CALIBRATING THE LIGO INTERFEROMETERS: THIRD TIME'S A CHARM

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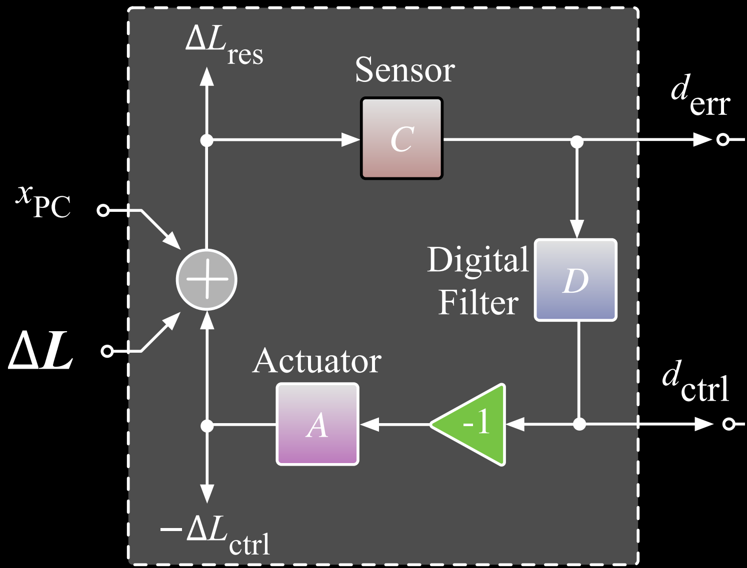
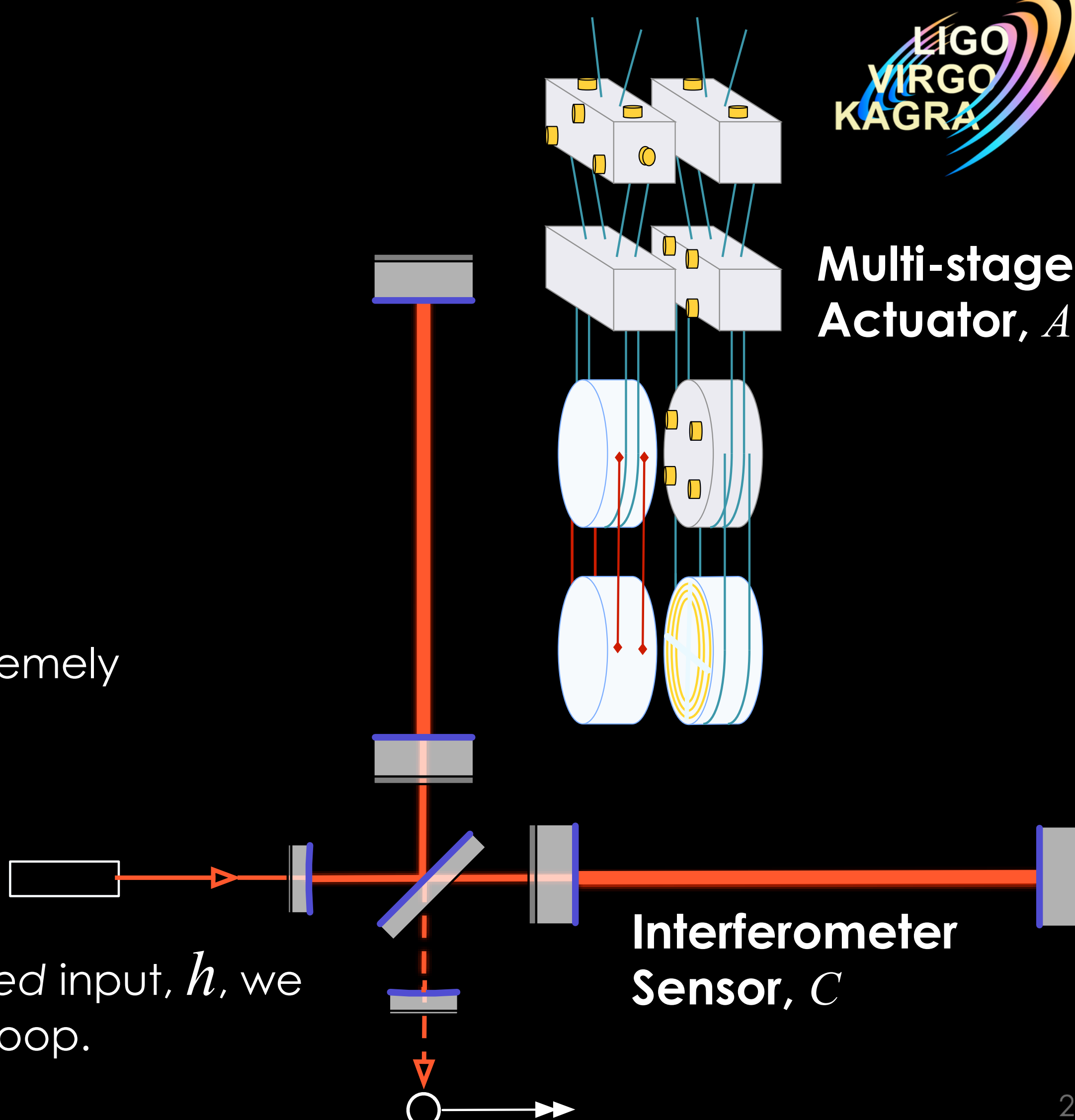
THIS MORNING: THE DARM LOOP





Interferometer is non-linear unless extremely well controlled.

Detector readout is fed to control differential arm lengths.

To produce an estimate of *uncontrolled* input, h, we must have an exquisite model of the loop.

THIS MORNING: ABSOLUTE REFERENCE

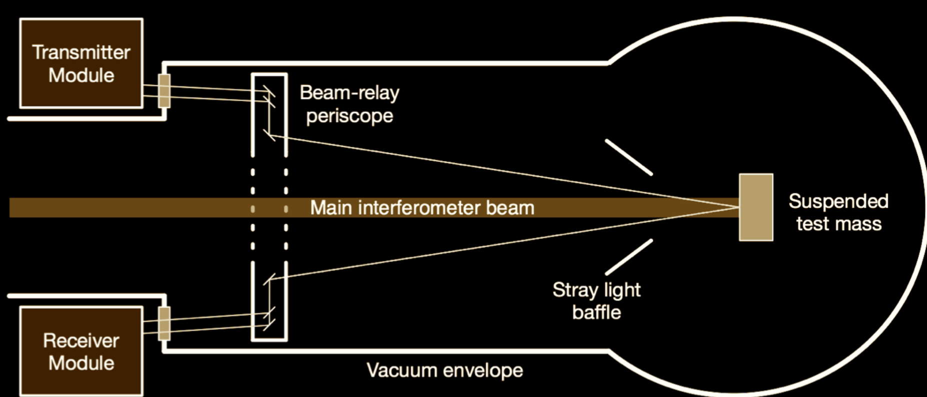
Let's assume that …

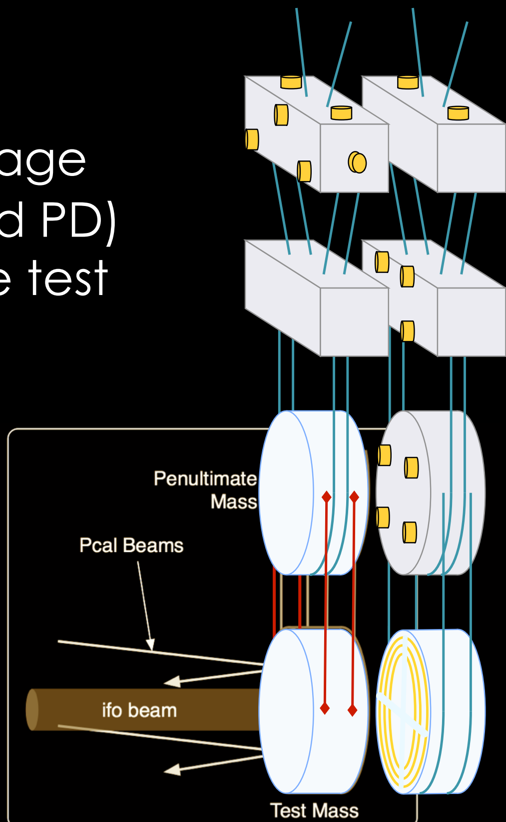
- Sudarshan told you everything about using **photon radiation pressure** as our absolute reference,

- the PCAL team has delivered us a perfect reference (no systematic error, but still with statistical uncertainty)

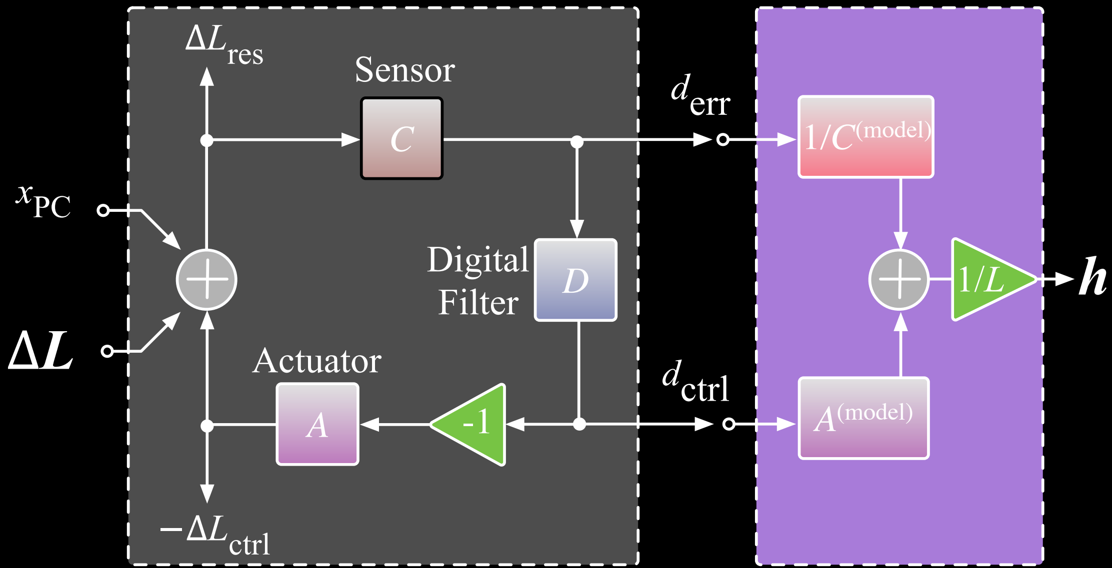
- we have digital signal, pd_{pc} (the digitized voltage from receiver module's integrating sphere and PD) that's been converted to displacement of the test mass, x_{pc} .

O2 Uncertainty = 0.76 %
O3 Uncertainty = 0.5%





OVERALL UNCERTAINTY



$$h = R \; d_{err}$$

$$R \equiv \frac{1}{L} \frac{1+G}{C}$$

$$G = A\,D\,C$$

$$h\,L = \frac{1}{C^{(model)}} d_{err} + A^{(model)} d_{ctrl}$$

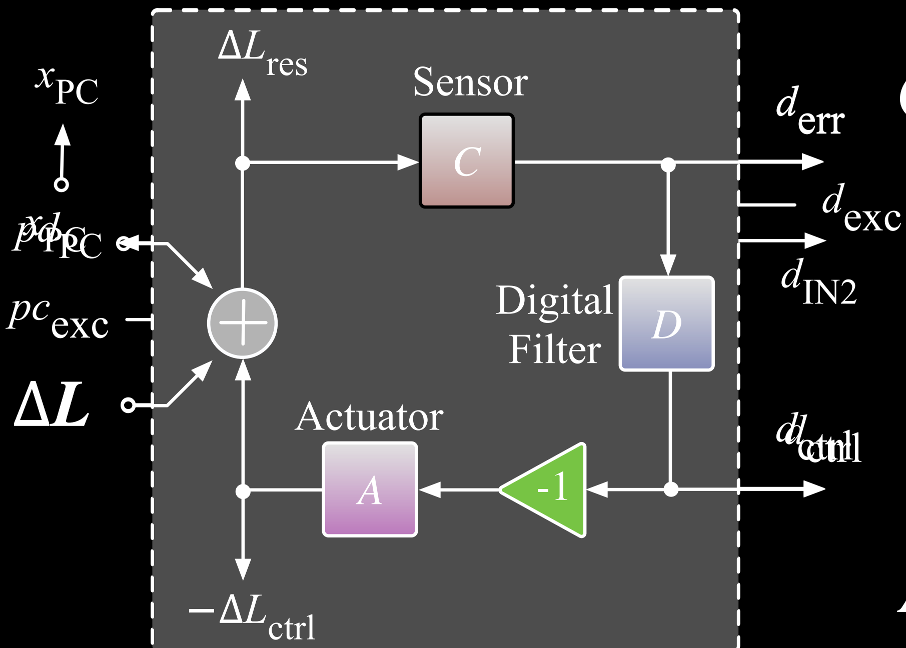
The **model**, and therefore the estimated detector input, has **<u>uncertainty and systematic error</u>**:

$$\partial h^2 \approx \partial R^2 \approx \left(\frac{1}{1+G}\right)^2 \left(\frac{\partial C}{C}\right)^2 + \left(\frac{G}{1+G}\right)^2 \left(\frac{\partial A}{A}\right)^2$$

+ Systematic Errors

THE PROBLEM GETS HARDER…

With all loops closed and the detector running at its best sensitivity, we request a series of **in-loop** excitations to obtain direct measurements of the sensor and actuator



$$C = \left(\frac{d_{exc}}{d_{IN2}}\right) \times \left(\frac{d_{err}}{pc_{exc}}\right) \times \frac{x_{pc}}{pd_{pc}}$$

Complex Transfer Functions w/ Full IFO

Absolute Scale Factor

$$A_i = \left(\frac{d_{err}}{a_{i,exc}}\right) \times \left(\frac{pc_{exc}}{d_{err}}\right) \times \frac{pd_{pc}}{x_{pc}}$$

Our measurements of C and A are *ratios of complex transfer functions*

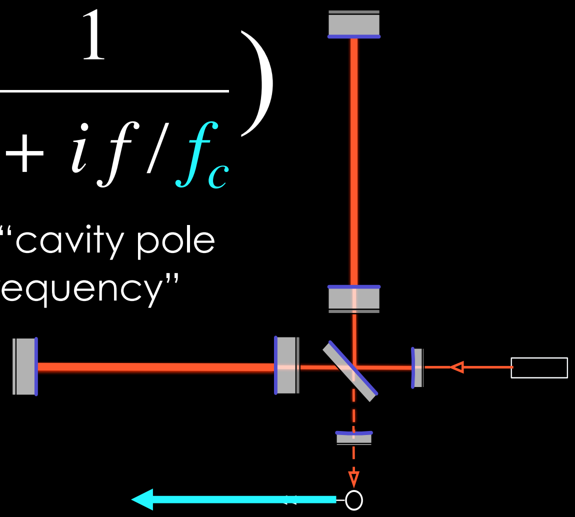
>> *the* ***frequency-dependent***, ***magnitude and phase*** *are important for A and C*

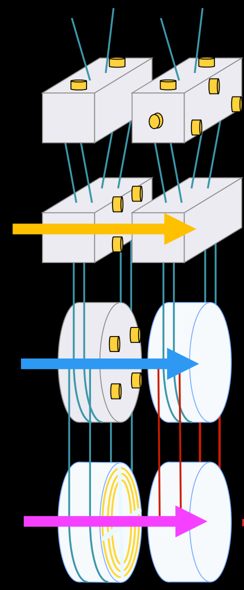
THE SIMPLE VERSION…

- Evan mentioned the "free" parameters that dominate the uncertainty in A and C.
- Here's what you normally tell people in a talk:

$$\text{gain} \times \left(\frac{1}{1 + i f / f_c}\right)$$

f_c ="cavity pole frequency"





gain x $1/f^6$

gain x $1/f^4$

gain x $1/f^2$

Why focus on these?

- (they're "easily" accessible to the "average" audience)
- They can **only** be measured with the interferometer running.

…BUT THERE'S MORE

IFO measurements contain more than just simple functions.

There are **WAY** more details in

- C's analog-to-digital conversation path
- A's digital-to-analog path

that must be measured and modelled, in order to obtain a "clean" result that can be fit for free parameters

For C (two paths of):

- Detuning of optical plant
- PD trans-impedance amplifiers
- analog whitening filters,
- analog anti-aliasing filters,
- computational delays,
- digital down-sampling filters

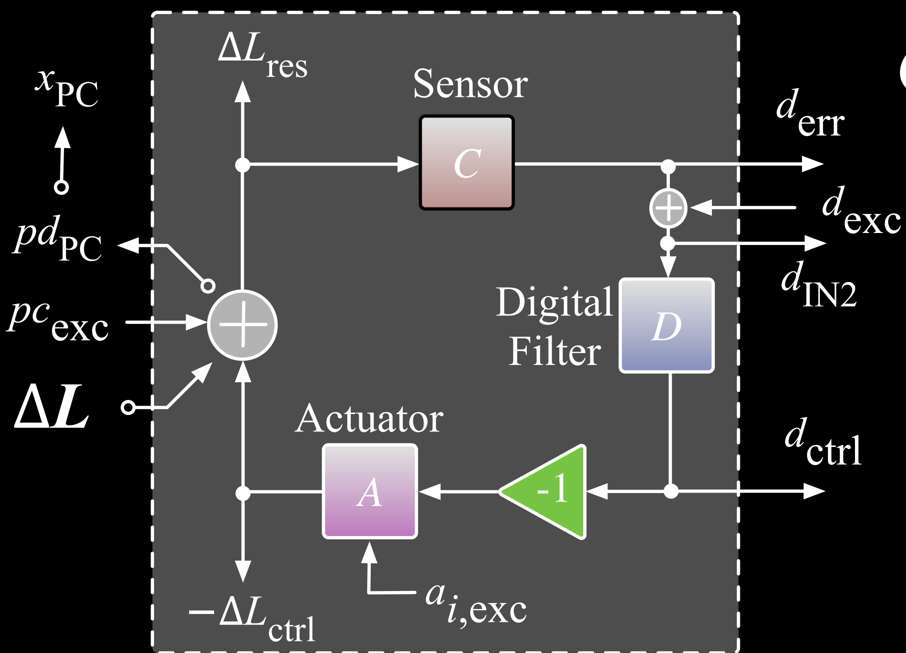
For A (three stages of…):

- digital up-sampling filters,
- computational delays
- analog anti-imaging filters
- analog low-pass filters
- trans-conductance/voltage drivers
- drive signal to force actuators
- non-trivial force to displacement dynamics of multi-stage pendula

… all must be included with **systematic error** ***quantifiably negligible*** in the model

ASSUMING WE'VE GOT THAT UNDER CONTROL…

Back to the full interferometer measurements…



$$C = \frac{d_{exc}}{d_{IN2}} \times \frac{d_{err}}{pc_{exc}} \times \frac{x_{pc}}{pd_{pc}}$$

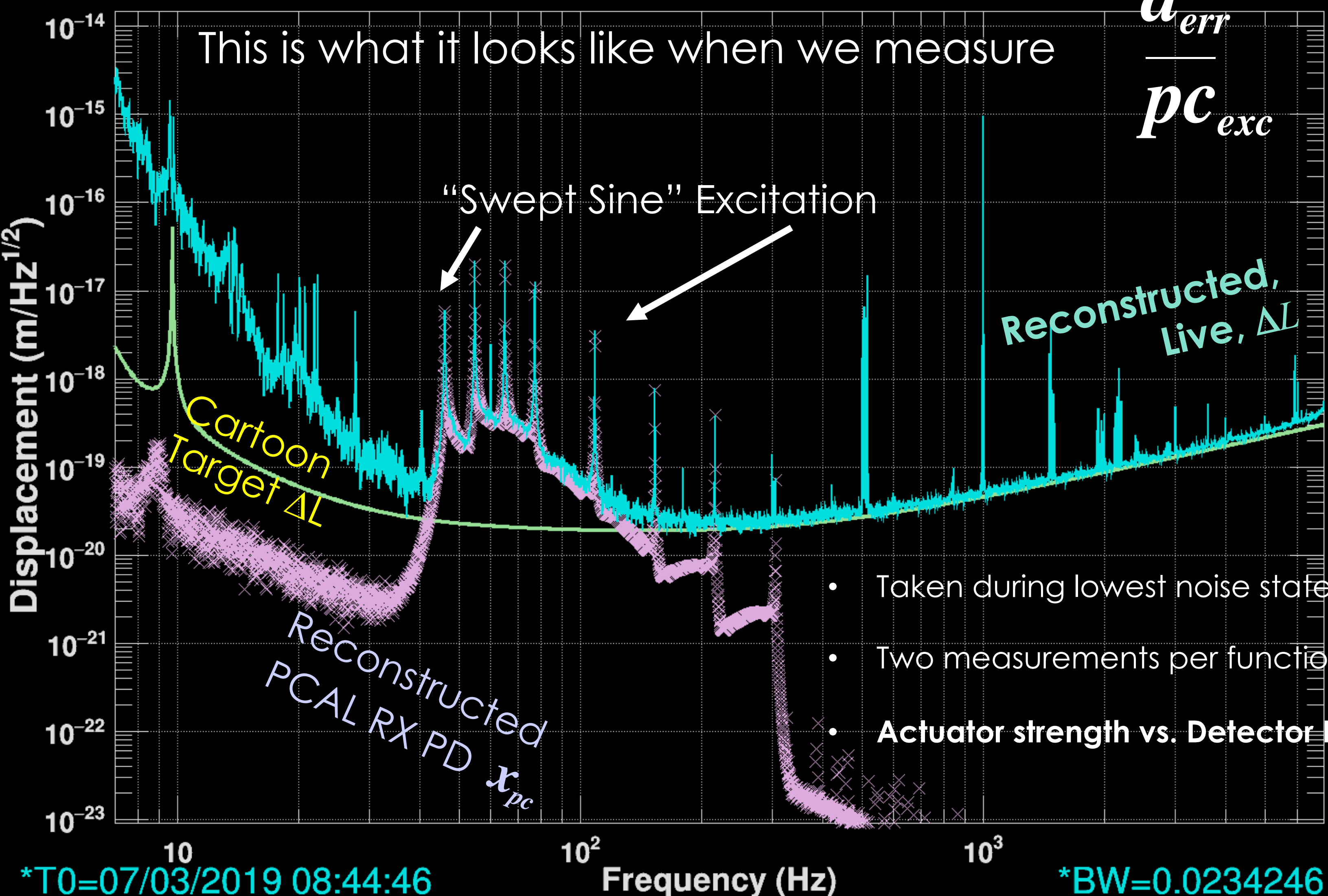
Complex Transfer Functions w/ Full IFO

Absolute Scale Factor

$$A = \frac{d_{err}}{a_{i,exc}} \times \frac{pc_{exc}}{d_{err}} \times \frac{pd_{pc}}{x_{pc}}$$

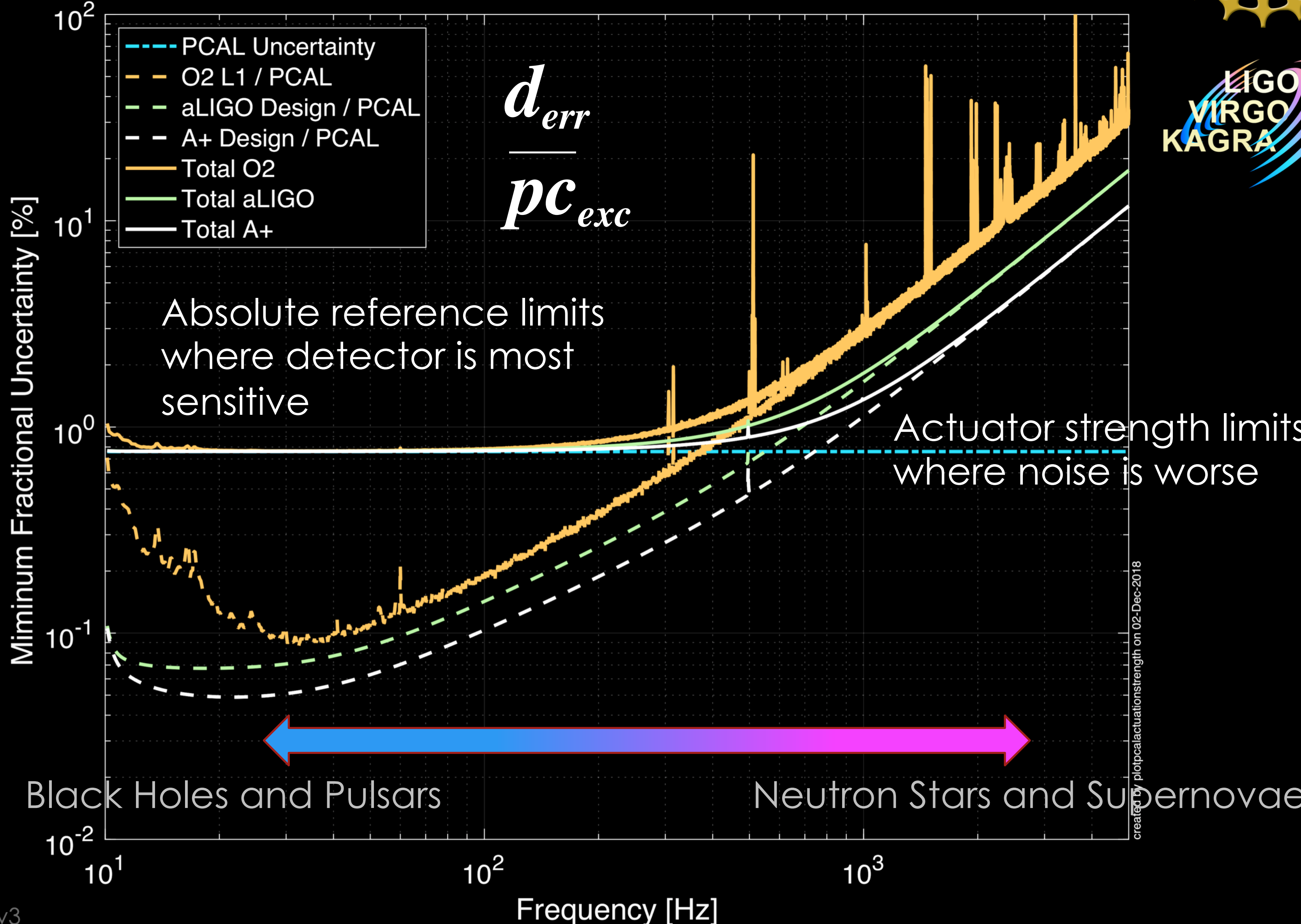
EXAMPLE MEASUREMENT

$$\frac{d_{err}}{pc_{exc}}$$

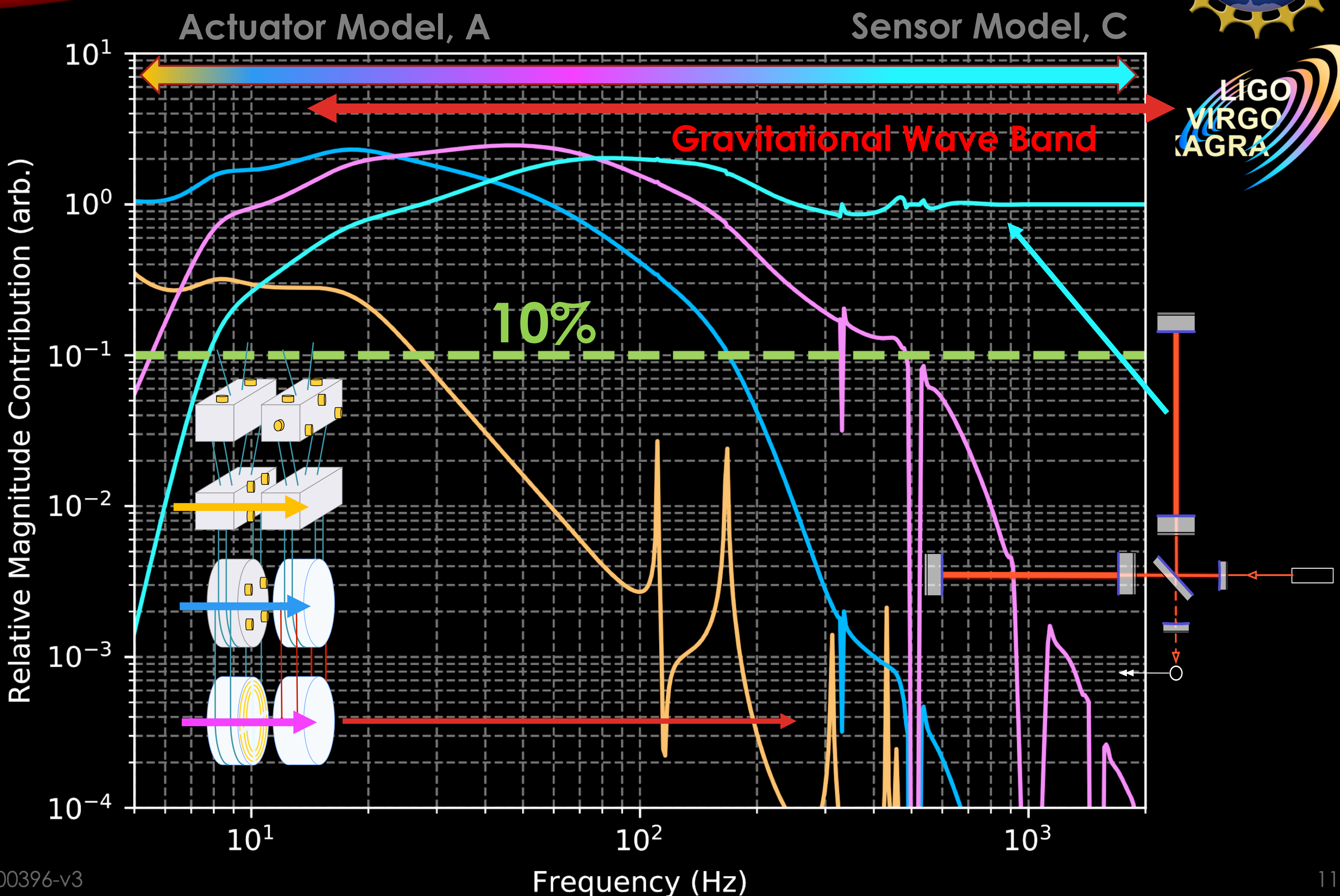


- Taken during lowest noise state, in loop
- Two measurements per function
- **Actuator strength vs. Detector Noise limited!**

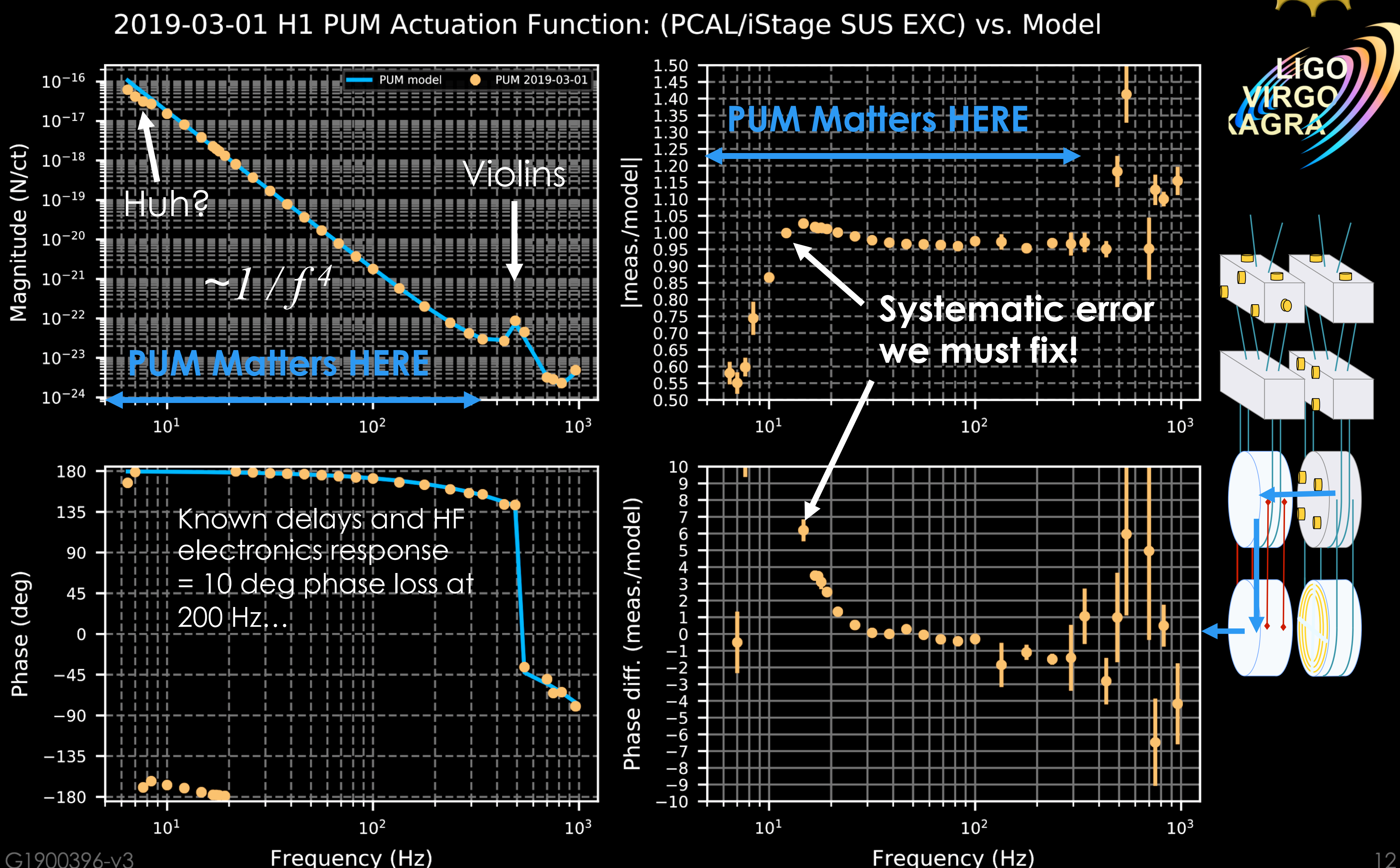
LIMITATIONS



UNCERTAINTY COMPONENT BREAKDOWN

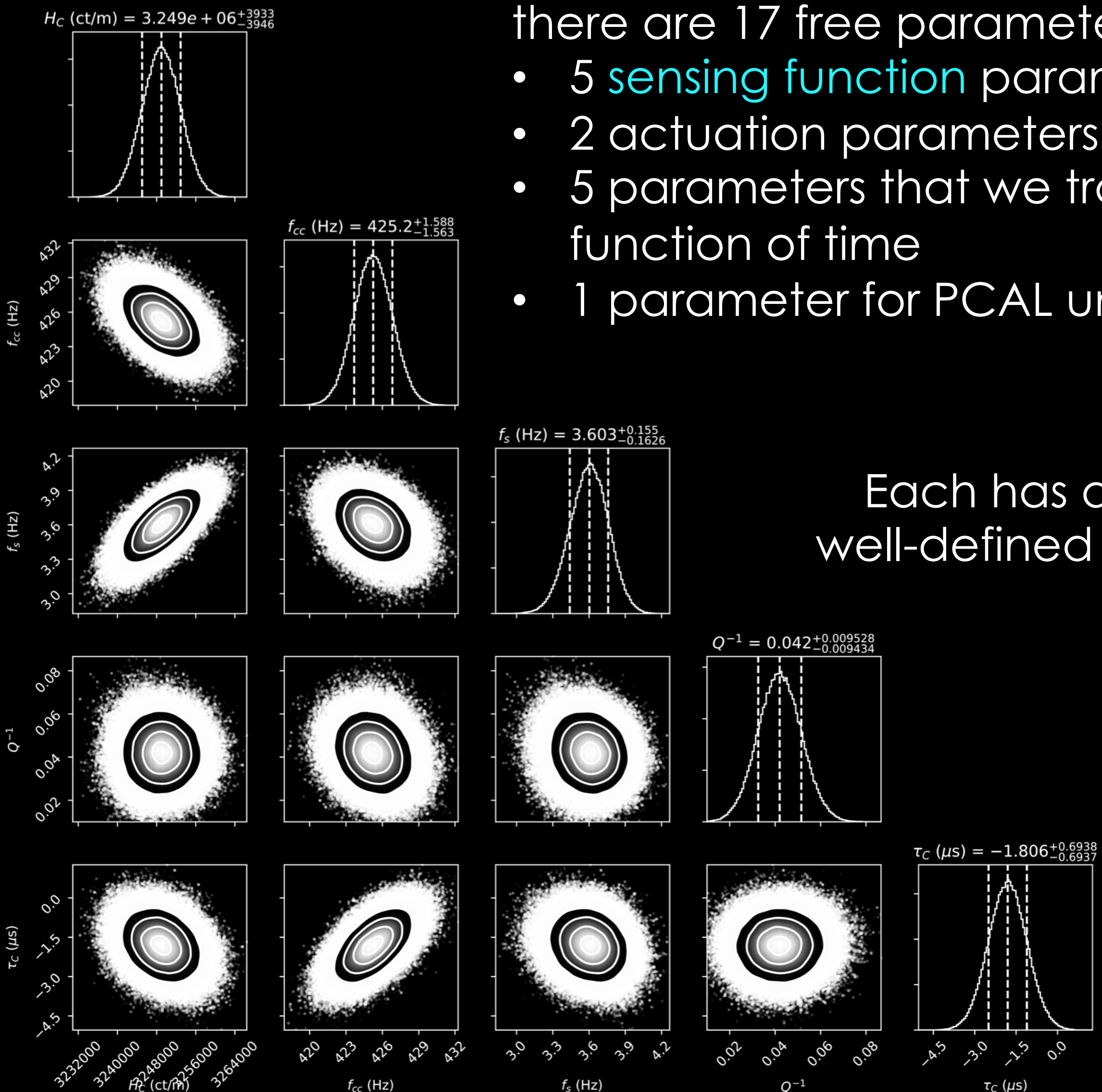


CURRENT STATUS, *A*



STATISTICAL UNCERTAINTY

there are 17 free parameters per IFO:
- 5 sensing function parameters
- 2 actuation parameters for each stage
- 5 parameters that we track as a function of time
- 1 parameter for PCAL uncertainty



example results for 5 sensing function parameters

Each has a ***posterior distribution***, and a well-defined contribution to the response

$$R = \frac{1}{L}\,\frac{1 + ADC}{C}$$

So, we ***numerically evaluate*** the uncertainty on $\boldsymbol{R}$

HOW TO HANDLE UNKNOWN SYSTEMATIC ERROR?

Instead of concatenating all the residual systematic errors from every single part of each function, we divide out the model from our measurement

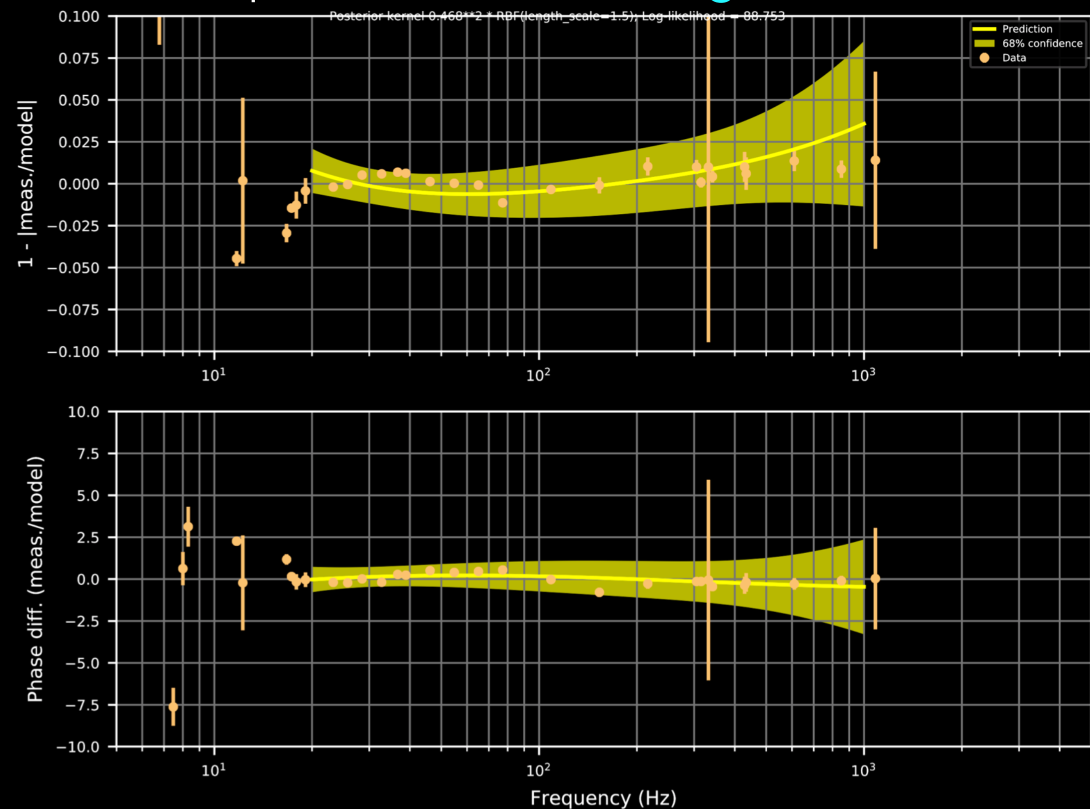
Remaining frequency dependence is **unknown systematic error**, for which we can fit

Gaussian Process Regression (GPR):

It's black magic, like any other transfer function fitting program

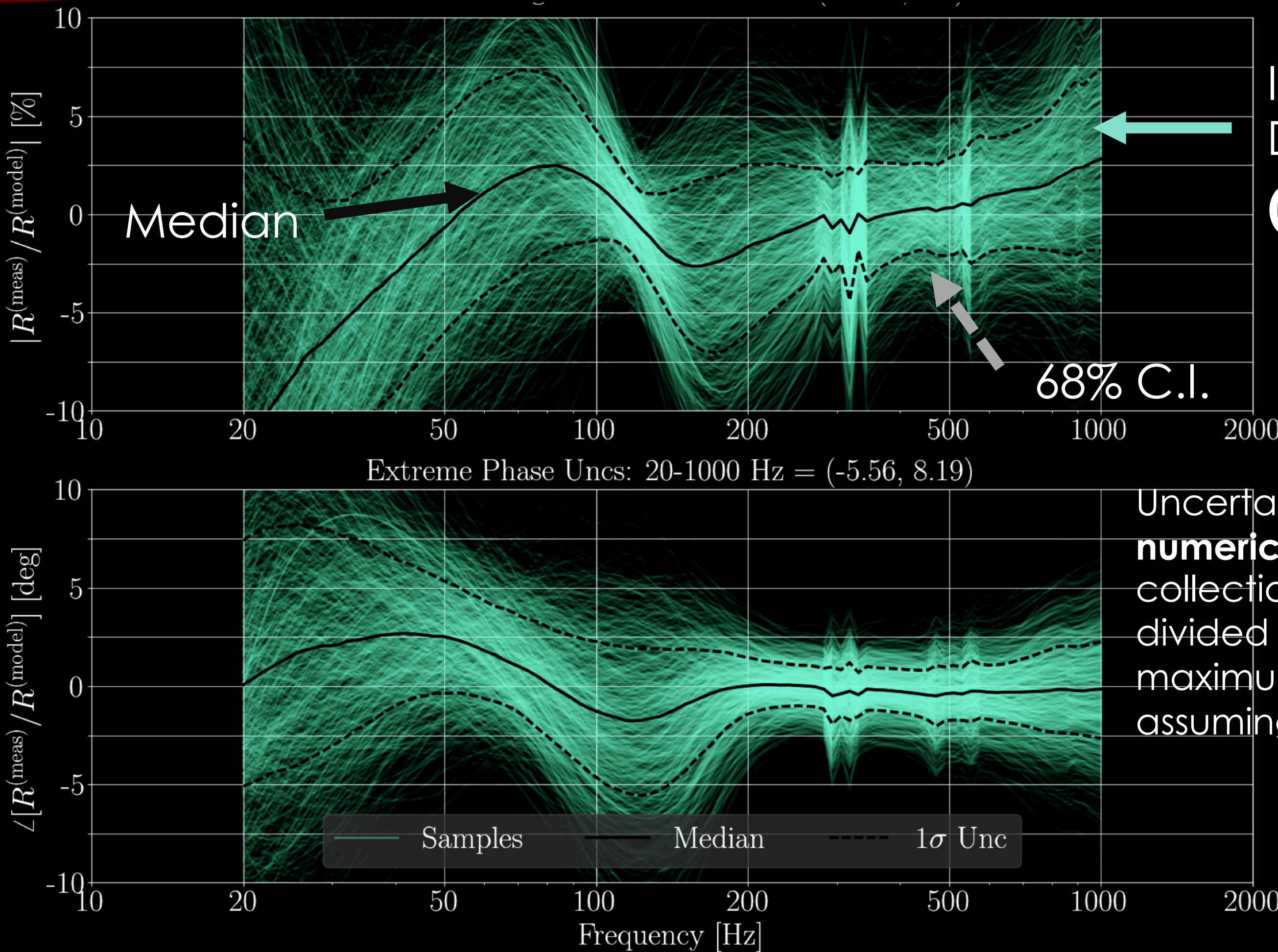
but importantly it gives you a **posterior distribution** of curves, do be sampled for numerical evaluation of total uncertainty in R

example GPR fit to Sensing Residual



4 more **distributions** of free parameter ***functions***…

THE FINAL ANSWER (EXAMPLE)



Individual Draws of $(R + \partial R)_i / R$

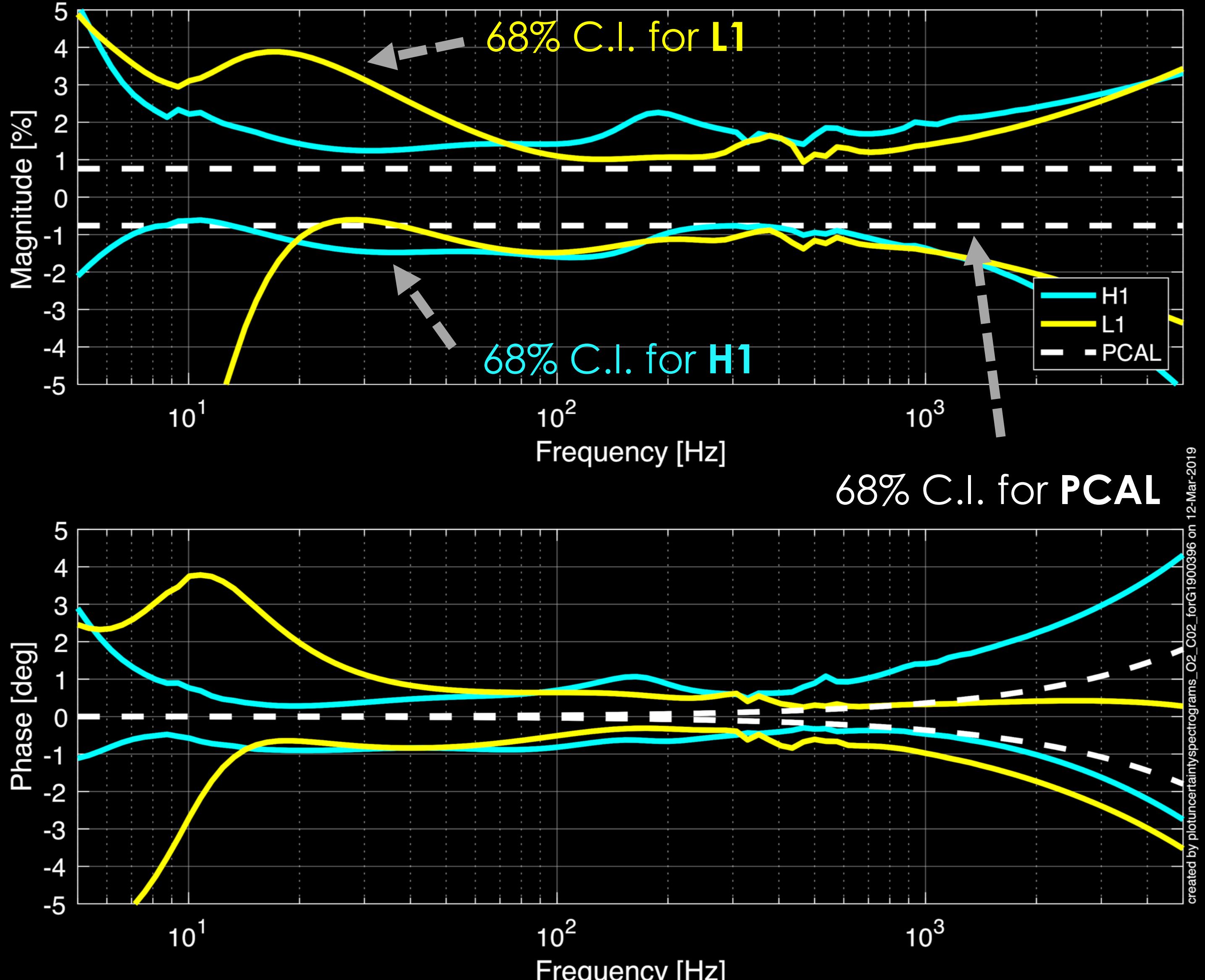
Uncertainty is informed by **individual, numerically evaluated draws** from the collection of **posterior distributions**, divided by "**nominal**" model (model of maximum a posteriori values, and assuming no systematic error)

$$\partial h^2 \approx \partial R^2 \approx \left(\frac{1}{1+G}\right)^2 \left(\frac{\partial C}{C}\right)^2 + \left(\frac{G}{1+G}\right)^2 \left(\frac{\partial A}{A}\right)^2$$

+ Systematic Errors

The **68% confidence interval** bounds at each frequency are ***the*** response function's **frequency dependent uncertainty and systematic error**.

WE WERE ABLE TO GET TO "FUNDAMENTAL" LIMIT LAST TIME…



WHERE TO?

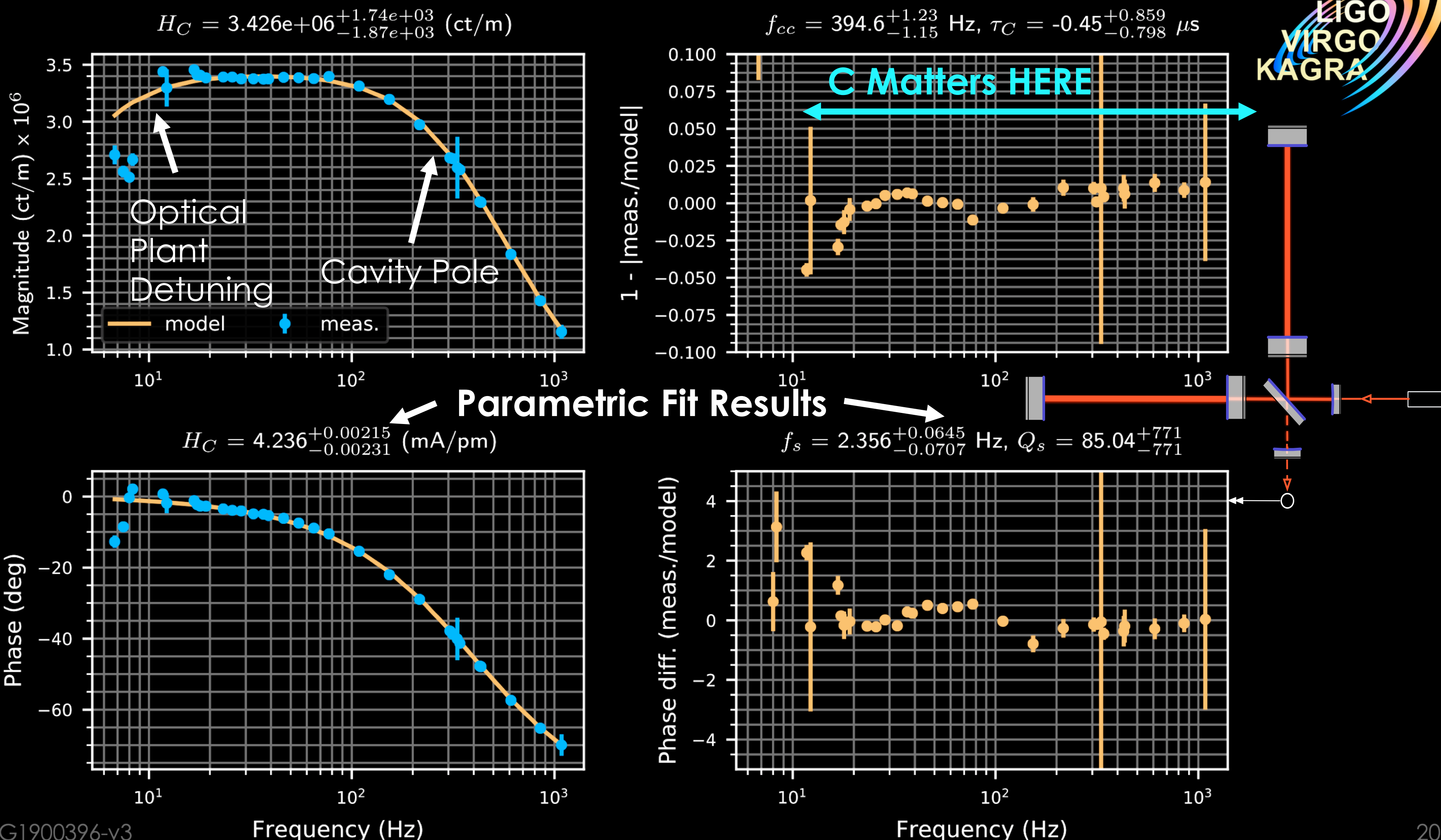
- We're using photon radiation pressure to great success.
- Improving PCAL uncertainty will **directly improve** interferometer response uncertainty, and thus **astrophysical parameter uncertainty**
- We're (on our 3rd round of) reducing as much of our other systematic error such that we can reach the PCAL's "fundamental" limit (again)
- Coordinate standards in the global GW Network
- Let us know if there's a systematic error in the estimate of the power…

THANK YOU

BONUS MATERIAL

CURRENT STATUS, *C*

H1 sensing function measurement: 2019-03-07



WHY IS IT DIFFERENT BETWEEN RUNS?

The interferometers are *constantly* evolving between (and during!) runs to improve the noise, and due to reality of experiment!

Change for Noise between O2 and O3	Consequence For Calibration
New Test Masses	new force to displacement dynamical model
Higher power for O3	more complex interferometer response
Loss on optics accrued from vent	more complex interferometer response
Better sensor electronics for sensitivity features	New measurement of electronics
Better actuator electronics to reduce impact of DAC noise	New measurement of electronics
One of the O2 actuators are broken	New actuator scheme model

The calibrator's job is never finished!

WHY BOTHER BEING FULLY BAYESIAN?

- In detection era, it was safe to just quote a maximum in magnitude and phase “XX % / YY deg,” because *detection* is relatively insensitive to calibration uncertainty

- In the observational era, astrophysical parameters depend ~proportionally to calibration uncertainty.

- **Astrophysical parameter estimation** is fully Bayesian, with its own set of 14 parameters

- New, on-going project: study the **subtle interactions** between **astrophysical parameters** and **physical interferometer parameters**.
  - Can better marginalize (integrate) over Gaussian distribution of real uncertainty, instead of naïve box-car distribution
  - Can retrace steps to ask “I want to improve estimation of astro-param M, so please improve estimation of interferometer-param N”

LIGO CALIBRATION REFERENCES

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