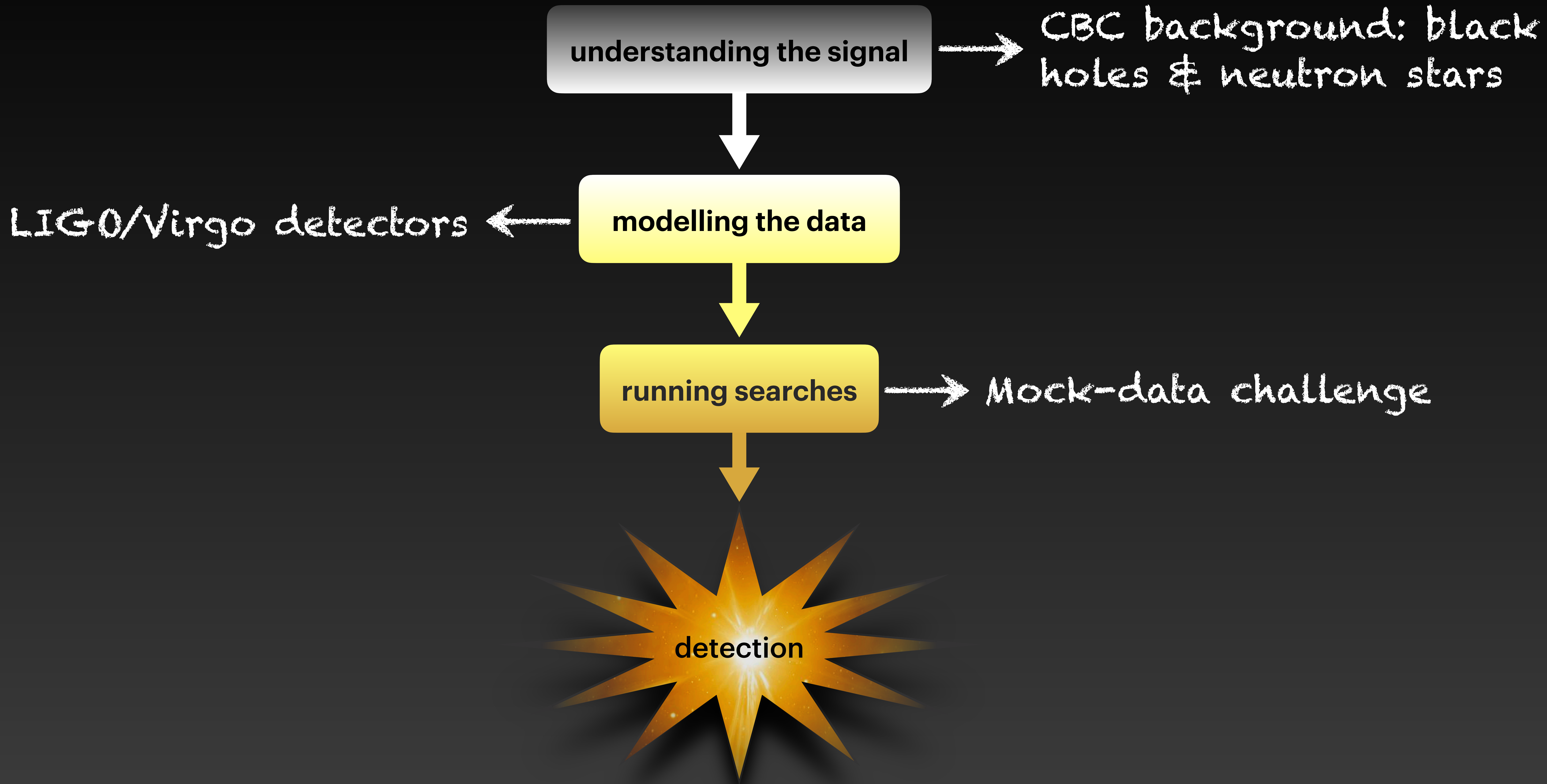
Detecting a stochastic gravitational-wave background in LVK

Are we there yet?

Arianna Renzini

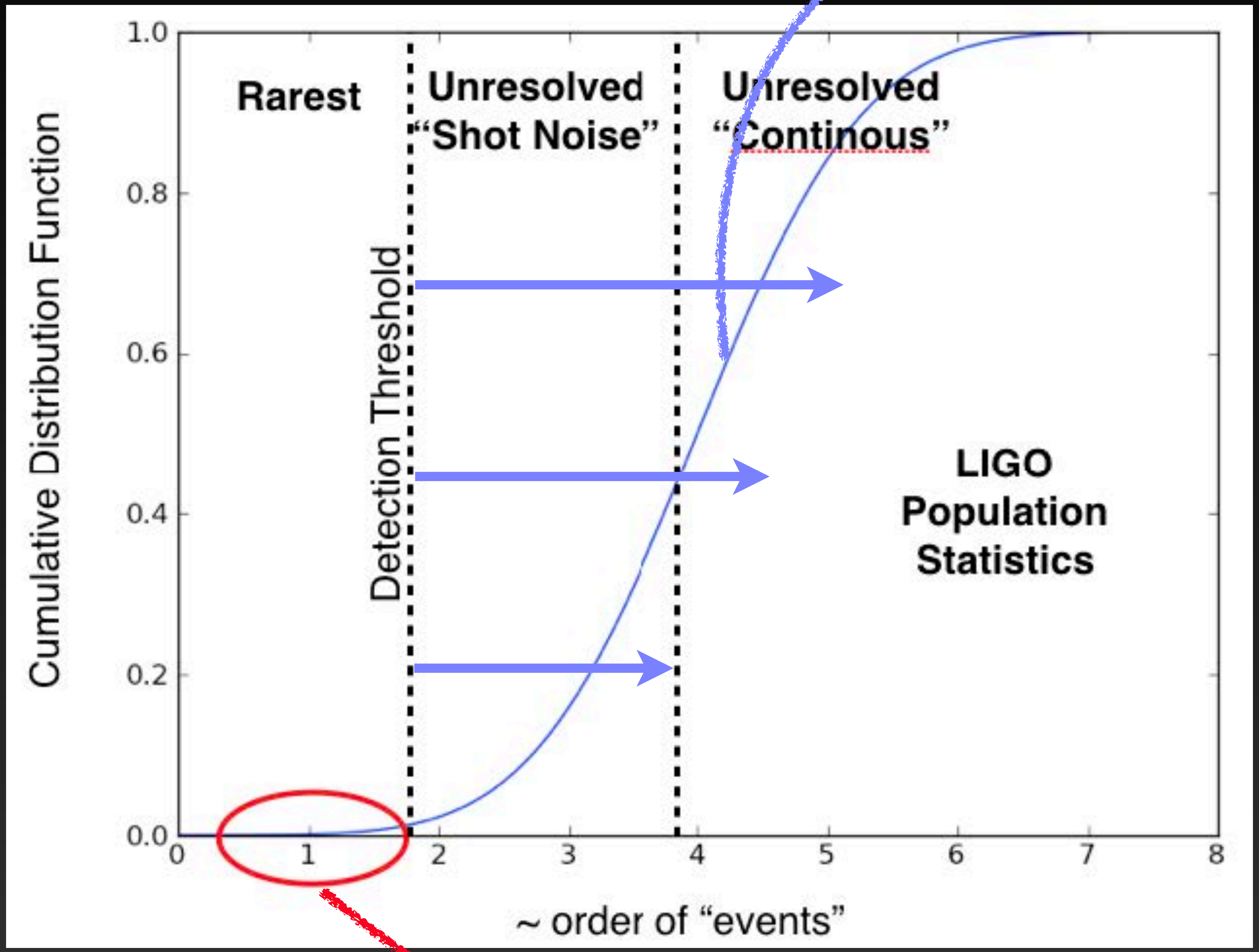
APS April meeting 2022 — DGRAV session X16

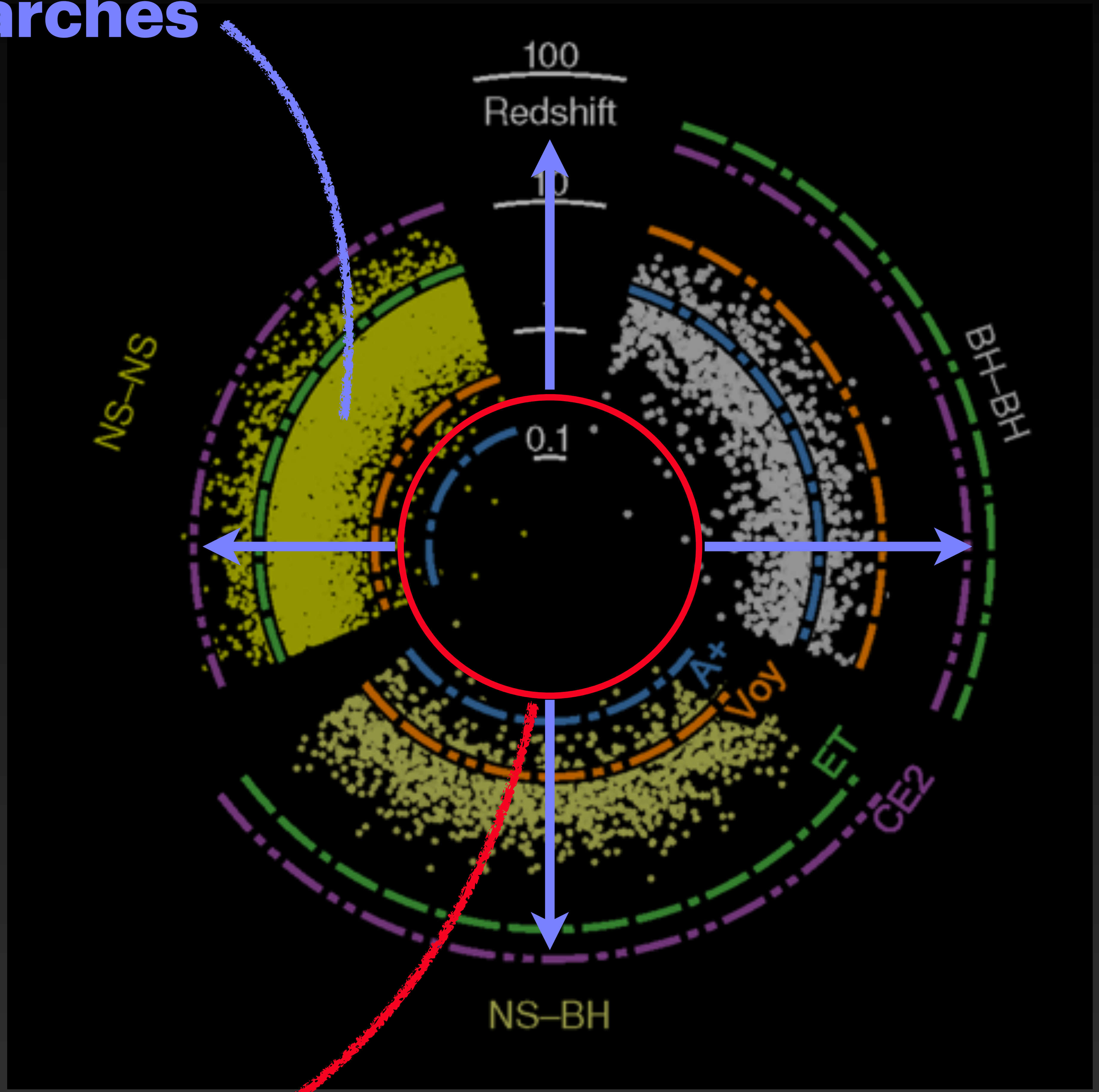
Detection roadmap



The CBC stochastic GW background in L/V

target for stochastic searches





Contaldi + AIR, ICL

target for match-filtered searches

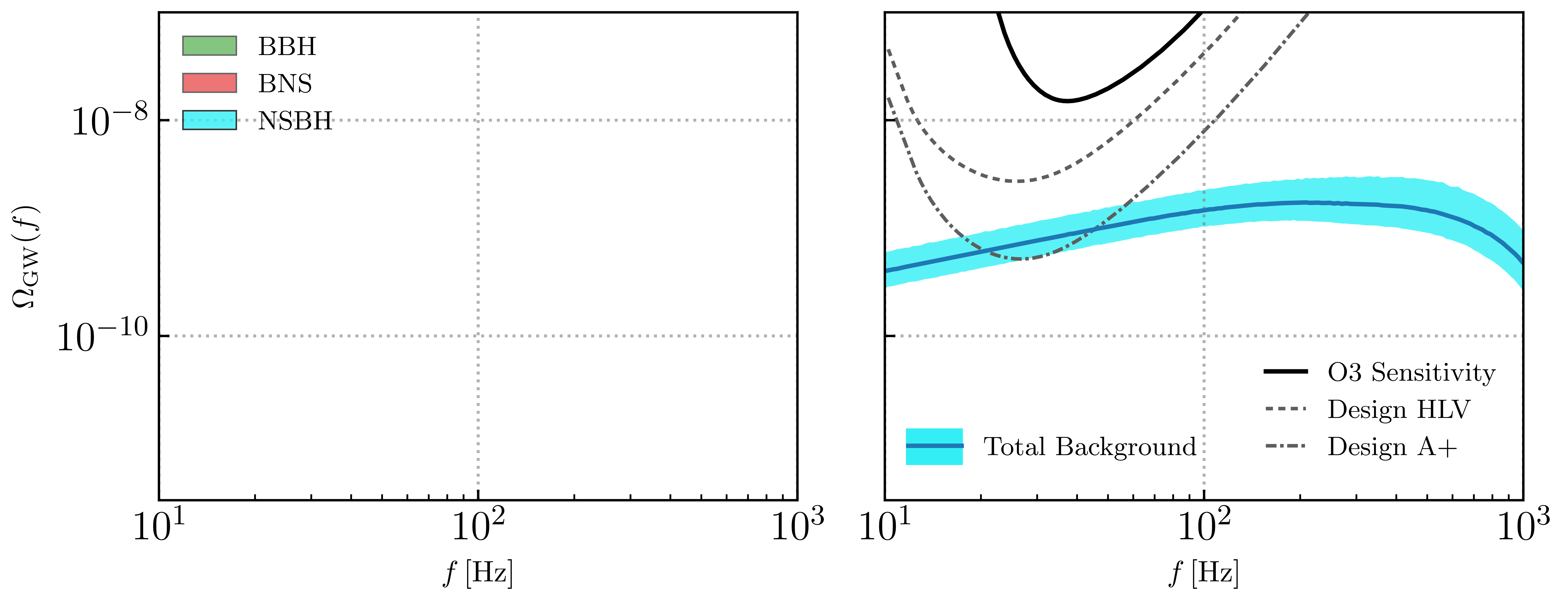
Hall + Vitale, MIT

incoherent superposition ⟶ unresolved ⟶ stochastic variables

Observing the background: the spectrum

fractional GW energy density

$$\Omega_{GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln f} = \frac{32\pi^3}{3H_0^2} f^3 I(f)$$



CBC SGWB from [LVK GWTC-3 populations paper]

How long until detection?→ *depending on sensitivity/methods*, before 3G!

The cross-correlation statistic

GW detectors collect timestream data which we assume:

$$d(t) = s(t) + n(t)$$

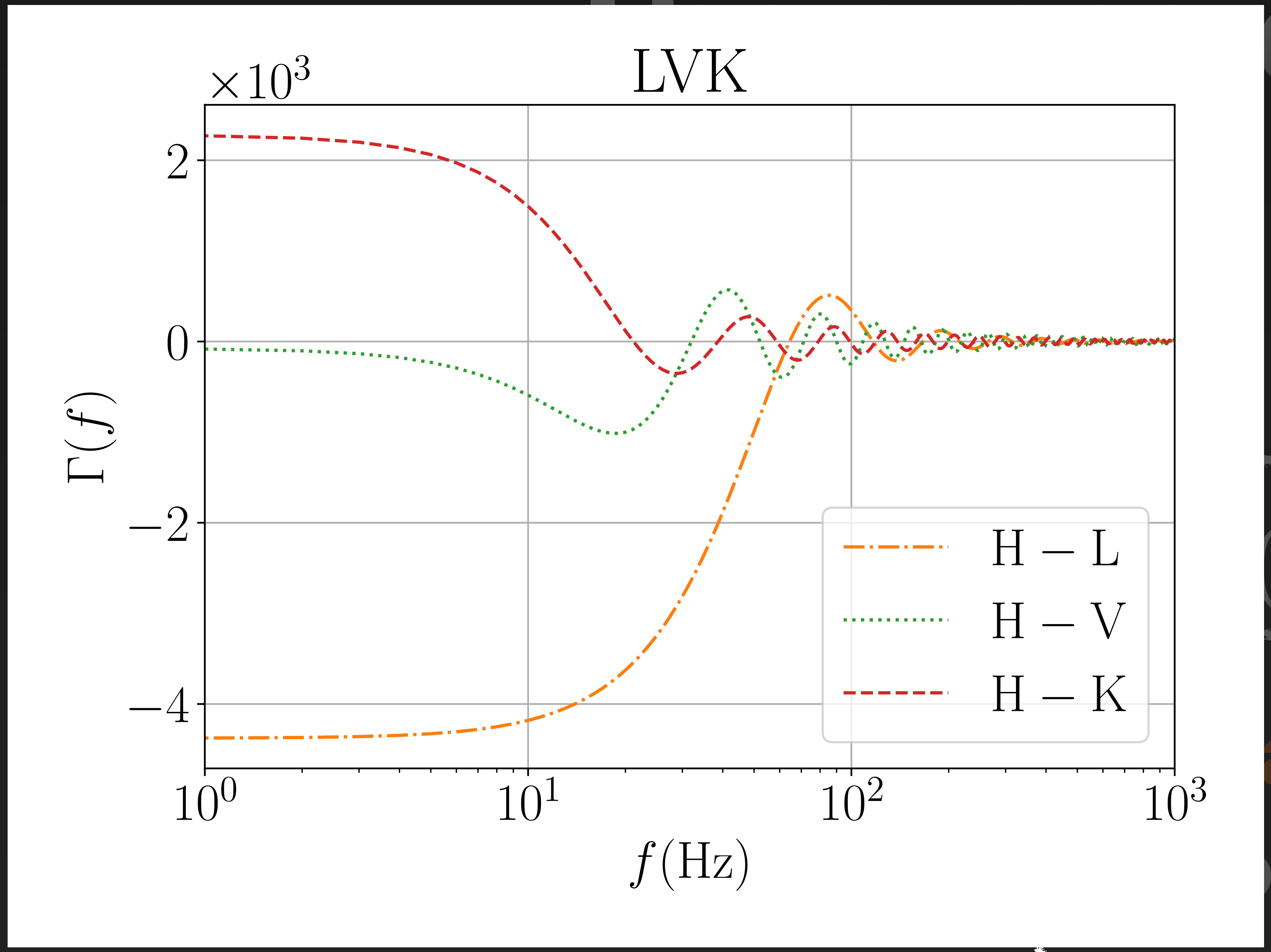
Assuming noise is uncorrelated between detectors, search for GWB with cross correlation: $C_{12}(f) = \tilde{d}_1(f)\, \tilde{d}_2^\star(f)$

$$\langle C_{12}(f) \rangle = R_1(f)\, R_2^\star(f)\, \langle \tilde{h}_1(f)\, \tilde{h}_2^\star(f) \rangle = T_{\rm obs}\, \Gamma_{12}(f)\, I_{\rm GW}(f)$$

detector responses

overlap reduction function

Th… correla…

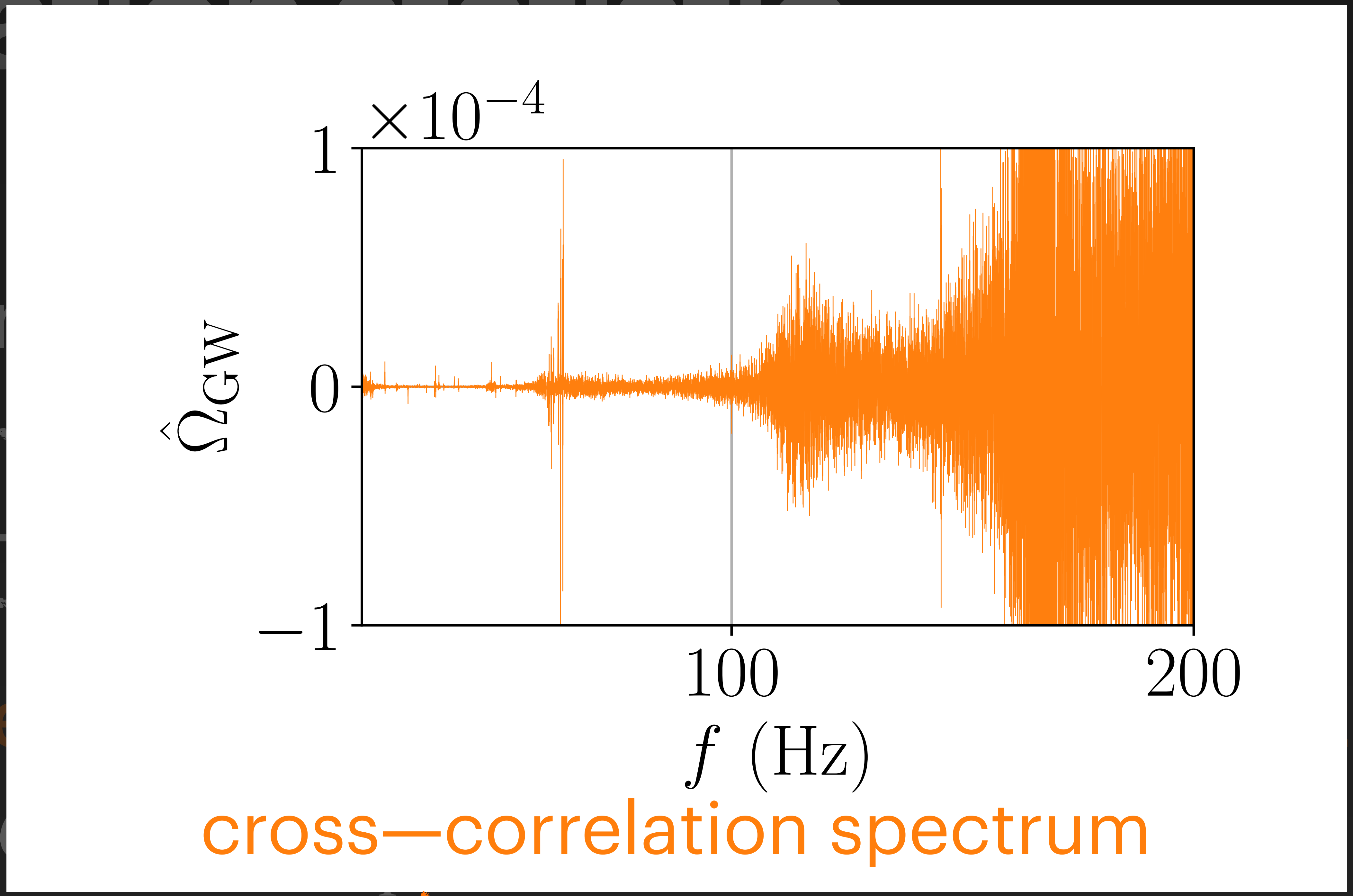


… timestrea…

…(t) = s(t) …

…ated betw… …h

…rrelation: …



$$\langle C_{12}(f)\rangle = R_1(f)\,R_2^\star(f)\,\langle \tilde{h}_1(f)\,\tilde{h}_2^\star(f)\rangle = T_{\rm obs}\,\Gamma_{12}(f)\,I_{\rm GW}(f)$$

$$\langle C_{12}(f)\rangle \propto T_{\rm obs}\,\Gamma_{12}(f)\,f^{-3}\,\Omega_{\rm GW}(f)$$

Stochastic searches: Bayesian model selection

Gaussian assumption: $\langle d \rangle = 0$

$$\mathscr{L}(d\,|\,I(f)) \propto \prod_{f,\tau} \frac{1}{|C|^{1/2}} e^{-\frac{1}{2} d^\dagger C^{-1} d}$$

low-signal limit

[Matas Romano '21]

TEST different MODELS

$$\log \mathscr{L}(\hat{\Omega}_{\rm GW}(f)\,|\,\Theta) \propto \frac{1}{2} \sum_{f,\tau} \left(\frac{\hat{\Omega}_{\rm GW}(f) - \Omega_{\rm M}(f\,|\,\Theta)}{\sigma^2_\Omega(f)} \right)^2$$

Stochastic searches: spectral weighting

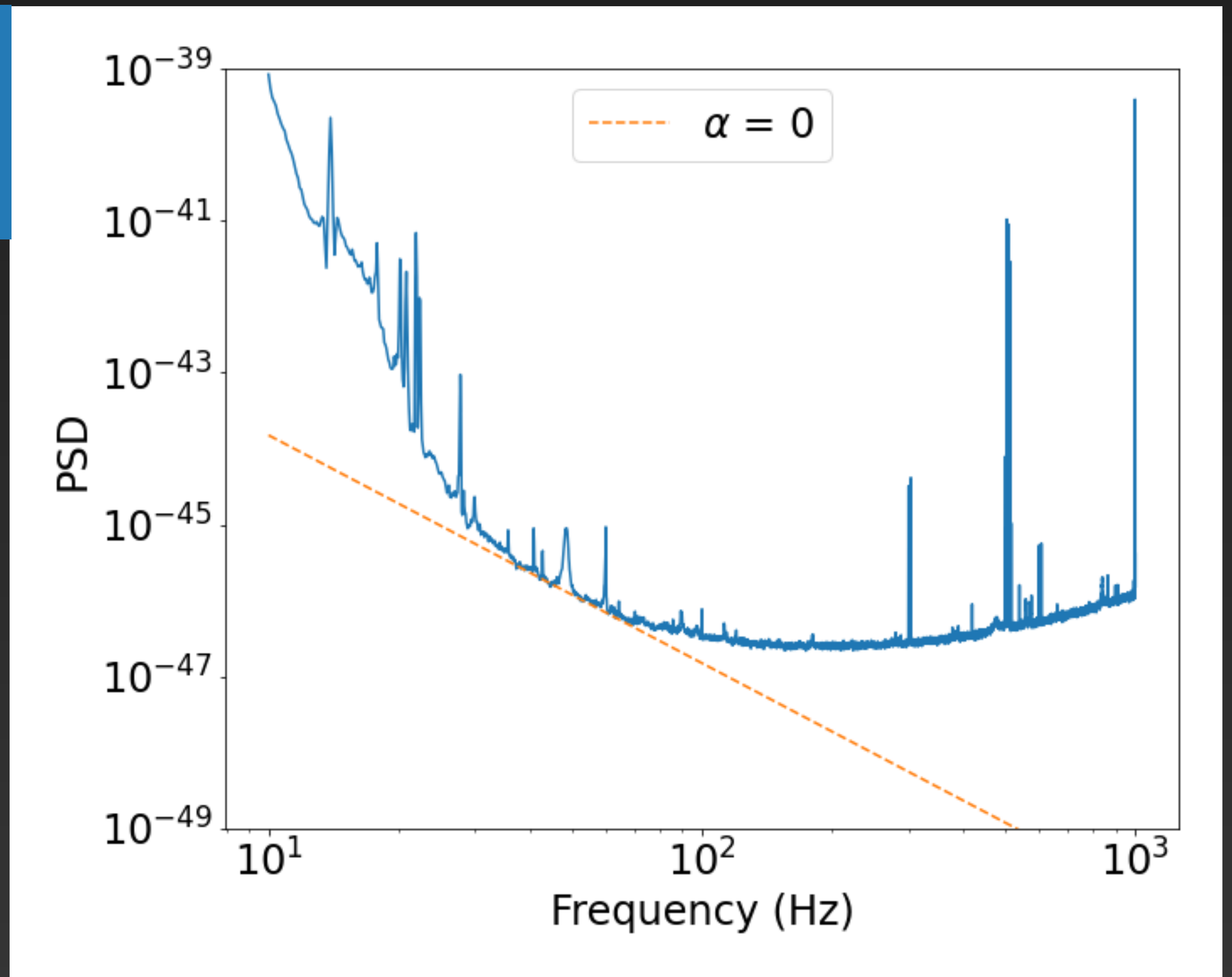
- Spectral shape usually assumed a power law:

$$\Omega_{GW} = \Omega_{GW}(f_{ref})\left(\frac{f}{f_{ref}}\right)^{\alpha}$$

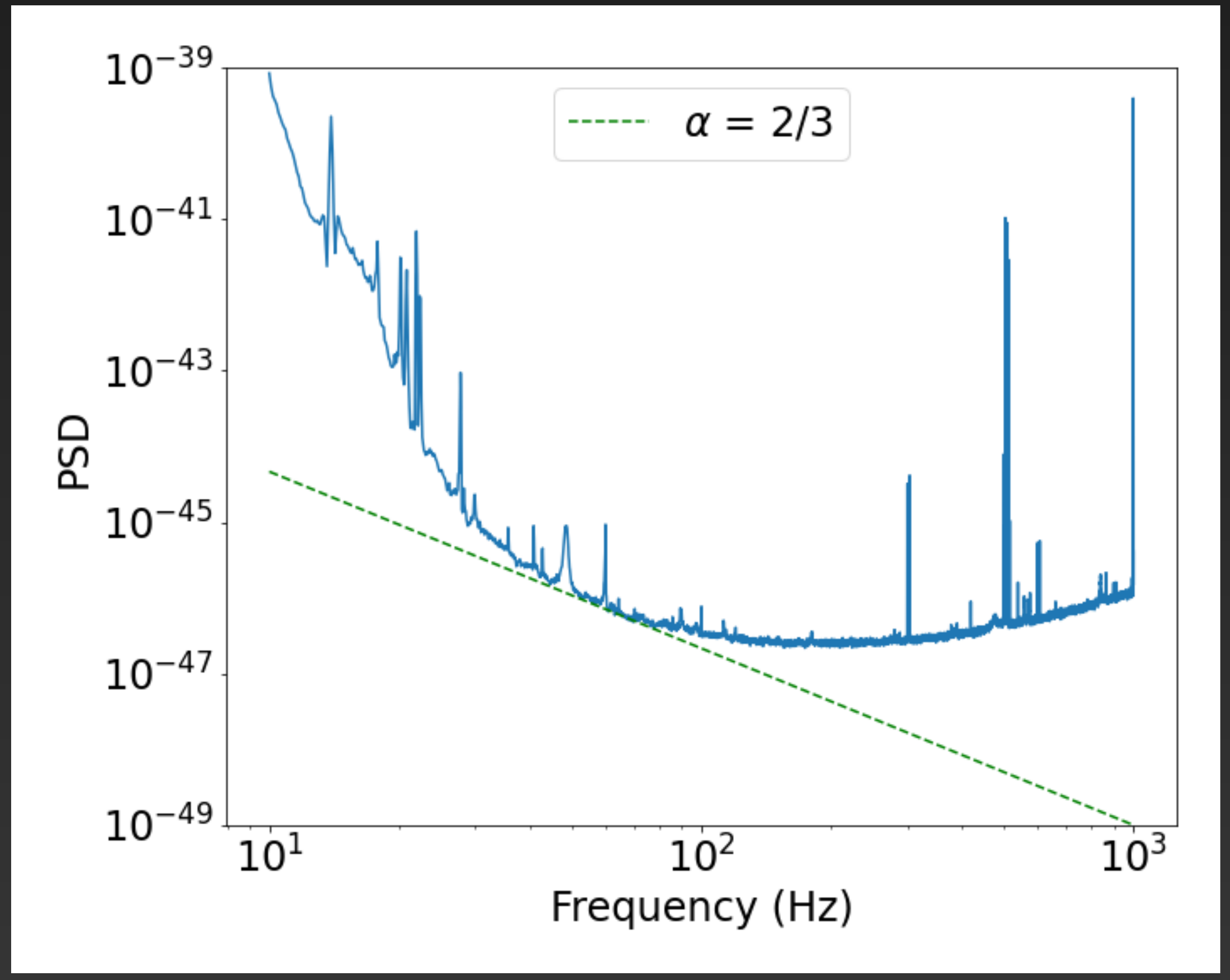
- Either fix α or parameter in **Bayesian fit**
- binned narrowband frequency fitting [(AIR, Contaldi '19)]

LIGO NOISE PSD

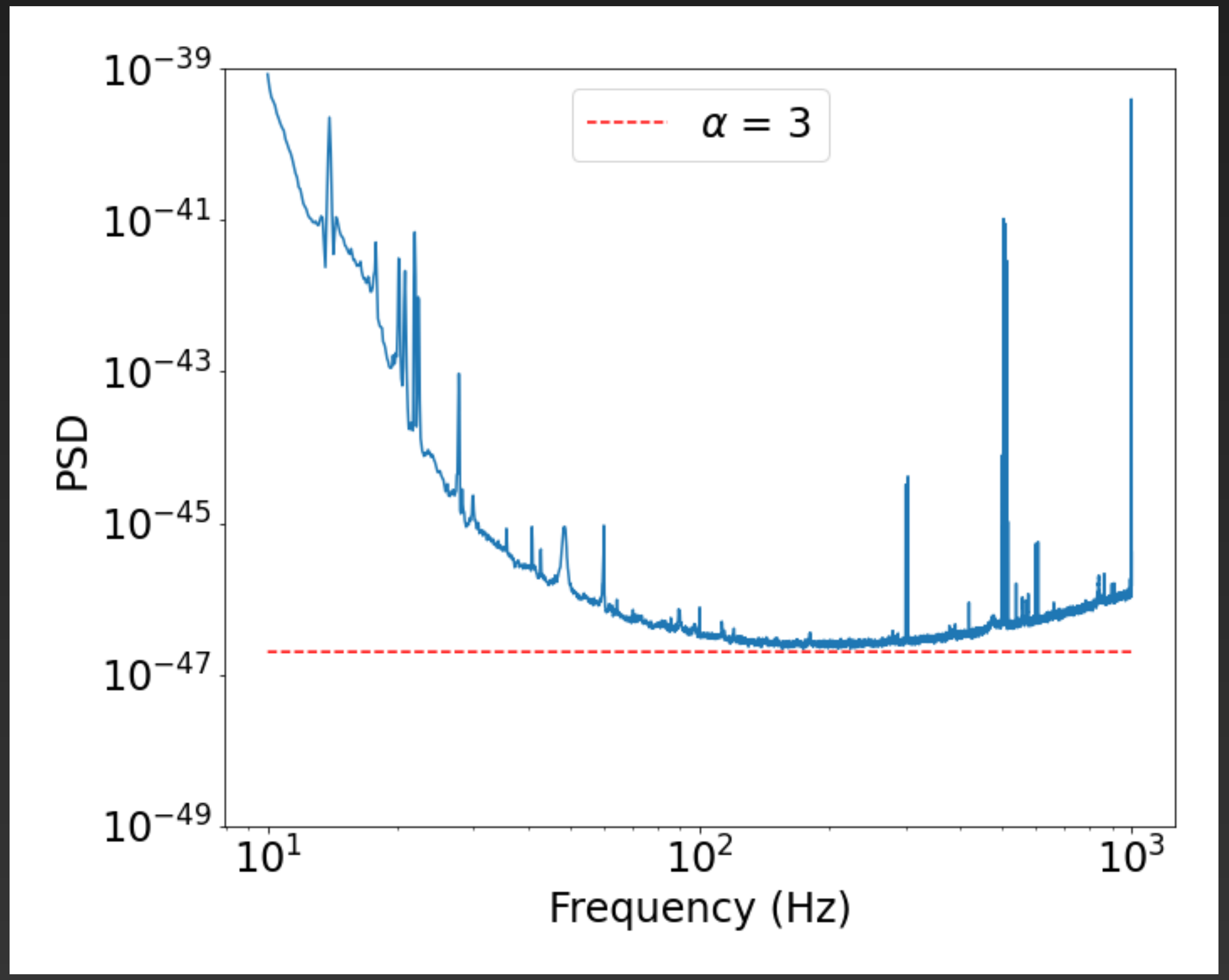
"cosmological"



"inspiral/astro"



"best fit"

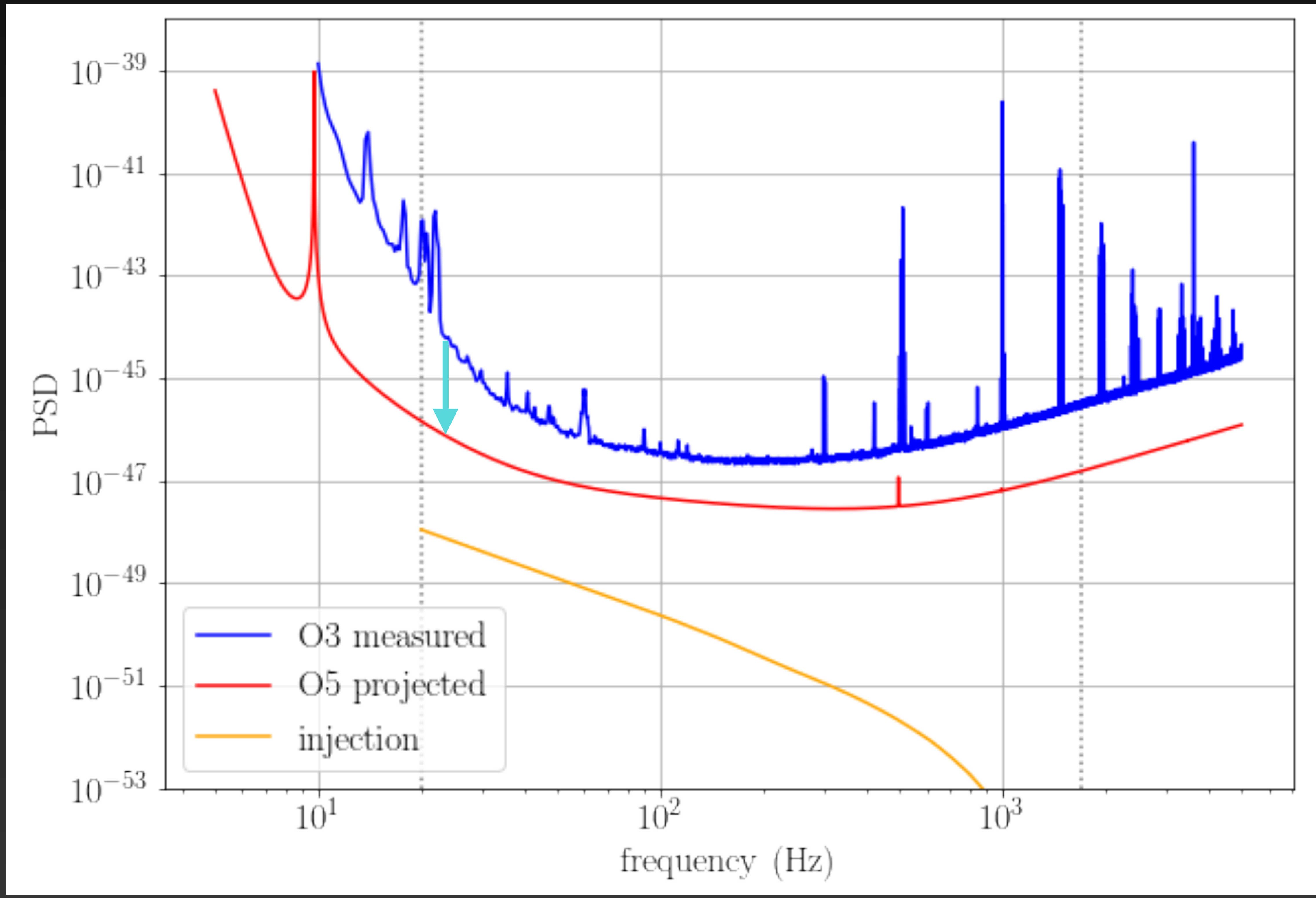


Ongoing mock-data challenge (MDC)

Testing the search on a *realistic* dataset

6 month dataset for 3 detector setup (H - L- V)

- **content: gaussian LIGO/Virgo noise + CBC injections**
  - ligo noise curve: Design O5 Ad-Ligo +
  - CBCs: 87% BNS, 13% BBH; 1 event every ~60 seconds (Christensen '19)
  - mass distribution: O3 power law plus peak for BBHs
- **signal slightly amplified to be detectable within 6 months (SNR=5) ⟶ corresponds to a SNR=5 detection in 4/5 years of data**

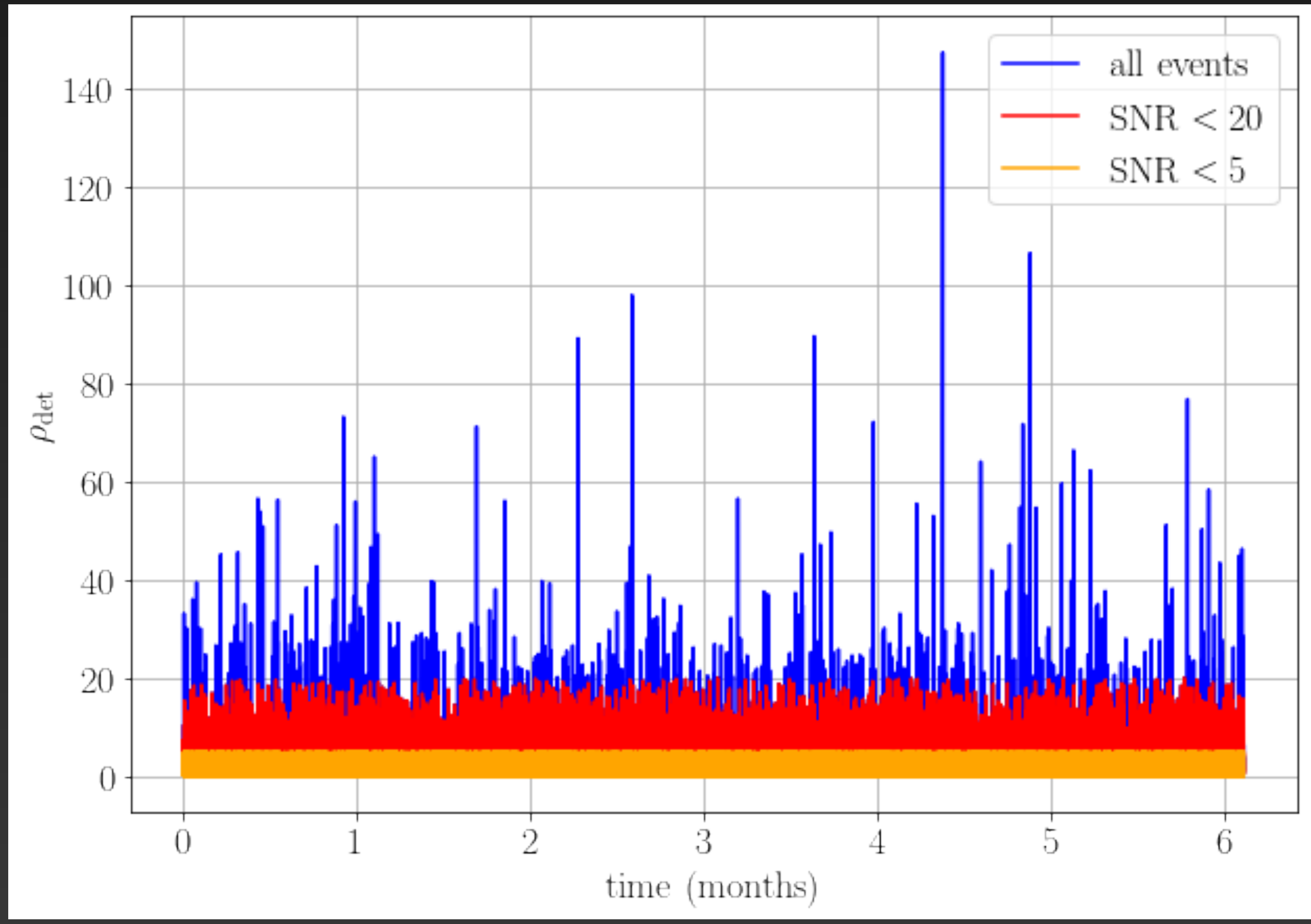


The LIGO/Virgo detectors: the “optimal” SNR

match-filtered SNR

$$\rho^2_{\text{det}} = 4 \int df \, \frac{\left| \sum_A F_A h_A(f) \right|^2}{P_n(f)}$$

noise in 1 detector



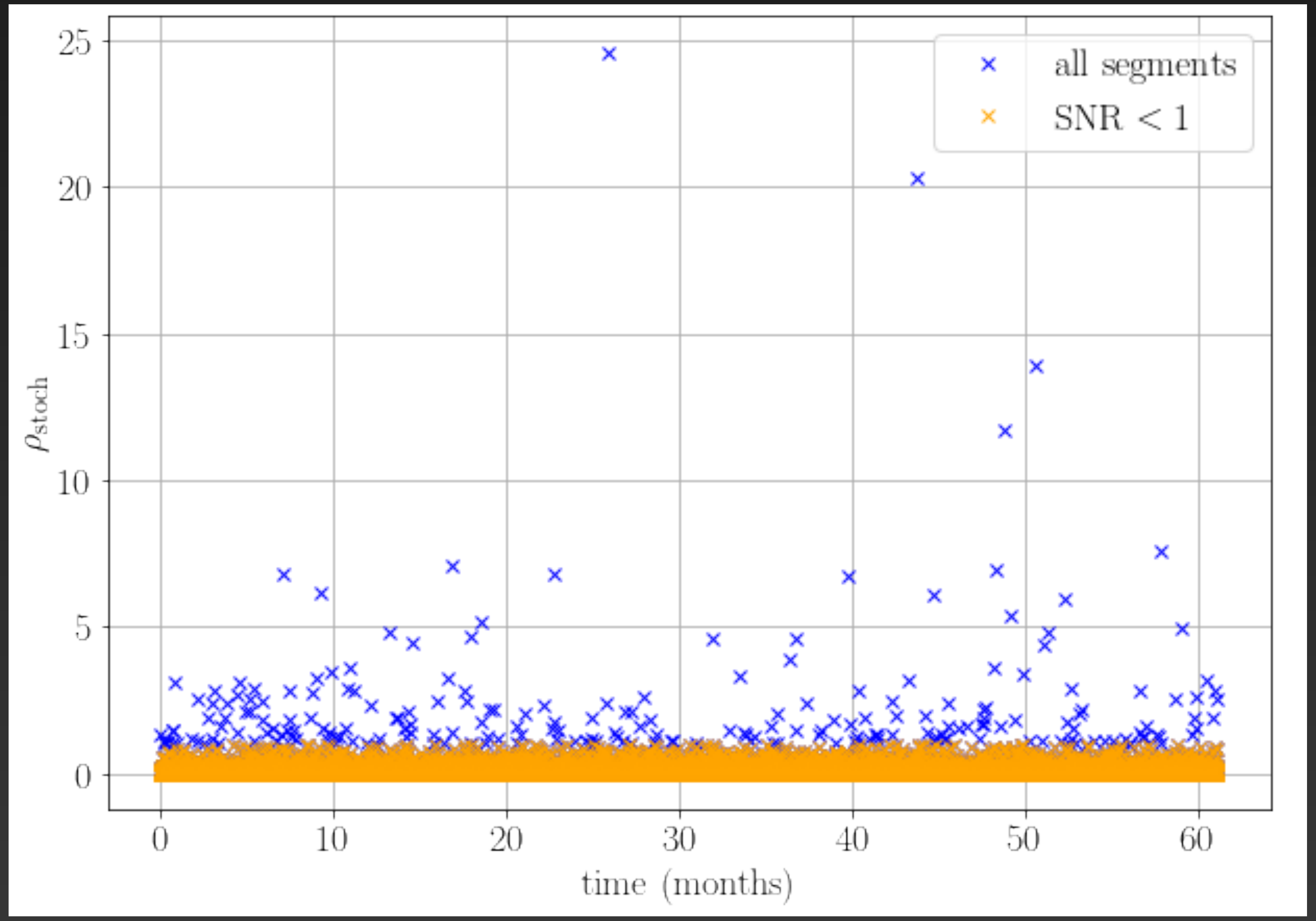
coherent

incoherent

stochastic SNR

$$\rho^2_{\text{stoch}} = T_{\text{obs}} \int df \, \frac{\Gamma^2(f) I^2_{\text{GW}}(f)}{P_{n1}(f) P_{n2}(f)}$$

noise in detectors 1 and 2



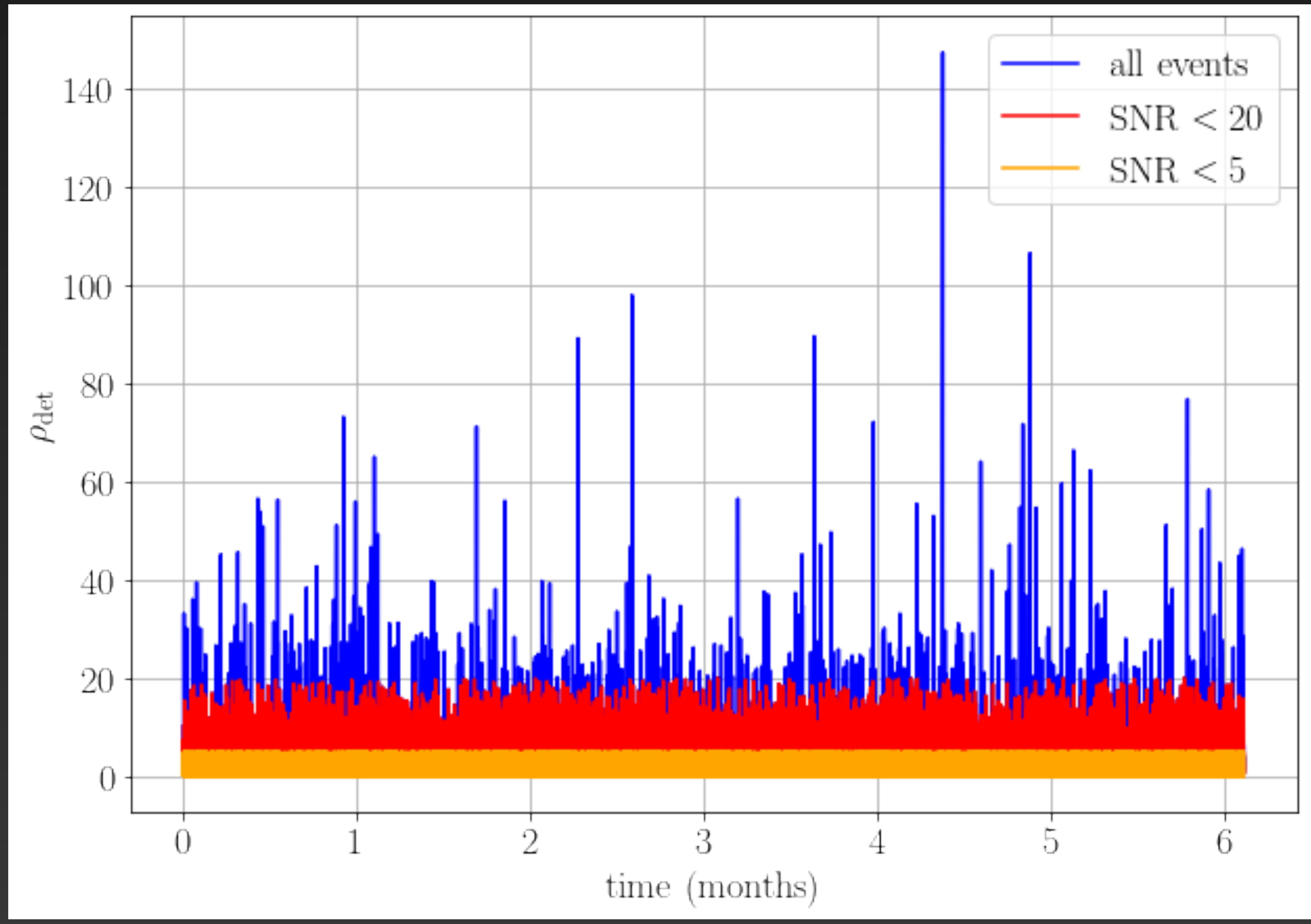
The LIGO/Virgo detectors: the "optimal" SNR

match-filtered SNR

$$\rho^2_{\rm det} = 4 \int df \frac{\left| \sum_A F_A h_A(f) \right|^2}{P_n(f)}$$

coherent

noise in 1 detector

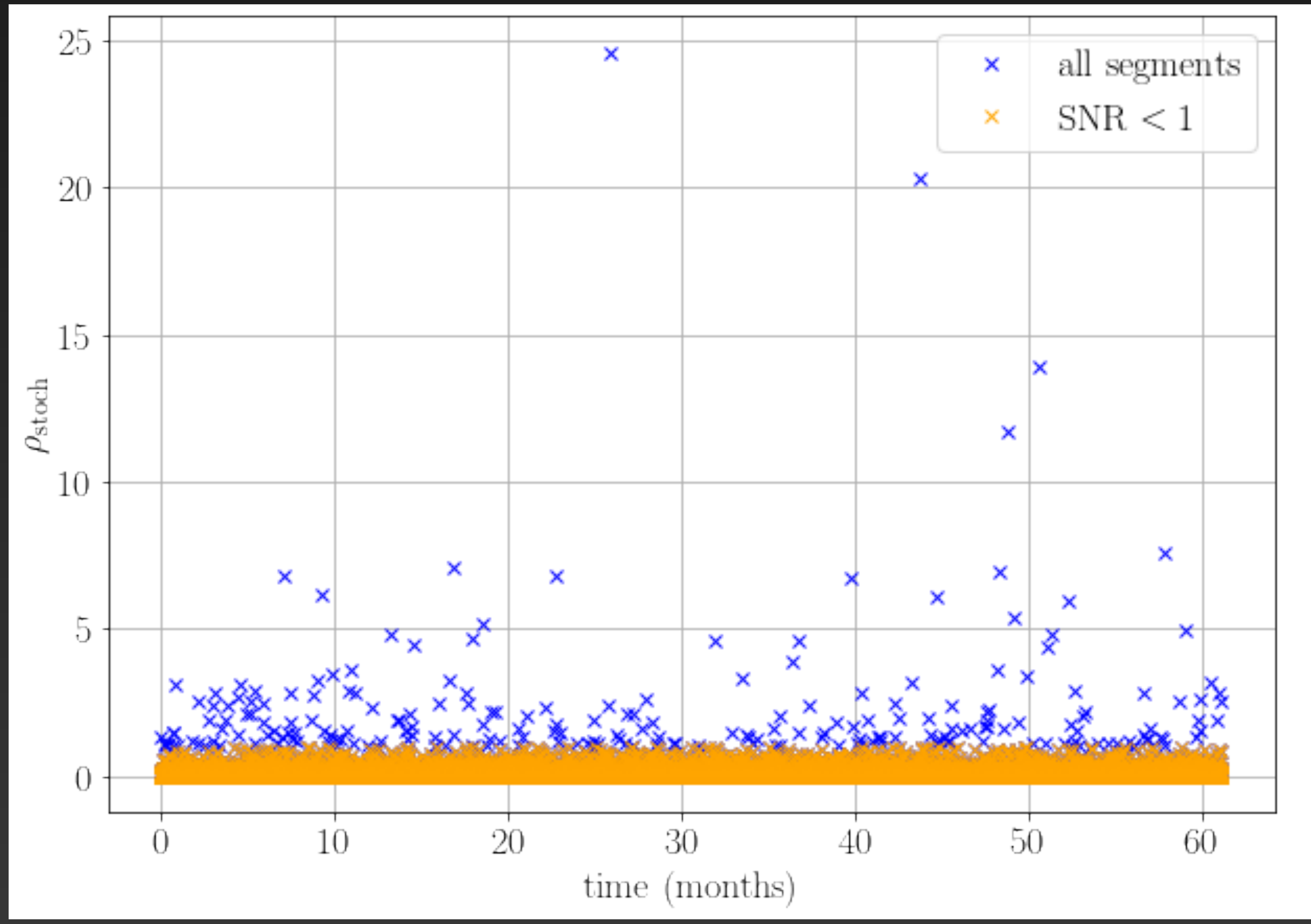


incoherent

stochastic SNR

$$\rho_{\rm stoch,\,tot} = \sqrt{N_{\rm seg}} \langle \rho_{\rm seg} \rangle$$

noise in detectors 1 and 2



MDC results: we can detect a signal!

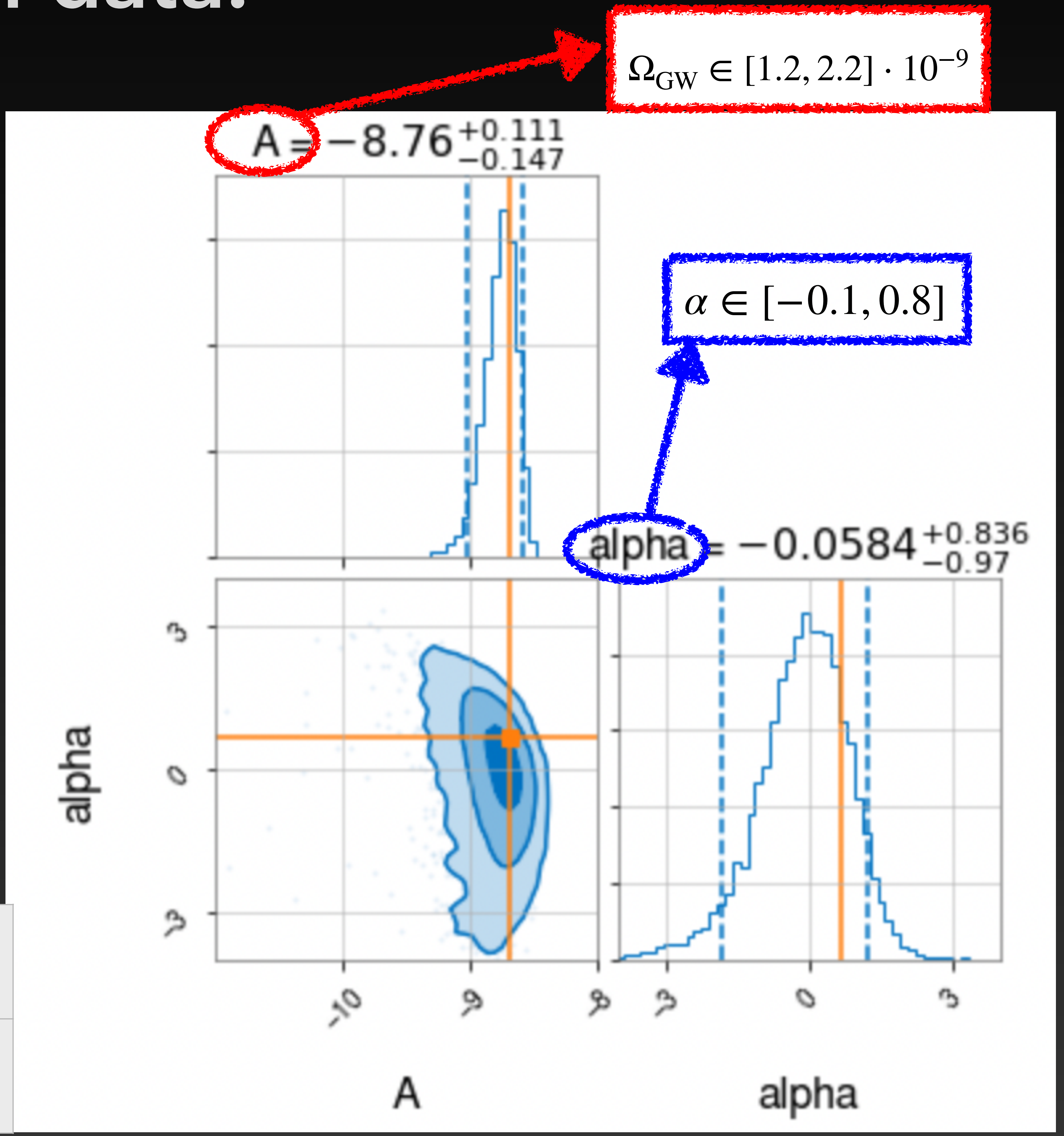
but we need a lot of data.

Cross-correlation statistic results:

Injected: $\Omega_{GW}(f_{ref} = 25\text{Hz}) = 2.05 \cdot 10^{-9}$
Recovered: $\Omega_{GW}(f_{ref} = 25\text{Hz}) = (1.7 \pm 0.4) \cdot 10^{-9}$

Bayesian Modelling results:

logBayes factor of signal vs noise: 6.67 +/- 0.05



$\log_{10}\Omega_{GW}$	Uniform: [-9, -6]
α	Uniform: [-4, 4]

New library to do this sort of analysis: **pygwb** — soon on conda!


... Thanks!

Extra Slides

The stochastic GW background

incoherent superposition/ stochastic generation ⟶ **unresolved** ⟶ **stochastic variables**

1 gravitational wave: $h_{ab}(t) = \sum_A h_A(t)\, \epsilon^A_{ab}$ → target for match-filtered searches

superposition of plane waves: $h_{ab}(t, x) = \int_f df \int_{S^2} d\hat{n}\, \tilde{h}_A(f, \hat{n})\, \epsilon^A_{ab}(\hat{n})\, e^{i2\pi f \hat{n}\cdot x}$

total GW intensity: $I(f) = \frac{1}{2} \sum_{\text{waves}} \sum_A |\tilde{h}_A(f)|^2$ → target for stochastic searches

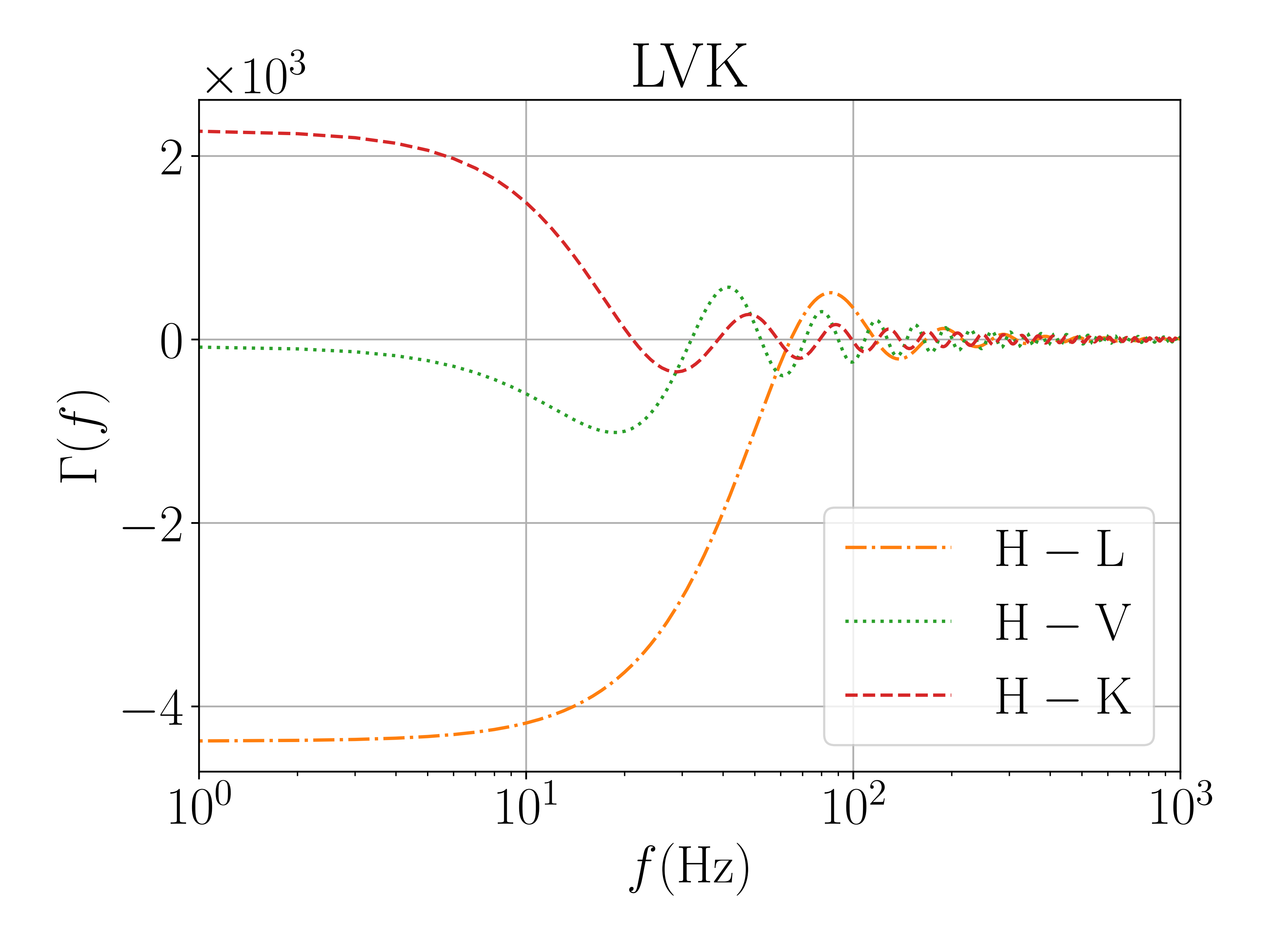
fractional GW energy density

$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d\ln f} = \frac{32\pi^3}{3H_0^2} f^3\, I(f)$$

from Allen & Ottewill '97

Overlap reduction function

$$d_A(f)\, d_B^\star(f) = \int_{S^2} d\hat{n}\, I(f,\hat{n})\, \gamma^I_{AB}(f,\hat{n})\, e^{i2\pi f \hat{n}\cdot x_{AB}} = I(f)\,\Gamma(f)$$



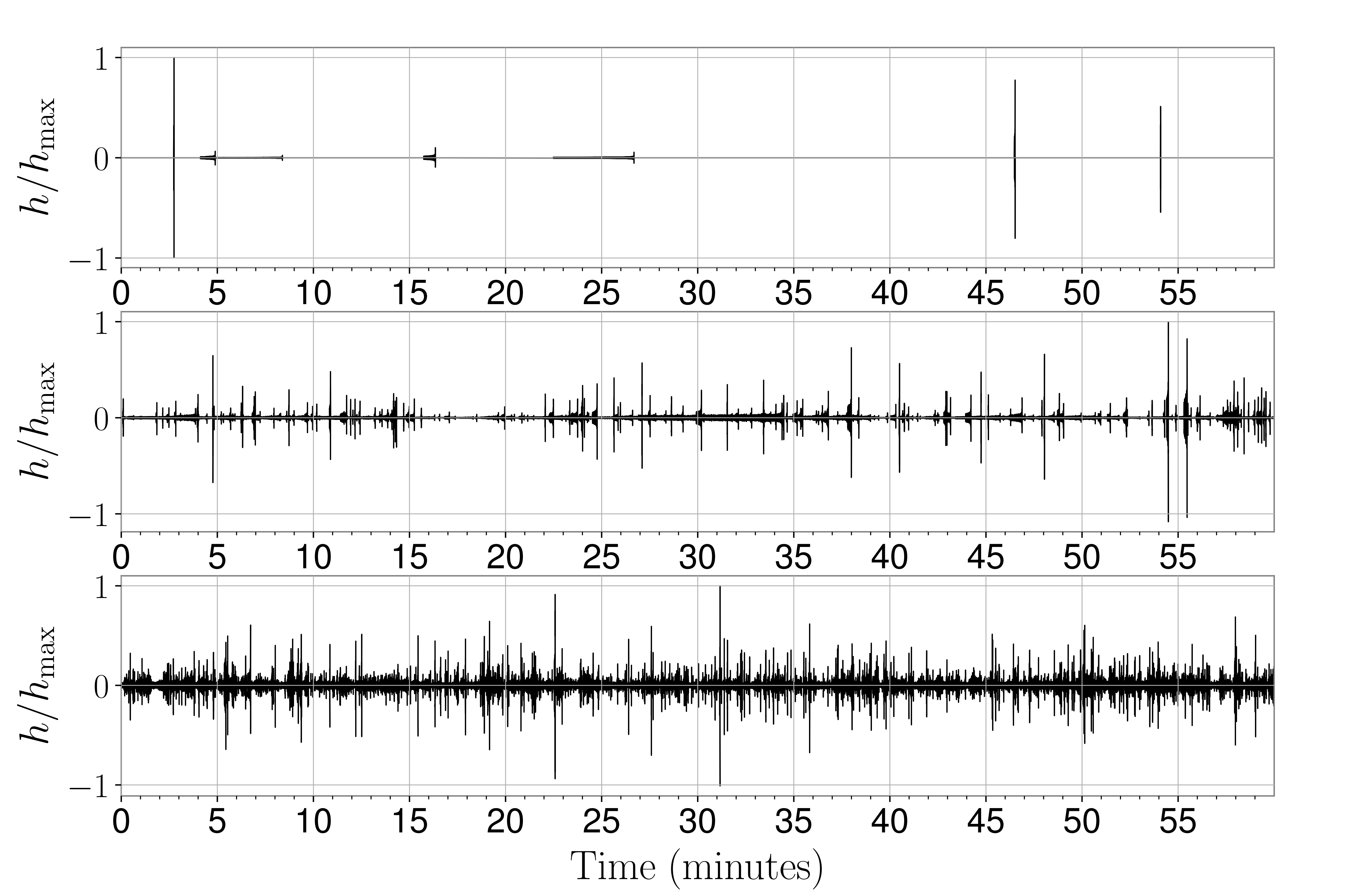
"small antenna":

$$L \ll c/f_{\rm GW}$$

arm transfer function is constant; modulations given by baseline length

Observing the background: the time domain

Gaussianity/non-Gaussianity of continuous/intermittent GWBs



We can use different methods to search for Gaussian/non backgrounds!

Stochastic searches: non-Gaussian signal

intermittent background of CBCs; "deterministic" CBC likelihood:

$$\mathscr{L}(d_i) \propto \prod_{f,\tau} \frac{1}{|C|^{1/2}} e^{\frac{1}{2}(d_i - h_i)\, C^{-1}\, (d_i - h_i)^\star}$$

gaussian noise + intermittent signal = Gaussian mixture model:

$$\mathscr{L}_{\rm full}(d_i) = \xi\, \mathscr{L}_s(d_i\,|\,h_i) + (1 - \xi)\, \mathscr{L}_n(d_i\,|\,0)$$

duty cycle: probability of there being a CBC signal in the data at any given time

Stochastic searches: non-Gaussian signal

intermittent background of CBCs; “deterministic” CBC likelihood:

$$\mathscr{L}(d_i) \propto \prod_{f,\tau} \frac{1}{|C|^{1/2}} e^{\frac{1}{2}(d_i - h_i)\, C^{-1}\, (d_i - h_i)^\star}$$

“deterministic” =
use CBC waveforms:
“The Bayesian Search”
Smith & Thrane ‘18

“deterministic”
GWB strain model?

~~“deterministic”~~, *stochastic* burst
search for correlated bursts of
GW energy, Drasco & Flanagan, '03, Lawrence+(AIR) in prep.

MDC results: we can detect a signal!

(O2 power law model for BBH masses)

Cross-correlation statistic results:

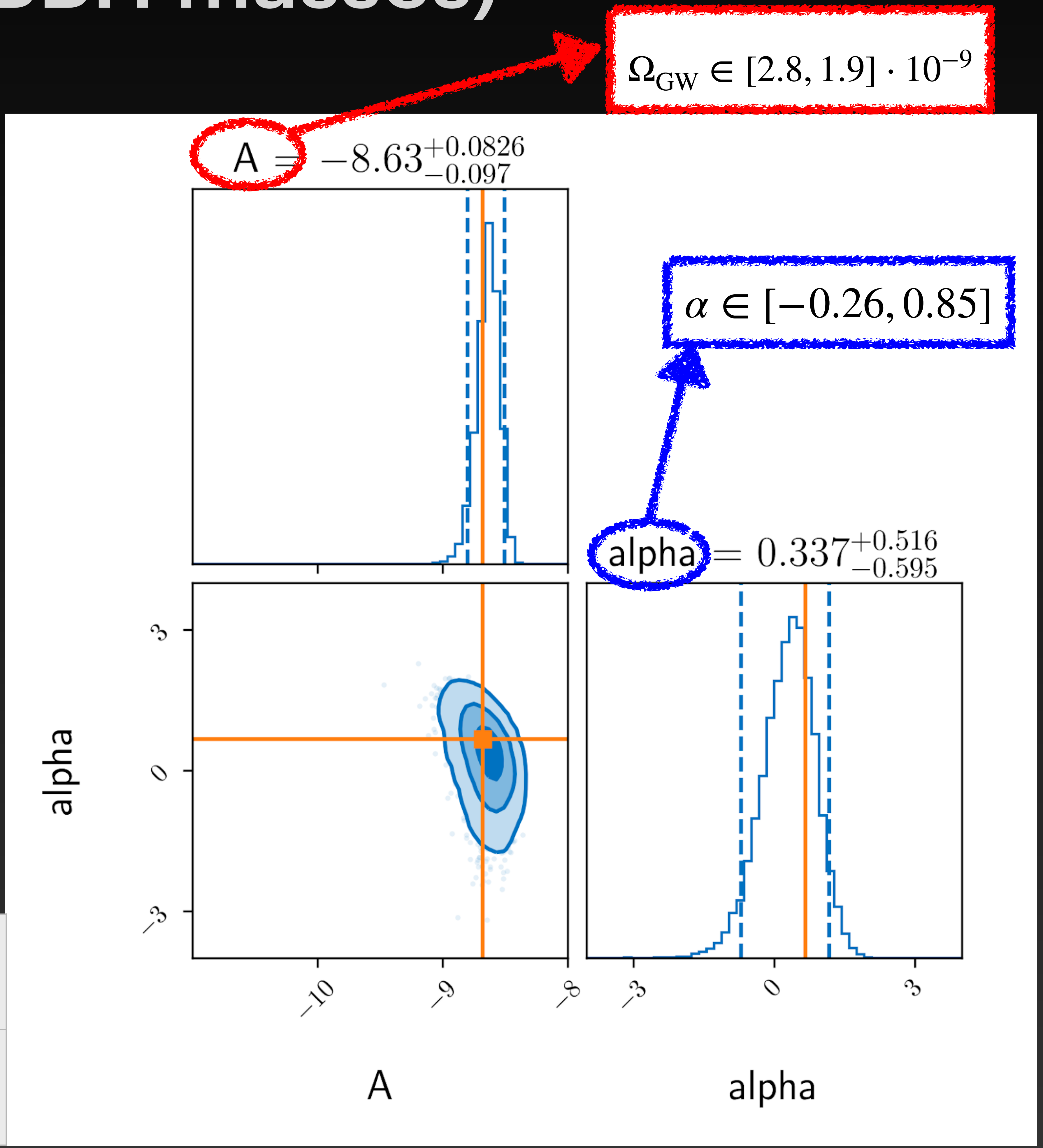
Injected: $\Omega_{GW}(f_{ref} = 25\text{Hz}) = 2.05 \cdot 10^{-9}$

Recovered: $\Omega_{GW}(f_{ref} = 25\text{Hz}) = (2.4 \pm 0.4) \cdot 10^{-9}$

Bayesian Modelling results:

Bayes factor of signal vs noise: 121309.52 +/- 0.05

$\log_{10} \Omega_{GW}$	Uniform: [-9, -6]
α	Uniform: [-4, 4]



New library to do this sort of analysis: `pygwb` — soon on conda!