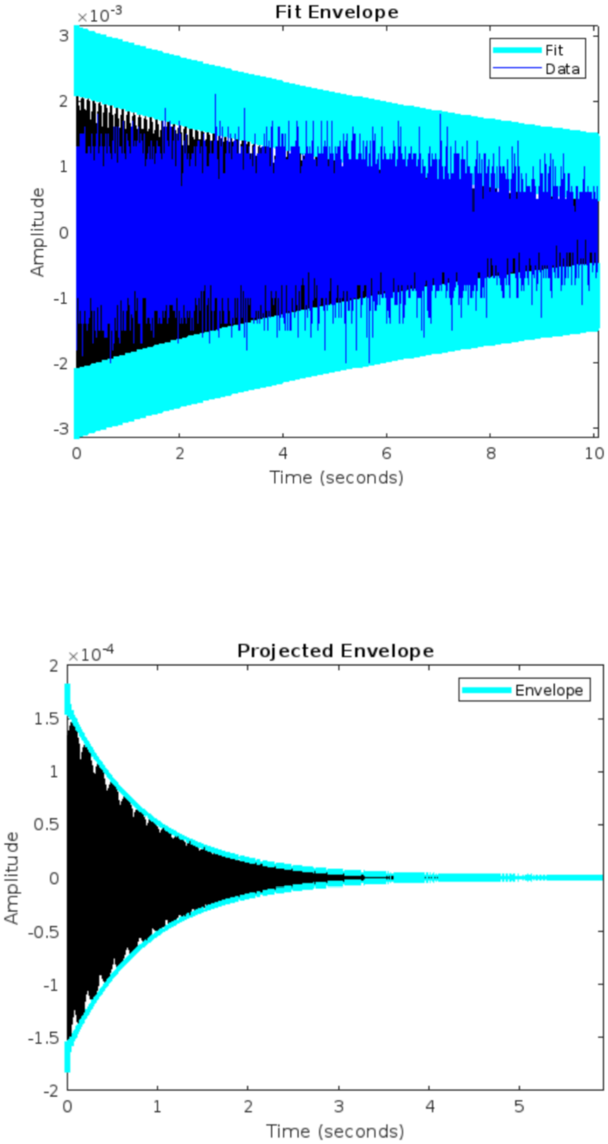

Figure 7: Ringdown fitted to exponentially decaying cosine wave

10 Ringdown Comparison

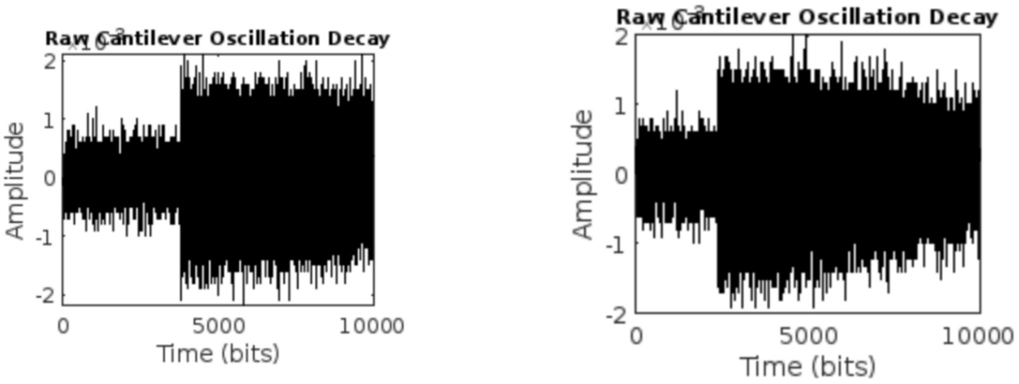

Figure 8: Regular cantilever ringdown data

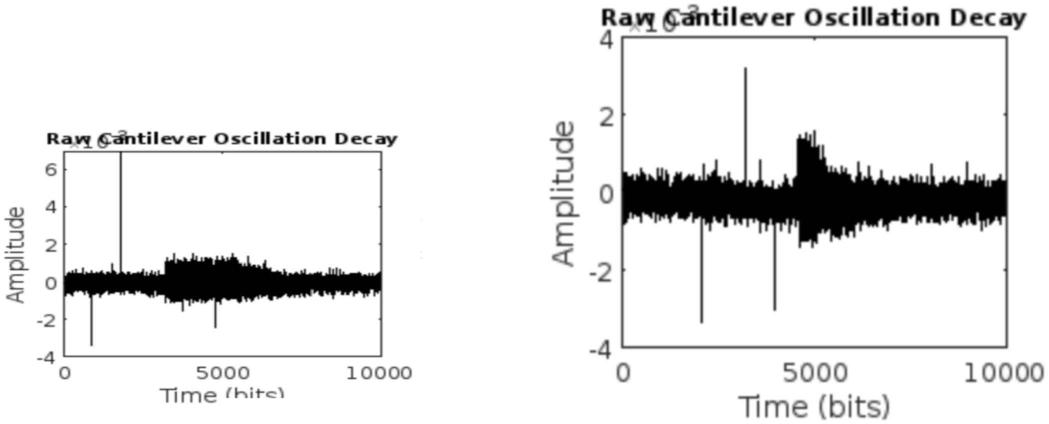

Figure 9: Optically contacted cantilever ringdown data

11 Error Analysis

To quantify the goodness of fit and adjust the code accordingly the Sum of Squares Error was used which is a sum of the residuals squared or the squared difference between each observation and its group's mean. The closer the sum of squares error value is to 0 the better the data matches the fit. The fit outputs a SSE around 100, so it can be accepted as a reasonable approximation.

$$SSE = \sum_{i=1}^{n} (x_i - x)^2$$

The residuals should look like random noise. If the residuals have a systematic pattern then the fit is poor. The residuals showed agreement between the fit and the data. Test data with random parameters and white gaussian noise was also generated to test the fit.

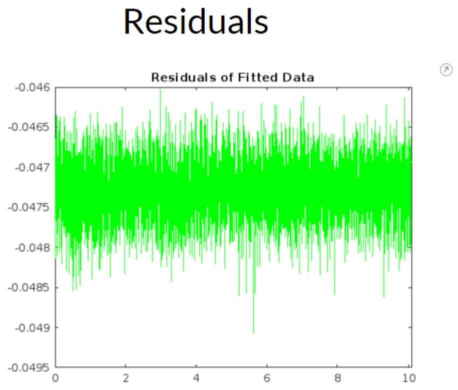

Figure 10: Test ringdown data with white gaussian noise

The chi-squared test was also used to measure the quality of the fit. The chi-squared test defines how great the statistical difference is between the data and the theory.

$$\chi_c^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

The lower the probability value, the smaller the statistical difference. The probability value should fall to the left of the significance level of 0.05 since it is a right tailed distribution. If the chi-squared statistic is greater than or equal to the critical value there is a significant statistical difference. The test ultimately determines if a null hypothesis can be rejected based on if it doesn't fall below the critical value. The chi-squared analysis of the ringdown fit yields that the null hypothesis can't be rejected. There isn't a significant statistical difference between the data and the fit, so the fit is adequate for Q factor calculations.

12 Experimental Conclusions

The cantilever ringdown experiment overall was an amazing experience and I gained lots of valuable skills throughout this project. Some recommendations for improvement would be that the ringdowns to compare different cantilever setups be done within the same timeframe, as variables can change over time affecting the signal you measure. A better oscilloscope The cantilevers should always be picked up with tweezers as they break very easily which doesn't bode well for the experiment. The code was a tedious part of the experiment since the fits calculations depend on where the data starts and

13 Ackniowledgements

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