

LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY  
- LIGO -  
CALIFORNIA INSTITUTE OF TECHNOLOGY  
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Technical Note	LIGO-T2100160-v1	2021/06/16
<b>Optimal State-Space Estimation of Interferometer Mode-Matching</b>		
Mark Nguyen		

**California Institute of Technology**  
**LIGO Project, MS 18-34**  
**Pasadena, CA 91125**  
Phone (626) 395-2129  
Fax (626) 304-9834  
E-mail: info@ligo.caltech.edu

**Massachusetts Institute of Technology**  
**LIGO Project, Room NW22-295**  
**Cambridge, MA 02139**  
Phone (617) 253-4824  
Fax (617) 253-7014  
E-mail: info@ligo.mit.edu

**LIGO Hanford Observatory**  
**Route 10, Mile Marker 2**  
**Richland, WA 99352**  
Phone (509) 372-8106  
Fax (509) 372-8137  
E-mail: info@ligo.caltech.edu

**LIGO Livingston Observatory**  
**19100 LIGO Lane**  
**Livingston, LA 70754**  
Phone (225) 686-3100  
Fax (225) 686-7189  
E-mail: info@ligo.caltech.edu

# 1 Introduction

In 1916, Albert Einstein theorized “ripples in space-time” known as gravitational waves in his paper on general relativity. It is well understood from this publication that Einstein’s definition of gravity differed from Issac Newton’s in that Einstein said that gravity is spacetime curvature as opposed to Newton stating that gravity is an attractive force or rather an interaction between two objects in only space. Further deviating from the Newtonian gravity explanation, Einstein proposed that gravity was actually the result of a wave of spacetime curvature propagating at the speed of light through not only the three dimensions of space as we know it, but through time as well. These waves would only be caused by the motion of massive astrophysical objects like colliding black holes, supernovae, and colliding neutron stars.

Although it might have been outlandish when it was first proposed and even shook the foundation of modern physics at the time, general relativity mathematics has proven to be correct with its role in increasing the precision of global positioning systems. It was not until an effort designed to detect these waves by the Laser Interferometer Gravitational-Wave Observatory (LIGO), which set out to find evidence of their existence, that Einstein’s theoretical predictions would be visually observed and recorded. The first gravitational wave event to be detected and measured was in the year 2015 as a result of the observation of two LIGO detectors during a merger of a pair of black holes into a single black hole. Since that historic event, LIGO has detected 49 more accounts of gravitational waves, some because of the collision of black holes like the first detection and others as a result of supernovae and colliding neutron stars. This effort to detect gravitational waves is no where near ending as work is being done on the current detectors to increase the rate and efficiency of detection by reducing surrounding unwanted noise.

LIGO has achieved a large feat by managing to detect gravitational waves at the sites of extreme astrophysical phenomena of the universe; however, astrophysics is not the only benefiter of these discoveries. Optical physics has been further advanced as a result. Although these extraordinary optical instruments have assisted with the numerous detections, they still have limitations that can further be improved. There are current systems in place to reduce these noise limitations and one of them is known as the thermal compensation system (TCS). As seen by the layout of Advanced LIGO in Figure 2 there are two actuators that “serve to correct dynamic changes in the ITM and ETM surface curvatures and substrate lenses and are also used to remove static lenses in the ITM substrates[...]” The limitation that exists with this system is that “one cannot actuate with the TCS actuators to affect the SRC mode without also affecting the PRC mode.” Furthermore, the true spatial modes of the different interferometer cavities are the hidden state of the system. The hidden state is the result of having measurements and diagnostic signals that are sensitive to the cavity modes but are not directly measured by the system. In an effort to mitigate this issue and gain information about the hidden states, the Kalman filter will be implemented to infer the hidden state.

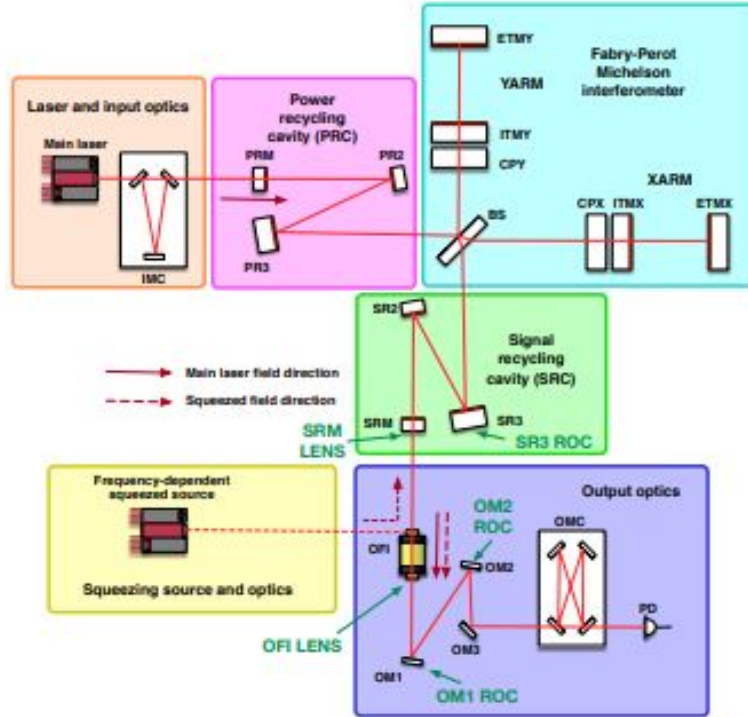


Figure 1: Thermal compensation system with a pair of actuators

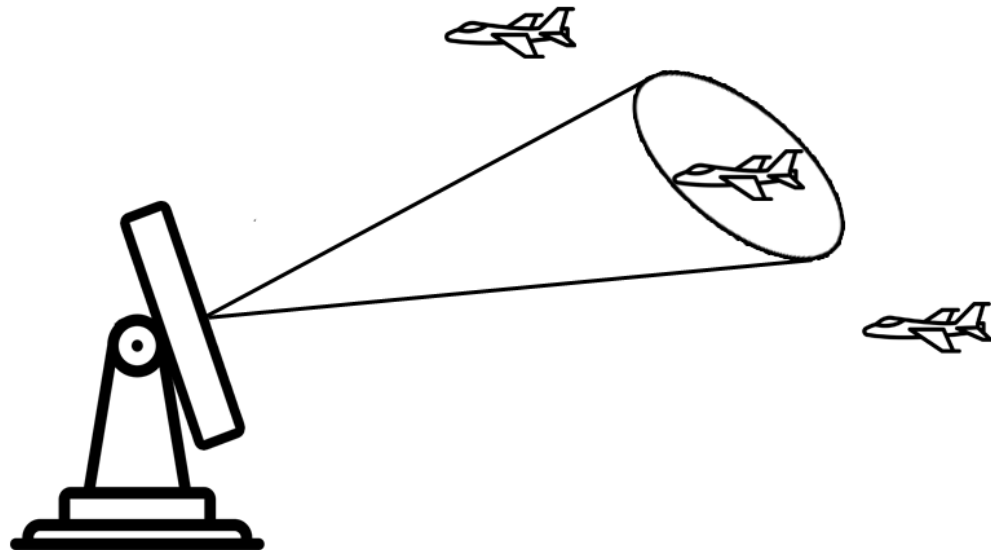


Figure 2: Kalman Filter used as a prediction algorithm in radar systems

$$\begin{cases} x = x_0 + v_{x0}\Delta t + \frac{1}{2}a_x\Delta t^2 \\ y = y_0 + v_{y0}\Delta t + \frac{1}{2}a_y\Delta t^2 \\ z = z_0 + v_{z0}\Delta t + \frac{1}{2}a_z\Delta t^2 \end{cases}$$

Figure 3: Newtons Equations of Motions in three dimensions

## 2 Kalman Filter

Kalman filters are not a new invention and in fact have been “the most widely used prediction algorithm”. The filter essentially takes in information about a current state and then predicts the next state based on this initial state. The Kalman filter’s role in “relatively simple state space estimation” has been applied to other physical systems. For example, in radar systems as shown in Figure 1, the use of Kalman filters can increase the accuracy in which an aircraft’s position and velocity can be monitored. In order to use a Kalman filter, the function must either be linear or differentiable. Relatively simple mathematical differentiable functions that may be used in such a radar system are shown in Figure 3 where “the target parameters  $[x, y, z, v_x, v_y, v_z, a_x, a_y, a_z]$  are called a System State.” Having the system of equations from Figure 3 which is also known as the “Dynamic Model” along with the current system state, an estimation about a future state can be predicted within a relative margin of error. It is well known that in the real world, these Newtonian equations of motion are not the only thing dictating an object’s behavior as there are external factors that contribute to the system state that affect its motion. To make an accurate prediction of a future state, it is necessary that these external factors, or what is referred to as noise in our case, be taken into account. The plan is to implement this same technology in gravitational wave detection systems to solve the aforementioned mystery of the hidden state.

The Kalman filter’s implementation is meant to be able to “optimally estimate the current state of the [LIGO] interferometer”. Specifically, an extended Kalman filter with “extended” referring “to the fact that the Kalman filter contains non-linear state propagation functions”, will be implemented for the sole purpose of mode-matching an input laser beam. The electric field of an optical beam is represented as follows where  $u_{nm}$  describes the “spatial properties of the beam”, “ $a_{jnm}$  as complex amplitude factors, and  $\omega_j$  is... the angular frequency and  $k_j = \omega_j/c$ ”. The intensity of this beam is unlike a plane wave and would actually resemble an intensity distribution. The goal is to have the curvature of the beam’s wavefront match the curvature of the optical cavity. Figure 4

$$E(t, x, y, z) = \sum_j \sum_{n,m} a_{jnm} u_{nm}(x, y, z) \exp(i(\omega_j t - k_j z)) \quad (1)$$

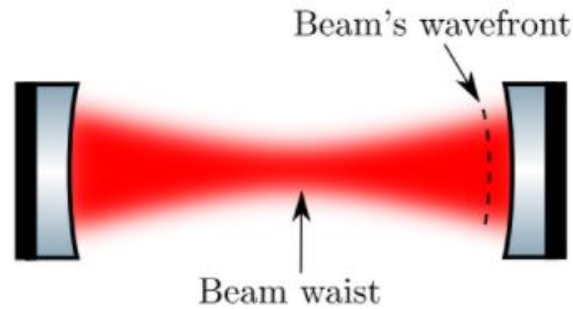


Figure 4: Laser beam front mode-matched with optical cavity

Similar to having the dynamic model along with the current state in the general use of a Kalman filter, “knowledge of the input beam and TEM00 mode combined with our model engine (state-propagation function) allows [...] an a priori estimate of the mode-matching.” The mode matching itself will be measured simultaneously with some uncertainty. The ideal estimate will be a combination of this direct mode matching measurement with the prediction and weight of the combination determined by the Kalman filter.

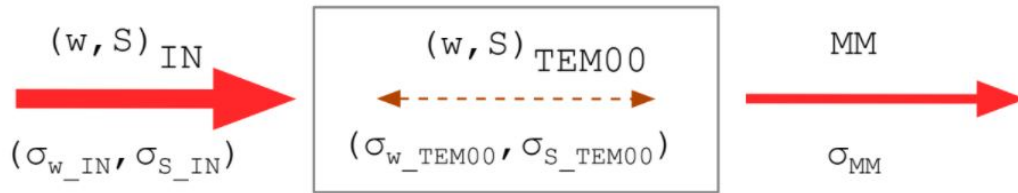


Figure 5: Input laser beam injected into a “TEM00 eigenmode” and the output mode-match is measured

### 3 Objective

The purpose of the project becomes apparent when examining the ability of the Kalman filter to fulfill the outstanding requirement of LIGO detectors. Following the relatively simple description of mode-matching with one laser beam in Figure 5, one can see that the plan is to eventually scale the potential of the Kalman filter up to a more complicated system like the one in Figure 6. Similar, yet more intricate compared to the structure of combining the measurements of the mode-matching and the prediction that is to be weighted by the Kalman filter, this more complex system will essentially take in multiple uncertain measurements.

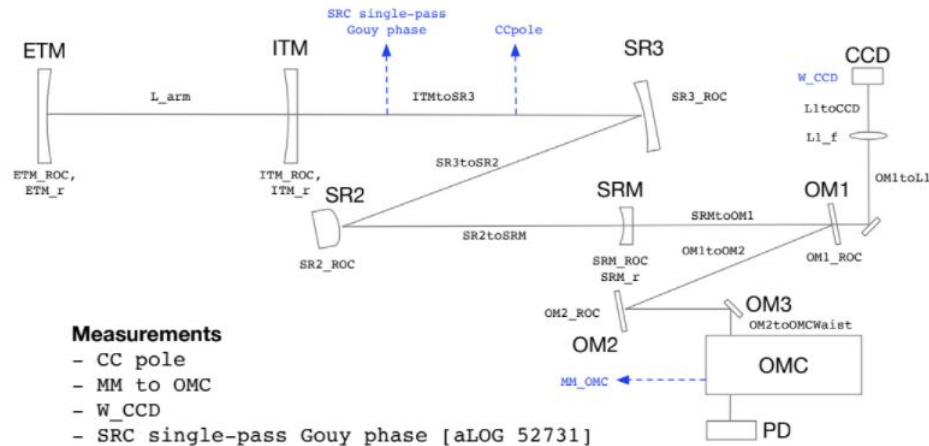


Figure 6: More complex Kalman filter application

## 4 Approach

The Kalman filter will be implemented in Python and will utilize the Finesse/PyKat simulation package for optical modeling. The first couple of weeks will be spent getting comfortable with the Finesse 2.1 and PyKat library packages. The third and fourth weeks will be used to implement the Kalman filter program on a simple system of optical cavities. After the successful implementation and testing of the software on this simple system, the program will be put to the test with real data from LIGO detectors. The program will then be tweaked to its final version until the eigenmode matching is favorable.

## References

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