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Bayesian factors analysis of LIGO detections

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ABSTRACT
LIGO (Laser Interferometer Gravitational Wave Observatory) detectors are capable of detecting gravitational waves created by collisions of massive stellar remnants moving at high accelerations (The IPAC Communications and Education team 2021). Collisions of massive stellar remnants create electromagnetic waves that can reveal many aspects of the remnants that collided. The EM community is attempting to use data from the LIGO instruments to infer the location of the sources and view the sources across all electromagnetic wavelengths as shown from the efforts from (LIGO Scientific Collaboration 2021). Gravitational wave detectors are poor at localizing mergers, making the discovery of counterparts a challenging task. We will compare the Bayes factor and the False Alarm Rate as metrics of classifying sources as astrophysical in low latency. We will improve the low latency data products that are provided to EM observers in order to aid in the discovery more counterparts in the future.

1 INTRODUCTION

1.1 Gravitational Waves

Gravitational waves (GWs) are disturbances in space-time caused by an accelerated mass, that propagate away from their source at the speed of light. Because GW are the result of a pair of massive objects in a decaying orbit around a shared center of mass, their observed signal strength increases as the objects approach one another.

The Laser Interferometer Gravitational Wave Observatory(LIGO) is an instrument used to observe propagations in space-time from colliding objects. The collisions that LIGO detects primarily results from three types of mergers, Binary Black Hole(BBH), Binary Neutron Stars(BNS), and Neutron Star - Black Hole (NS-BH).

1.2 LIGO Detectors

LIGO detectors are Michaelson Interferometers The IPAC Communications and Education team (2021),but the detectors are made of one laser, a power and signal recycling mirror, a photon collector, a beam splitter and many other components as shown in fig(1).

The beam splitter splits the beam of light from the laser into two beams of light orthogonal to each other, but the two beams remain parallel to the surface of Earth. The two light beams split from the beam splitter oscillate approximately 300 times individually between mirrors that are separated by 4km for each arm; this distance is equivalent to 1200km.

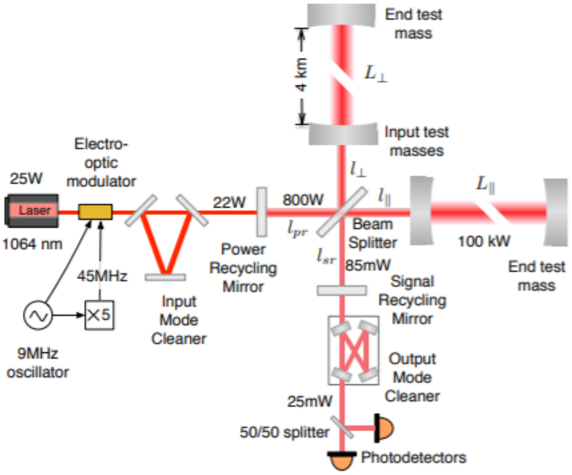

Figure 1. The design for Advanced LIGO detectors from (Martynov et al. 2016)

1.3 Detecting Sources

LIGO detectors aim to detect these disturbances using its powerful laser (750KWatt) and effective length (1200km)The IPAC Communications and Education team (2021). When both beams of light in the arms of LIGO reaches a length of 1200km the beams transmit through the mirrors and interfere destructively with one another. If GWs happen to pass through Earth the arms will no longer be the same length which causes a phase difference between the light beams, resulting in the beams not interfering destructively. The photon collector can be used to find the intensity of the GW that causes the strain on the detector.

1.4 Classifying Sources

When LIGO detects a GW, we infer properties of the merger based on the observed signal. After sources are obtained, the FAR(False Alarm Rate) is used to to determine if the source is astrophysical or not. If the analysis from FAR yields an astrophysical result, we then analyze the signal to determine what

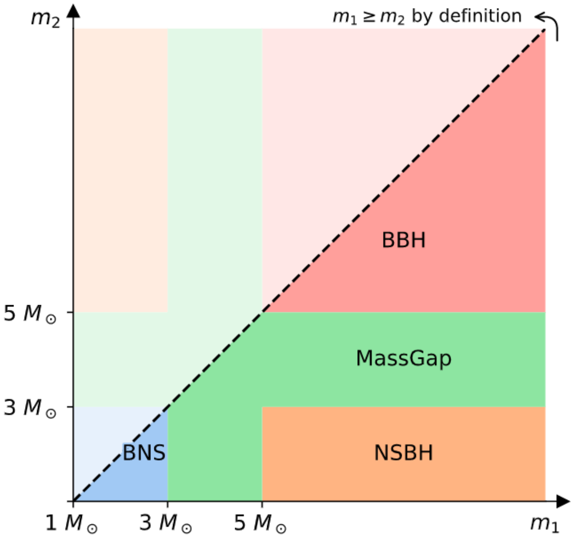

Figure 2. Mass relationship used to classify collided objects with $m_1 >= m_2$ detected by LIGO. Also known as p-astro.
From (LIGO Scientific Collaboration 2021)

two objects collided to create the GW. From strain caused by a GW on the LIGO lasers, one can directly determine the chirp mass, this is a parameter which is a combination of the masses of the two objects
The chirp mass is defined as Chen & Shen (2019):

$$M = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} (Kapadiaet\ al.\ 2020) \quad (1)$$

M is the chirp mass with m_1 and m_2 being the mass of the two objects. The chirp mass allows us to use the mass from sources to classify them depending on their mass.[1]

As shown in fig(2) the ratio of mass between the two colliding objects reveals the types of objects that collided. There are five classifications for sources.

BNS Mergers are consist of a Binary Neutron Star(BNS) system, as shown on fig(2) they are relatively low in mass, with masses between 1-3 solar masses[2]

A NSBH merger consists of a Black Hole and a Neutron Star colliding with one another. These collisions are classified from mid-sized mass detections that are greater than 5 Solar Masses for the most massive object and between 1-3 solar masses for the less massive object.

BBH[3] mergers consist of two black holes colliding. These collisions are typically between two massive objects. Both the low mass and high mass object have masses that are greater than 5 Solar Masses.

MassGap mergers consists of at least one intermediate mass object with a mass in a range of 3-5 solar masses. When an object is in this mass range we are unsure of what to label it.

Finally we have the classification of Terrestrial.If a detection is classified as Terrestrial, it is not astrophysical and does not cause gravitational waves. Terrestrial signals could be a result of the noise artifacts in the LIGO instrument or something in the environment.

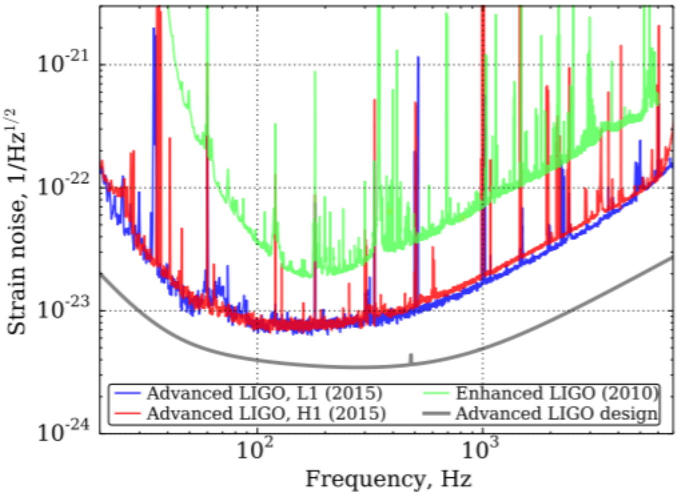

Figure 3. The gray curve is the noise level for the Advanced LIGO detector from (Martynov et al. 2016)

1.5 Validity of Sources

The process explained in the previous sections, in practice, is difficult, because anything that can cause a vibration to the mirrors will result in the photon collector collecting light. To counter this a considerable amount of noise must be filtered out of the detectors. The LIGO detector itself will generate noise; to counter this we search for GW waves in frequencies where the detector does not generate a substantial amount of noise.
The False Alarm Rate(FAR) is used to determine if a detected source is astrophysical or not. FAR is defined as:

$$FAR(\rho) = \int_{\rho}^{\rho_{max}} \Lambda_n p_n(\rho')(d\rho') \quad (2)$$

Where ρ is the SNR(Signal to Noise Ratio), Λ_n is the mean Poisson rate of signal and noise triggers(Callister et al. 2017),p_n is the probability density describing the distribution of detection statistics of ρ. The False Alarm Rate(FAR) is equal to 10^-2yr^-1 and is the current method used to check if an object is astrophysical. source(Callister et al. 2017)

1.6 Localizing Sources

Localization maps are created by using GW source parameters and antenna patterns of the LIGO detectors. When LIGO detectors detect a GW assuming it is astrophysical, the time delay between detections from different observatories can locate what direction the GW came from. The timing accuracy for a given source is:

$$\sigma_t = \frac{1}{2\pi\rho\sigma_f} \quad (3)$$

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[1] LIGO is most sensitive to a combination of ¿10 solar masses
[2] A Solar Mass is the mass of the sun
[3] Binary Black Hole

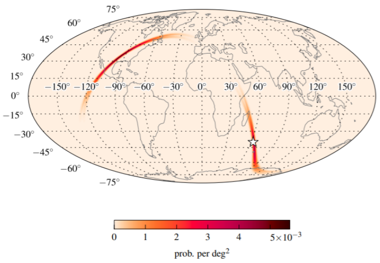

Figure 4. A localization map created from a source detected by LIGO from (Singer et al. 2014)

ρ[4] and σ_f is the effective bandwidth. The localization will depend on the time delay as shown in:

$$p(r|R) = p(r) * e^{-\frac{(D\cdot(r-R))^2}{2(\sigma_1^2+\sigma_2^2)}} \tag{4}$$

D is the distance between detectors, r is the reconstructed location, and R is the position of the source. When localizing a source we find the smallest region of the sky that contains a source:

$$\frac{Area(.90)}{4} = 3.3\frac{\sqrt{\sigma_1^2+\sigma_2^2}}{D} \tag{5}$$

This gives the distribution of time observed with, t being the observed time, T being the time of arrival at the sites. The result is a localization map as shown in fig(4).

By decreasing the size of the sky maps created by detections, optically viewing sources of gravitational waves will be possible allowing further research into gravitational waves sources to be done.

2 OBJECTIVE AND APPROACH

2.1 Objective

In the project I am proposing, I will use Bayesian statistics and the FAR method to determine which process is the most effective low latency method to determine if an object is astrophysical. I will do this by viewing O3 catalogue Prestegard (2021) detections from LIGO and comparing the results from both methods to see which result closely resembles the final and more robustly determined FAR.

If Bayesian statistics results in a more accurate data then I will add these to the low latency packets.

If Bayesian statistics yields no useful results, I will look into p-astro fig(2) to increase accuracy in a shorter amount of time.

If I finish my project early I will assist a fellow SURF student on their project focusing on likely properties of EM counterparts.

2.2 Bayes factors analysis

$$P(H_i|\vec{d}, I) = \frac{P(H_i|I)P(\vec{d}|Hi, I)}{P(\vec{d}|I)} \tag{6}$$

Bayesian inference, the probability of a hypothesis H_i when given a set of observational data d and prior information I is given by eqn(6). $P(H_i|I)$ is the prior probability of H_i, $P(d|H_i, I)$ is the likelihood function of the data, given that H_i is true and:

$$P(\vec{d}|I) = \sum P(\vec{d}|H_i, I) \tag{7}$$

eqn(7) is the minimum probability of the data set d. We can compare models by calculating probabilities in the form of "posterior odds ratio":

$$O_{ij} = \frac{P(H_i|I)P(\vec{d}|H_i, I)}{P(H_j|I)P(\vec{d}|H_j, I)} = \frac{P(H_i|I)}{P(H_j|I)}B_{ij} \tag{8}$$

$P(d|I)$ is the normalization factor, and it cancels out giving:

$$B_{ij} = \frac{P(\vec{d}|H_i, I)}{P(\vec{d}|H_j, I)} \tag{9}$$

The equation above eqn(9) will be used to determine if a detected source is astrophysical.Veitch & Vecchio (2010)

2.3 Editing P-astro

[5] If Bayes factor is less effective than low latency FAR, I will focus on a more effective way to classify sources. To achieve this I will work with the creators of the code to find any potential caveats that can be altered resulting in a refined classification system.

The statement below from (Fairhurst 2009) raises a potential issues in the classification of sources:
"However, the confusion between low (high) mass NSBH signals and BNS (BBH) signals, would be a concern, if this significantly changed the counts of these types of signals"

(Chatterjee et al. 2020) also suggests that some false negatives could potentially be in the current data recognized as BBH, due to a strict threshold.

$$p(HasRemnant) = 1 = p(HasNS) = SOURCE \tag{10}$$

and

$$0 = p(HasRemnant) = NoSource = p(HasNS) = 0 \tag{11}$$

This would suggest that for a binary system to have a NS there must be a remnant, which is not true for NSBH systems as the black-hole may "swallow" the NS but leave some mass around itself.

This threshold can allow software to ignore NSBH collisions and consider them as BBH collisions or even terrestrial as (Chatterjee et al. 2020) suggests. Lowering the threshold and comparing its results to the O3 catalogue data will be helpful in knowing if the current threshold could be responsible for false negatives.

2.4 EM counterparts

If I manage to finish my project early I will assist my peer in their project focusing on EM counterparts to detections. I can assist my peer by looking for potential caveats in the current methods and attempt to refine them.

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[4] Signal to Noise Ratio

[5] P-astro is fig(2)

The first caveat that I will look into will be how the equation of state was defined for NS detected.

In many cases during the paper (Chatterjee et al. 2020) mentioned that their ”point-estimate” is erroneous, for NSBH mergers because the merger and ring-down in the signal were not considered. Using a more accurate waveform would yield much more sufficient data for EM counter parts. Finally, we will examine how knowing the chirp mass of the binary affects the mass ejected from the merger and its relation to the expected light-curve properties of an associated optical counterpart.

3 SCHEDULE

In the first three weeks I will use Bayesian statistics to determine is past detections are astrophysical and compare it to the success rate of low latency FAR results(weeks 1-3)

In the following four weeks I will classify the objects the Bayesian statistics deemed astrophysical(weeks 3 - 7)

I will compare Bayesian and low latency FAR results to view the most accurate low latency method. (weeks 6-8)

I will implement Bayesian statistics to low latency packets if it yields positive results, if it yields negative results I will use Fig. 2 to increase accuracy in low latency packets (weeks 7-9)

If I finish early I will assist a peer with their project. Their project involves looking at the likely properties of EM counterparts.(weeks 8-9)

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