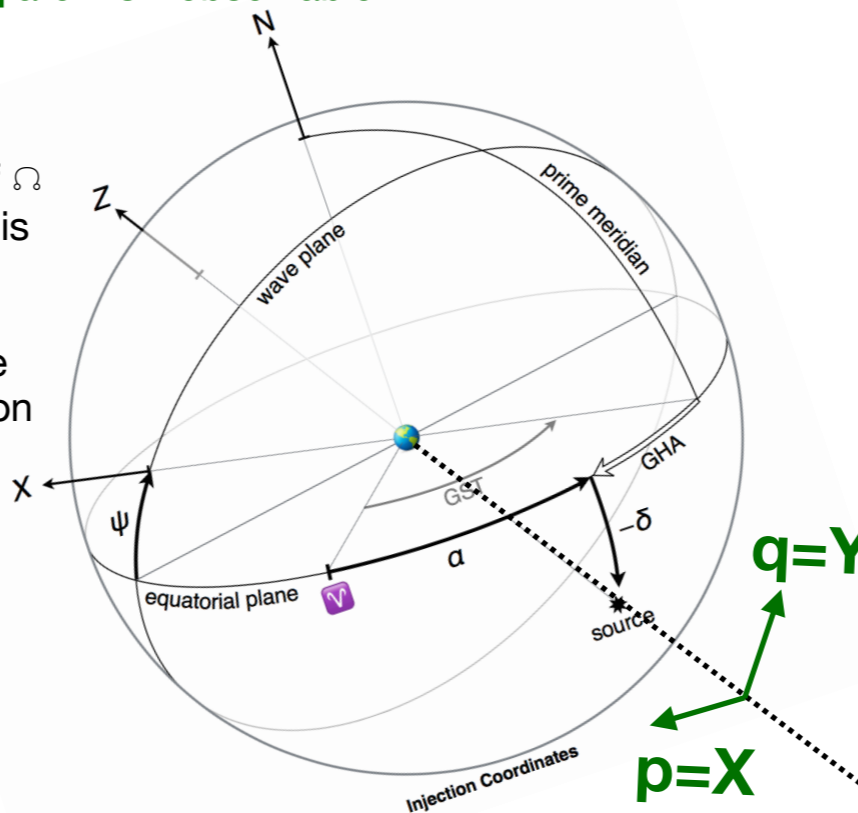


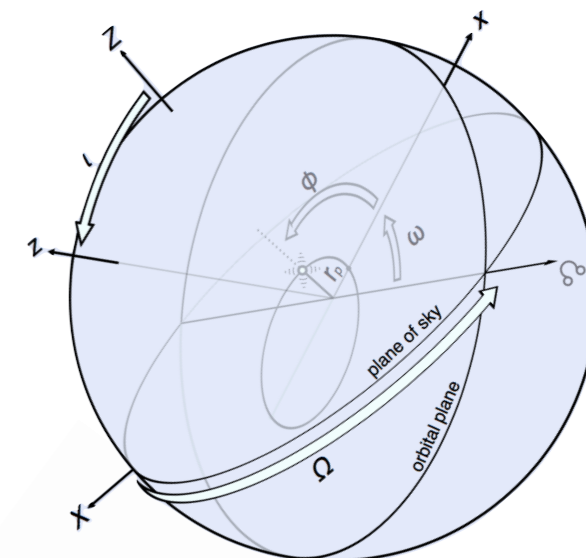
\mathbf{p} , \mathbf{q} parameterise plane orthogonal to \mathbf{N} (plane of the sky), and define how h_{ij} is decomposed into h_+ and h_x . \mathbf{p} and \mathbf{q} are NOT observable.

- The line Ω indicates semi-major axis of orbital trajectories, projected onto the sky
- The sum $\Psi + \Omega$ indicates the overall rotation of Ω relative to the equatorial plane of the Earth. This *is* observable, even though Ψ and Ω individually are not.
- If $\Psi + \Omega = 90^\circ$, then the semi-major axis of the projected orbital ellipse is parallel to the rotation axis of the Earth, i.e. points north-south



lal_inference_o2

- spins defined in source frame (x', y', z')
- $S_x' = \mathbf{S} \cdot \hat{\mathbf{x}} = \text{direction of periaapsis}$ [link]
 $S_z = \mathbf{S} \cdot \hat{\mathbf{z}} = \mathbf{L}'$
- Usually $\mathbf{X}' = \mathbf{x}' = \Omega$, i.e. $\Omega' = \omega' = 0$ and $\mathbf{z}' \in \text{span}(\mathbf{Y}', \mathbf{Z})$. Then S_x', S_y' depend on choice of \mathbf{X}' , i.e. on (some of) $\{a, \delta, \Psi\}$
- For [ChooseTD] 'phiRef = GW phase at end', i.e. $\Phi' = ???$



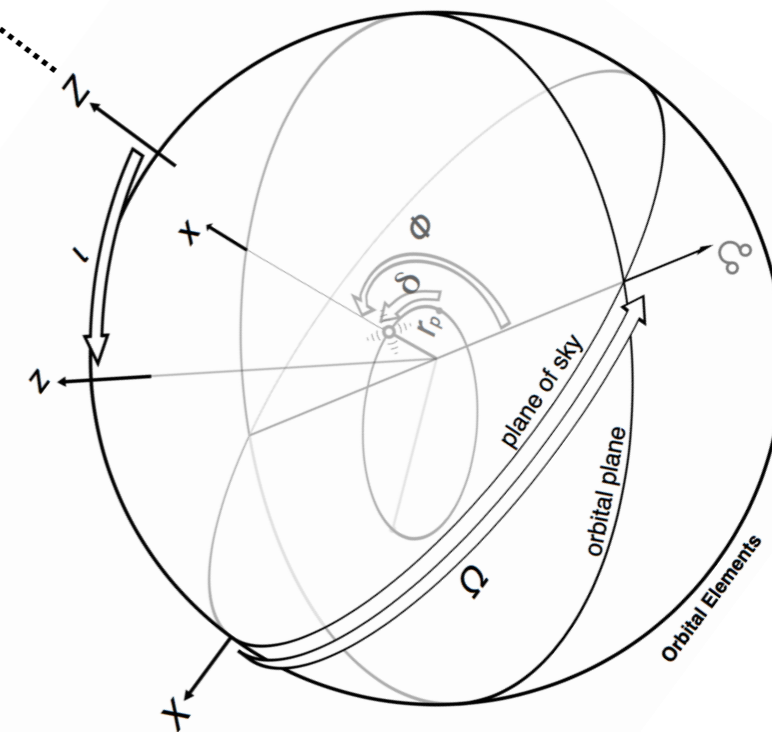
https://git.ligo.org/lscsoft/lalsuite/blob/lal_inference_o2/lalsimulation/src/lalsiminspiral_orbitelements.svg
https://git.ligo.org/lscsoft/lalsuite/blob/lal_inference_o2/lalsimulation/src/LALSimInspiral.h

master vs lal_inference_o2

- $\mathbf{Z} = \mathbf{Z}'$, $\mathbf{L} = \mathbf{L}'$, $\Omega = \Omega'$
- $\mathbf{X}' = \Omega = \mathbf{Y}$, i.e. $\Omega' = 0$, $\Omega = \pi/2$, i.e. different polarization basis $\{\mathbf{p}, \mathbf{q}\}$
- $\Psi' = \Psi + \pi/2$
 - change in $\mathbf{p}, \mathbf{q} \Rightarrow$ change in polarization
 - preserves $\Psi' + \Omega' = \Psi + \Omega$
- $\{x, y\}$ & $\{x', y'\}$ are linear combinations of each other:
 - Expand $\Omega = \mathbf{x}' = \mathbf{Y}$:
 - $\mathbf{x}' = \cos\Phi \mathbf{x} - \sin\Phi \mathbf{y}$, $\mathbf{y}' = \sin\Phi \mathbf{x} + \cos\Phi \mathbf{y}$
- S_x, S_y are same linear combinations
 - $S_x' = \cos\Phi S_x - \sin\Phi S_y$
 - $S_y' = \sin\Phi S_x + \cos\Phi S_y$
- Because $\omega = 0$, $\Phi = \Phi'$. But Φ' inconsistently defined by lalinf_o2 waveform models, so must verify for each model whether same phiRef yields $\Phi = \Phi'$.
- To backport code on master to lal_inference_o2:
 1. Set $S_x = \cos\Phi S_x' + \sin\Phi S_y'$,
 $S_y = -\sin\Phi S_x' + \cos\Phi S_y'$, and $S_z = S_z'$
 2. Call master's ChooseTDWaveform w/ longAscNodes=0 (i.e. $\Omega = \pi/2$)
 3. Un-rotate polarizations by $\pi/2$, $(h_+, h_x) = -(h_+, h_x)$

master

- spins defined in source frame (x, y, z)
- $S_x = \mathbf{S} \cdot \hat{\mathbf{x}} = \text{body2} \rightarrow \text{body1}$ [link]
 $S_z = \mathbf{S} \cdot \hat{\mathbf{z}} = \mathbf{L}$
- $\Omega = \text{longAscNode} + \pi/2$,
i.e. for longAscNodes=0, $\mathbf{Y} = \Omega$, $\mathbf{z} \in \text{span}(\mathbf{X}, \mathbf{Z})$
[this choice was already implemented by many approximants, despite its inconsistency with the 'old' documentation]
- $\mathbf{Z} = \sin(i)\sin(\Phi) \mathbf{x} + \sin(i)\cos(\Phi) \mathbf{y} + \cos(i) \mathbf{z}$
 $= \hat{\mathbf{r}}(\theta=i, \varphi=\pi/2-\Phi)$ [xyz-spherical coords]
- For [ChooseTD] phiRef= Φ
- ChooseTD defines location of periastron by mean-anomaly = $2\pi(t - T_{\text{last peri}})/(T_{\text{next peri}} - T_{\text{last peri}})$, and not by angle δ .



https://git.ligo.org/lscsoft/lalsuite/blob/master/lalsimulation/lib/lalsiminspiral_orbitelements.svg