LIGO India Observatory (LIO) coordinate system for GW analyses

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The purpose of this document is to provide LIGO India Observatory (LIO) global and local coordinate details needed for GW searches. We note here that at this point these are the most likely design values and may change once the detector construction is completed. For this reason, these values are purely for academic purpose and are not vetted by the LIGO-India project. We plan to update these values as and when new information reaches us.

The design GPS coordinates of the beam splitter and the two end-mirrors, as well as their heights, † are given in Table 1,

Location	Latitude	Longitude	Height (m)
Beam splitter	$19^\circ\,36'\,47.9017''$ N	$77^\circ\,01'\,51.0997''$ E	440.0
End-X	$19^\circ\,38'\,43.1430''$ N	$77^\circ\,00'\,47.4656''$ E	440.0
End-Y	$19^\circ\,35'\,47.5998''$ N	$76^\circ\,59'\,49.4969''$ E	440.0

Table 1: GPS coordinates of main optics

Based on the above values below we obtain the following detector constants for LIO to be used in the LALDetectors.h [1].

```
LAL_LIO_4K_DETECTOR_NAME                "LIO_4k"
LAL_LIO_4K_DETECTOR_PREFIX              "A1"
```

†These are design Above Mean Sea Levels (AMSL) of floor of beam tube enclosures (BTE) at the middle of the arms. We haven't accounted for the distance between the BTE floor and beamline, and also any effect of curvature of the earth. So these height values expected to have systematic uncertainties of a few meters.

```
LAL_LIO_4K_DETECTOR_LONGITUDE_RAD        1.34444215058
LAL_LIO_4K_DETECTOR_LATITUDE_RAD         0.34231676739
LAL_LIO_4K_DETECTOR_ELEVATION_SI         440.0
LAL_LIO_4K_DETECTOR_ARM_X_AZIMUTH_RAD    5.80120119264
LAL_LIO_4K_DETECTOR_ARM_Y_AZIMUTH_RAD    4.23039066080
LAL_LIO_4K_DETECTOR_ARM_X_ALTITUDE_RAD   0.0
LAL_LIO_4K_DETECTOR_ARM_Y_ALTITUDE_RAD   0.0
LAL_LIO_4K_DETECTOR_ARM_X_MIDPOINT_SI    2000.00000000000
LAL_LIO_4K_DETECTOR_ARM_Y_MIDPOINT_SI    2000.00000000000
LAL_LIO_4K_VERTEX_LOCATION_X_SI          1.34897115479e+06
LAL_LIO_4K_VERTEX_LOCATION_Y_SI          5.85742826577e+06
LAL_LIO_4K_VERTEX_LOCATION_Z_SI          2.12756925209e+06
LAL_LIO_4K_ARM_X_DIRECTION_X             0.38496278183
LAL_LIO_4K_ARM_X_DIRECTION_Y             -0.39387275094
LAL_LIO_4K_ARM_X_DIRECTION_Z             0.83466634811
LAL_LIO_4K_ARM_Y_DIRECTION_X             0.89838844906
LAL_LIO_4K_ARM_Y_DIRECTION_Y             -0.04722636126
LAL_LIO_4K_ARM_Y_DIRECTION_Z             -0.43665531647
```

Further details of those individual constants are given below.

Detector prefix:

```
LAL_LIO_4K_DETECTOR_PREFIX           "A1"
```

The official prefix for LIO is "A1" (Aundha is the town near which the LIO is going to be built). Currently this prefix is used by ALLEGRO, so it needs to be properly changed.

Detector location:

```
LAL_LIO_4K_DETECTOR_LONGITUDE_RAD        1.34444215058
LAL_LIO_4K_DETECTOR_LATITUDE_RAD         0.34231676739
```

These are longitude and latitude of the beam splitter location in radians. The location of the beam splitter is sometimes considered as the detector location.

Detector elevation:

```
LAL_LIO_4K_DETECTOR_ELEVATION_SI          440.0
```

This is the height of the vertex floor from sea level. In principle this height parameter should also include the distance from floor to the center of the beam splitter ($\sim 2-3$ meter). Currently that is not included.

Azimuths of arms:

```
LAL_LIO_4K_DETECTOR_ARM_X_AZIMUTH_RAD    5.80120119264
LAL_LIO_4K_DETECTOR_ARM_Y_AZIMUTH_RAD    4.23039066080
```

These are angles of X and Y arms measured clock-wise (east of north) from the true north. This is usually measured. But one can also calculate the azimuth of a line from the GPS coordinates of the two ends of the line. This is an estimate and expected to approach the true value when the model of the earth used is perfect. For this estimate we have used WGS 84 model (latest) of the earth. These values were calculated using the website `https://geodesyapps.ga.gov.au/vincenty-inverse`

Altitude of arms:

```
LAL_LIO_4K_DETECTOR_ARM_X_ALTITUDE_RAD   0.0
LAL_LIO_4K_DETECTOR_ARM_Y_ALTITUDE_RAD   0.0
```

These are angles that the arms make with the local horizontal. By design, they arms are expected to be flat for LIO and hence these angles are set to zero.

Midpoint of arms:

```
LAL_LIO_4K_DETECTOR_ARM_X_MIDPOINT_SI    2000.00000000000
LAL_LIO_4K_DETECTOR_ARM_Y_MIDPOINT_SI    2000.00000000000
```

By design they are expected to be half of the length of each arm (which is 4000m).

Detector coordinate in earth-fixed frame:

LAL_LIO_4K_VERTEX_LOCATION_X_SI	1.34897099624e+06
LAL_LIO_4K_VERTEX_LOCATION_Y_SI	5.85742757731e+06
LAL_LIO_4K_VERTEX_LOCATION_Z_SI	2.12756900034e+06

These are the coordinates of the vertex in the earth-fixed Cartesian coordinate system. In an oblate ellipsoidal model of the earth, they are defined by the equations [2],

$$X_{\mathrm{E}} = (R + h)\ \cos(\phi)\ \cos(\lambda)$$

$$Y_{\mathrm{E}} = (R + h)\ \cos(\phi)\ \sin(\lambda)$$

$$Z_{\mathrm{E}} = \left(\frac{b^2}{a^2}\, R + h\right)\ \sin(\phi)$$

where a is the semi-major axis, b is the semi-minor axis and R is the local radius of curvature given by the expression,

$$R = \frac{a^2}{\sqrt{a^2 \cos\phi^2 + b^2 \sin\phi^2}}$$

In the latest WGS 84 earth model, $a = 6378137$ m and $b = 6356752.314$ m.

Orientation vectors of arms:

LAL_LIO_4K_ARM_X_DIRECTION_X	0.38496278183
LAL_LIO_4K_ARM_X_DIRECTION_Y	-0.39387275094
LAL_LIO_4K_ARM_X_DIRECTION_Z	0.83466634811
LAL_LIO_4K_ARM_Y_DIRECTION_X	0.89838844906
LAL_LIO_4K_ARM_Y_DIRECTION_Y	-0.04722636126
LAL_LIO_4K_ARM_Y_DIRECTION_Z	-0.43665531647

These are unit orientation vectors of the arms in the earth-fixed Cartesian frame. These are given by the equations [3],

$$
\begin{aligned}
\mathrm{X_{arm}} = [&-\cos(\psi_x)\cos(\omega_x)\sin(\lambda) - \sin(\psi_x)\cos(\omega_x)\cos(\lambda)\sin(\phi) + \sin(\omega_x)\cos(\phi)\cos(\lambda), \\
&\cos(\psi_x)\cos(\omega_x)\cos(\lambda) - \sin(\psi_x)\cos(\omega_x)\sin(\lambda)\sin(\phi) + \sin(\omega_x)\cos(\phi)\sin(\lambda), \\
&\sin(\psi_x)\cos(\omega_x)\cos(\phi) + \sin(\omega_x)\sin(\phi)]
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Y_{arm}} = [&-\cos(\psi_y)\cos(\omega_y)\sin(\lambda) - \sin(\psi_y)\cos(\omega_y)\cos(\lambda)\sin(\phi) + \sin(\omega_y)\cos(\phi)\cos(\lambda), \\
&\cos(\psi_y)\cos(\omega_y)\cos(\lambda) - \sin(\psi_y)\cos(\omega_y)\sin(\lambda)\sin(\phi) + \sin(\omega_y)\cos(\phi)\sin(\lambda), \\
&\sin(\psi_y)\cos(\omega_y)\cos(\phi) + \sin(\omega_y)\sin(\phi)]
\end{aligned}
$$

where $\psi_{x,y}$ are angles arms make with the (North of) East and $\omega_{x,y}$ are angles arms make with local horizontal. ψ is related to azimuth by the expression ψ = mod(450 - azimuth, 360), and $\omega_{x,y}$ = altitude in radian. For LIO,

$$\psi_x = 117.6157^\circ$$

$$\psi_y = 207.6165^\circ$$

and

$$\omega_{x,y} = 0$$

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References

[1] https://lscsoft.docs.ligo.org/lalsuite/lal/_l_a_l_detectors_8h_source.html

[2] LIGO Document - T980044 (https://dcc.ligo.org/LIGO-T980044/public)

[3] LIGO Document - T010110 (https://dcc.ligo.org/LIGO-T010110/public)