LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Technical Note **LIGO-T2000338–v1** 2020/05/26

Project Proposal: Interferometer Optimization and Evaluation of Multi-Mode Squeezing

Swadha Pandey
Mentors: Jonathan Richardson, Rana Adhikari

California Institute of Technology
LIGO Project, MS 18-34
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project, Room NW22-295
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
Route 10, Mile Marker 2
Richland, WA 99352
Phone (509) 372-8106
Fax (509) 372-8137
E-mail: info@ligo.caltech.edu

LIGO Livingston Observatory
19100 LIGO Lane
Livingston, LA 70754
Phone (225) 686-3100
Fax (225) 686-7189
E-mail: info@ligo.caltech.edu

http://www.ligo.caltech.edu/

1 Introduction

Gravitational waves are ripples in space-time, which means they contract and expand the distance between two points in space-time. Caused by accelerating masses, they propagate away from their source at the speed of light. They were first predicted by Albert Einstein in 1916, but were not detected until 2015 - when the Laser Interferometer Gravitational-Wave Observatory (LIGO) detectors detected gravitational waves coming from the merging of two black holes 1.3 billion light-years away.

The LIGO detectors achieved this by measuring the disturbances made in space-time as the gravitational waves passed through the earth. These disturbances are ten thousand times smaller than a proton, and hence sensitivity is of the utmost importance. Currently, LIGO's sensitivity is limited by quantum noise (above 200 Hz). Squeezed states are being used to reduce this quantum noise. A squeezed state is one in which the quantum uncertainty is unevenly distributed between the amplitude and phase quadratures, in order to reduce the uncertainty in one of the quadratures while maintaining Heisenberg's uncertainty relation. Squeezed states of light are the key to improving the sensitivity of the LIGO detectors. However, the amount of squeezing is limited by optical losses in the interferometer. The major cause for these losses are mode-mismatches between cavities caused by deviations from the design parameters. A laser beam is said to be in perfect resonance with an optical cavity if its spatial mode is the fundamental spatial mode of the cavity. Any misalignment or perturbation in the apparatus results in mode-mismatch with the incoming laser beam and coupling with higher-order spatial modes, i.e., loss of power from the fundamental mode to the higher-order modes [1]. In Enhanced LIGO, this accounted for $25 \pm 5\%$ of the losses [2]. The goal of the project is to re-design the LIGO set-up in order to minimise its sensitivity to these perturbations. .

We will consider mode-mismatches due to perturbations in the positions and radii of curvature of the optical instruments used in the Advanced LIGO set-up, shown in Figure 1. We will consider these perturbations for the two sets of ITM (Input Test Mass) and ETM (End Test Mass) mirrors as well as the mirrors in the SR (Signal Recycling) cavity shown in the figure.

We will approach this problem analytically as well as numerically. First we will analytically determine the power loss to higher-order modes due to these perturbations for specific cases of consecutive Fabry-Perot cavities. Then we will study and minimise the sensitivity of the set-up to these perturbations using the Jacobian matrix, thus optimising the set-up. The proposed formalism is outlined in Section 3.

In the numerical approach, we will be modelling the aLIGO set-up on the interferometer simulation software, Finesse [3]. We will then numerically calculate and minimise the Jacobian using Markov chain Monte Carlo methods. The analytical model will provide a theoretical check for the Finesse model and also reduce the burn-in time of the MCMC simulations.

In the final part of the project, we will use this optimised set-up to quantify the benefits of multi-mode squeezing. We will quantify how much sensitivity to the perturbations in the design parameters is reduced by two-mode squeezing. This will help answer whether the potential benefit of multi-mode squeezing is worth the technical R&D required to develop it.

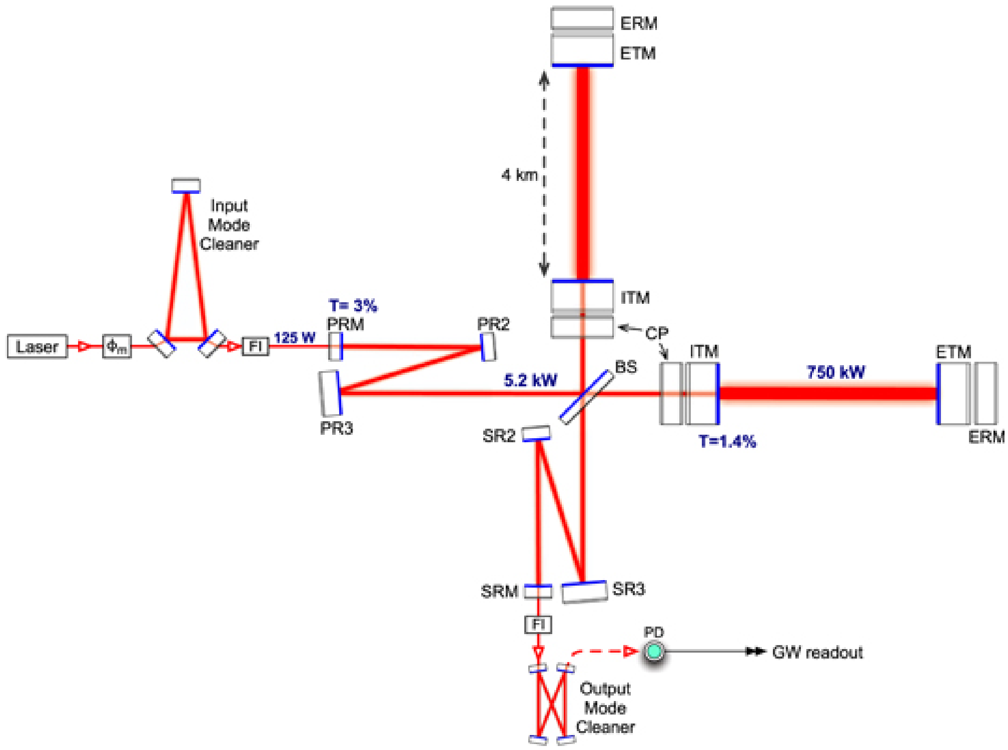

Figure 1: The Advanced LIGO optical layout, reproduced from Aasi et al.(2015) [4].

The project ultimately aims to reduce the above problems in future LIGO detectors like the Aundha detector.

2 Objectives

1. Optimise the design parameters of a LIGO-like interferometer:
   (a) Re-design the set-up to minimize sensitivity to errors in the design parameters of the optical instruments, such as radii of curvature and positions.
   (b) Better avoid higher-order mode co-resonances.
2. Quantify how much sensitivity to the perturbations in the design parameters is reduced by two-mode squeezing.

3 Background

3.1 Laguerre-Gaussian Modes and Mode-Mismatch

This section is based on the work of Kogelnik and Li (1966) [5] and Bond et. al. (2017) [1].

An optical beam produced by a laser has some spatial intensity distribution. Consider such a beam moving along the z-direction. We can then write

$$E(t,x,y,z) = \sum_j \sum_{n,m} a_{jnm}\, u_{nm}(x,y,z)\, exp(i(\omega_j t - k_j z)) \tag{1}$$

where u_{nm} are the functions describing the spatial properties of the beam, a_{jnm} are the complex amplitude factors, and ω_j is the angular frequency. For simplicity we take $t = 0$. We then solve for $u_{nm}(x,y,z)$ by using the Helmholtz equation

$$\nabla^2 \mathbf{E} - \frac{\ddot{\mathbf{E}}}{c^2} = 0 \tag{2}$$

Solving this for $E(x,y,z) = u(x,y,z)\, exp(-ikz)$ and applying the paraxial wave approximation, we get that $u_{nm}(x,y,z)$ can be described by the set of Hermite-Gauss or Laguerre-Gauss polynomials, both of which are complete sets of orthonormal functions. The choice between the Hermite-Gaussian (HG) and Laguerre-Gaussian (LG) modes is equivalent to a basis change as they are both solutions of the Helmholtz equation. The u_{00} polynomial in both cases is purely Gaussian. Hence the $E(x,y,z) = u_{00}(x,y,z)\, exp(-ikz)$ is known as the fundamental mode, denoted by TEM_{00} mode for the HG basis. The remaining modes are known as the higher-order modes.

An optical beam produced by a laser is in the fundamental Gaussian mode, and can be described by the Gaussian beam parameter $q_(z)$. At each optical element that the beam passes through, the beam parameter is transformed from $q_1(z)$ to $q_2(z)$. This transformation depends on the element.

A cavity eigenmode is defined as the optical field with spatial properties such that the field after one round-trip through the cavity remains the same. For an optical cavity with spherical mirrors, this eigenmode is a Gaussian mode with a beam parameter $q_{cav}(z)$.

Now a Gaussian beam with beam parameter $q_{inj}(z)$ injected into a cavity with beam parameter $q_{cav}(z)$ is said to be in perfect resonance if: (a) the optical axes of the beam and the cavity are the same, and (b) $q_{inj}(z) = q_{cav}(z)$. If these conditions are not met, there is said to be a (a) misalignment or (b) mode-mismatch.

In case of a misalignment or mode-mismatch, the fundamental LG (or HG) mode is coupled with/scattered into higher-order LG (or HG) modes, and power is lost to these higher-order modes.

3.2 Scattering-Matrix Formalism

Consider the electric field of an optical beam represented by a 2×1 vector, where the first element represents the amplitude and phase of the TEM_{00} mode and the second element represents the aggregate amplitude and phase of the higher-order modes. Initially, we start with a purely Gaussian beam represented by

$$E_{in} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3}$$

Say the beam goes through an optical element which couples it to the higher-order modes. We represent this set-up by a 2×2 scattering matrix, say A, such that

$$E_{out} = A \times E_{in} \tag{4}$$

If the beam goes through multiple consecutive elements with the scattering matrices A_0, A_1, A_2..., we have

$$E_{out} = A_0 \times A_1 \times A_2 \times ...E_{in} \tag{5}$$

Now we consider the specific case of reflection from a distorted mirror that scatters power into the higher-order modes. The scattering matrix is then given by [6]

$$A_{mir} = \begin{pmatrix} r & 0 \\ 0 & r \end{pmatrix} \begin{pmatrix} \sqrt{1-a^2} & -a \\ a & b\sqrt{1-a^2} \end{pmatrix} \tag{6}$$

where r is the amplitude reflectivity coefficient, a is the aggregate complex amplitude coupling into the higher-order modes, and b is the aggregate field transmission of the higher-order modes in the arm cavities.

In this way we use this formalism to keep track of the power losses to the higher-order modes.

3.3 Mode-Mismatch due to Radius of Curvature Perturbation

Let us consider a two-mirror optical cavity with two spherical mirrors of radii of curvature (ROC) R_1 and R_2, and cavity eigenmode having Gaussian beam parameter q_{cav}. Consider a mode-matched and aligned Gaussian beam injected into this cavity, propagating in the z-direction, with Gaussian beam parameter q_{inj}. Then we have $q_{cav} = q_{inj}$.

Now we consider small perturbations in the ROC R_1 and R_2. This changes the cavity eigenmode parameter from q_{cav} to q'_{cav}. Now to calculate the aggregate amplitude coupling into the higher-order modes, we calculate the overlap of the injected beam with the new cavity eigenmode. From [8], this overlap $OL(q_{inj}, q'_{cav})$ is given by

$$OL(q_{inj}, q'_{cav}) = \int\int E(q_{inj}; x, y) E^*(q'_{cav}; x, y)\, dx\, dy \times \int\int E^*(q_{inj}; x, y) E(q'_{cav}; x, y)\, dx\, dy \tag{7}$$

where $E(q; x, y)$ is the fundamental HG mode having the Gaussian beam parameter q. Thus the aggregate amplitude coupling into the higher-order modes, a is given by

$$a = \sqrt{1 - OL(q_{inj}, q'_{cav})} \tag{8}$$

3.3.1 Example: Radius of Curvature Perturbation for the Two-Cavity Case

Let us consider the setup given in Figure 2. Let us consider the two cavities made by mirrors M1 and M2 (cavity 1), and M3 and M4 (cavity 2), separated by a considerable distance (i.e., they don't share any mirrors).

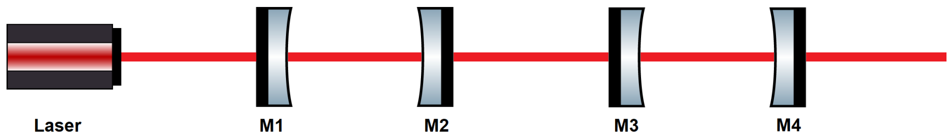

Figure 2: Setup for two-cavity mode-mismatch

For a cavity of length L (say cavity 1), consisting of spherical mirrors M1 and M2 with radii of curvature $R1$ and $R2$, the Gaussian eigenmode is given by the beam waist, ω_0 and position of the waist (distance from M1), z_0 where

$$\omega_0^2 = \frac{L\lambda}{\pi}\sqrt{\frac{g_1 g_2(1-g_1 g_2)}{(g_1+g_2-2g_1 g_2)^2}} \tag{9}$$

and

$$z_0 = \frac{g_2(1-g_1)}{g_1+g_2-2g_1 g_2}L \tag{10}$$

such that

$$g_1 = 1-\frac{L}{R_1},\ \ g_2 = 1-\frac{L}{R_2} \tag{11}$$

Initially, the two cavities are mode-matched, i.e., their Gaussian eigenmodes have the same beam waist size and position. We introduce a deviation in the radius of curvature of a mirror in cavity 1, say M1. If initially its radius of curvature was $R1$, we now have the radius $R1+\Delta R1$. We now want to calculate the internal loss in this system, which is the amount of mode-mismatch between the two cavities. Substituting $R1+\Delta R1$ in the above formulae, we find the overlap between the mode-mismatched cavities using the formula

$$\begin{aligned} OL(cav_1, cav_2) = &\int\int u_{00}(L, R_1, R_2; x, y) u_{00}^*(L, R_1+\Delta R1, R_2; x, y)\, dx\, dy \\ &\times \int\int u_{00}^*(L, R_1, R_2; x, y) u_{00}(L, R_1+\Delta R1, R_2; x, y)\, dx\, dy \end{aligned} \tag{12}$$

Calculating upto first order in $\Delta R1$, we get

$$OL(cav_1, cav_2) = 1-\Delta g_1\left(\frac{1}{2g_1}-\frac{g_2}{2(1-g_1 g_2)}+\frac{1-2g_2}{g_1(1-2g_2)+g_2}\right) \tag{13}$$

where

$$\Delta g_1 = \Delta R_1 \frac{L}{R_1^2} \tag{14}$$

3.4 How Losses Affect the Level of Squeezing

This section is based on the work of E. Oelker et. al. (2014) [9].

The effective level of squeezing in a real interferometer depends not only on the level of squeezing injected into the system, but also by the losses that occur in the system. Losses replace some of the squeezed light with ordinary vacuum, thus reducing the overall level of squeezing.

These losses are mainly the optical losses due to mode-mismatch and alignment fluctuations. In LIGO H1 squeezing demonstrations, optical losses of 56% were observed, limiting the squeezing enhancement to 2.1 dB [10]. On the other hand, in tabletop squeezing demonstrations, total losses have been reduced to less than 10%, giving a squeezing enhancement of more than 10dB [11]. Thus reducing the optical losses is essential to improve the level of squeezing in the interferometer.

4 Methods

4.1 Analytic Method

Using the formalism described in Section 3, we will analytically determine the power loss to the higher-order modes due to perturbations in the design parameters of the apparatus for specific cases of consecutive Fabry-Perot cavities.

The Jacobian matrix for this loss will give us the sensitivity of the set-up to these perturbations. We will then analytically minimise this Jacobian and thus optimise the design parameters for the set-up.

4.2 Numerical Modeling

Finesse is an interferometer simulation software, and PyKat is a wrapper for using Finesse in Python. Using this, we will numerically approach our problem.

We will simulate the cavities and compute the Jacobian for power loss to the higher-order modes due to perturbations in the design parameters. Then we will minimise it using Markov chain Monte Carlo methods, thus optimising the design parameters for the set-up.

Below are some of our simulations in Finesse that will be useful for the project.

4.2.1 Example 1: Mode Cleaner

A laser beam which is an equal sum of the the TEM_{00} and TEM_{10} modes is passed through a triangular cavity which is resonant for the TEM_{00} mode. The cavity thus acts as a mode cleaner - transmitting the TEM_{00} mode and reflecting the TEM_{10} mode, as shown in Figure 3.

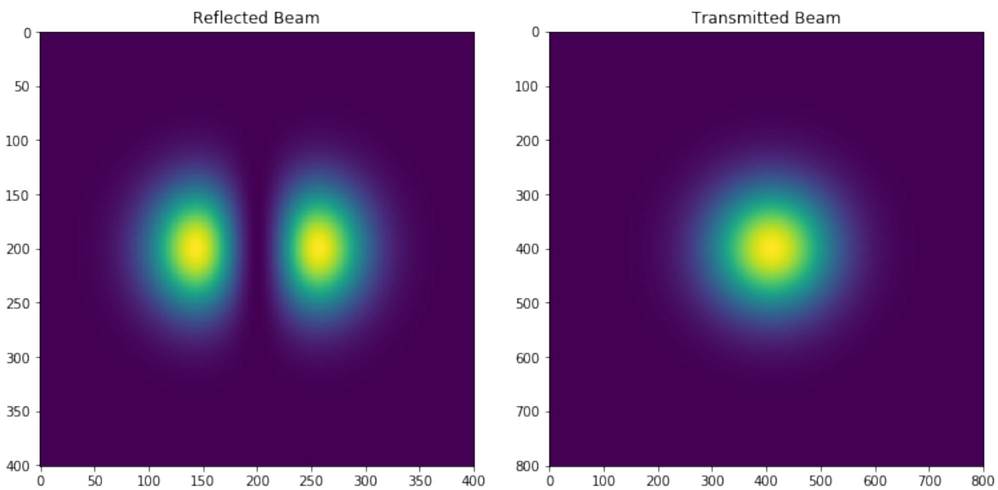

Figure 3: Reflected and transmitted beam from mode cleaner

4.2.2 Example 2: Circulating Power in a Misaligned Cavity

A misaligned cavity injected with a TEM_{00} beam is scanned for higher order spatial modes, as shown in Figure 4.

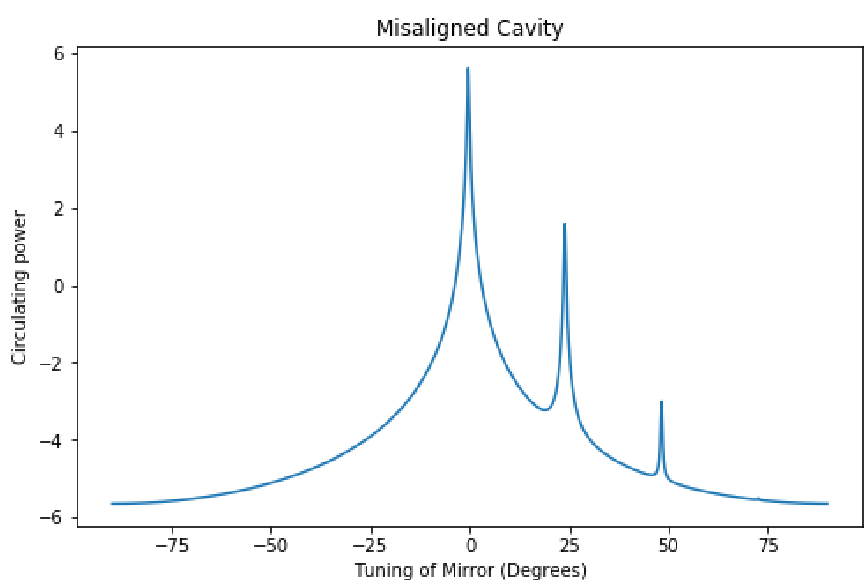

Figure 4: Higher order modes created by misalignment

4.2.3 Example 3: Optimization of Frequency Dependent Squeezing Input Angle

For a dual recycled Michelson model, we plot the vacuum noise sensitivity and the full quantum noise limited sensitivity and observe that the quantum noise is larger at the detuning frequency (Figure 5, top). After minimising the ratio of shot noise and quantum noise with respect to the squeezing input angle at each frequency, we see a considerable reduction in the noise (Figure 5, bottom).

5 Timeline

- **Week 1-2:** Develop formalism and analytically determine and minimise power losses for cases of consecutive Fabry-Perot cavities.
- **Week 3-4:** Repeat the above using simulations in Finesse, verifying the analytical result using Markov Chain Monte Carlo methods for a 3 mirror cavity.
- **Week 5-6:** Extend the numerical analysis to a complete LIGO-like set-up, and optimise the design using Markov chain Monte Carlo methods.
- **Week 7-10:** Quantify how much sensitivity to the perturbations in the design parameters is reduced by two-mode squeezing.

References

[1] Bond et. al. (2017), *Interferometer techniques for gravitational-wave detection, Ch 9-11.* doi: 10.1007/s41114-016-0002-8

[2] Aasi et al. (2013), *Enhanced sensitivity of the ligo gravitational wave detector by using squeezed states of light.* Nature Photon, vol. 7.

[3] Freise A. (2015), *FINESSE: an interferometer simulation* http://www.gwoptics.org/finesse

[4] Aasi et. al. (2015), *Advanced LIGO.* Classical Quantum Gravity 32, 074001

[5] H. Kogelnik and T. Li (1966), *Laser Beams and Resonators.* doi:10.1109/PROC.1966.5119

[6] Aidan Brooks (2020), *Formulae for scattering into HOM in FP cavities*

[7] Dana Z. Anderson (1984), *Alignment of resonant optical cavities.* Appl. Opt. 23, 2944-2949

[8] Perreca et. al. (2020), *An analysis and visualization of the output mode-matching requirements for squeezing in Advanced LIGO and future gravitational wave detectors.* arXiv:2001.10132v1 [physics.ins-det]

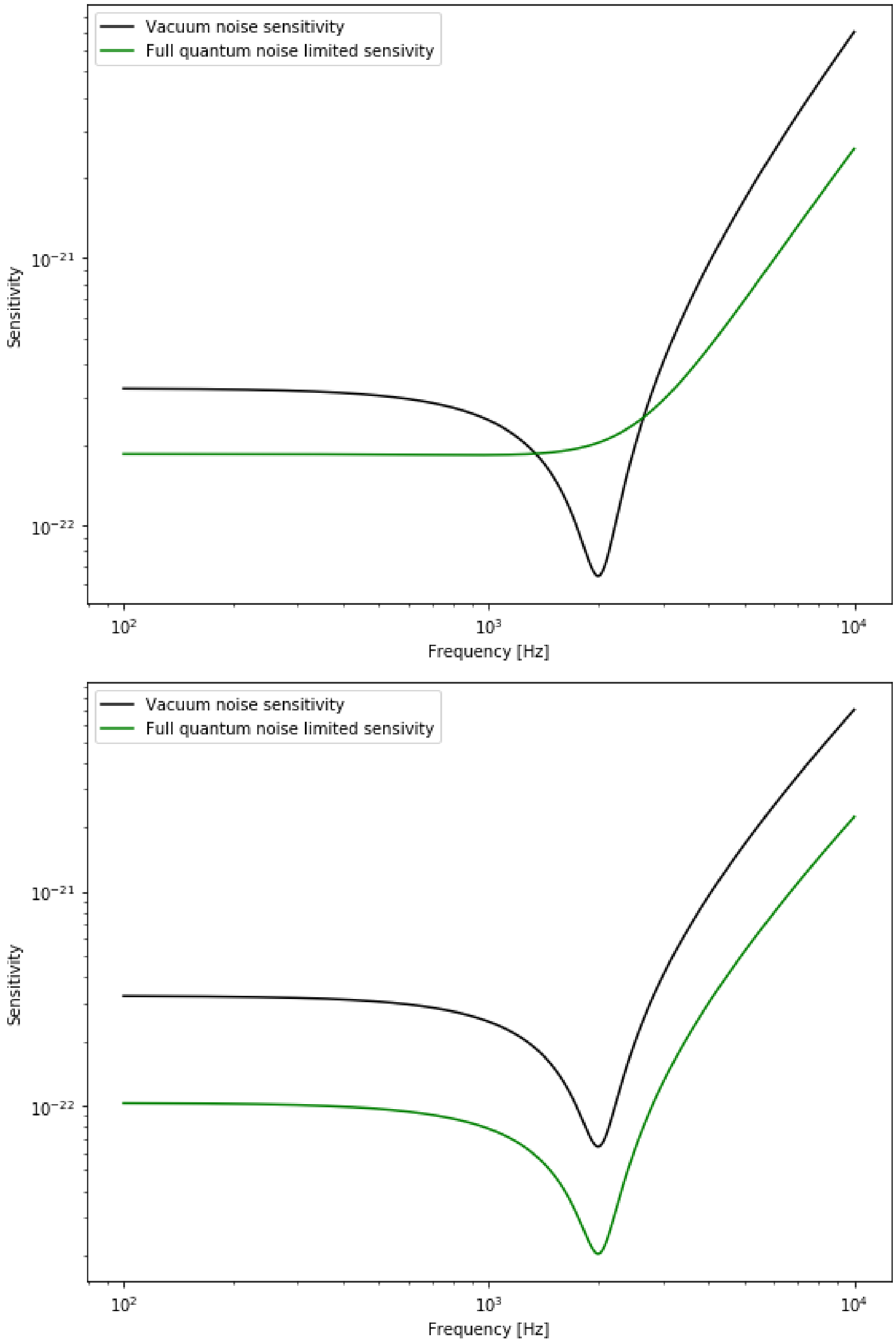

Figure 5: Sensitivity vs. Frequency before (top) and after (bottom) optimisation

[9] E. Oelker et. al. (2014) *Squeezed light for advanced gravitational wave detectors and beyond*. Opt. Express, vol. 22, no. 17

[10] The LIGO Scientific Collaboration (2013), *Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light*. Nature Photon. 7, 613–619

[11] T. Eberle et. al. (2010), *Quantum enhancement of the zero-area sagnac interferometer topology for gravitational wave detection* . Phys. Rev. Lett. 104, 251102