Mode Mismatching Analysis

#beaminthehole

LIGO SURF 2022
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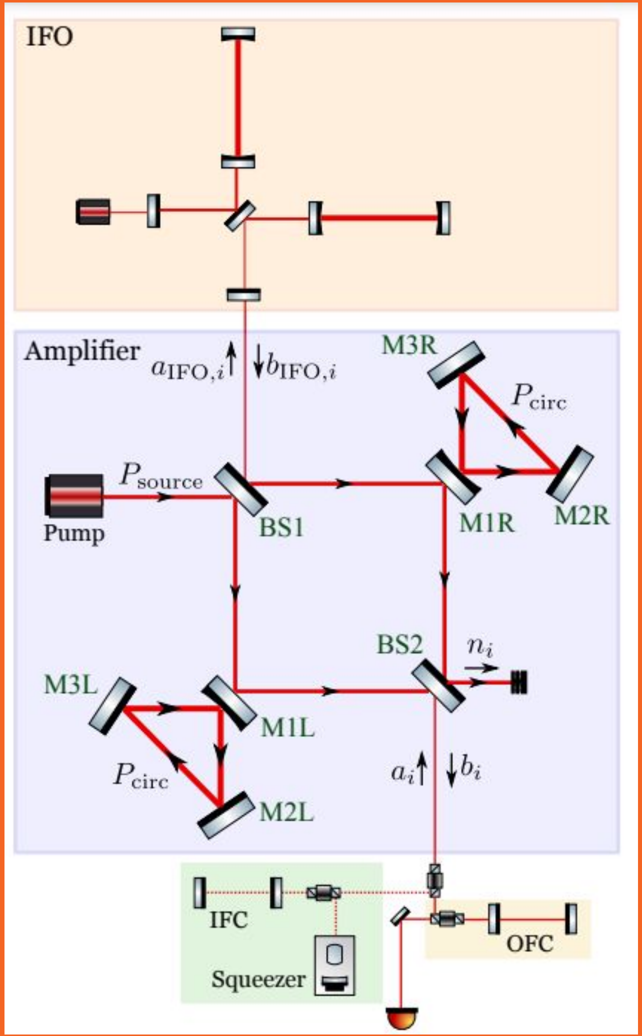
Project Overview:

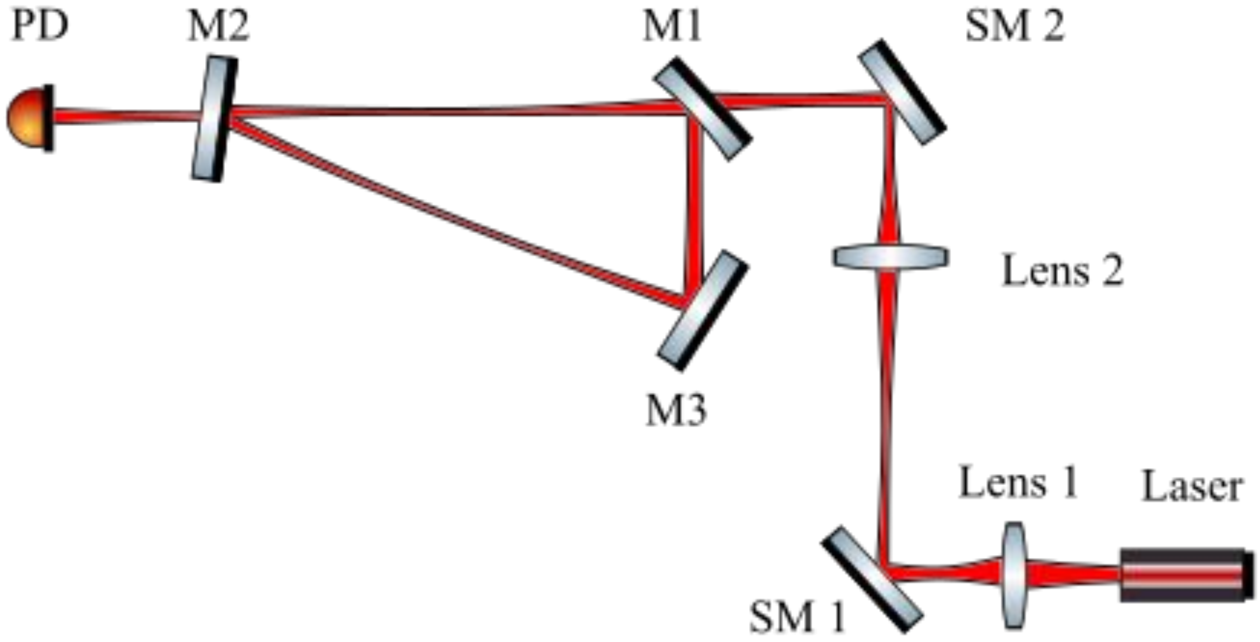
- Original project motivations
- Background on mode mismatch
- Thick lens analysis and lens aberrations
- Modeling with Gaussian beam mode coupling
- Future plans

Original Mission

- Phase Sensitive Optomechanical Amplifier (PSOMA) experiencing mode mismatching.
- Test thick lens analysis.
- Consider lens aberrations creating higher order modes



Tabletop design



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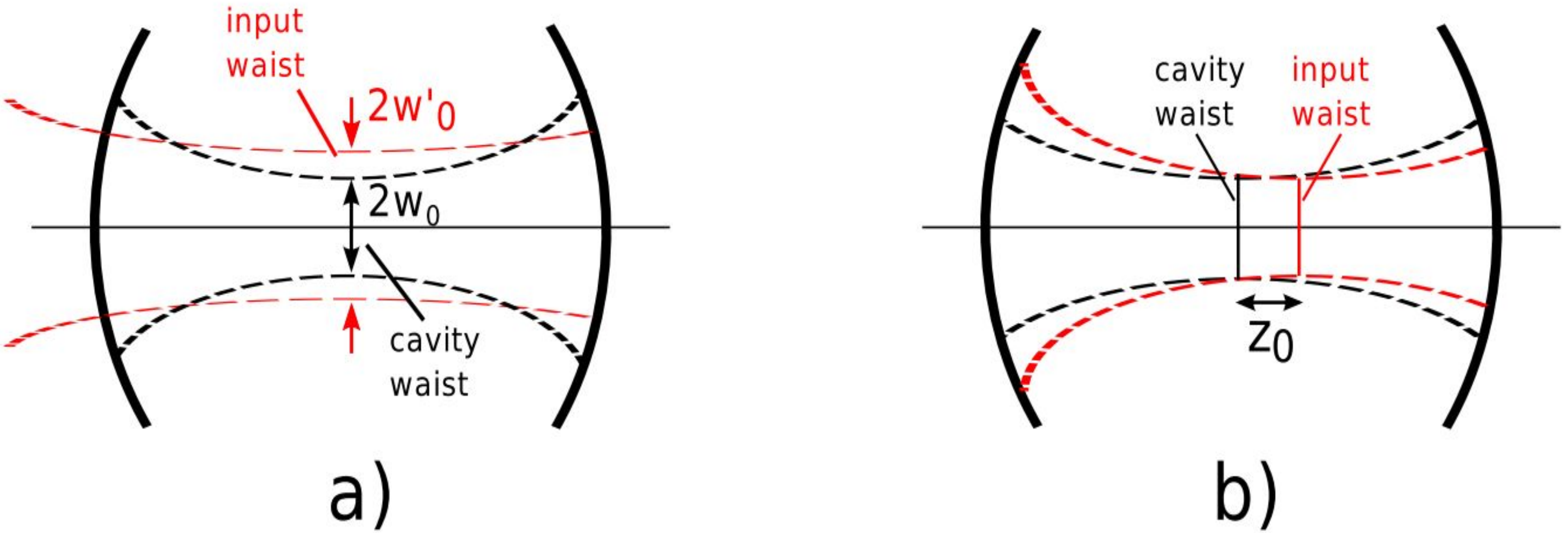
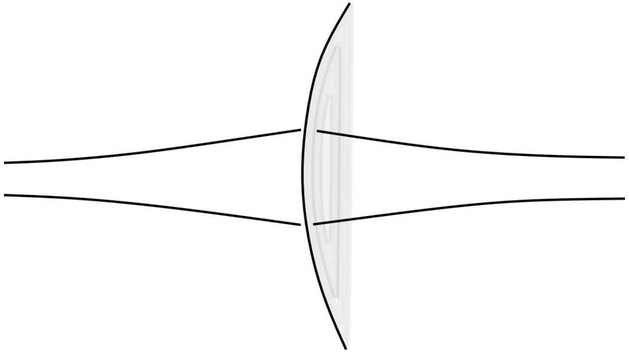


a) b)

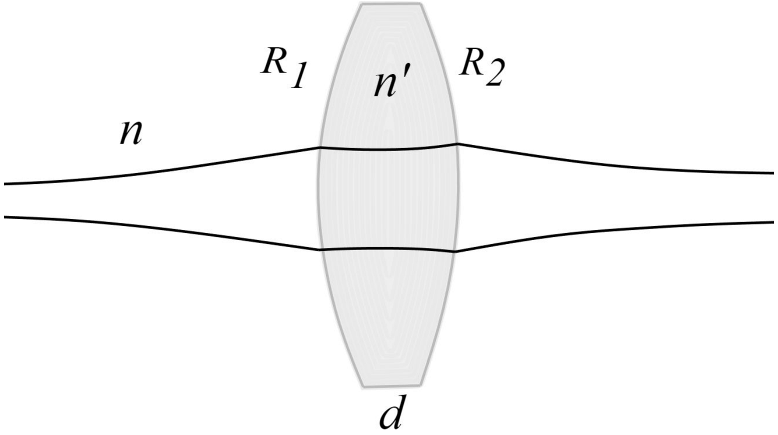
Through Thin and Thick

Thin lens

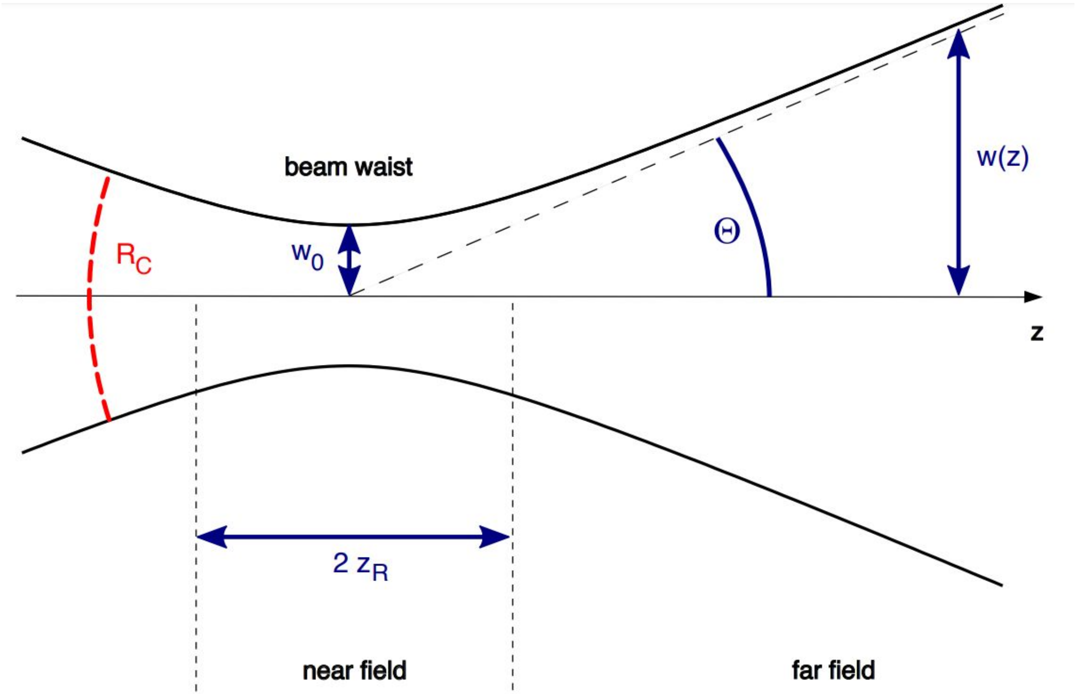


$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

Thick lens



$$\begin{pmatrix} 1 & 0 \\ \dfrac{n'-n}{R_2} & 1 \end{pmatrix}\begin{pmatrix} 1 & \dfrac{d}{n'} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \dfrac{n-n'}{R_1} & 1 \end{pmatrix}$$



Rc = Beam front radius of curvature

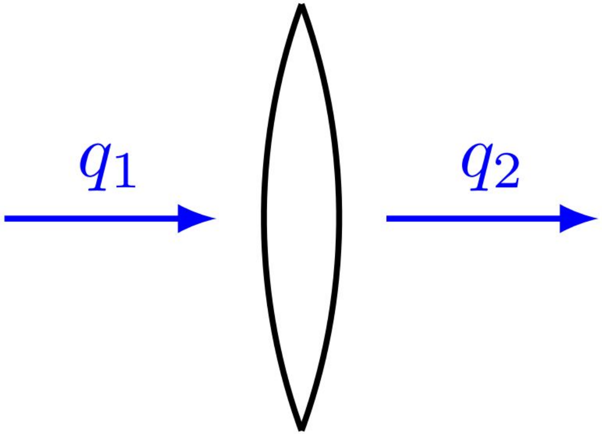
w_0 = Beam waist

w(z) = Width of beam at some z position

Z_R = Rayleigh range

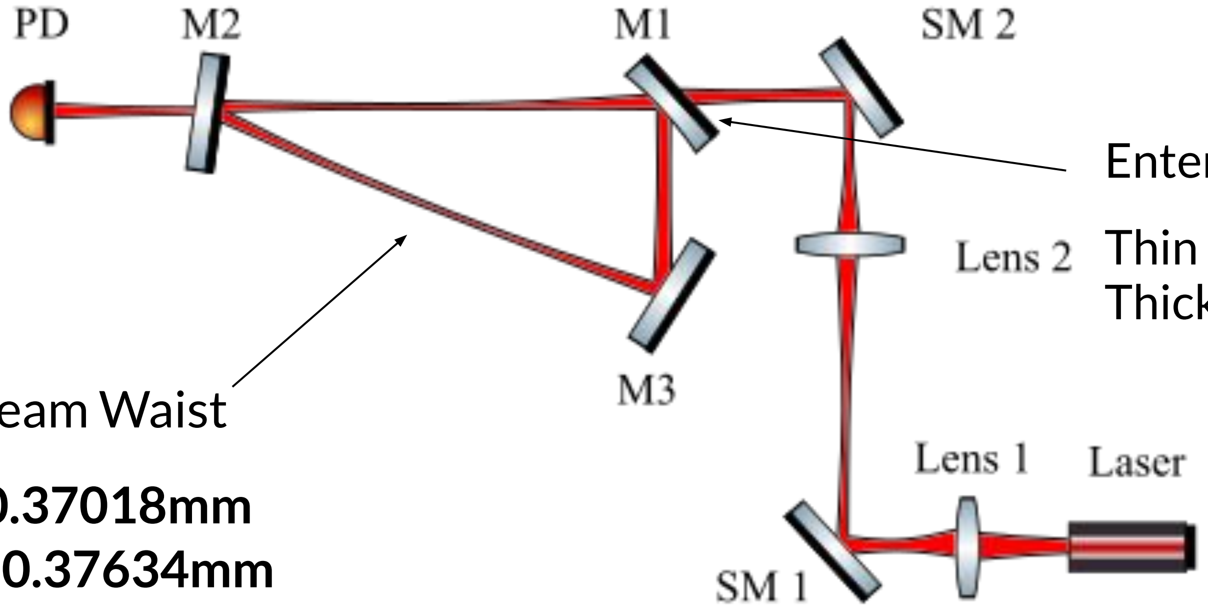
$$q(z) = \left(\frac{1}{R_c} - i\frac{\lambda}{\pi\omega^2(z)}\right)^{-1}$$

$$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}\cdots = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$



$$\frac{Aq_1 + B}{Cq_1 + D} = q_2$$

Results from Thick Lens Calculation



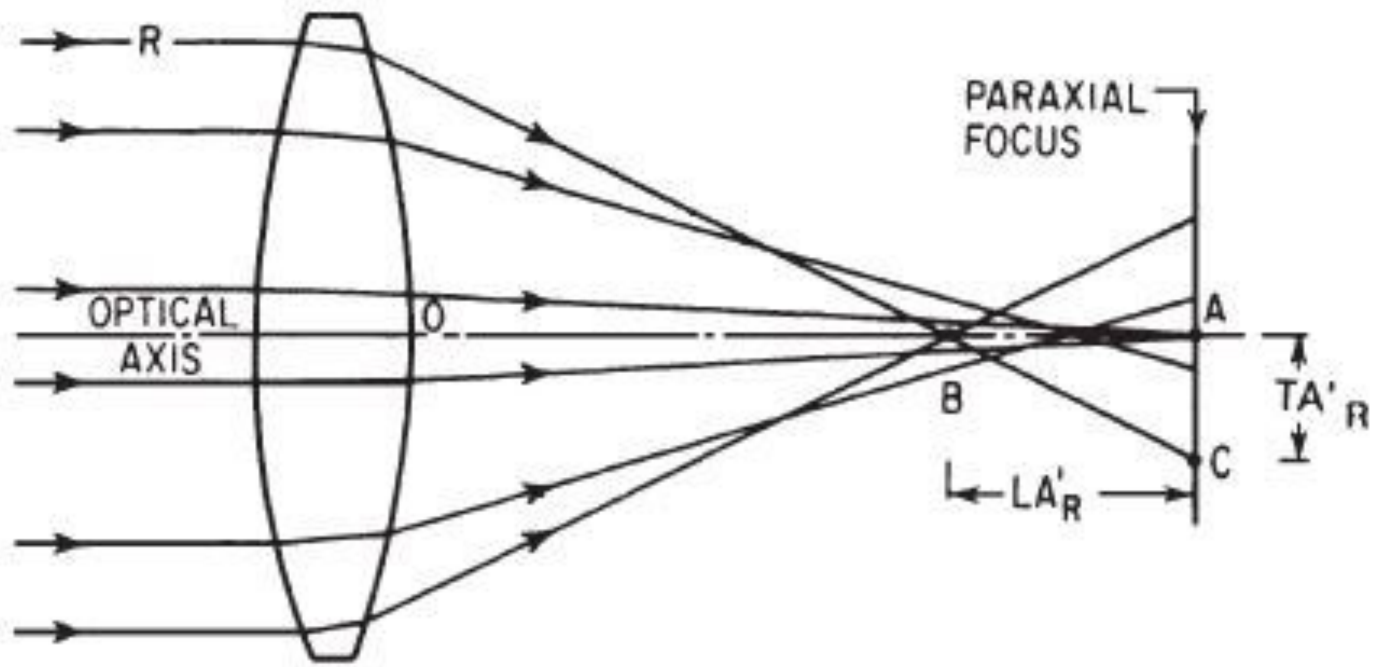
Entering first mirror

Thin Lens: **0.54431mm**
Thick Lens: **0.5475mm**

At Cavity Beam Waist

Thin Lens: **0.37018mm**
Thick Lens: **0.37634mm**

Lens Aberrations with Ray Tracing



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Ray Tracing to Gaussian Beams

A purely Gaussian laser can scatter into Higher Order Modes.

This is the Gaussian version of lens aberrations.

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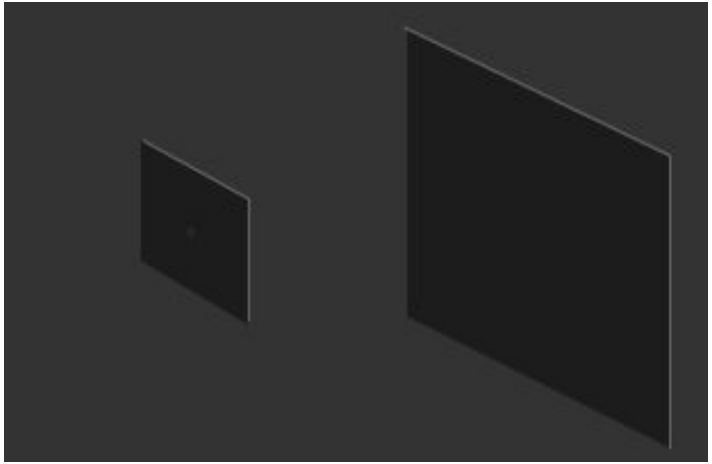
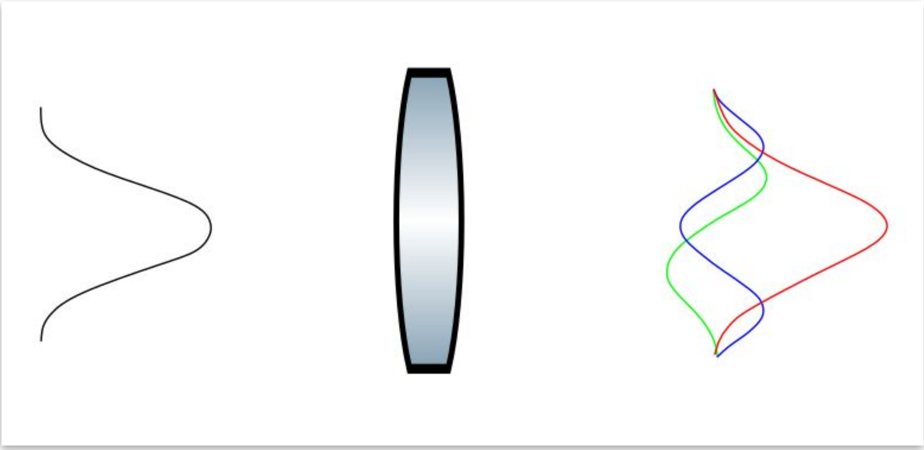




Quick Math Behind Gaussian Beams

$$u(x,y,z) = \sqrt{\frac{2}{\pi}}\frac{1}{\omega(z)}e^{\left(-ik\frac{x^2+y^2}{2R(z)}-\frac{x^2+y^2}{\omega(z)}\right)}e^{i\psi(z)}$$

Equations to get Hermite Gauss patterns

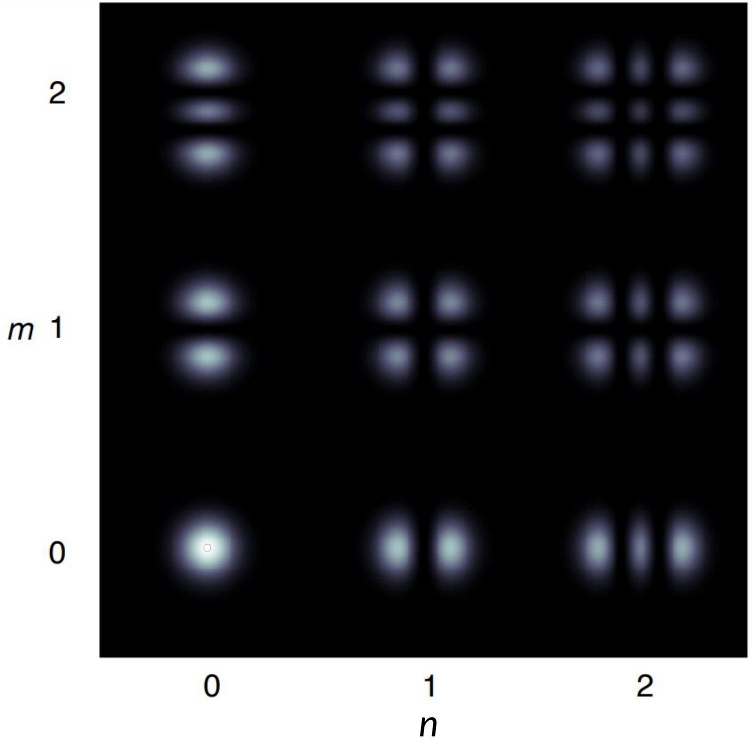
$$u_{nm} = \left(2^{n+m-1}n!m!\pi\right)^{-1}\frac{1}{\omega(z)}e^{(i(n+m+1)\psi(z))}H_n\frac{\sqrt{2}x}{\omega(z)}H_m\frac{\sqrt{2}y}{\omega(z)}e^{\left(-ik\frac{x^2+y^2}{2R(z)}-\frac{x^2+y^2}{\omega(z)}\right)}$$

Equations to get Laguerre Gauss patterns

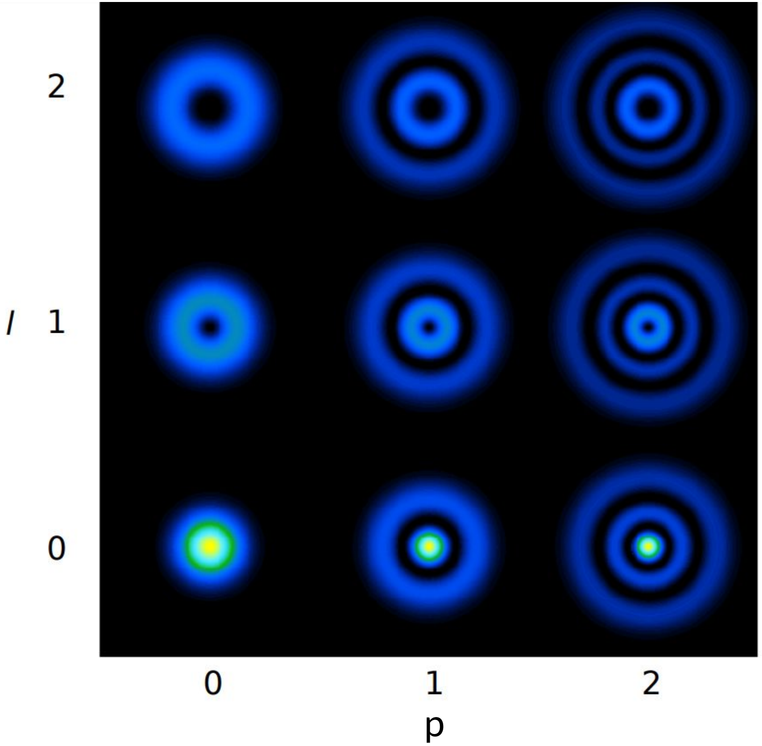
$$u_{pl} = \frac{1}{\omega(z)}\sqrt{\frac{2p!}{\pi(|l|+p)!}}e^{(i(2p+|l|+1)\psi(z))}\left(\frac{\sqrt{2}r}{\omega(z)}\right)^{|l|}L_p^{|l|}\left(\frac{2r^2}{\omega(z)^2}\right)e^{\left(-ik\frac{r^2}{2q(z)}+il\phi\right)}$$

Meet the Families of Modes

Hermite Gauss Modes

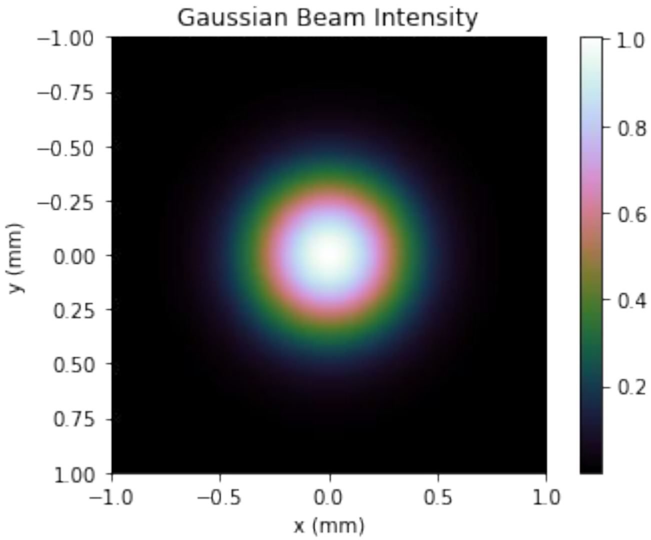


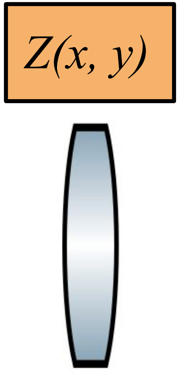
Laguerre Gauss Modes

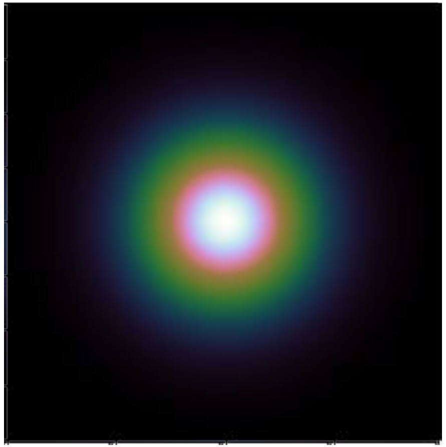


Just add them together #beaminthehole

$$c_{nmn'm'} = \iint_{-\infty}^{\infty} u_{nm} u^*_{n'm'} e^{(2ikZ(x,y))} dx dy$$

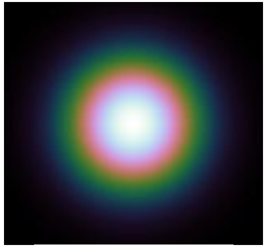






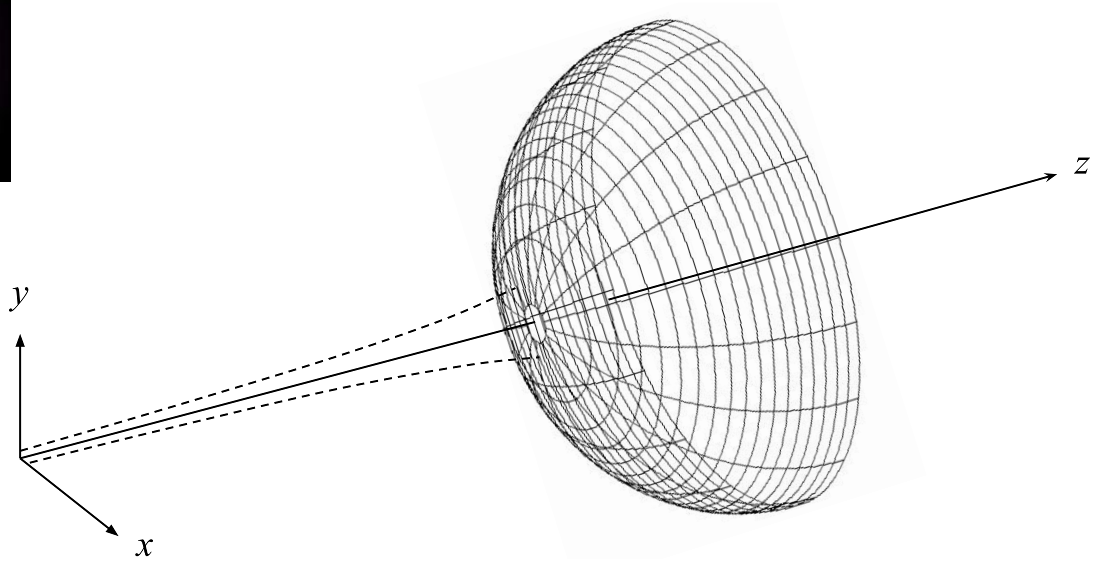
$$Z(x, y) = \sqrt{R^2 - x^2 - y^2}$$

We start with a pure
Gaussian beam.
p&l = 0

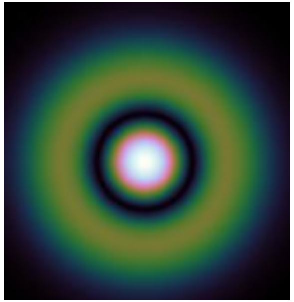


u_{00}

$$Z(x,\ y)\ =\ \sqrt{R^2 - x^2 - y^2}$$



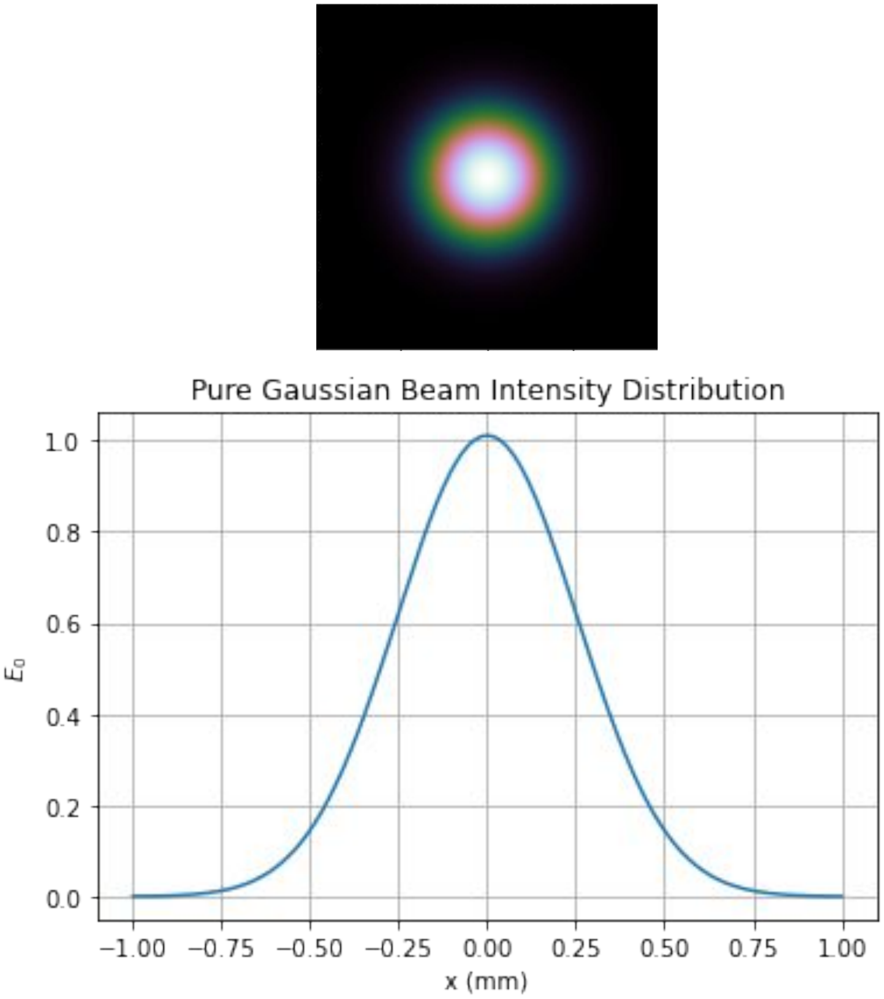
The first higher
order mode.
p' = 1, *l'* =0

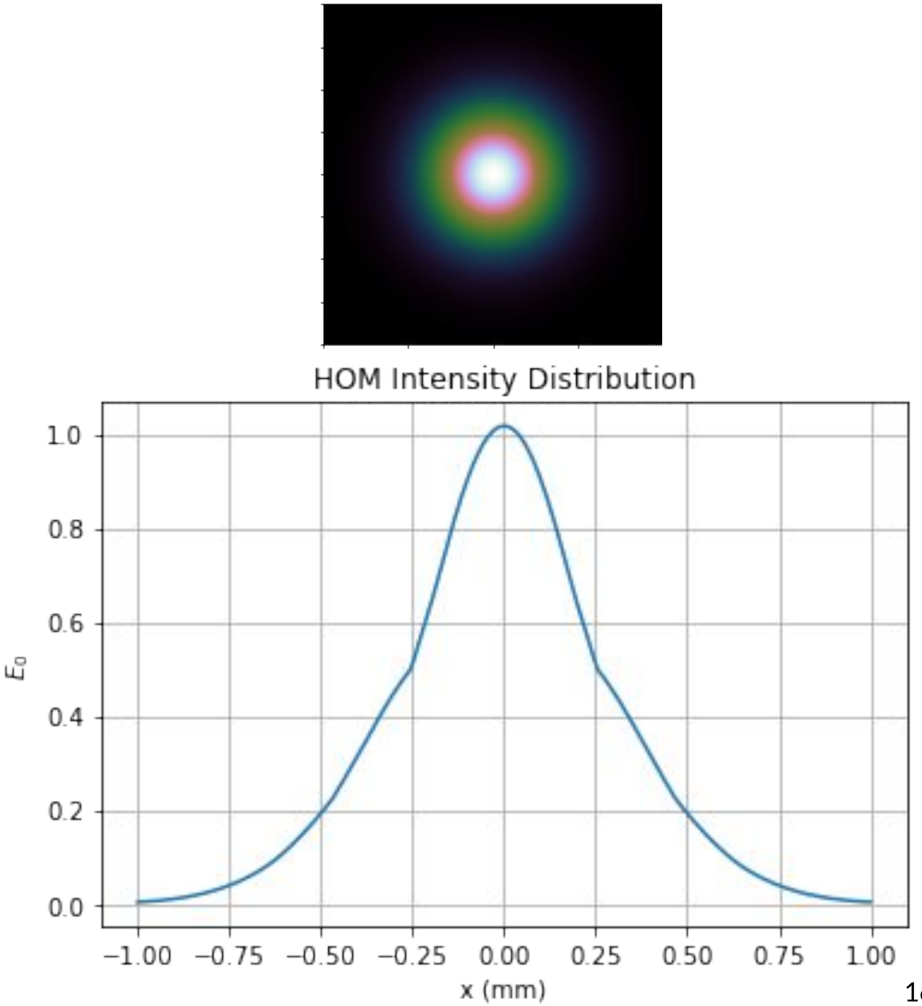


$|c_{10}|^2 u_{10}$

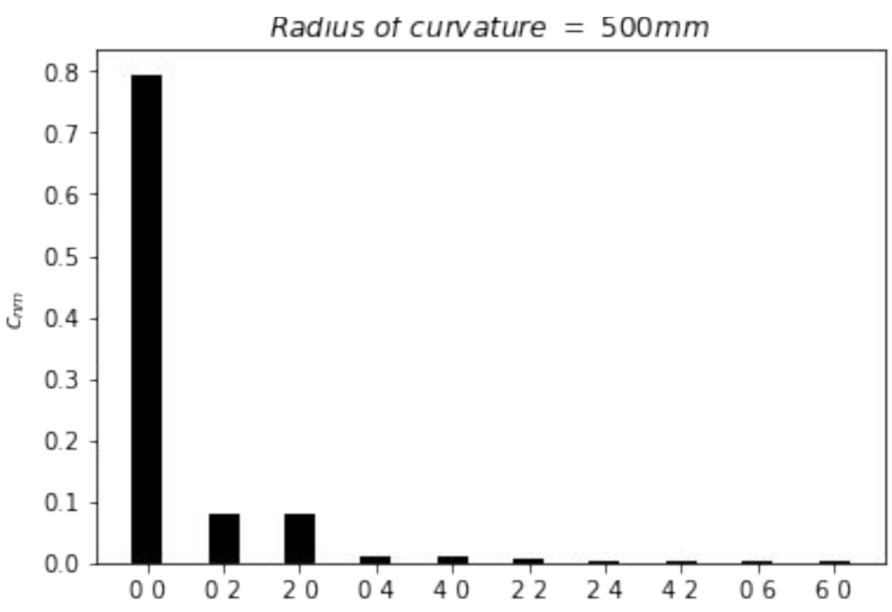
A coefficient
between 0 and 1

$$|c_{0,0}|^2,\ |c_{1,0}|^2,\ |c_{2,0}|^2\ ,\ .\ .\ .\ |c_{n,m}|^2 \leq 1$$


Pure Gaussian Beam Intensity Distribution
E_0
1.0
0.8
0.6
0.4
0.2
0.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x (mm)

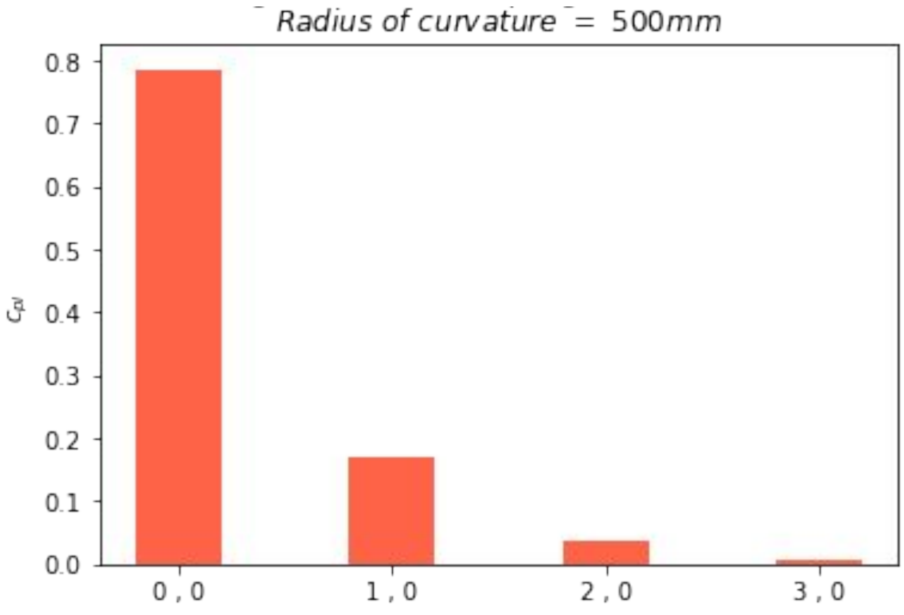

HOM Intensity Distribution
E_0
1.0
0.8
0.6
0.4
0.2
0.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x (mm)

Hermite Gauss Coupling Coefficients



n, m

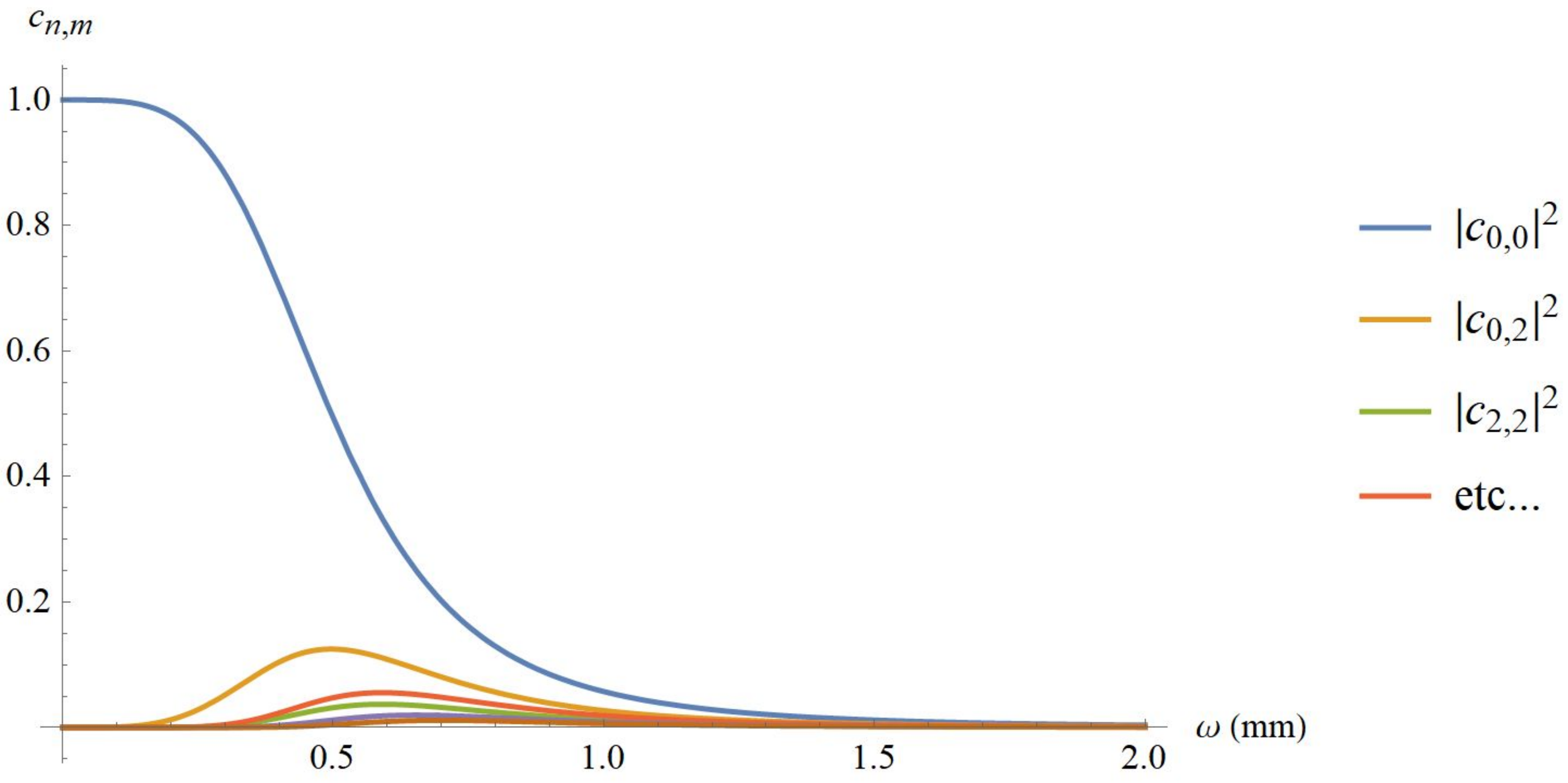
Laguerre Gauss Coupling Coefficients



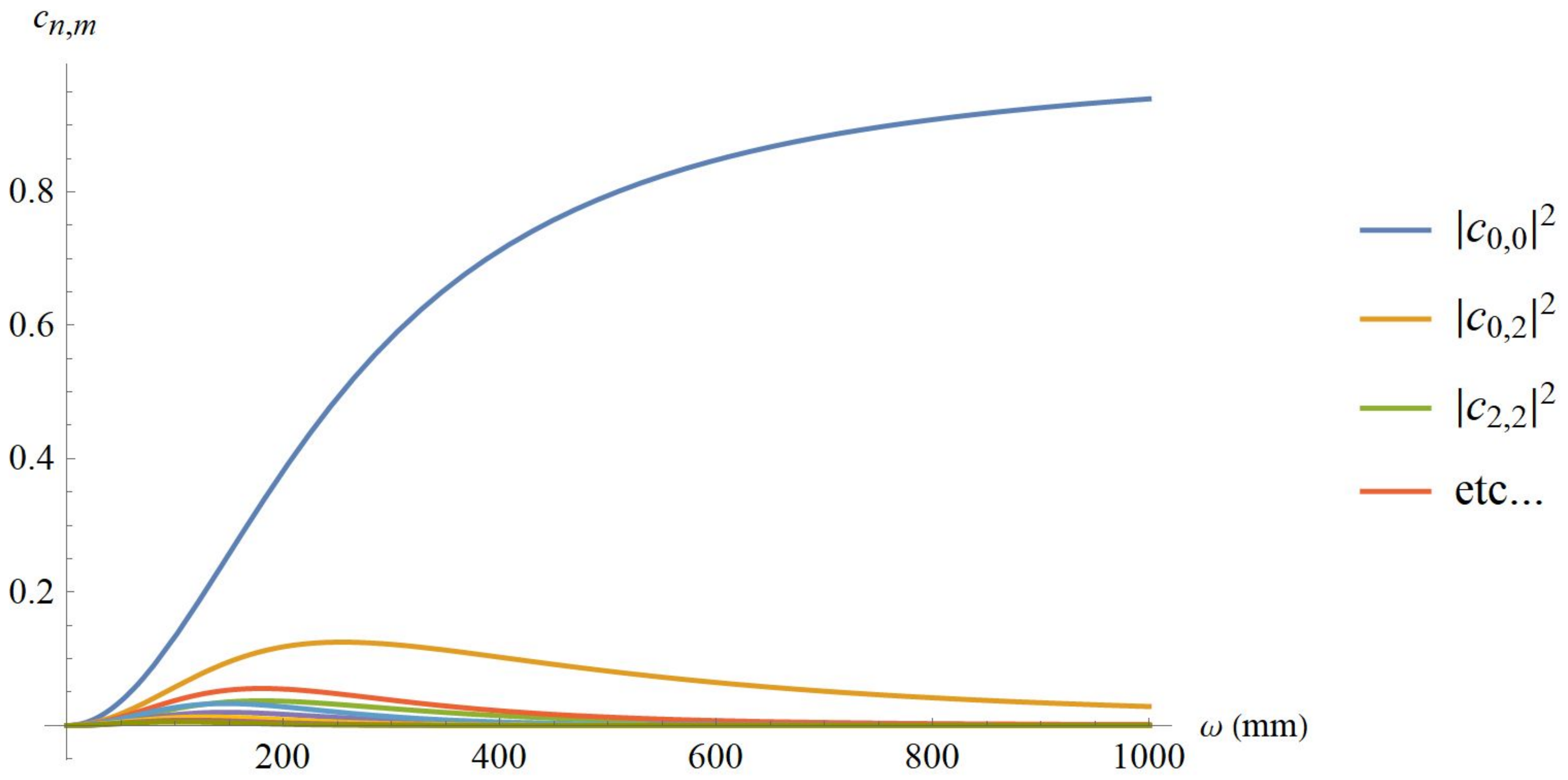
p, l

Coupling Coefficients vs. Beam Width

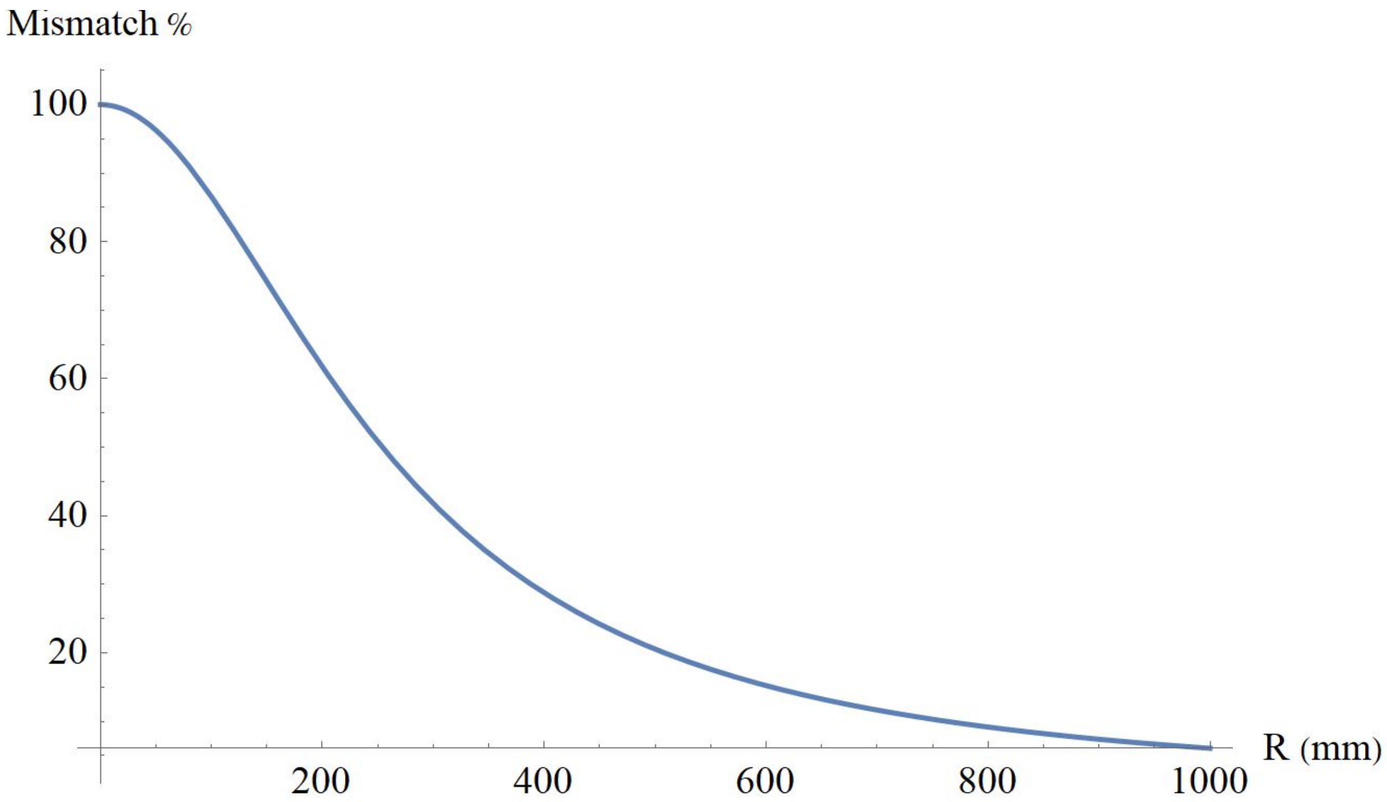
Fixed Radius of curvature R at 500mm



Coupling Coefficients vs. Z(x,y) Surface Radius of Curvature



Mode Mismatch vs. Z(x,y) Surface Radius of Curvature



The Work Continues

- Further research on how mode mismatching affects quantum squeezing loss.
- Test different Z(x,y) functions to see how different surfaces affect beams.
- Simulate some radius R, find the q parameter and compare with ABCD matrix approach.
- Simulate a two lens system with integration method for mode matching losses.
- Find best parameters for mode matching with numerical integration.
- If work yields good results, try modeling for LIGO optics

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