Spring Scaling Calculations - T2100281

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1 Introduction

This is a simple document holding a few variations on the basic blade spring scalings shown in Calum's thesis and Mike Plessy's paper. The key difference is that I replace the spring width with a width to length ratio r. We see that the spring length is uniquely determined by the the ratio, the load, the spring stiffness and the material properties. This makes it easier to understand the various scalings we face when making ISI upgrades or brainstorming about suspension updates. The calculations here are all for ideal triangular blades, operated when they are flat.

2 Basic spring equations

For a triangular spring of length l, base width b, thickness h and elastic modulus E, the stiffness k is

$$k = \frac{E\, b\, h^3}{6\, l^3} \tag{1}$$

and the stress σ on the top and bottom planes of the spring when you apply a load P is

$$\sigma = \frac{6\, P\, l}{b\, h^2} \tag{2}$$

we define the ratio r to be the ratio

$$r \equiv {}^{b}\!/\!_{l}, \text{ so that } b = r\, l \tag{3}$$

and so the stiffness becomes

$$k = \frac{E\, r\, h^3}{6\, l^2} \tag{4}$$

and the stress is

$$\sigma = \frac{6\, P}{r\, h^2} \tag{5}$$

so we solve for the thickness

$$h^2 = \frac{6\, P}{r\, \sigma} \tag{6}$$

and replace that into the equation for the stiffness

$$k = \frac{E\, r\, \left(\frac{6\, P}{r\, \sigma}\right)^{3/2}}{6\, l^2} \tag{7}$$

We rewrite this as k^2 and simplify to

$$k^2 = \frac{6\,E^2\,P^3}{r\,\sigma^3\,l^4} \tag{8}$$

and so we see that the spring length is

$$l^4 = \frac{6\,E^2\,P^3}{r\,\sigma^3\,k^2} \tag{9}$$

. We note a few things. Wide springs (large r) are slightly shorter than narrow springs. The material properties scale like E^2/σ^3, high stress materials like maraging steel do well, but other materials such a Titanium Beta-C, also known as grade-19, might be interesting to consider, see section XXX. If we use multiple springs n to achieve the same total stiffness and load, and replace $P \to P_{total}/n$ and $k \to k_{total}/n$

$$l^4 = \frac{6\,E^2\,P_{total}^3}{r\,n\,\sigma^3\,k_{total}^2} \tag{10}$$

It's interesting to note that the total amount of metal used for all the springs is independent of how many springs are used. The total volume V of the springs is

$$\begin{aligned} V &\propto n\,l\,b\,h \\ &\propto n\,r\,l^2\,h \\ &\propto n\,r\,\frac{1}{\sqrt{n\,r}}\,\frac{1}{\sqrt{n\,r}} \\ &\propto \text{ neither } n \text{ nor } r \end{aligned} \tag{11}$$