Physical-Statistical Analysis of Scatter in Febry-Pérot Arm Cavity of aLIGO

Wenxuan Jia, LIGO SURF ’18

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Talk Outline

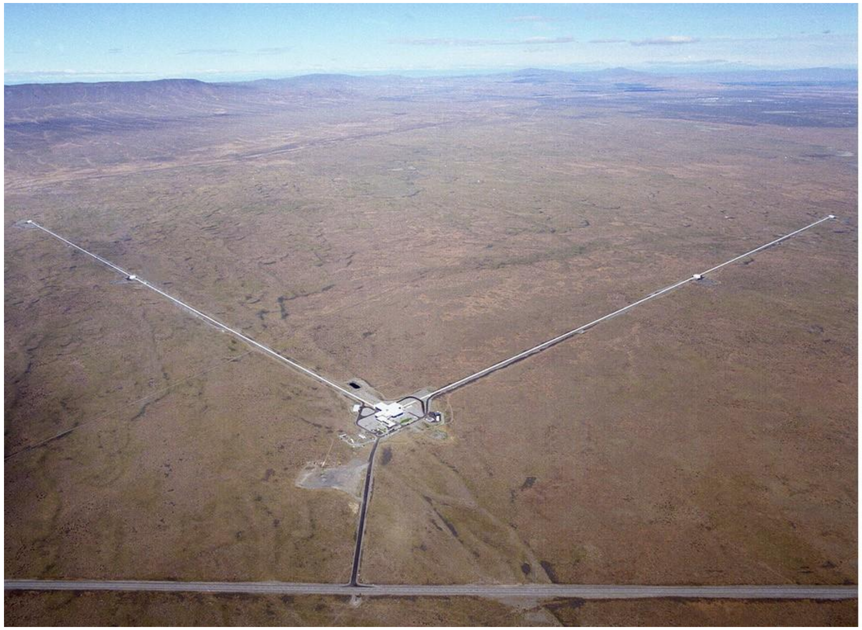
- Introduction
  - Advanced LIGO
  - Scatter characterization in Fabry-Perot arm cavity
  - Scatter measurements
- Analysis of Steady-State Scatter
  - Scatter during locks
  - Scatter change through Observation Run 2 (11/30/16 – 8/25/17)
  - Comparison with simulation
- Analysis of Transient Scatter
  - Coupling with beam position
- Regression Analysis
  - Train model to predict future beam positions

LIGO

Laser Interferometer Gravitational-Wave Observatory:

- Measuring the space-time ripple (strain $\sim 10^{-21}$)
- Twin detectors

LIGO Hanford, WA

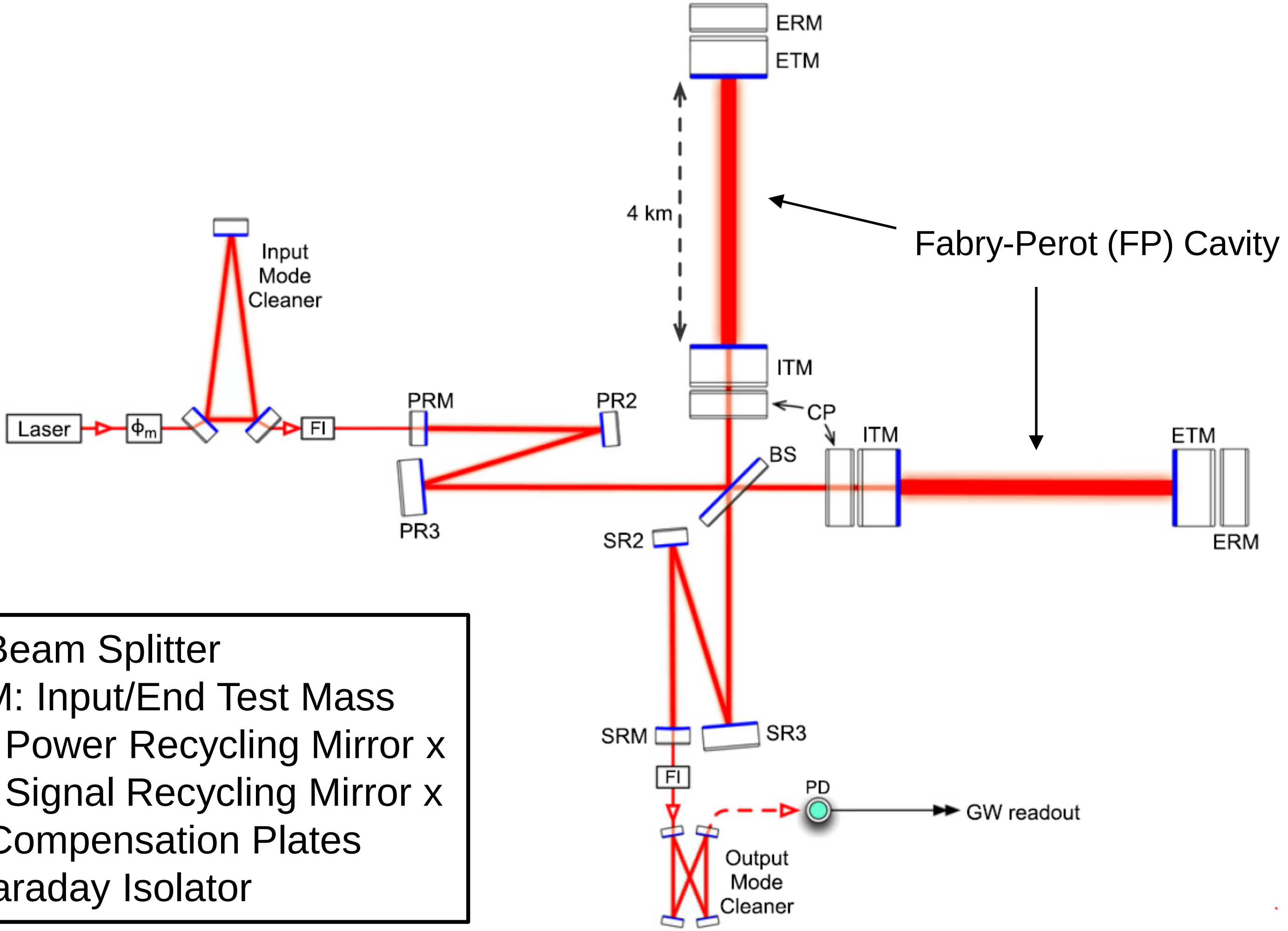


LIGO Livingston, LA

(*Credit: LIGO Lab*)

Advanced LIGO (aLIGO)

A dual-recycled interferometer:

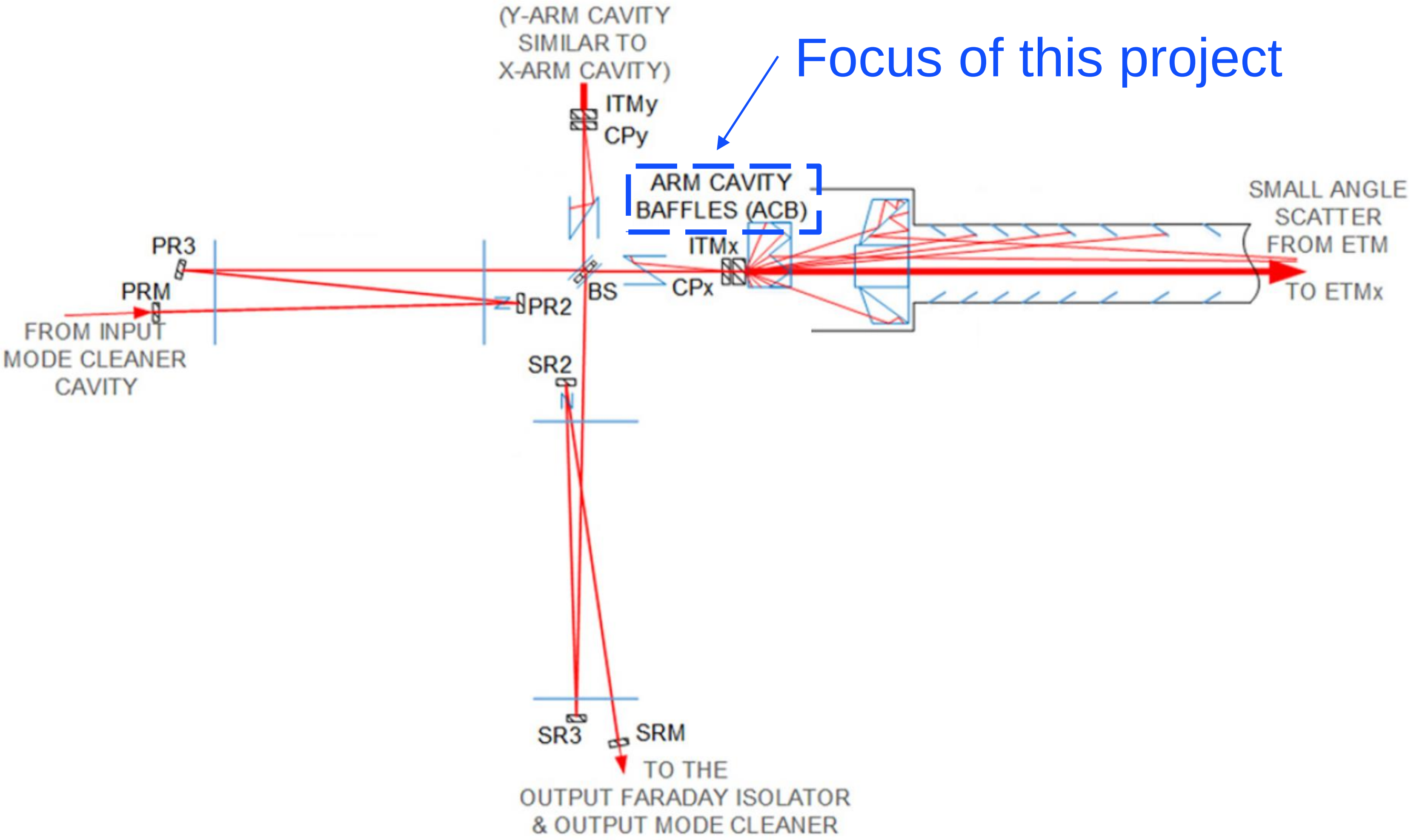


BS: Beam Splitter
I/ETM: Input/End Test Mass
PRx: Power Recycling Mirror x
SRx: Signal Recycling Mirror x
CP: Compensation Plates
FI: Faraday Isolator

(*Credit: LIGO Scientific Collaboration*)

Scattered Light Baffles in aLIGO

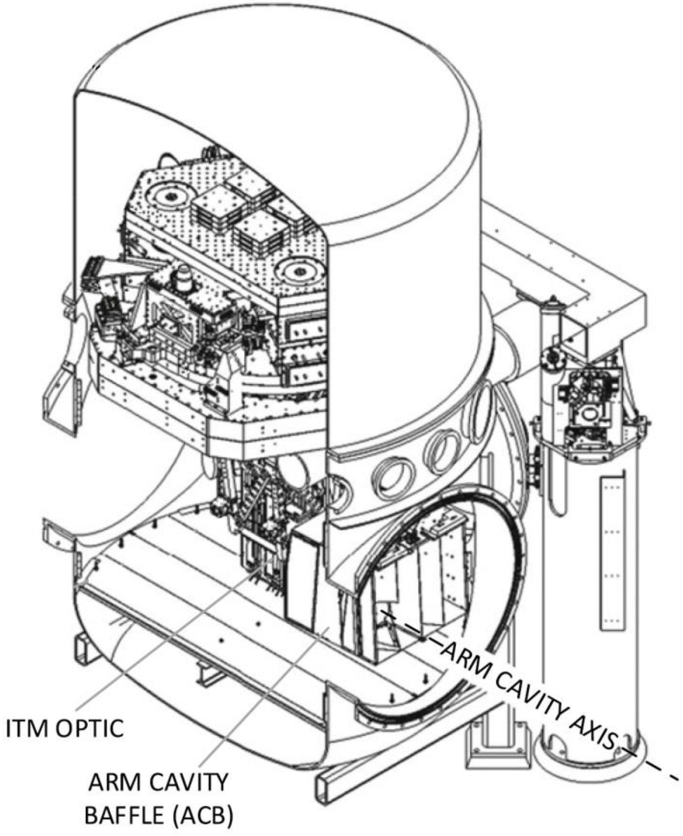
Various baffles are designed to block and measure scatter.

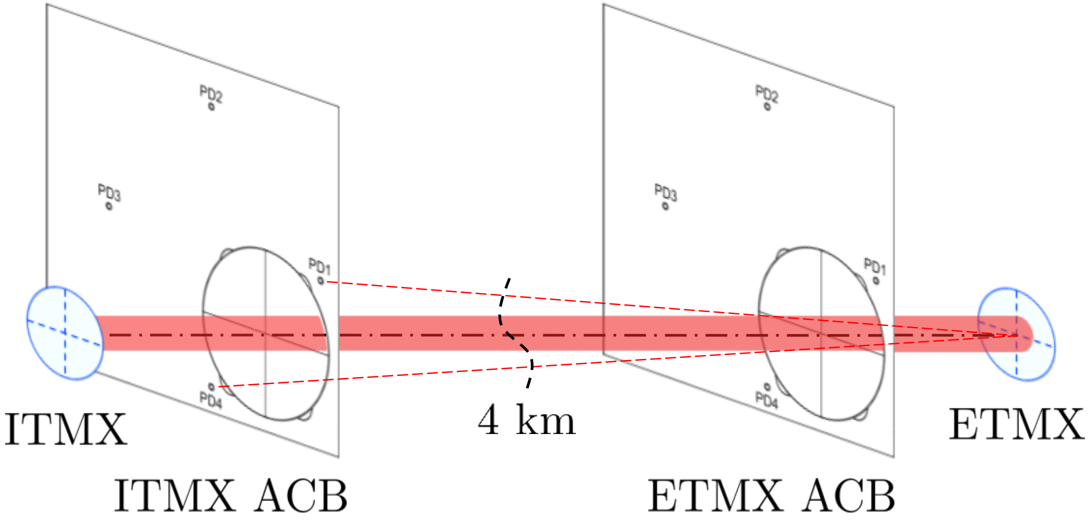


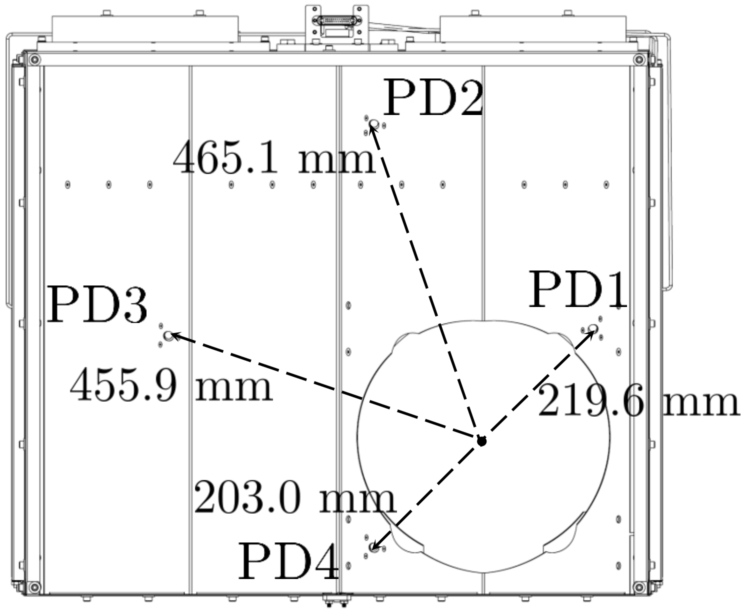
(*Credit: LIGO Scientific Collaboration*)

Scatter Measurement

ITMX chamber (cut view)

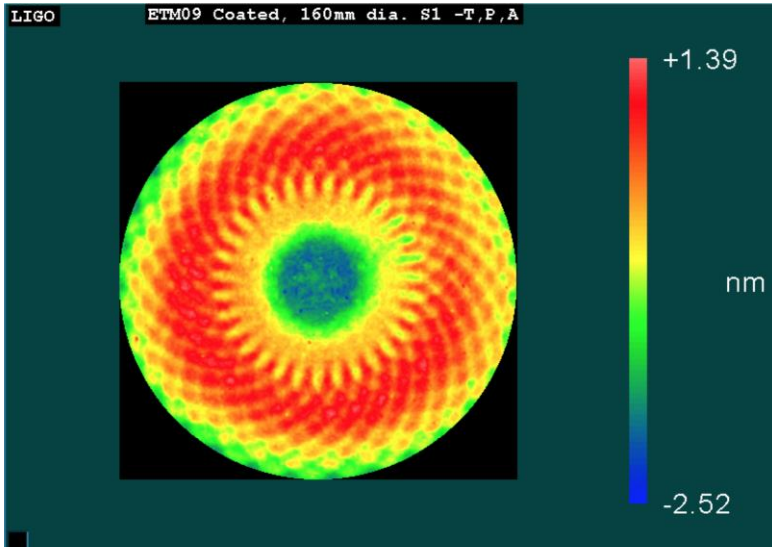


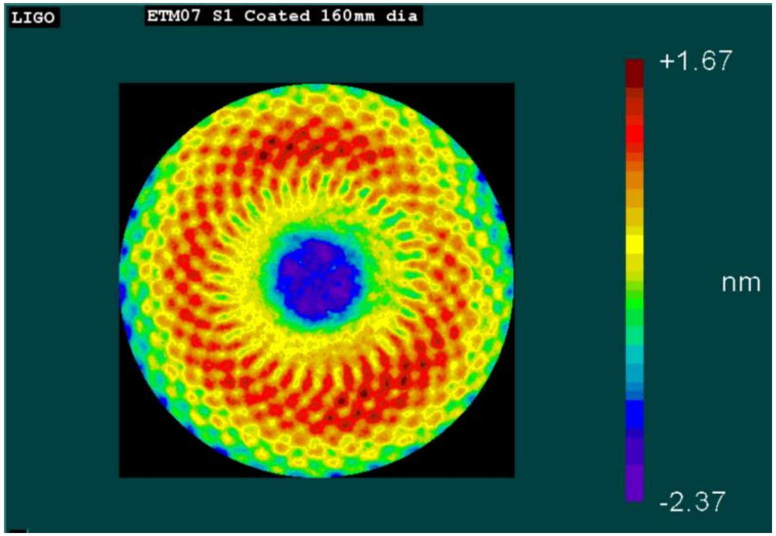


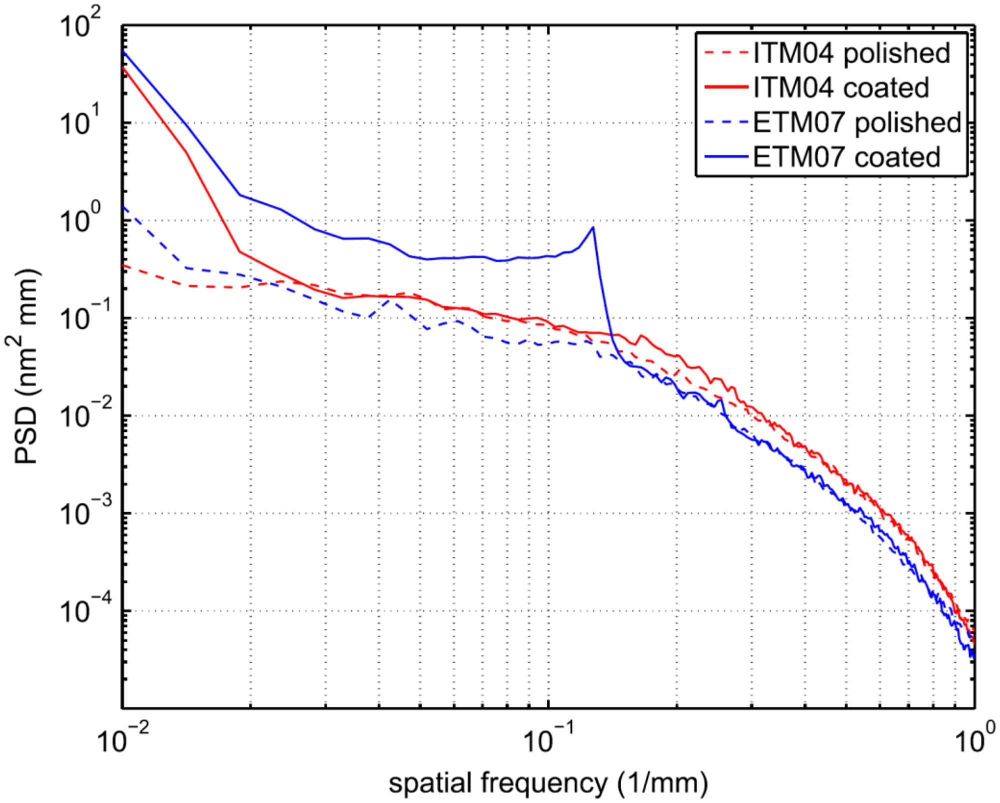


Imperfection of ETM

Spiral patterns on the end test mass (ETM):



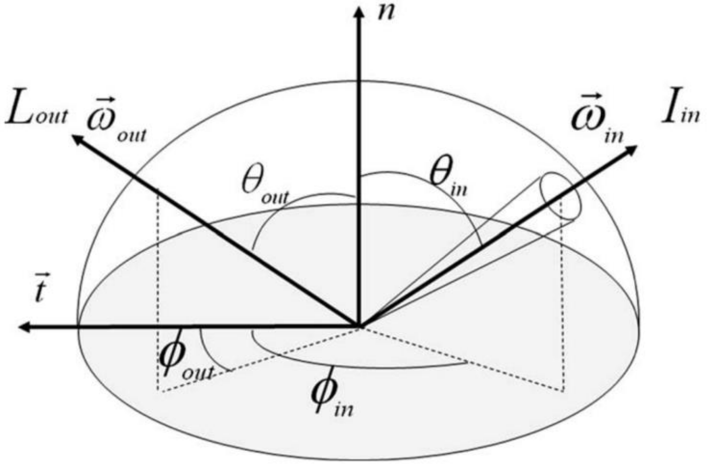




(*Credit: GariLynn, Hiro*)

Scatter Characterization

Bidirectional Reflectance Distribution Function (BRDF): a method to characterize scatter on the surface

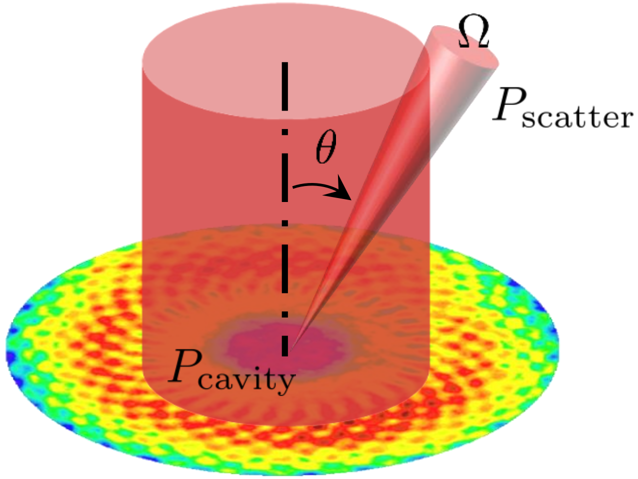


$$\mathrm{BRDF}(\theta_{in}, \theta_{out}, \phi_{in}, \phi_{out}) = \frac{dL_{out}(\theta_{out}, \phi_{out})}{dI_{in}(\theta_{in}, \phi_{in})}$$

L_{out}: Outgoing radiance $[\mathrm{W}/(\mathrm{sr}\cdot\mathrm{m}^2)]$

I_{in}: Incoming irradiance $[\mathrm{W}/\mathrm{m}^2]$

(Credit: Duck Bong Kim)

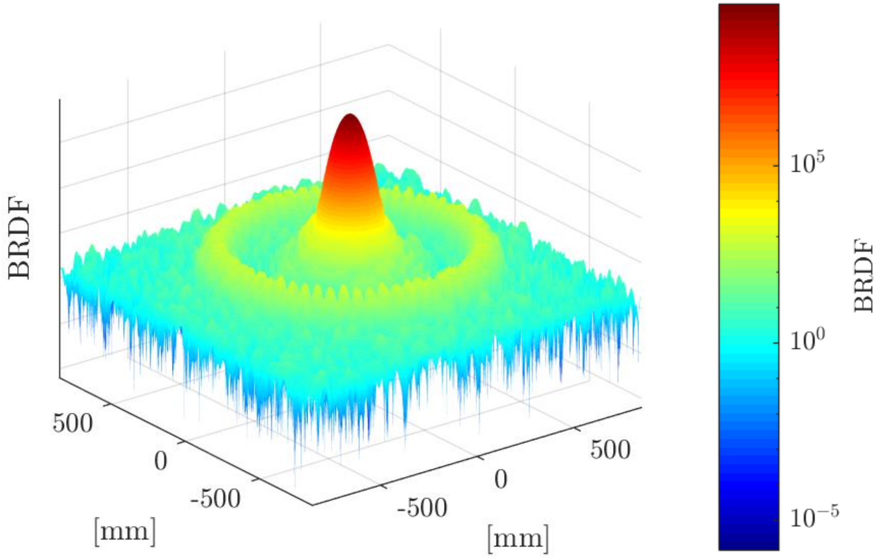


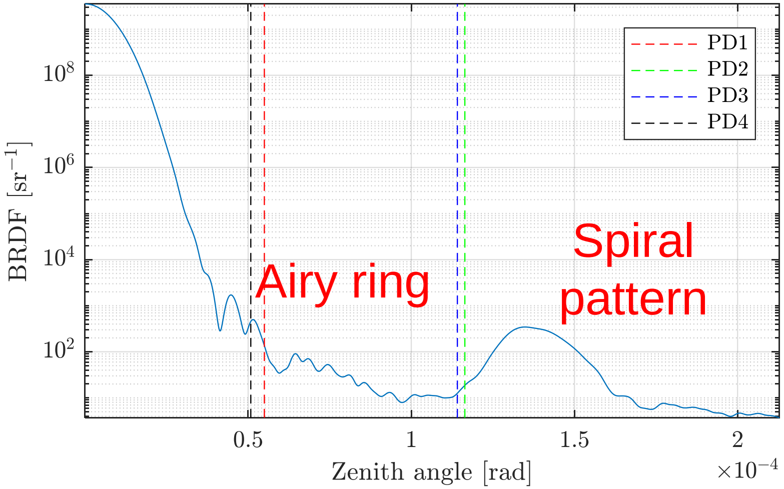
In LIGO FP arm, it can be simplified:

$$\mathrm{BRDF}(\theta) = \frac{P_{\mathrm{scatter}}}{P_{\mathrm{cavity}}\Omega}$$

BRDF Simulation

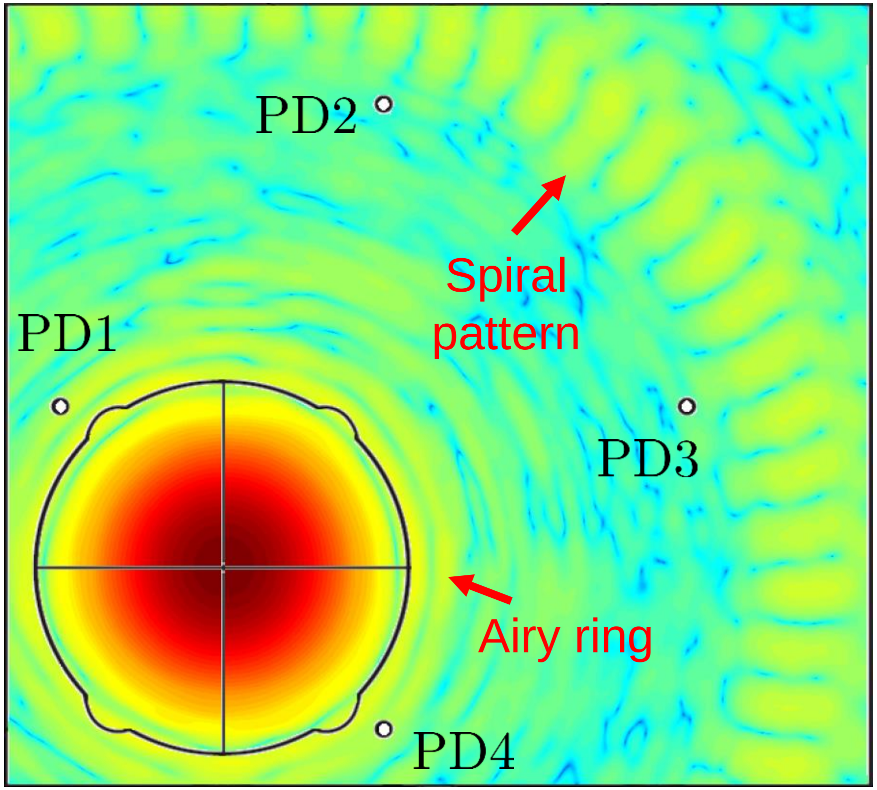
Estimation of ETMX BRDF from SIS (scatter on ITMX)





Airy ring

Spiral pattern



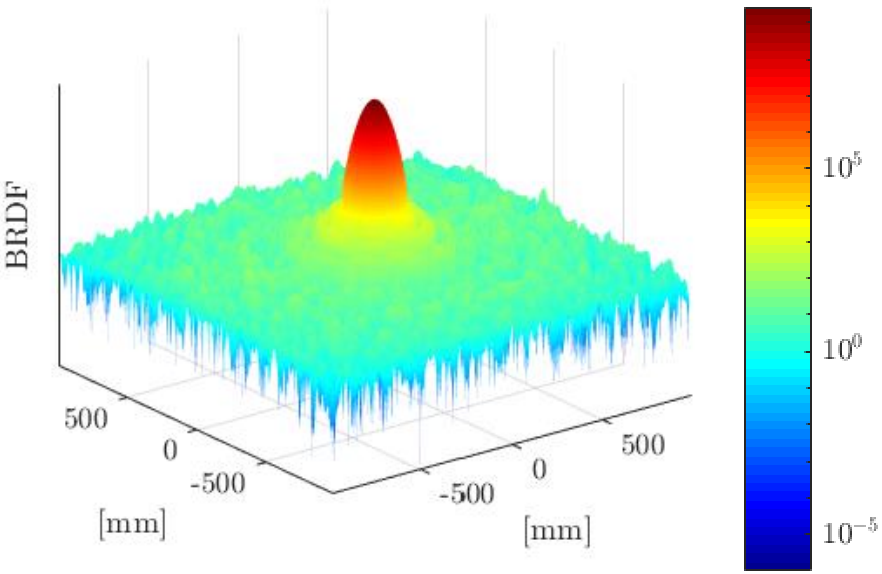
Spiral pattern

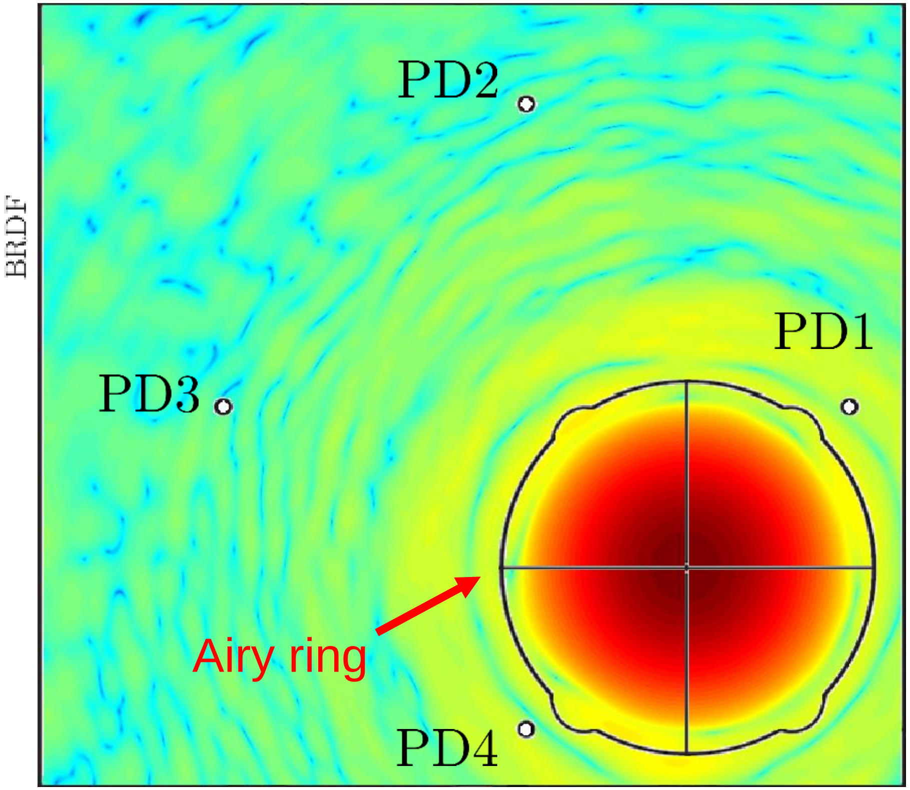
Airy ring

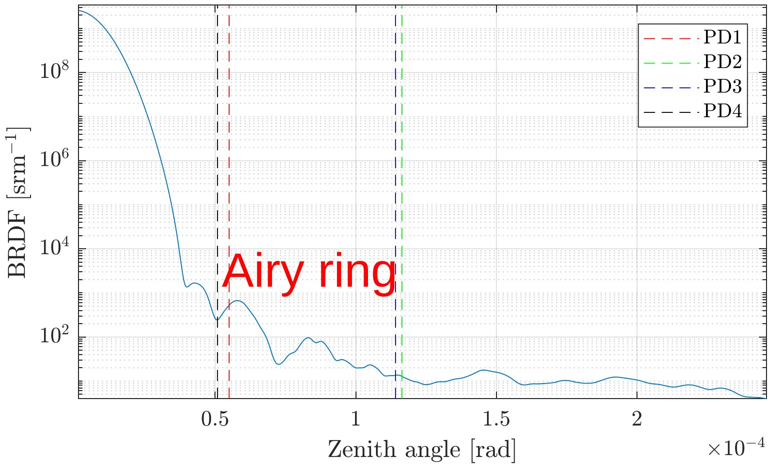
(*Credit: Hiro Yamamoto*)

BRDF Simulation

Estimation of ITMX BRDF from SIS (scatter on ETMX)



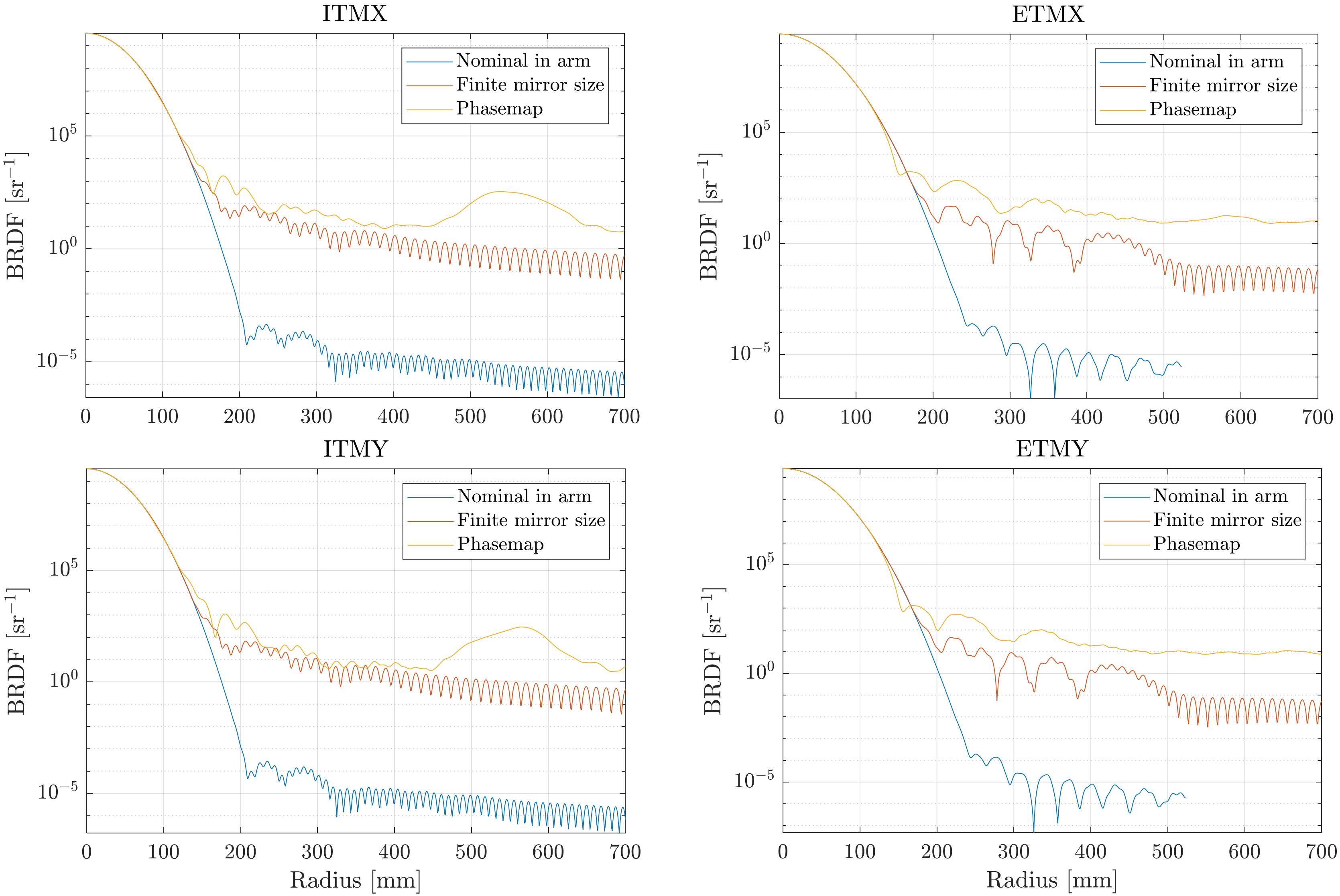




(Credit: Hiro Yamamoto)

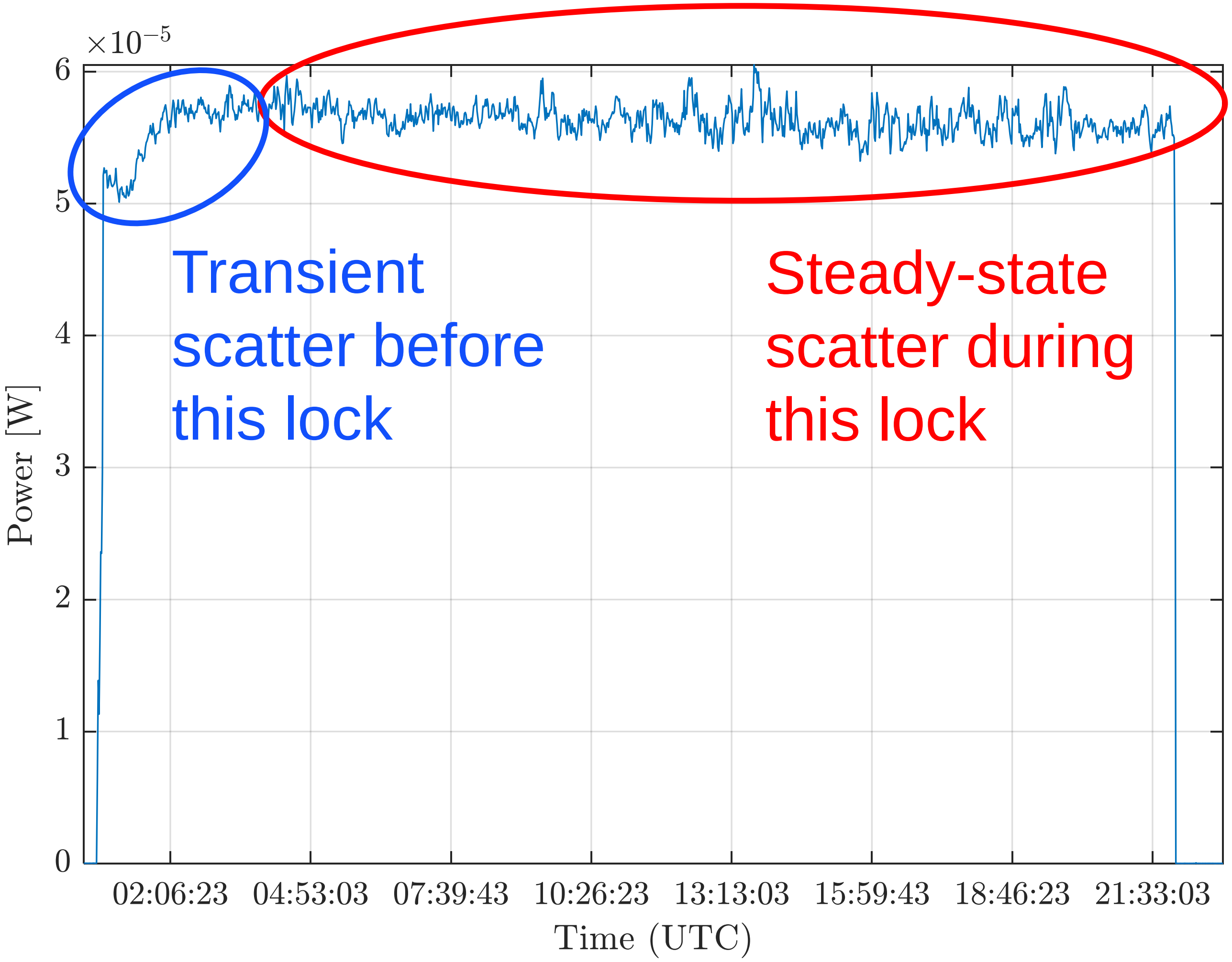
Static Interferometer Simulation

Effect of finite mirror aperture and phasemap:



Scatter Measurement

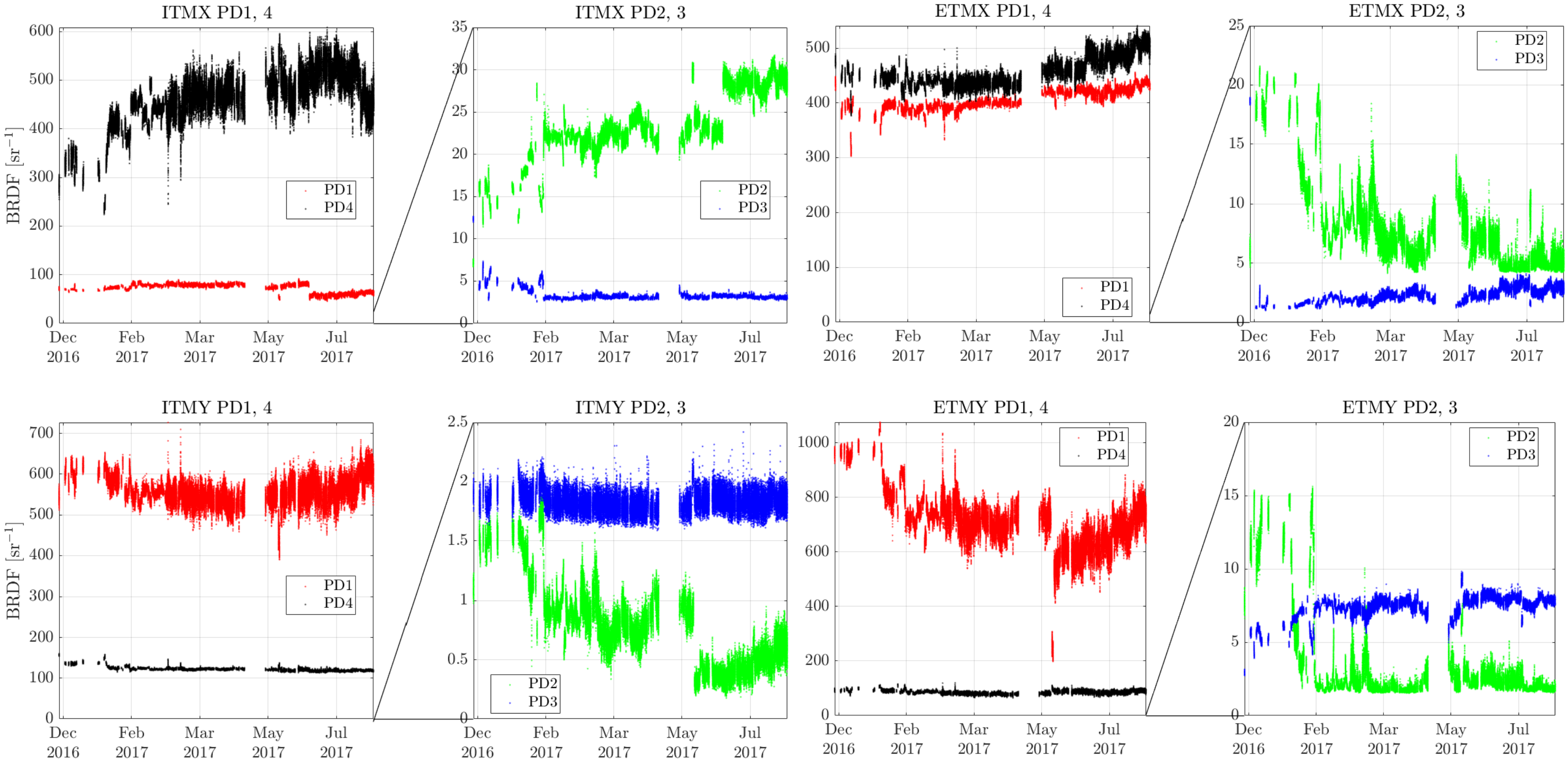
Sample minute-trend data of ITMX PD1 on 7/1/17:



Transient scatter before this lock

Steady-state scatter during this lock

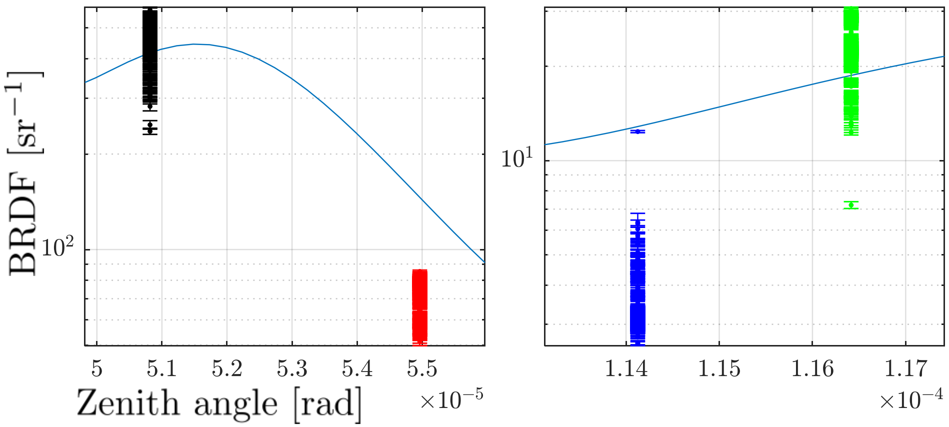
Overall Scatter Data in O2



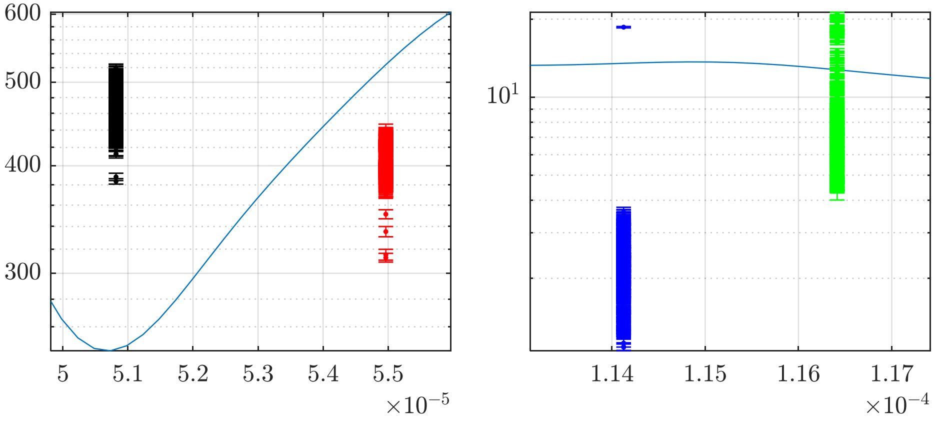
[R G B K] corresponds to PD 1,2,3,4

Steady-state BRDF

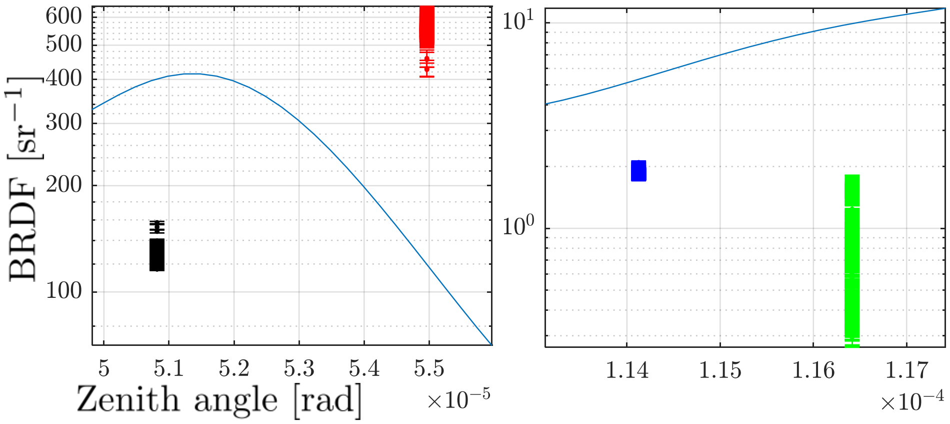
ITMX PD



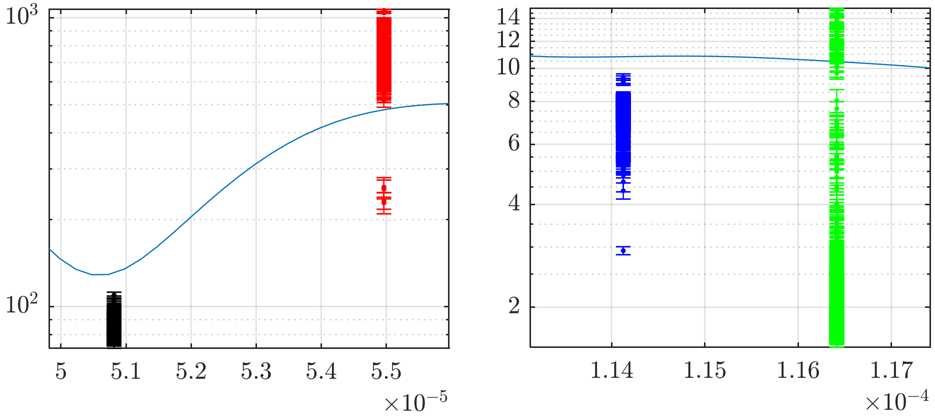
ETMX PD



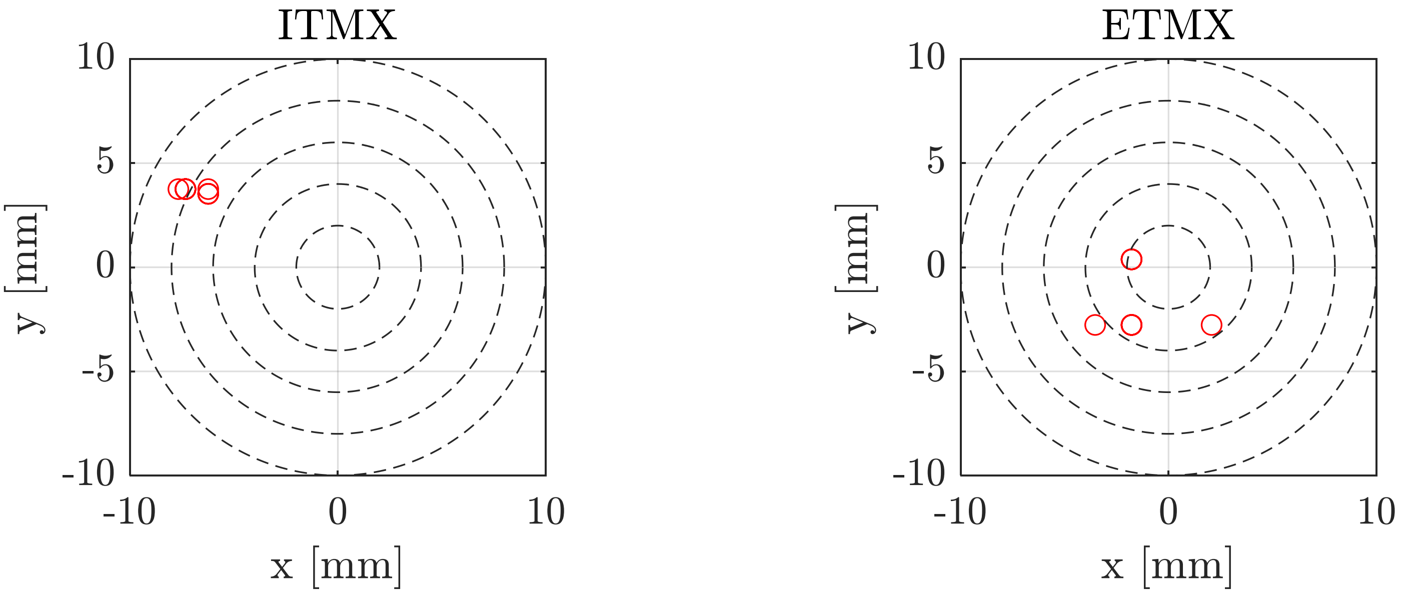
ITMY PD

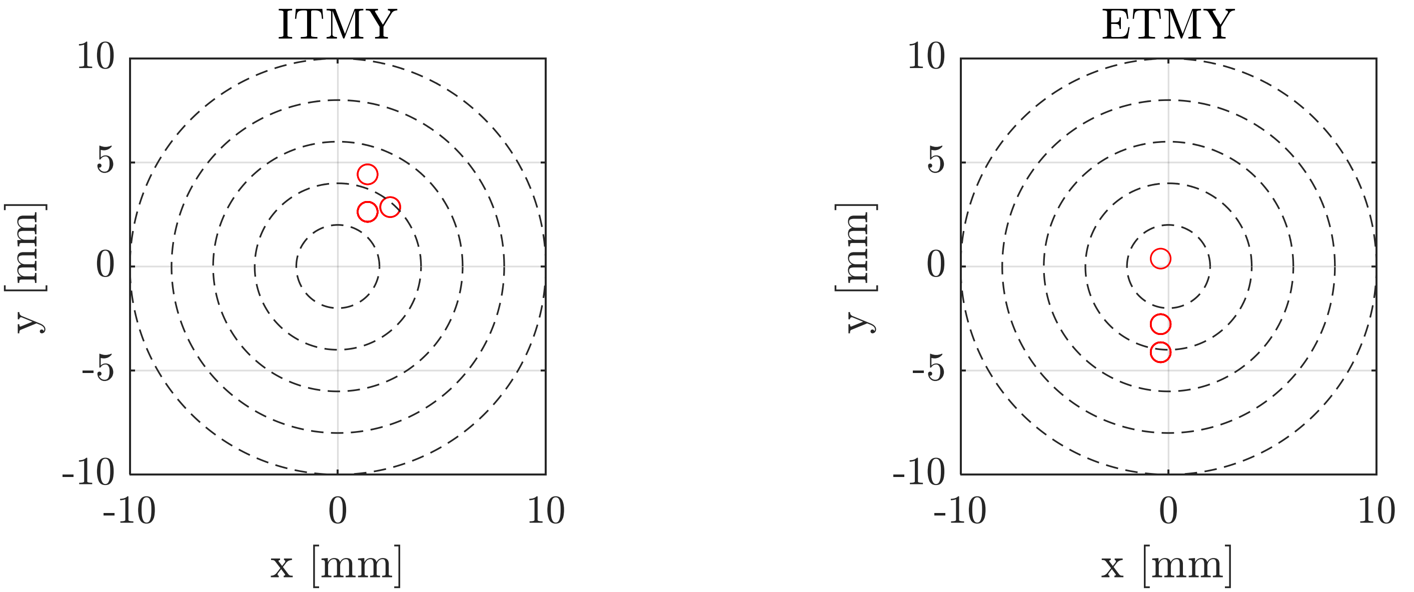


ETMY PD



Beam Position in O2 Locks

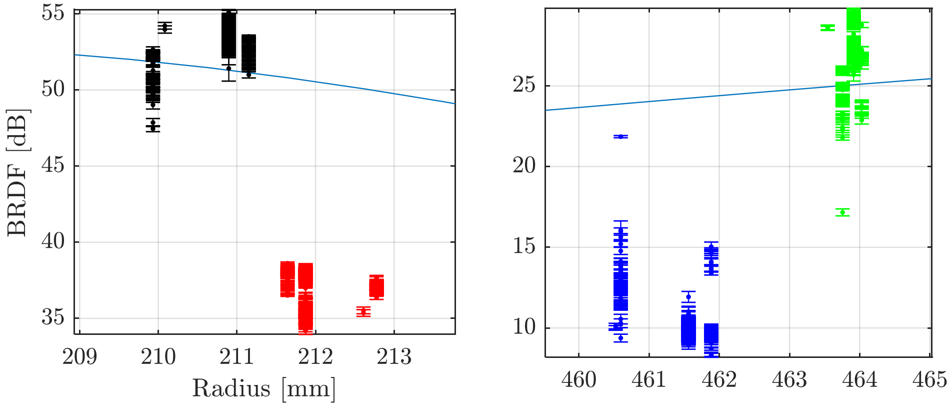




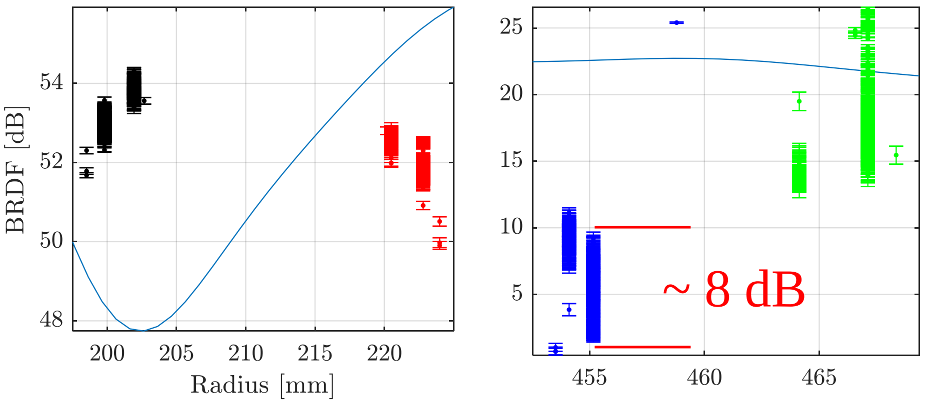
Scatter with Beam Positions in O2

Scatter simulation (all beams at center) versus scatter data.

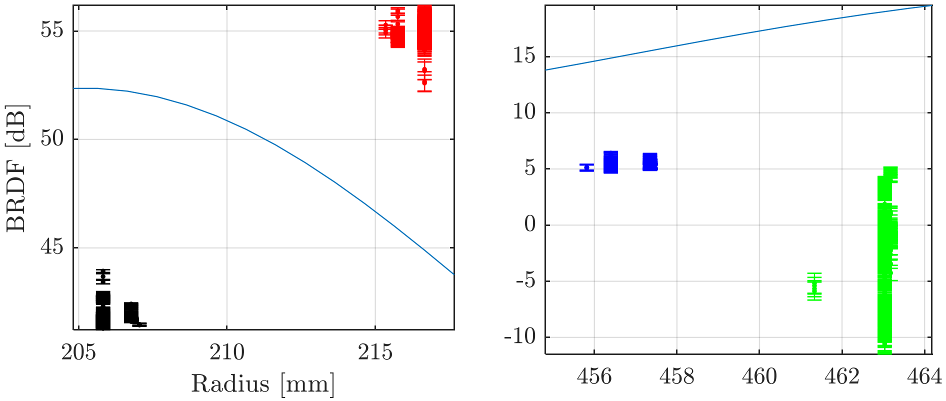
ITMX PD



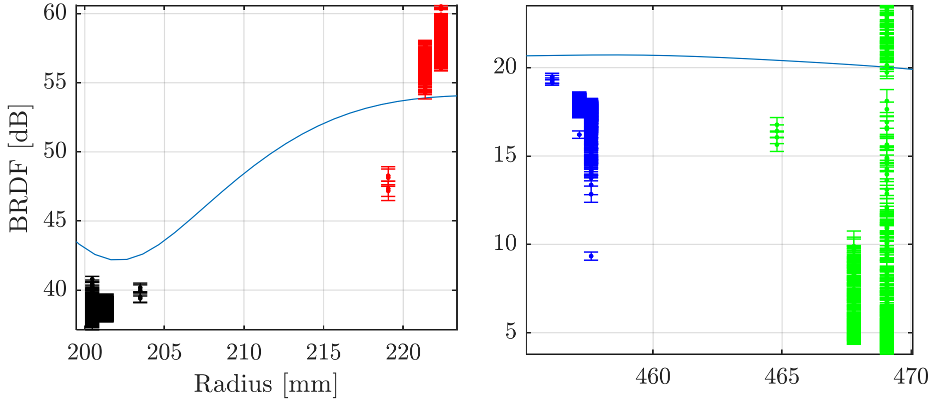
ETMX PD



ITMY PD



ETMY PD



Scatter Growth with Time

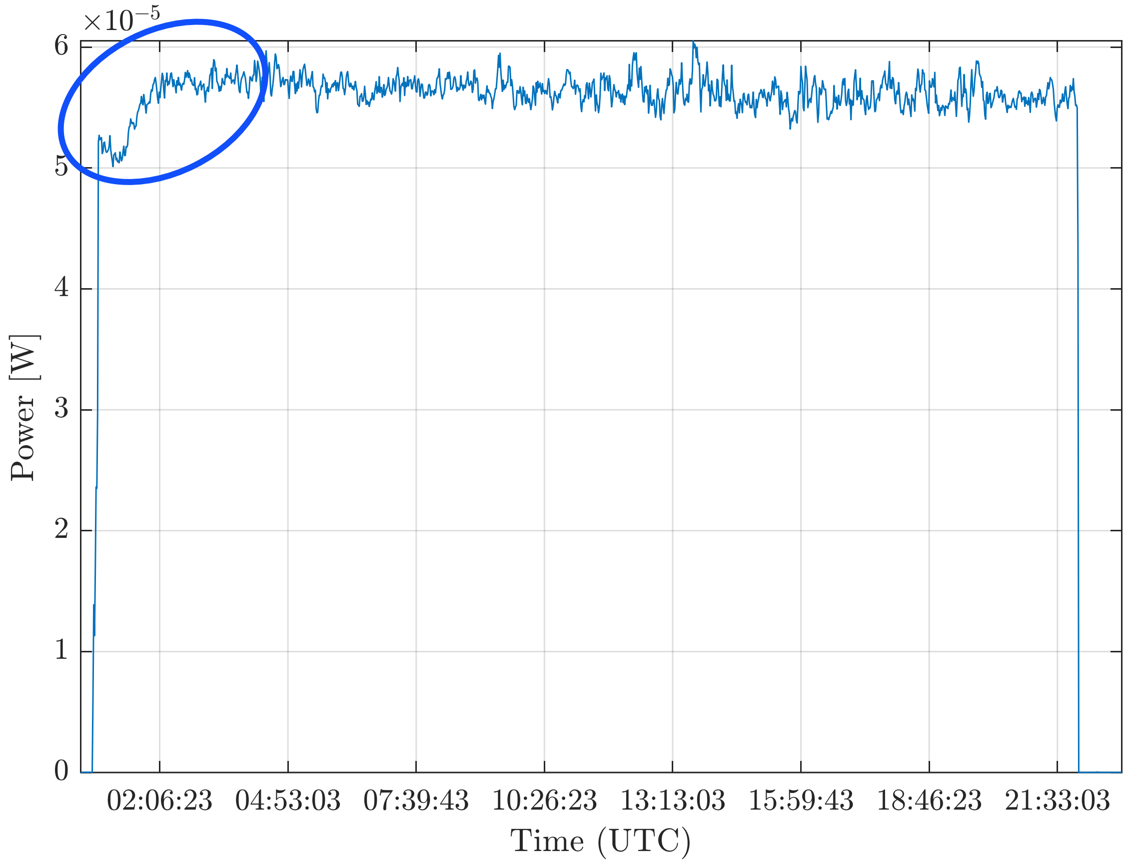
- Choose a longest period (2/25-6/6, 3 month) when beam positions are controlled.

	ITMX	ETMX	ITMY	ETMY
PD1	-16±1%	21.7±0.5%	-8±2%	-13±2%
PD4	33.6±0.2%	13.7±0.5%	-4±7%	-20±20%

- Difficult to conclude overall scatter growth.
Something else is changing to cause scatter change.

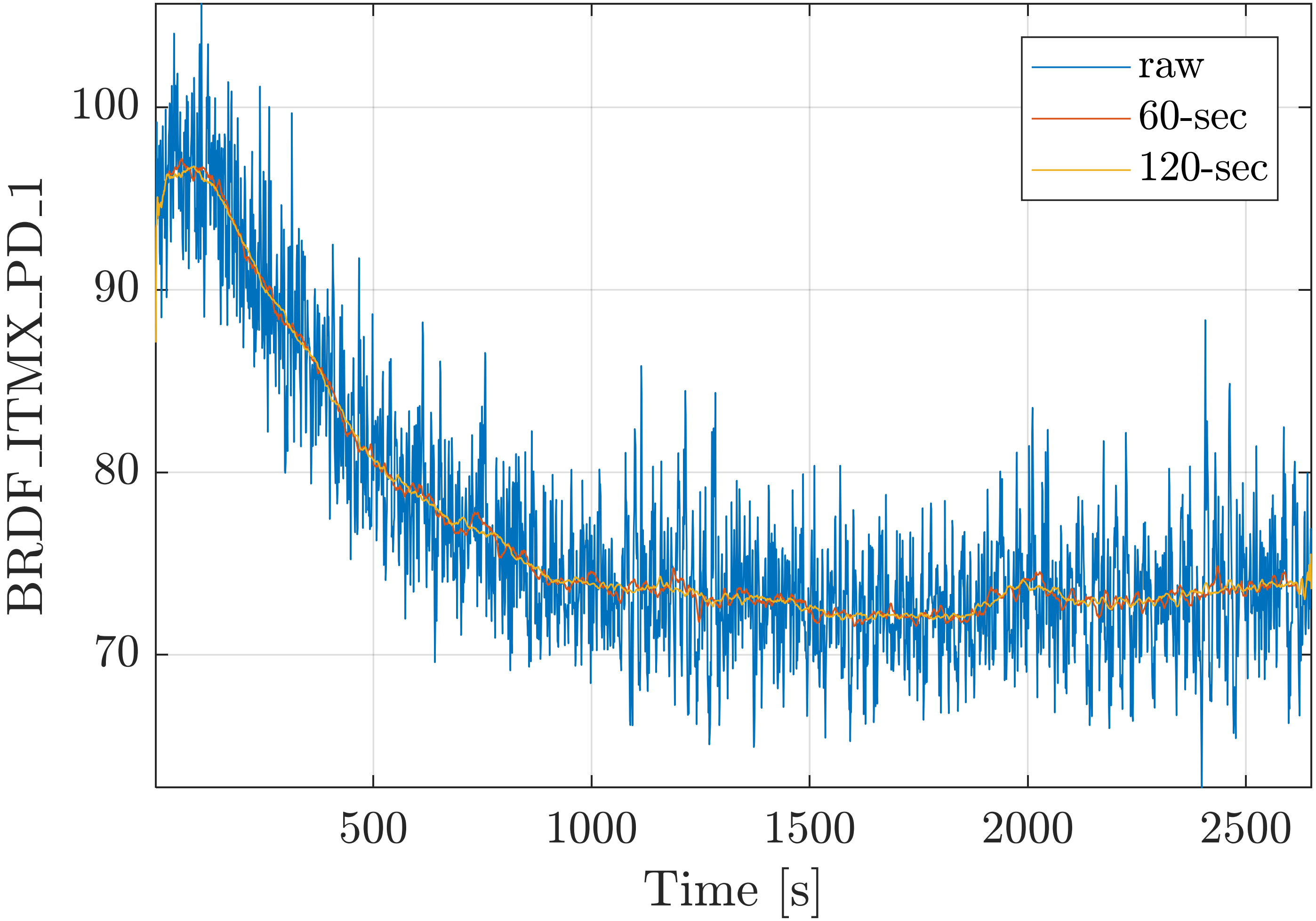
Transient Scatter

- More data on beam positions and scatter coupling during transience of lock. (ADS beam alignment)



Scatter Data Processing

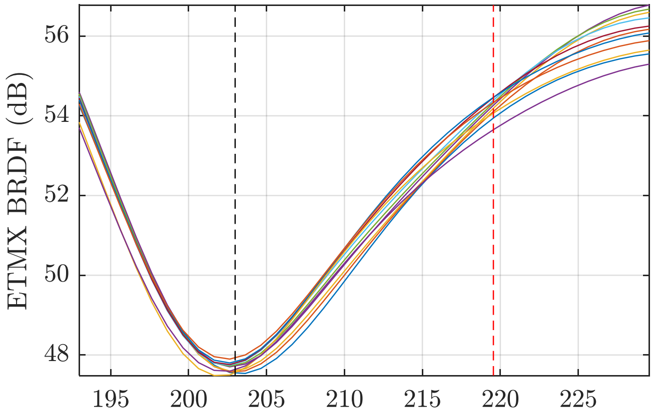
- Moving-average filter (sample transience on 3/29/17)



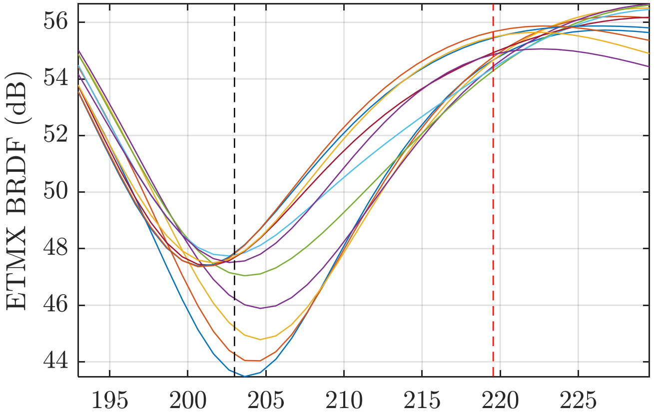
Scatter versus Beam on Both Sides

Range of change: [-10:2:10] mm

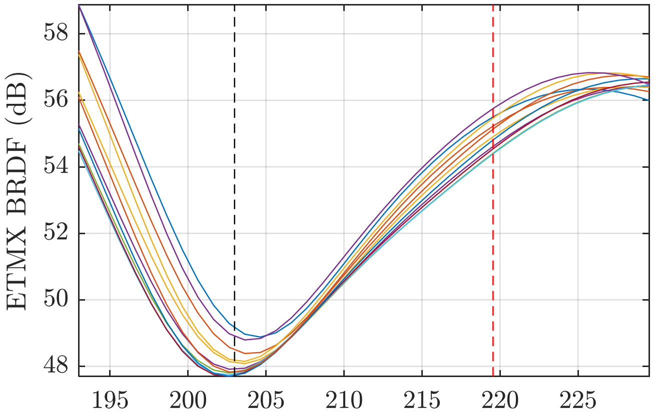
ITMX Position (x,0)



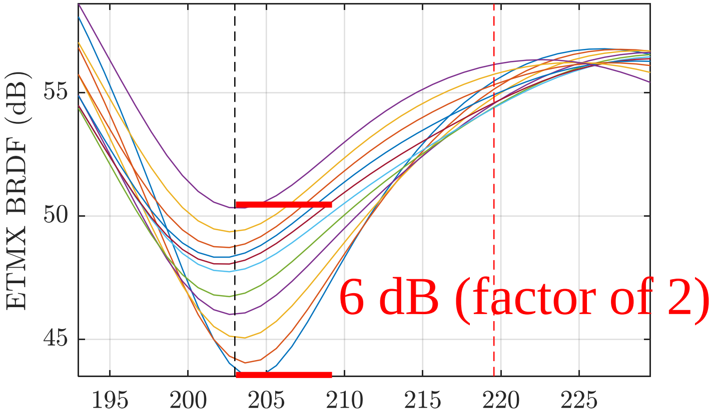
ITMX Position (0,y)



ETMX Position (x,0)

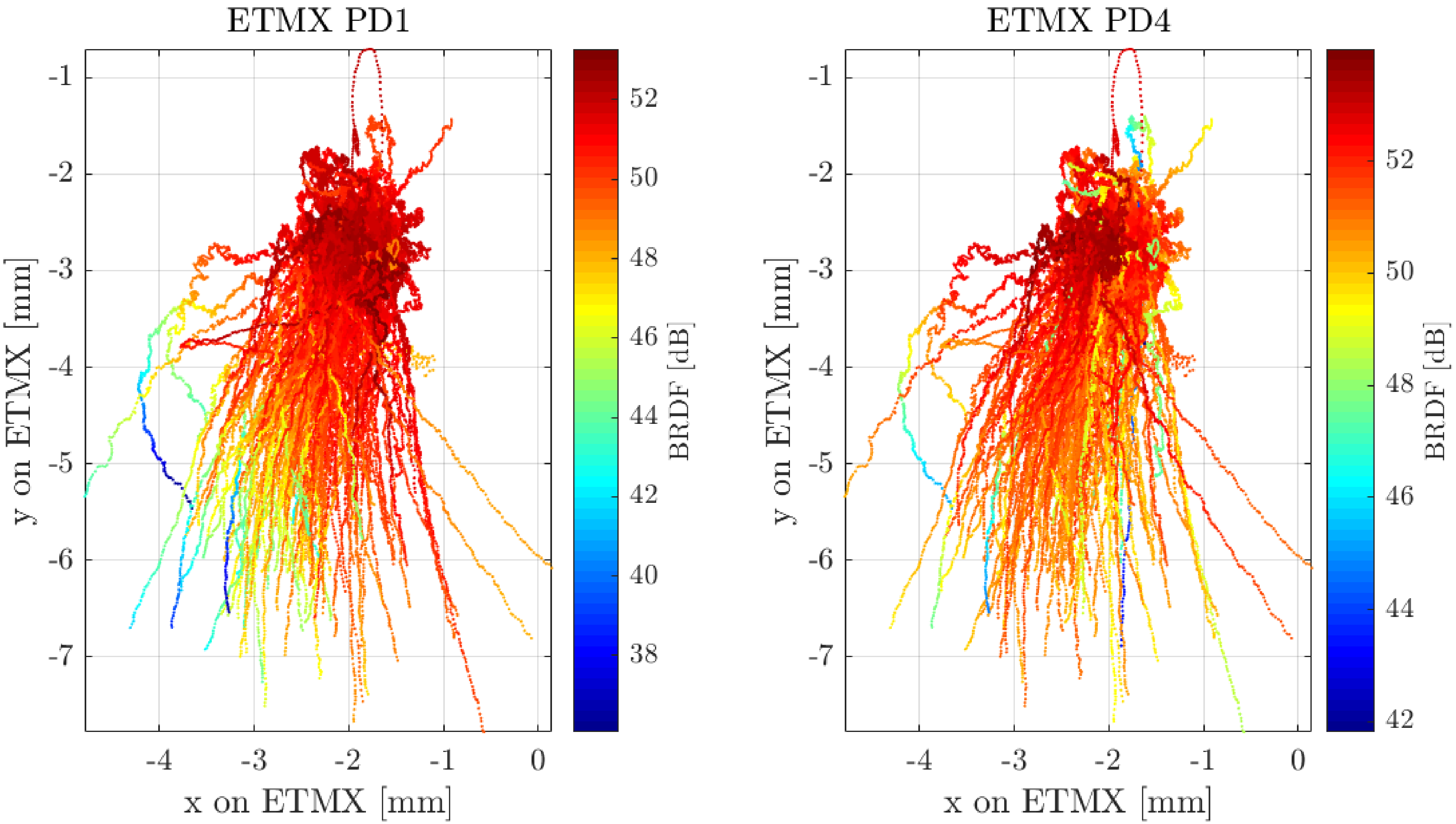


ETMX Position (0,y)



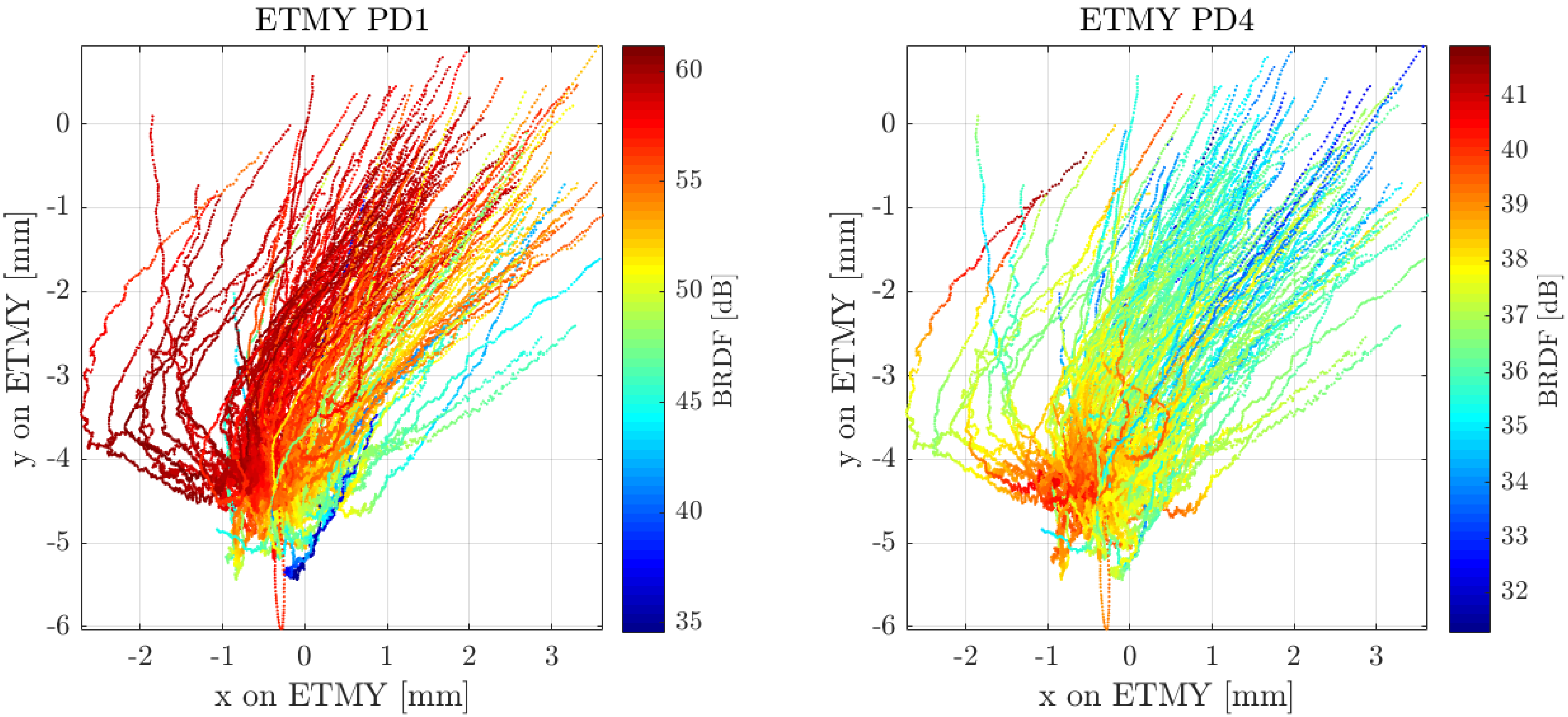
Transient Scatter in LSD

ETMX PD measurements (120-s MA filtered) through each alignment process, in LSD (2/25/17 - 6/6/17)



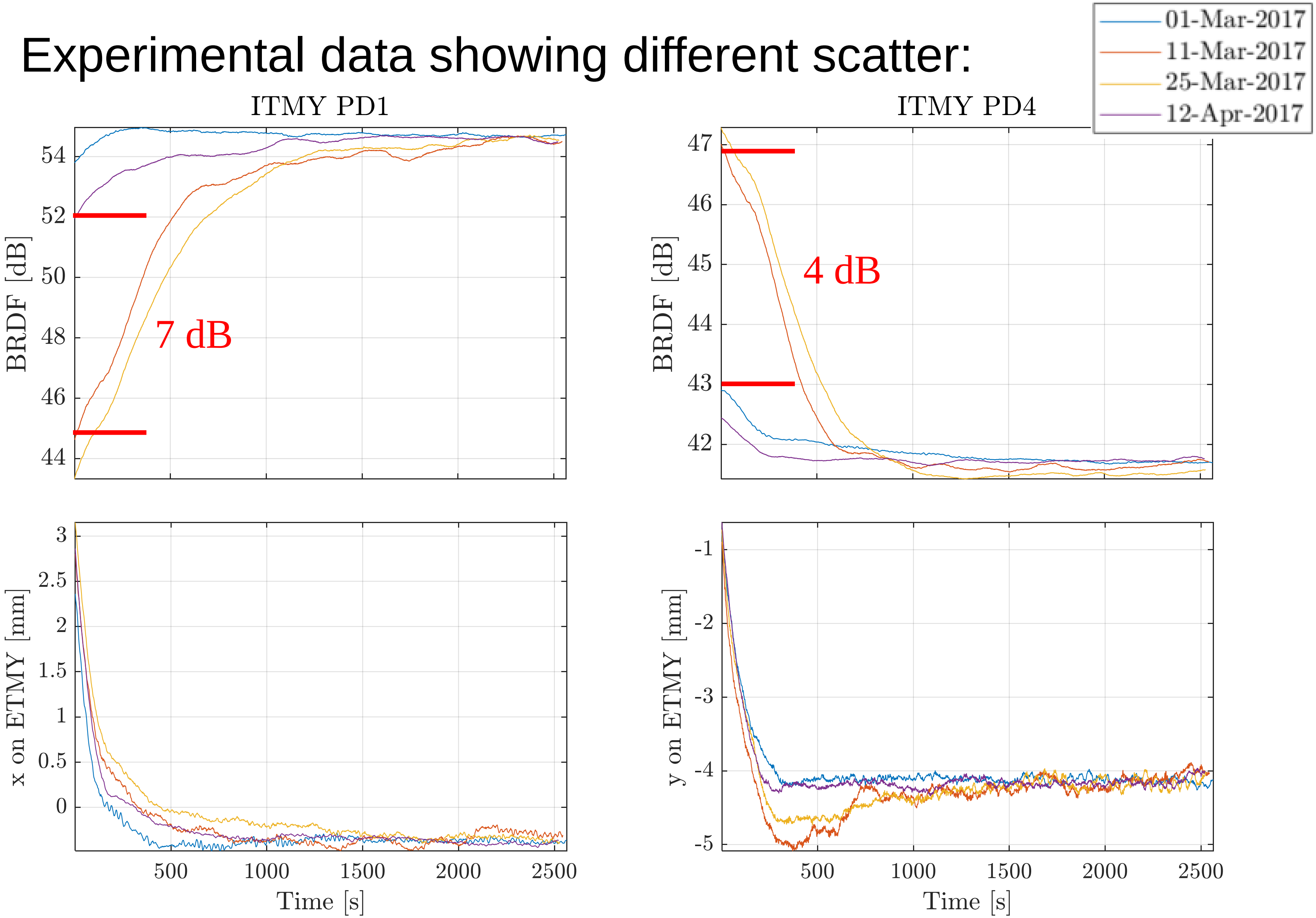
Transient Scatter in LSD

ETMY PD measurements (120-s MA filtered) through each alignment process, in LSD (2/25/17 - 6/6/17)



Scatter versus Beam on Both Sides

Experimental data showing different scatter:



Scatter versus Beam on Both Sides

- SIS doesn’t agree with the measured scatter at the precision of ~3 dB and beam positions of ~5 mm.

- The SIS result depends on the beam position on both test mass.

- Use regression model to predict beam positions from scatter data.

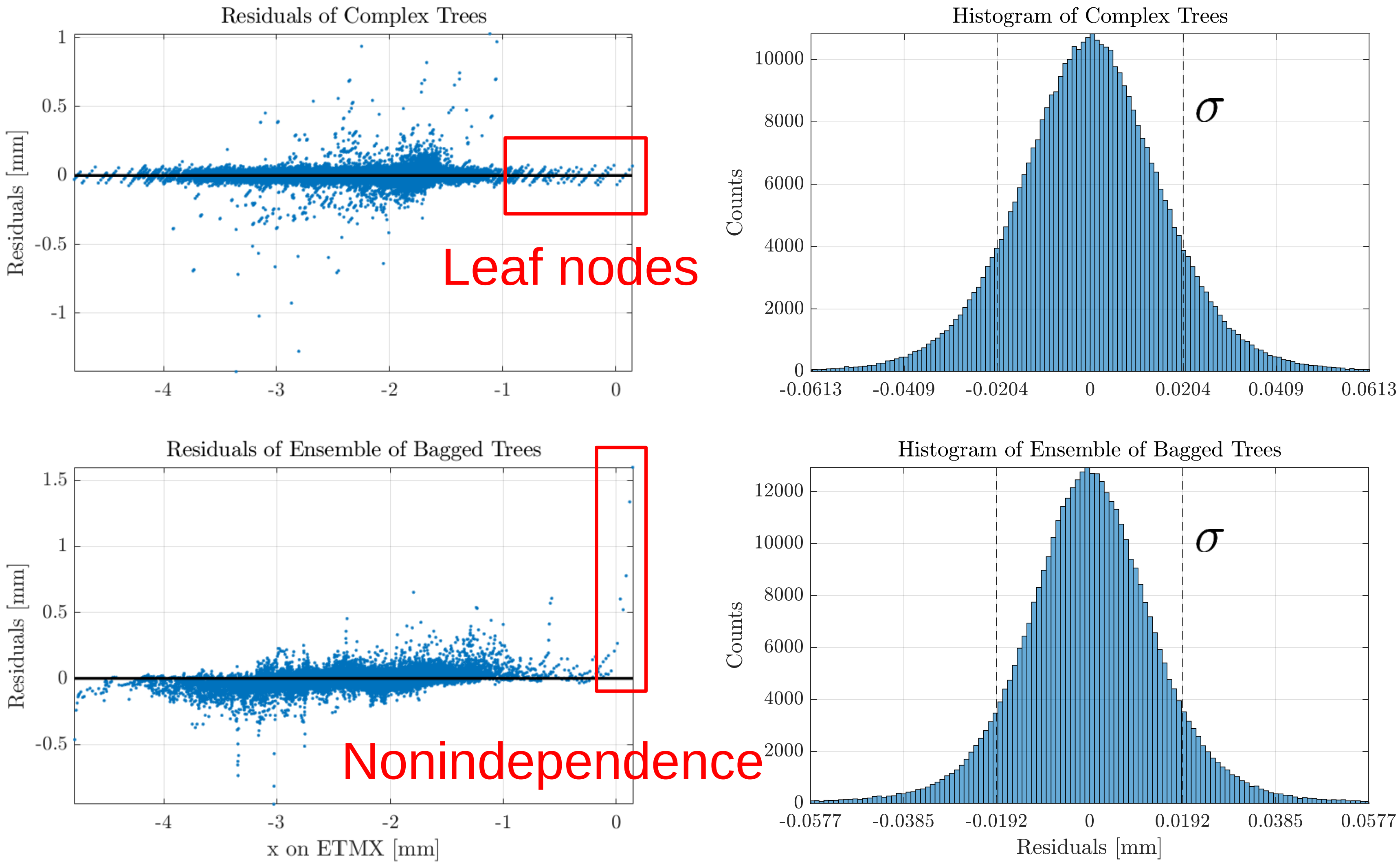
Ensemble Methods

- Used measured scatter of 4 ETMX PD to predict beam positions measured by ADS error signal.
- Train the model with all 3-month data

Model	R^2	RMSE	Prediction Speed
Stepwise Linear (order of 5)	0.64	0.17	~1.6e6 obs/s
Stepwise Interaction Linear (order of 5)	0.73	0.15	~1.3e6 obs/s
Simple Tree (> 32 obs on node)	0.94	0.07	~1.7e6 obs/s
Complex Tree (> 4 obs on node)	0.98	0.04	~1.6e6 obs/s
Ensemble of Boosted Trees	0.62	0.17	~2.2e5 obs/s
Ensemble of Bagged Trees	0.99	0.03	~6.7e4 obs/s

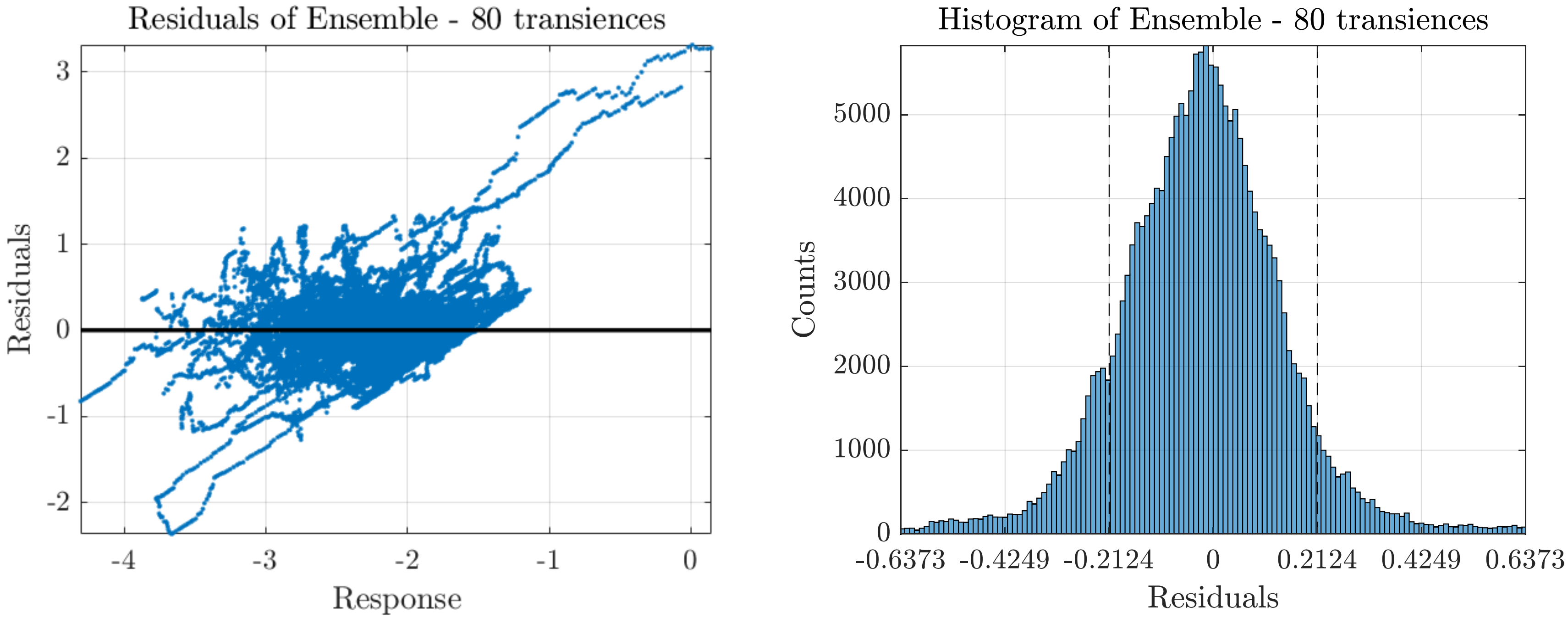
Trees versus Ensemble

Residuals plot of the two best models with highest R^2



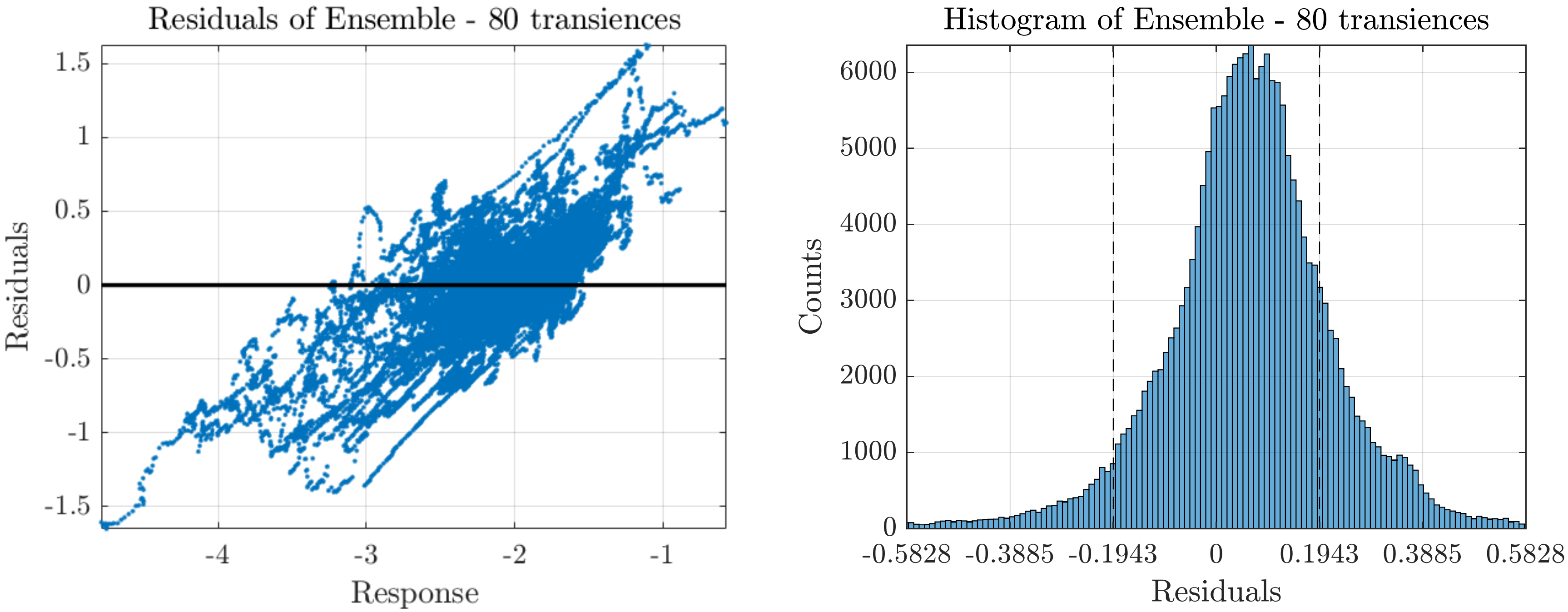
Model Training

- In practical, we won't have $\sim 10^5$ data and 159 transience data before training model in O3 scatter.
- Test ensemble method with first half of data, and test it with another half of data.



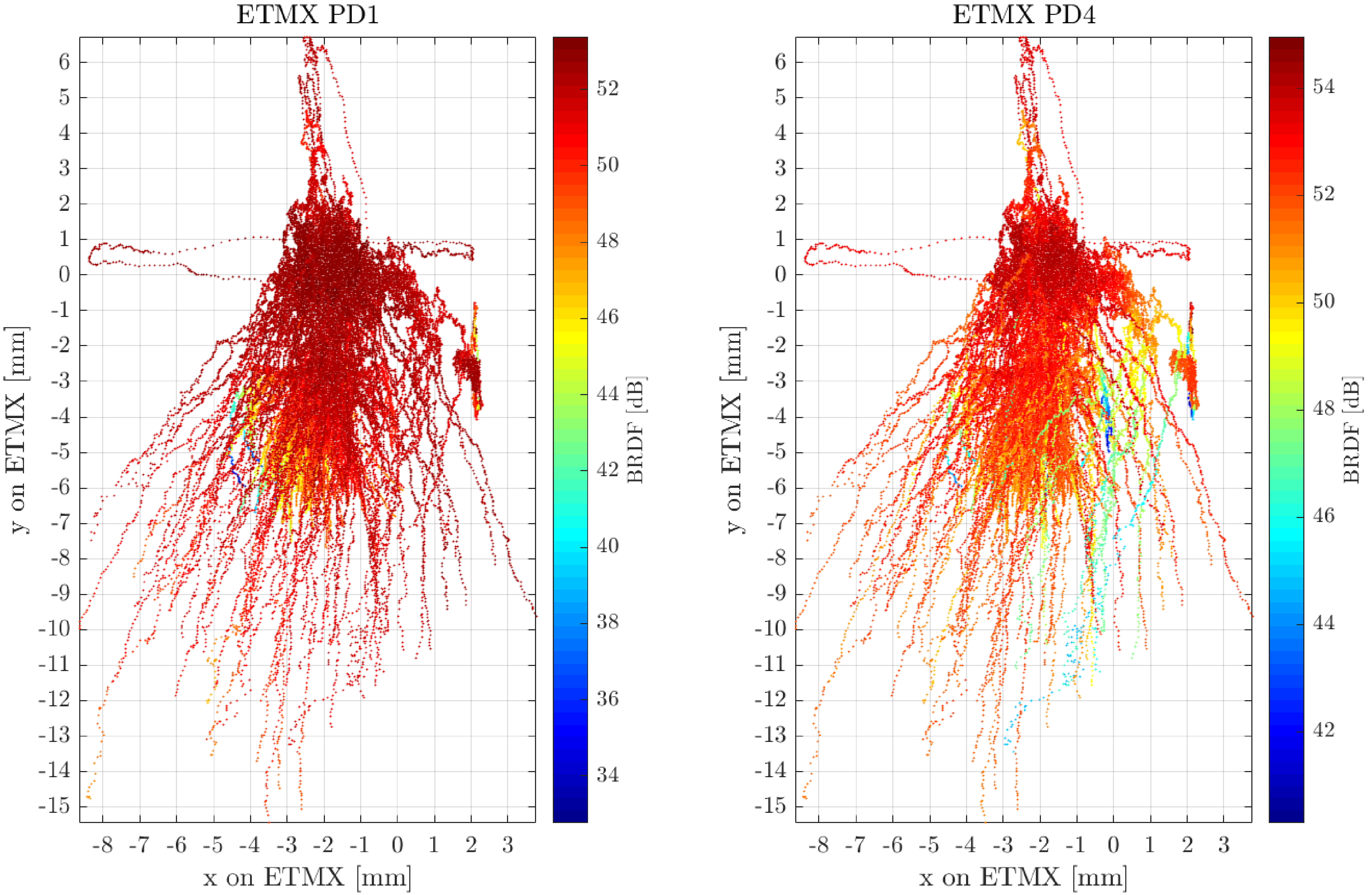
Model Training

- Training the other way around: use last-half to train and predict first-half data

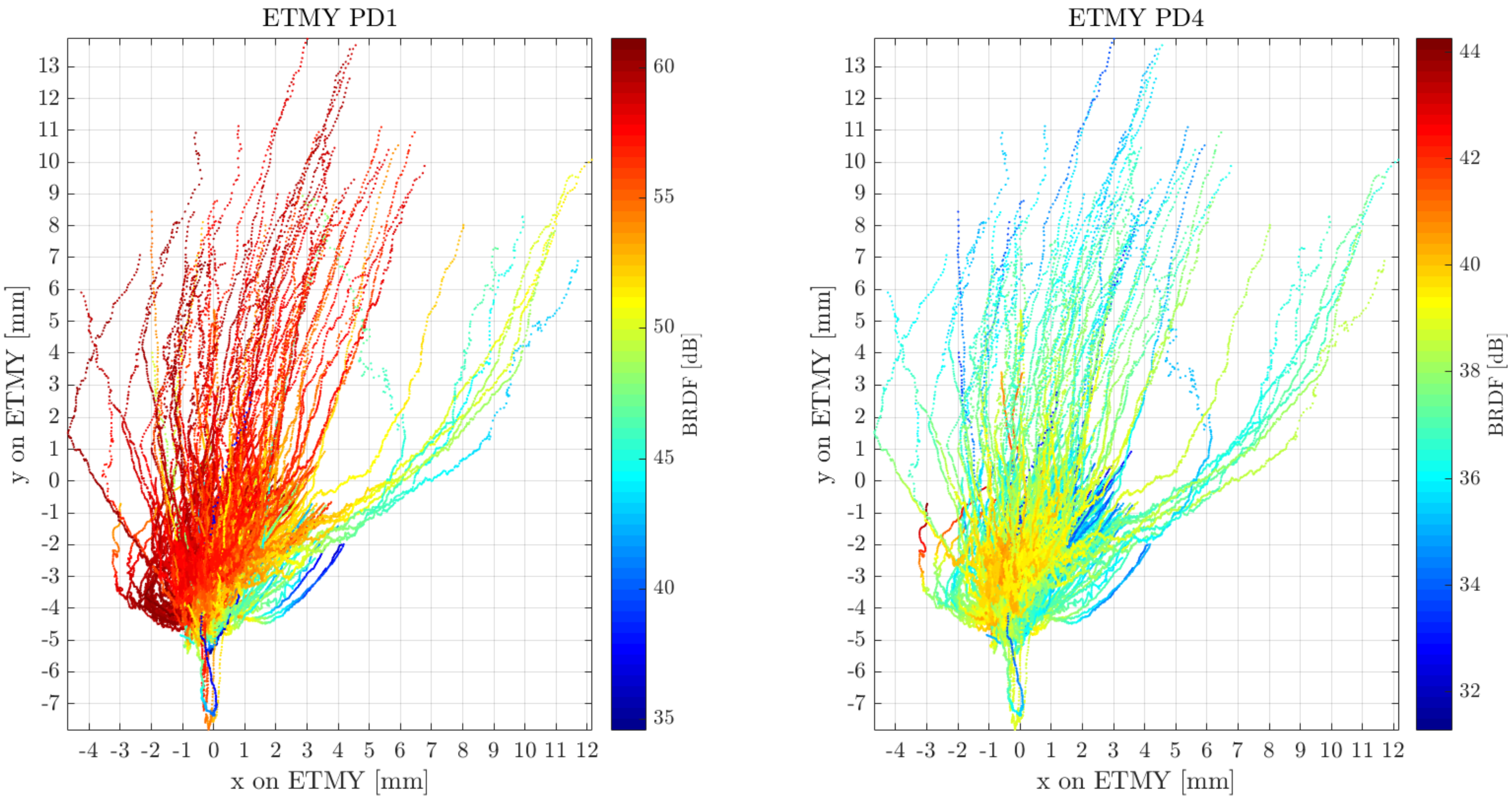


- Use diverse data to have better model (avoid selecting same path)

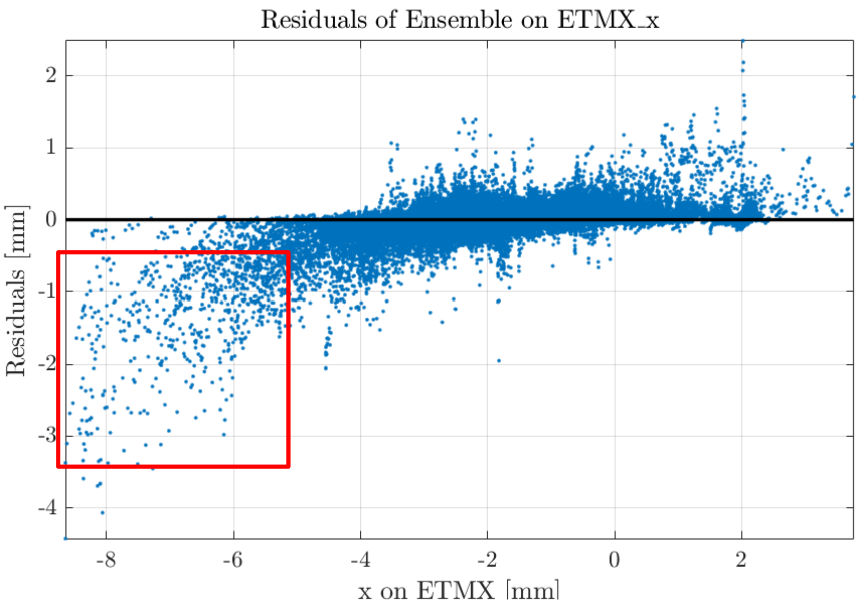
Transient Scatter in O2

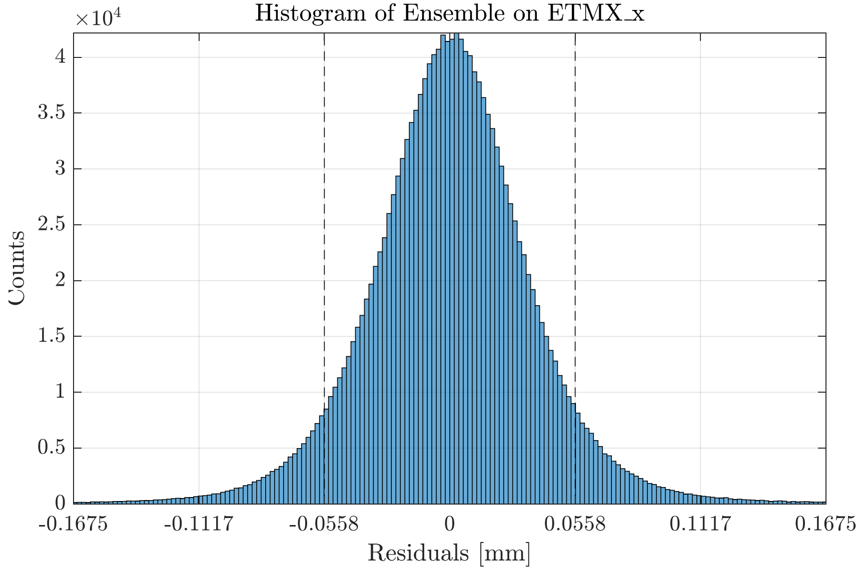


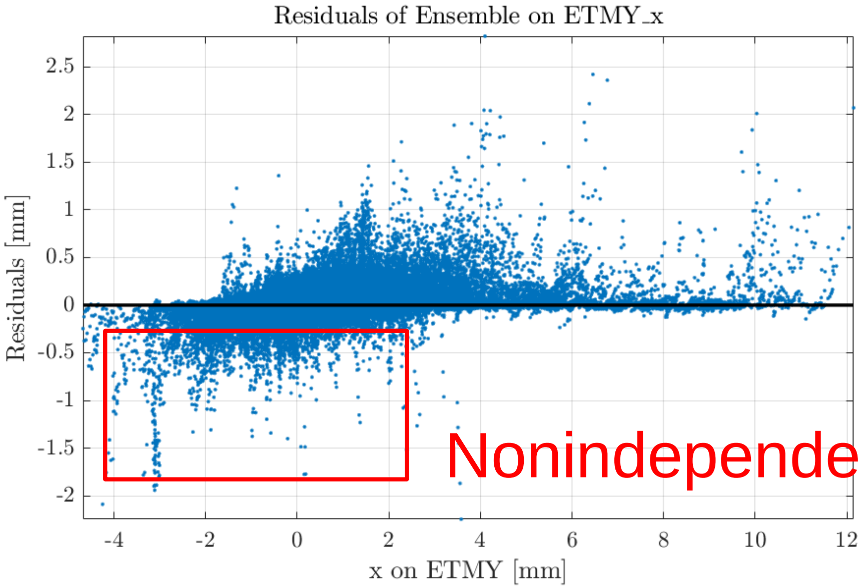
Transient Scatter in O2

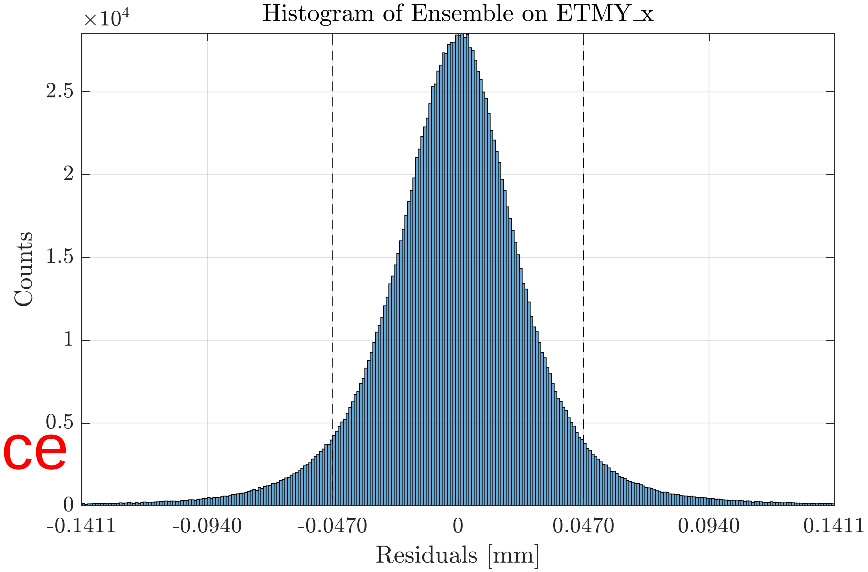


Ensemble on O2 Data


Residuals of Ensemble on ETMX_x


Histogram of Ensemble on ETMX_x


Residuals of Ensemble on ETMY_x


Histogram of Ensemble on ETMY_x

Nonindependence

LIGO

Conclusion

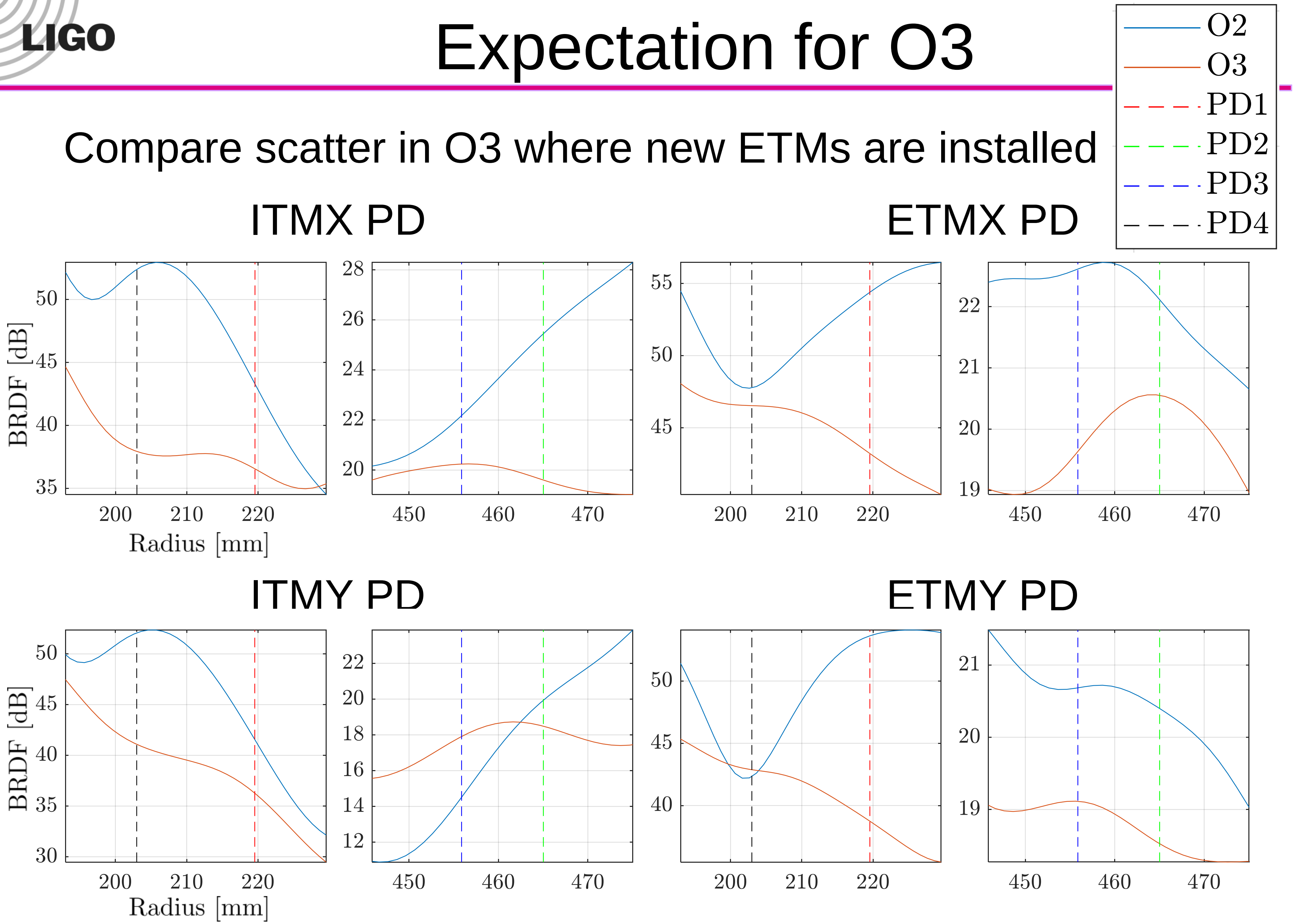
- Compare measured scatter to the simulation. The difference is at most 10 dB.

- Compare scatter with respect to beam position using transient data in the scale of millimeter and 5 dB. It is shown that the scatter depends upon beam positions on test masses on both sides.

- Proposed regression model to predict beam positions using scatter. The standard deviation of error is within 0.5 mm. We are still working on solving nonindependence problem.

Expectation for O3

Compare scatter in O3 where new ETMs are installed



Acknowledgement

I am very grateful to Anamaria Effler and Valera Frolov for patient mentoring, Hiro Yamamoto for consistent help on SIS, and all LLO fellows for answering questions! Special thanks to California Institute of Technology and NSF to make SURF happen!

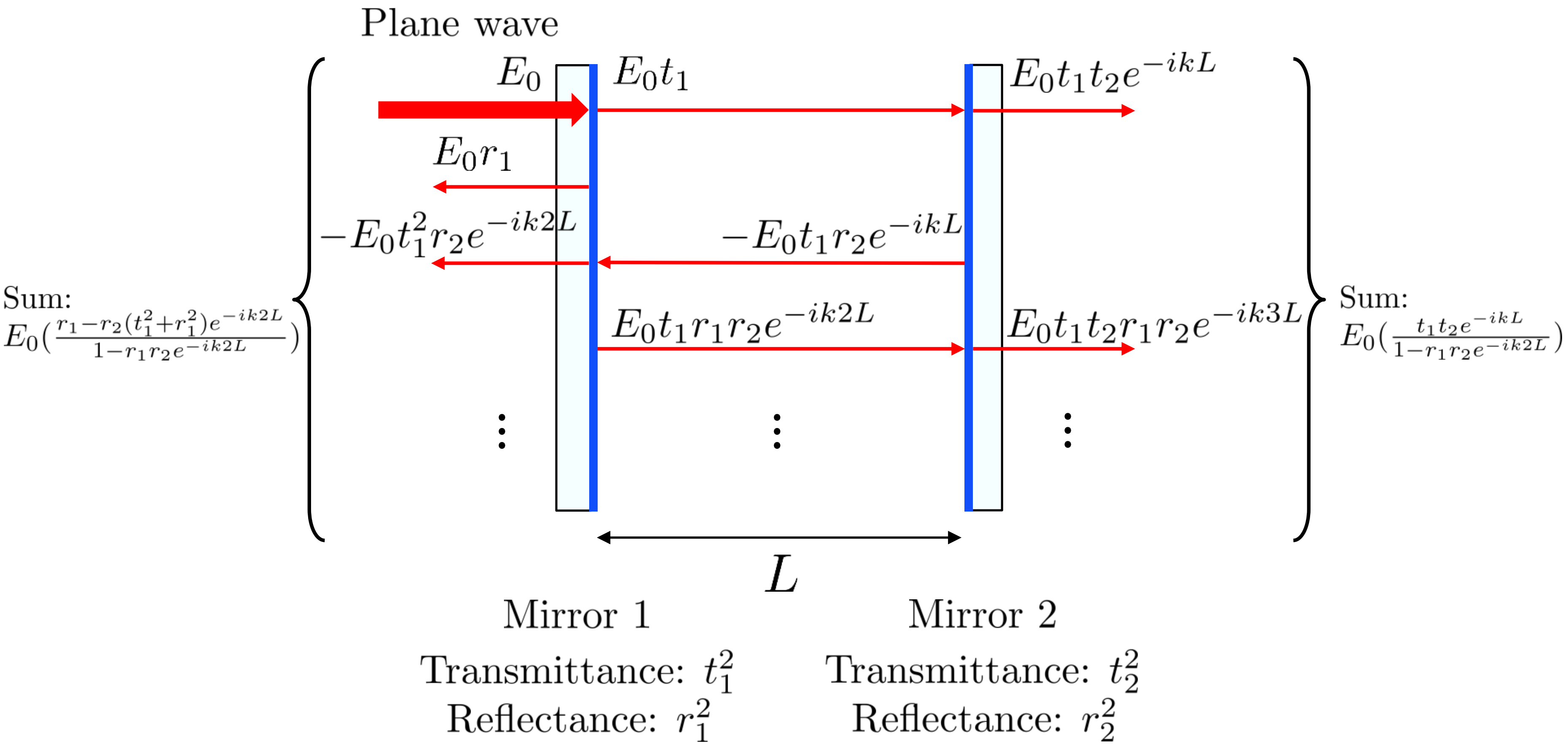
Thank you for listening!

Q&A

Backup Slides

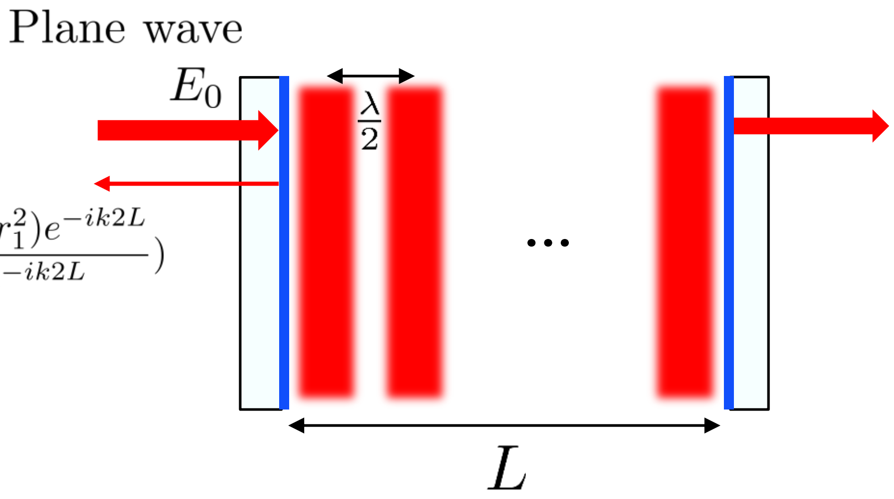
Simple Fabry-Perot Cavity

Fabry-Perot (FP) is composed of two highly reflective mirrors. A standing-wave is maintained in resonance.



Simple Fabry-Perot Cavity

Fabry-Perot (FP) is composed of two highly reflective mirrors. A standing-wave is maintained in resonance.

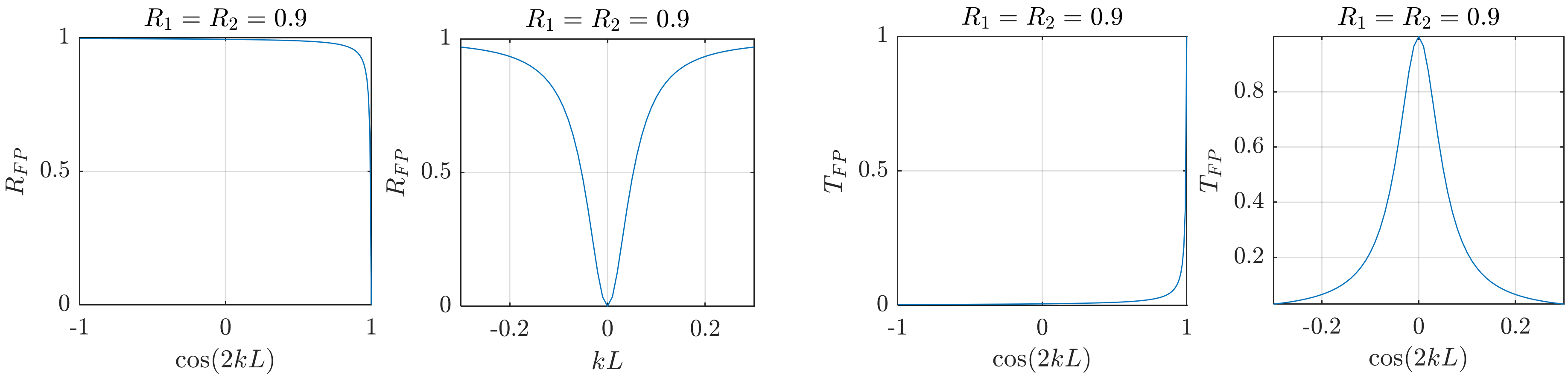


$$E_{\mathrm{tran}} = E_0\left(\frac{t_1 t_2 e^{-ikL}}{1 - r_1 r_2 e^{-ik2L}}\right)$$

$$P_{\mathrm{tran}} = |E_{\mathrm{tran}}|^2 = E_0^2 T_{\mathrm{FP}}$$

$$E_{\mathrm{refl}} = E_0\left(\frac{r_1 - r_2(t_1^2 + r_1^2)e^{-ik2L}}{1 - r_1 r_2 e^{-ik2L}}\right)$$

$$P_{\mathrm{refl}} = |E_{\mathrm{refl}}|^2 = E_0^2 R_{\mathrm{FP}}$$

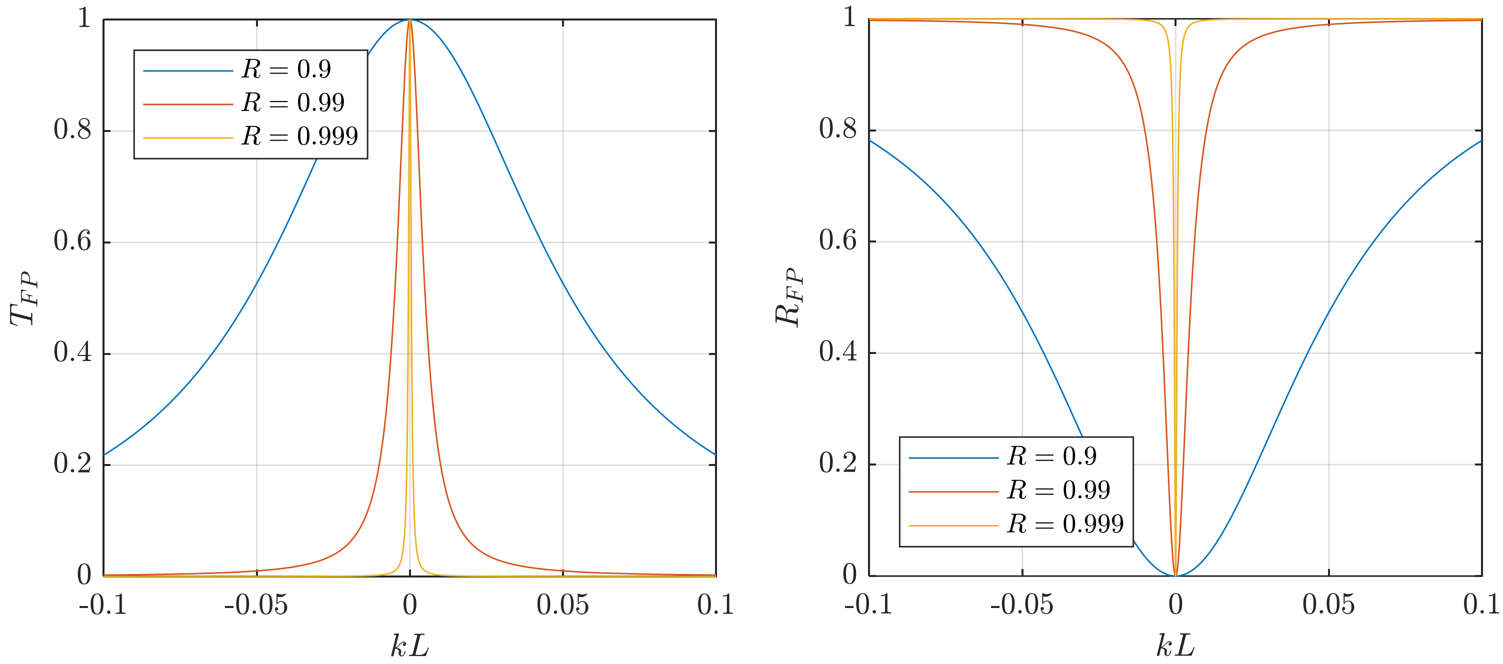


Simple Fabry-Perot Cavity

The role of transmittance and reflectance in FP cavity:

For simplicity, let $r_1 = r_2 = \sqrt{R}$:

$$R_{FP} = \frac{2R(1-\cos 2kL)}{1+R^2-2R\cos 2kL} \qquad T_{FP} = \frac{(1-R)^2}{1+R^2-2R\cos 2kL}$$



aLIGO FP Cavity

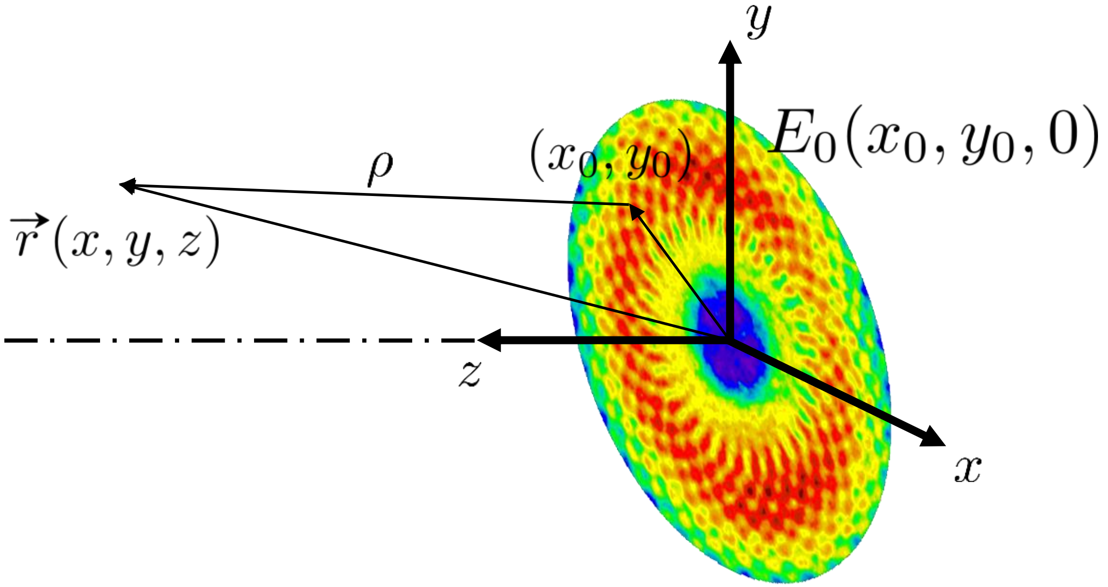
To achieve the nominal FP cavity, incredible engineering has been made:

- Stabilized laser at $\lambda = 1064\ \mathrm{nm}$
  - Relative intensity noise $< 2 \times 10^{-9}\ 1/\sqrt{\mathrm{Hz}}$ at 10 Hz.
  - Frequency noise $< 1 \times 10^{-3}\ \mathrm{Hz}/\sqrt{\mathrm{Hz}}$ at 100 Hz.
- Ultra-high vacuum in cavity: $< 4 \times 10^{-7}\ \mathrm{Pa}$
- Perfect optics (test mass):
  - Smoothness: peak-to-peak $\sim 1\ \mathrm{nm}$ (ITM), $\sim 4\ \mathrm{nm}$ (ETM)
  - Low transmission coating: ~ 0.014 (ITM), $\sim 5 \times 10^{-6}$ (ETM)
- Vibration isolation:
  - Longitudinal noise $\sim 1 \times 10^{-19}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$ at 10 Hz.

Field Propagation in Free Space

Huygen’s Principle:

Every point on the wavefront is the source of spherical wavelets.



$$\Delta x = x - x_0 \quad \Delta y = y - y_0$$

$$\rho = \sqrt{(\Delta x)^2 + (\Delta y)^2 + z^2}$$

$$E(\vec{r}) = \frac{i}{\lambda} \iint_{map} dx_0 dy_0 E_0(x_0, y_0) \frac{e^{-ik\rho} z}{\rho^2}$$

Field Propagation in Free Space

Fresnel approximation:

$$\rho = \sqrt{(\Delta x)^2 + (\Delta y)^2 + z^2} \approx z\left(1 + \frac{(\Delta x)^2 + (\Delta y)^2}{2z^2}\right)$$

So

$$E(\vec{r}) = \frac{i}{\lambda}\iint_{map} dx_0 dy_0 E_0(x_0, y_0)\frac{e^{-ikz}}{z}e^{-ik\left((\Delta x)^2+(\Delta y)^2)\right)/(2L)}$$

$$= \frac{ie^{-ikz}}{\lambda z}\iint_{map} dx_0 dy_0 E_0(x_0, y_0)e^{-ik\left((x-x_0)^2+(y-y_0)^2\right)/(2L)}$$

Convolution!

Field Propagation in Free Space

- Optics can be understood as a space-invariant linear system.
- The diffraction is the convolution of starting field with paraxial diffraction kernel K

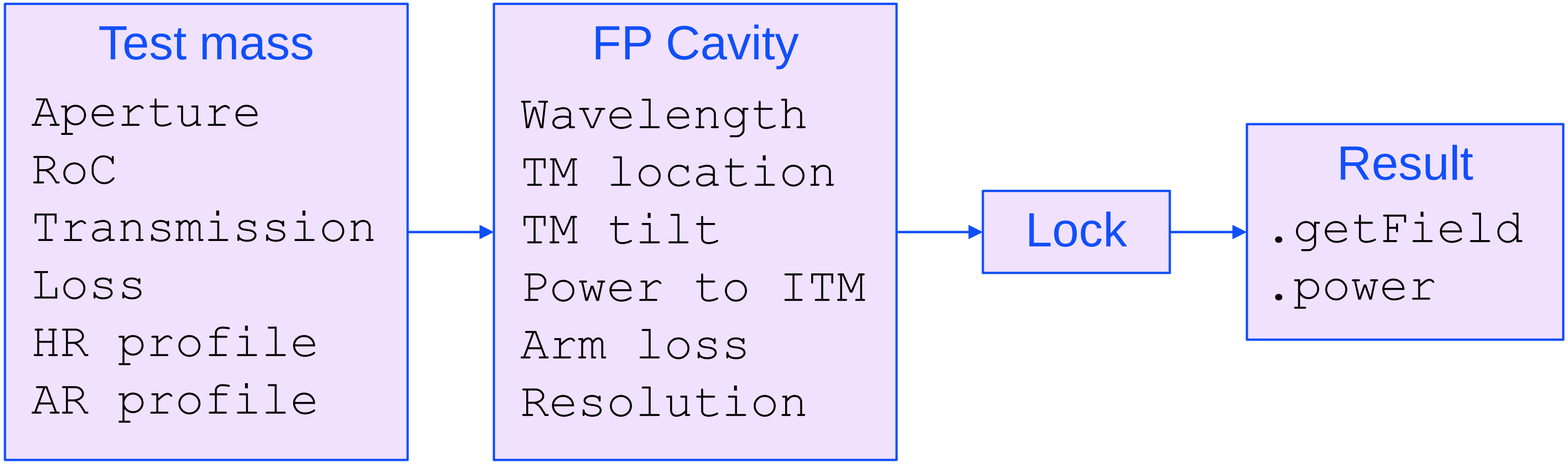
$$\tilde{E}_0(f_x, f_y) = \iint E_0(x, y) e^{-i2\pi(f_x x + f_y y)}\, dx\, dy$$

$$\tilde{K}(f_x, f_y) = \frac{ie^{-ikz}}{\lambda z} \iint e^{-ik\left(x^2+y^2\right)/(2L)} e^{-i2\pi(f_x x + f_y y)}\, dx\, dy$$

$$\tilde{E}(\vec{r}) = \tilde{E}_0 \cdot \tilde{K}$$

Static Interferometer Simulation

Given the dimensional and optical properties as well as the FP cavity configuration, the field amplitude distribution can be computed with a FFT-based tool: SIS.



Test mass

Aperture
RoC
Transmission
Loss
HR profile
AR profile

FP Cavity

Wavelength
TM location
TM tilt
Power to ITM
Arm loss
Resolution

Lock

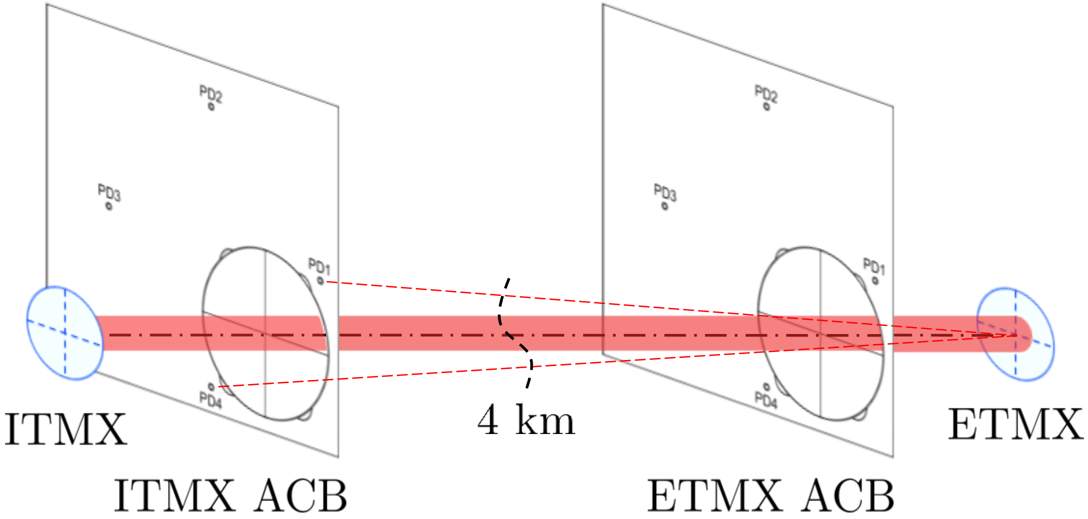
Result

.getField
.power

(*Father of SIS: Hiro Yamamoto*)

Scatter Measurement

Quadrant photo diode:
Excelitas YAG-444-AH



$$P = \frac{V}{RTG}$$

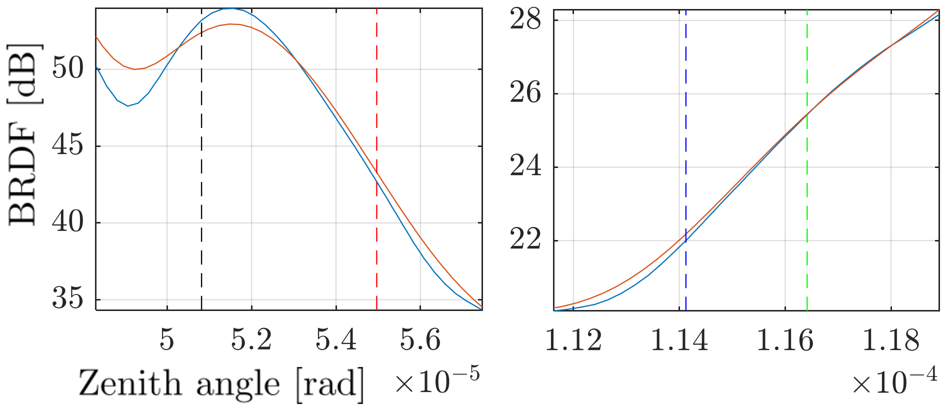
R: Responsivity (0.47 A/W)
T: Transimpedance (20 kΩ)
G: Analog gain

2 arms ⇨ 4 baffles ⇨ 16 PD

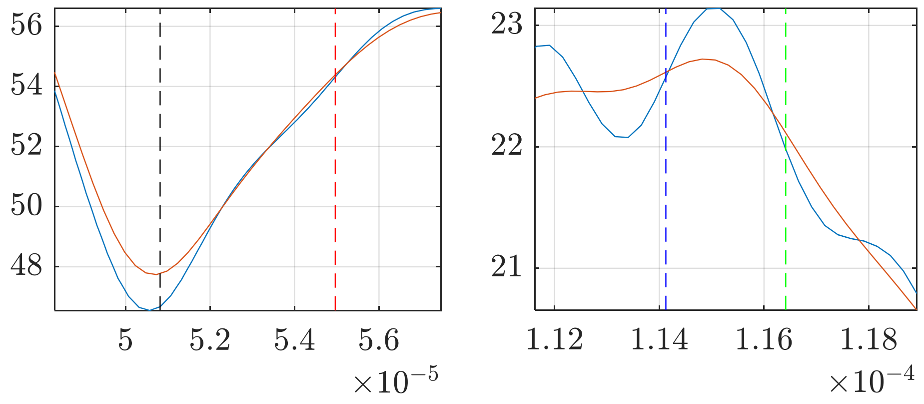
Radial Integration of SIS

Compute BRDF measured by PD:

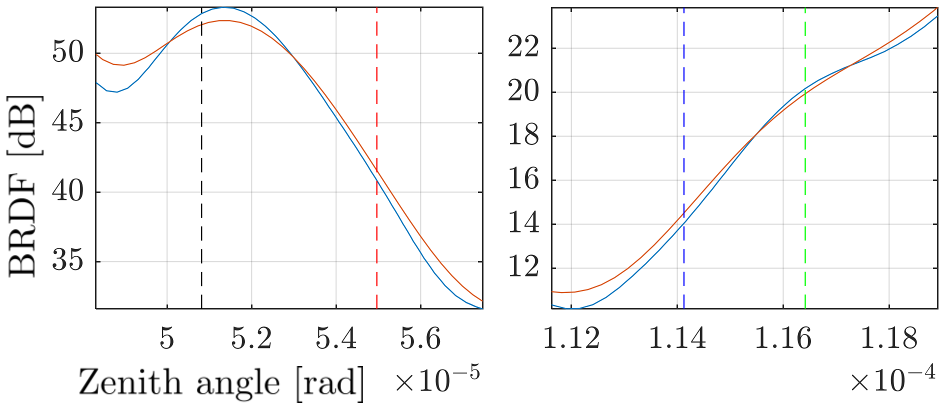
ITMX PD



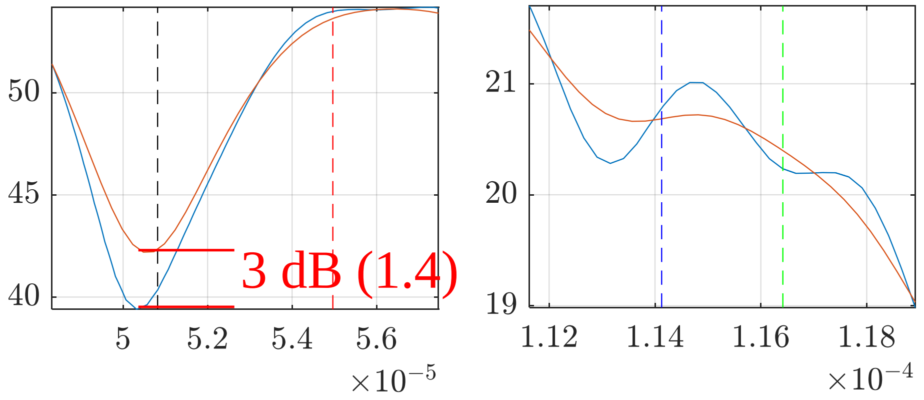
ETMX PD



ITMY PD



ETMY PD

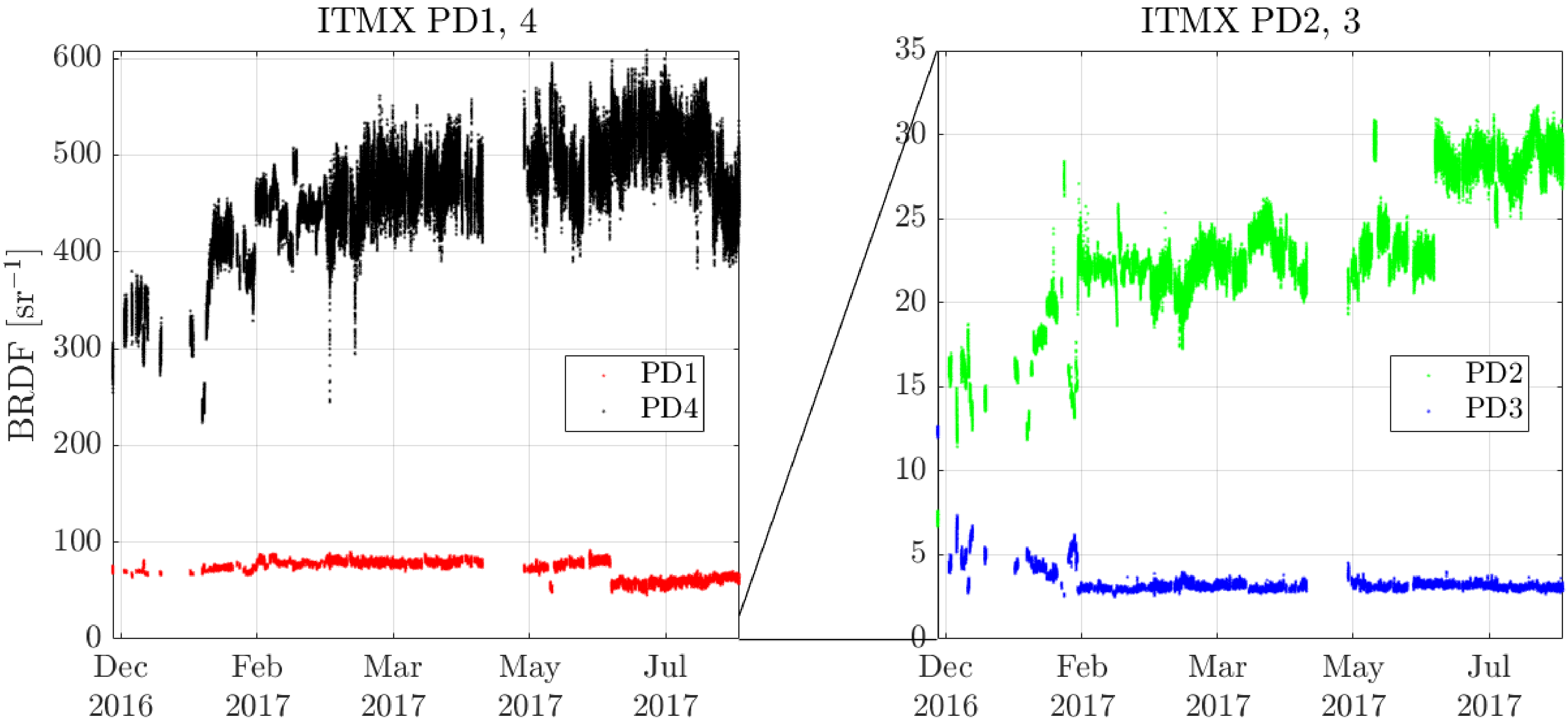


Scatter Data Processing

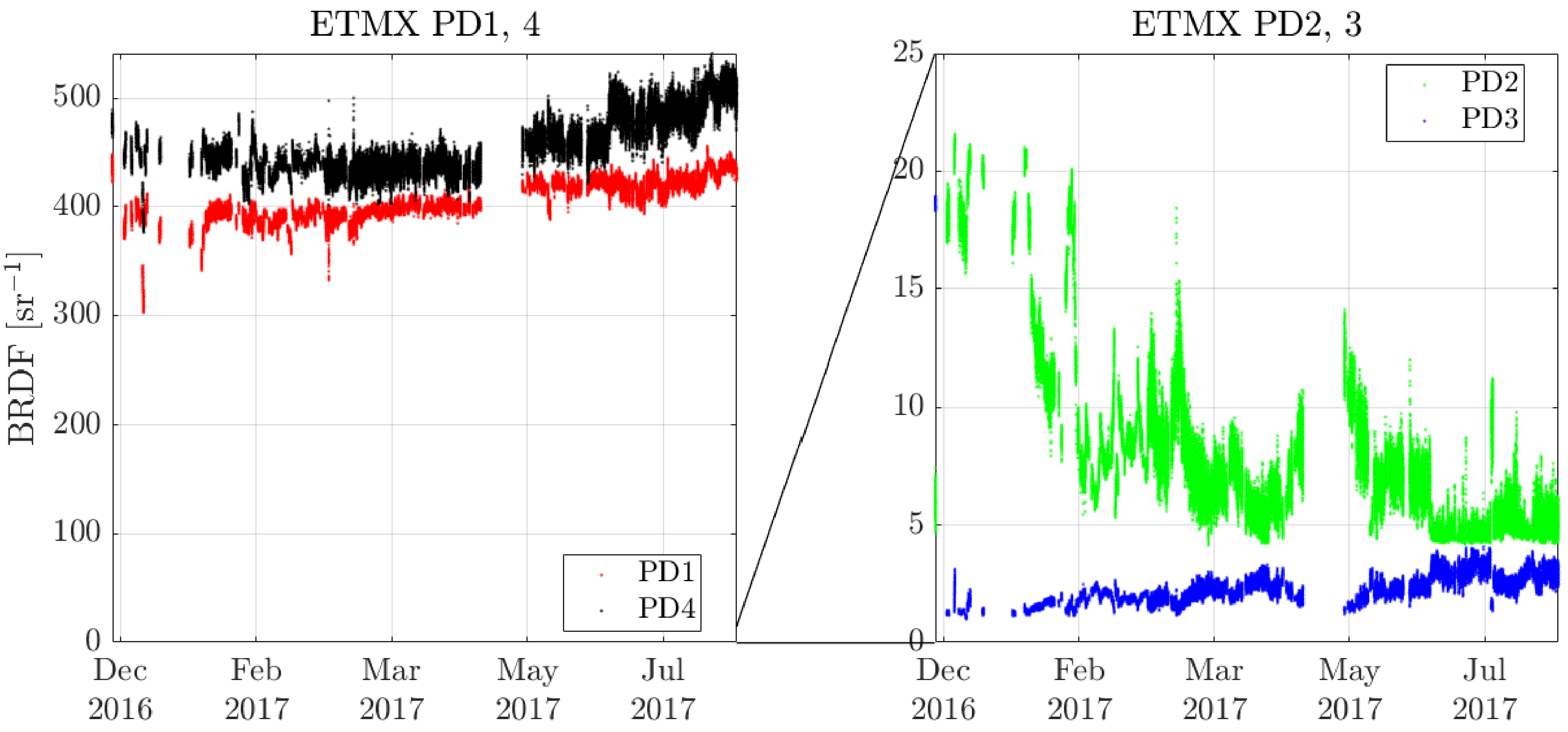
Factors	Channel Names
Lock State	*L1:GRD-ISC_LOCK_STATE_N*
Science Run Mode	*L1:ODC-OPERATOR_OBSERVATION_READY*
Power in PRC	*L1:LSC-POP_A_LF_OUTPUT*
Test Mass Attitude	*L1:SUS-*TM*_L2_DRIVEALIGN_P2L_GAIN* *L1:SUS-*TM*_L2_DRIVEALIGN_Y2L_GAIN*
PR2 Attitude	*L1:SUS-PR2_M3_DRIVEALIGN_P2L_GAIN* *L1:SUS-PR2_M3_DRIVEALIGN_Y2L_GAIN*

- Remove 3600 second transience on lock state, science run and all drivealign periods.
- Select locks with remaining time longer than 1 hour.

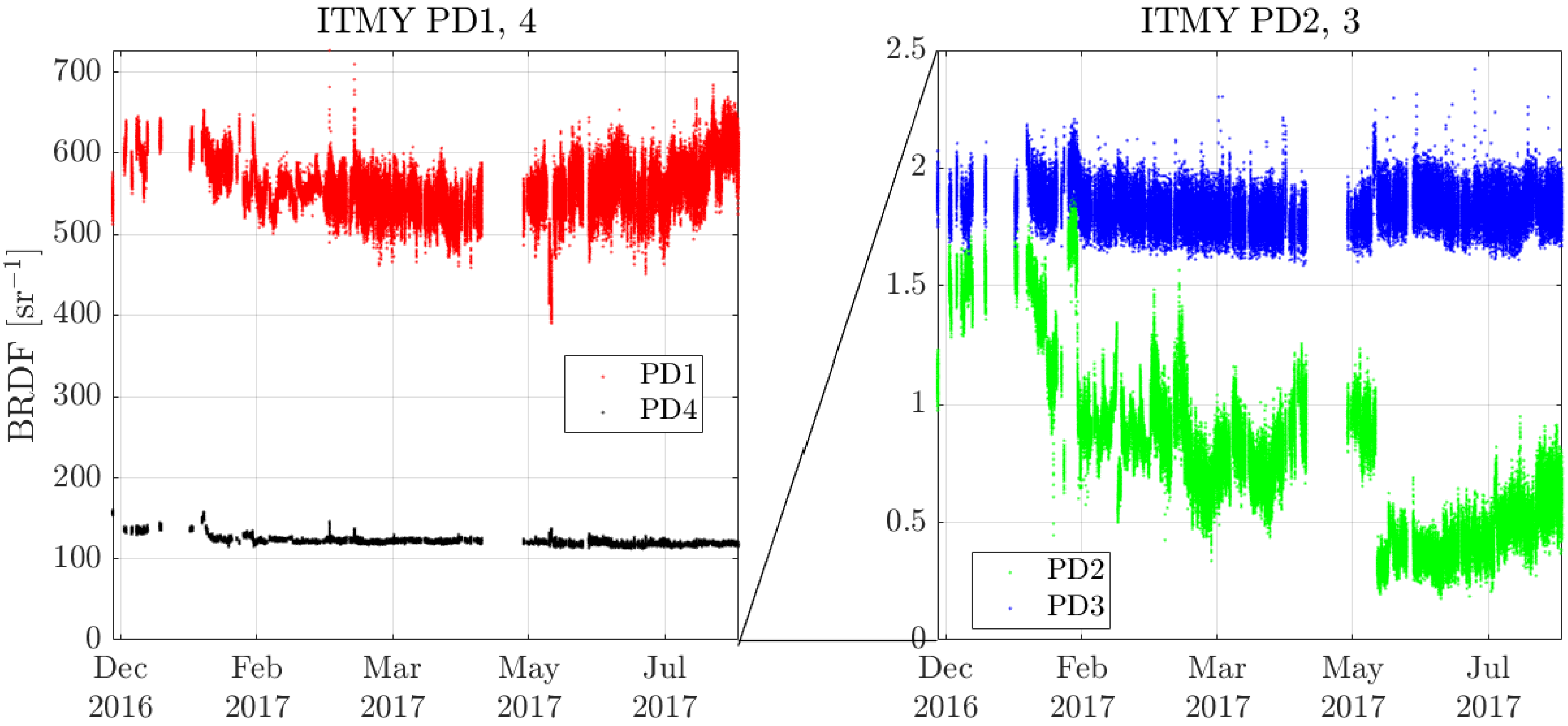
Scatter on ITMX



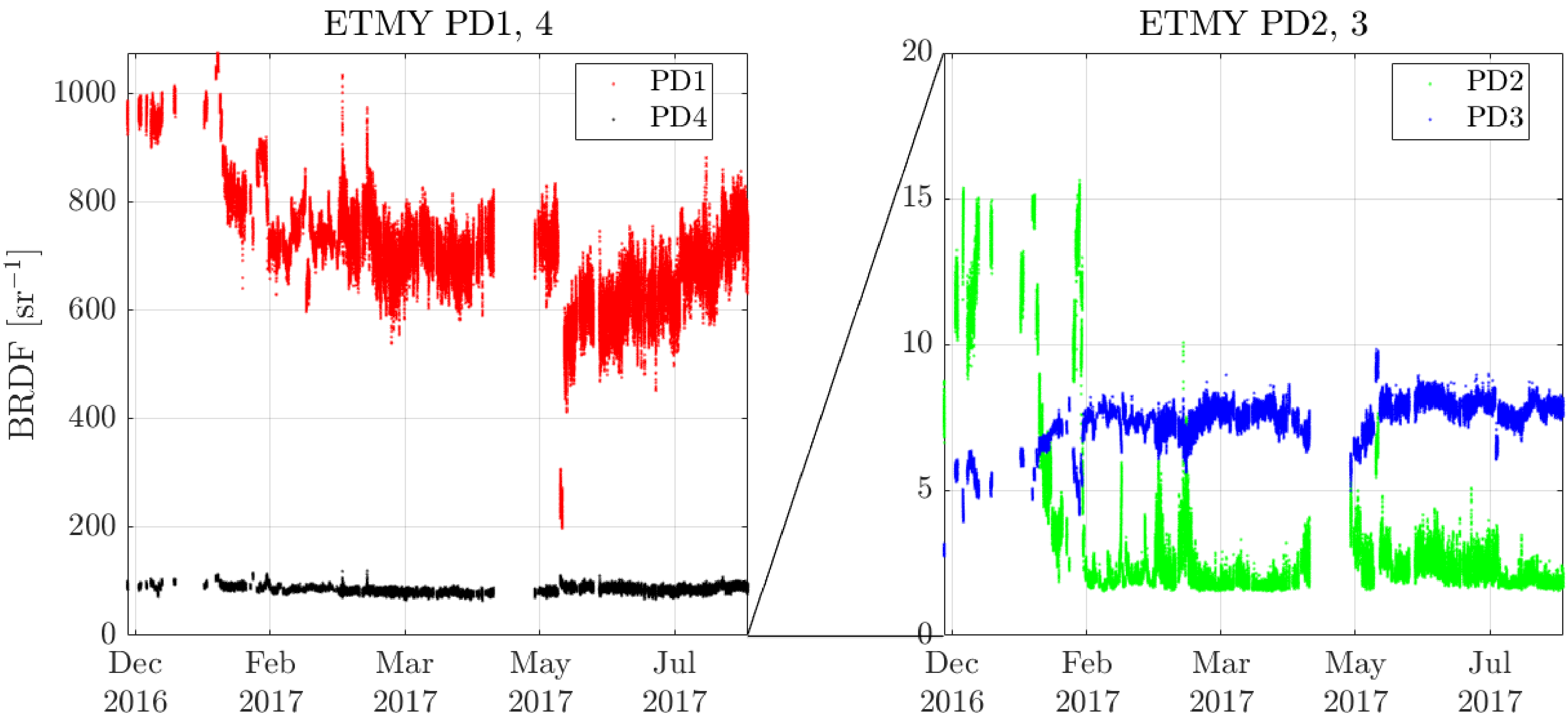
Scatter on ETMX



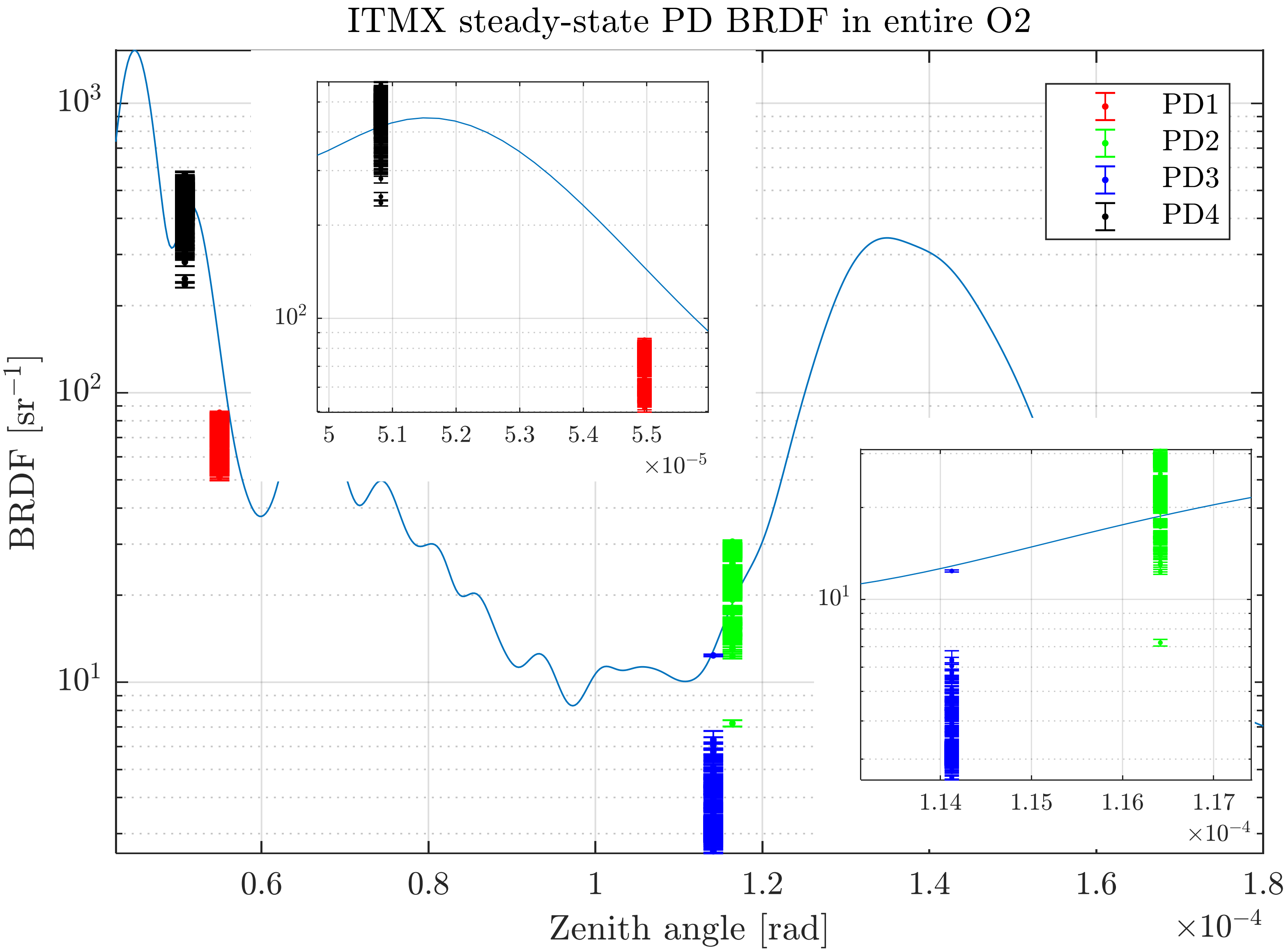
Scatter on ITMY



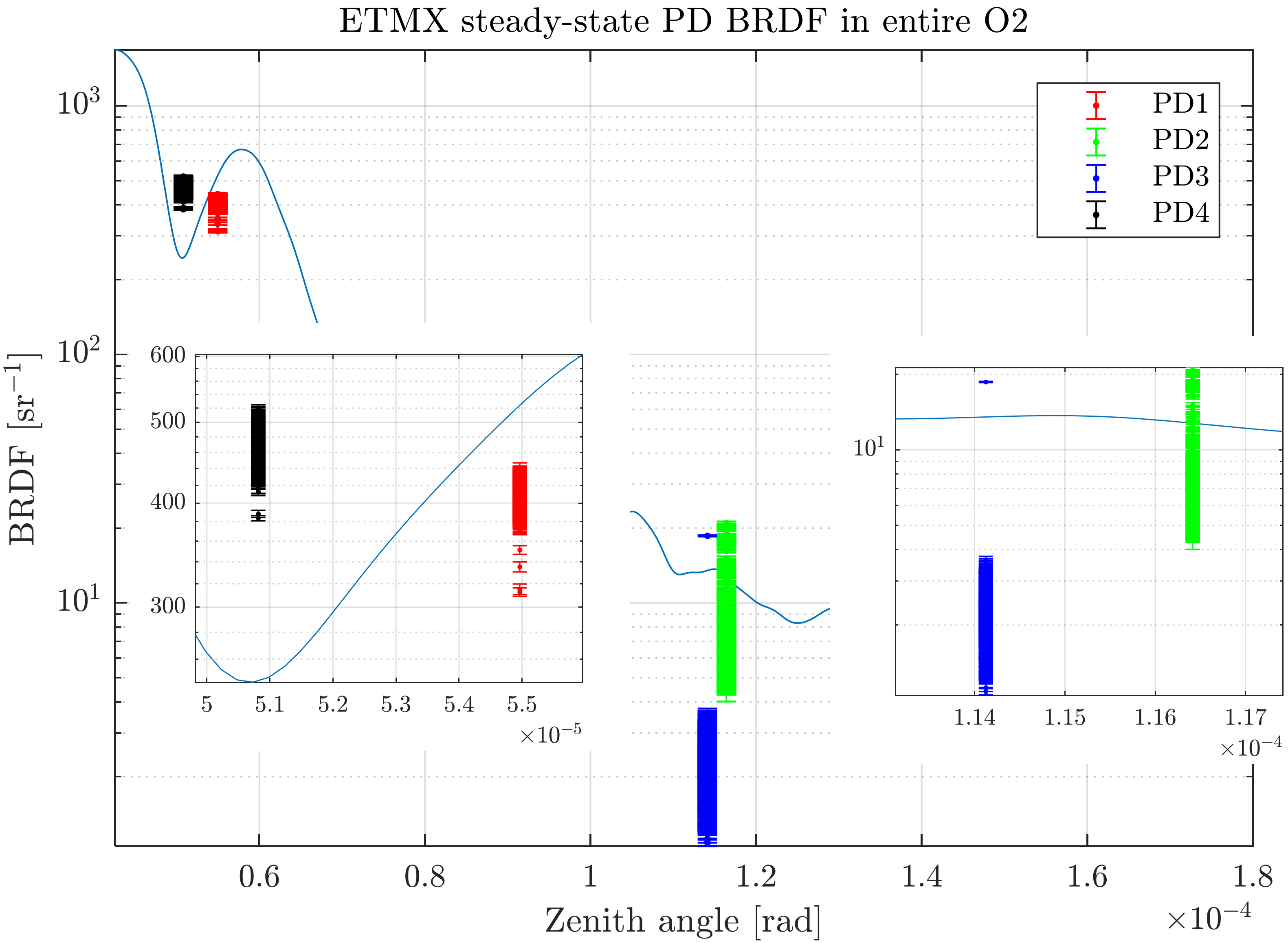
Scatter on ETMY



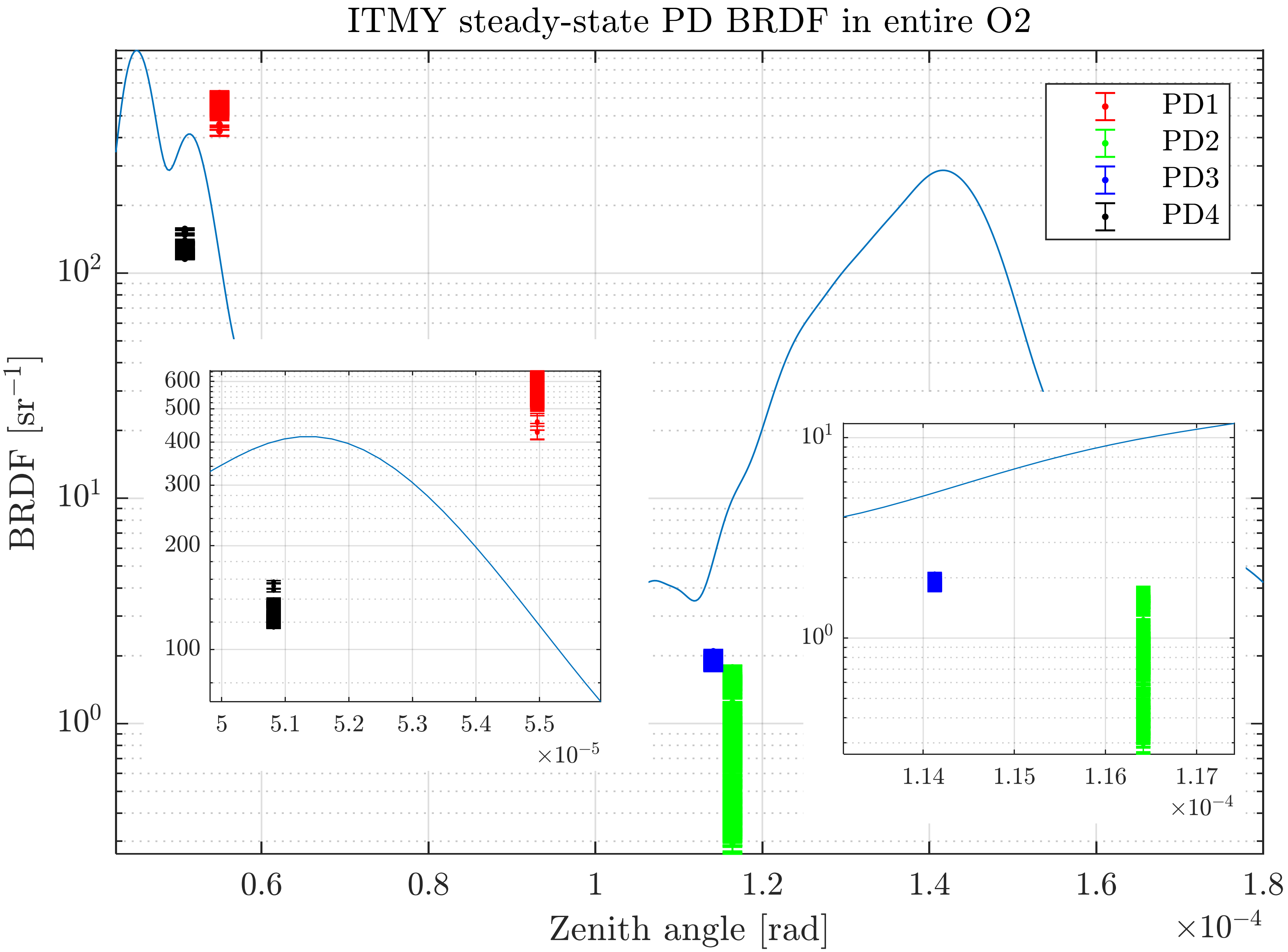
Steady-state BRDF



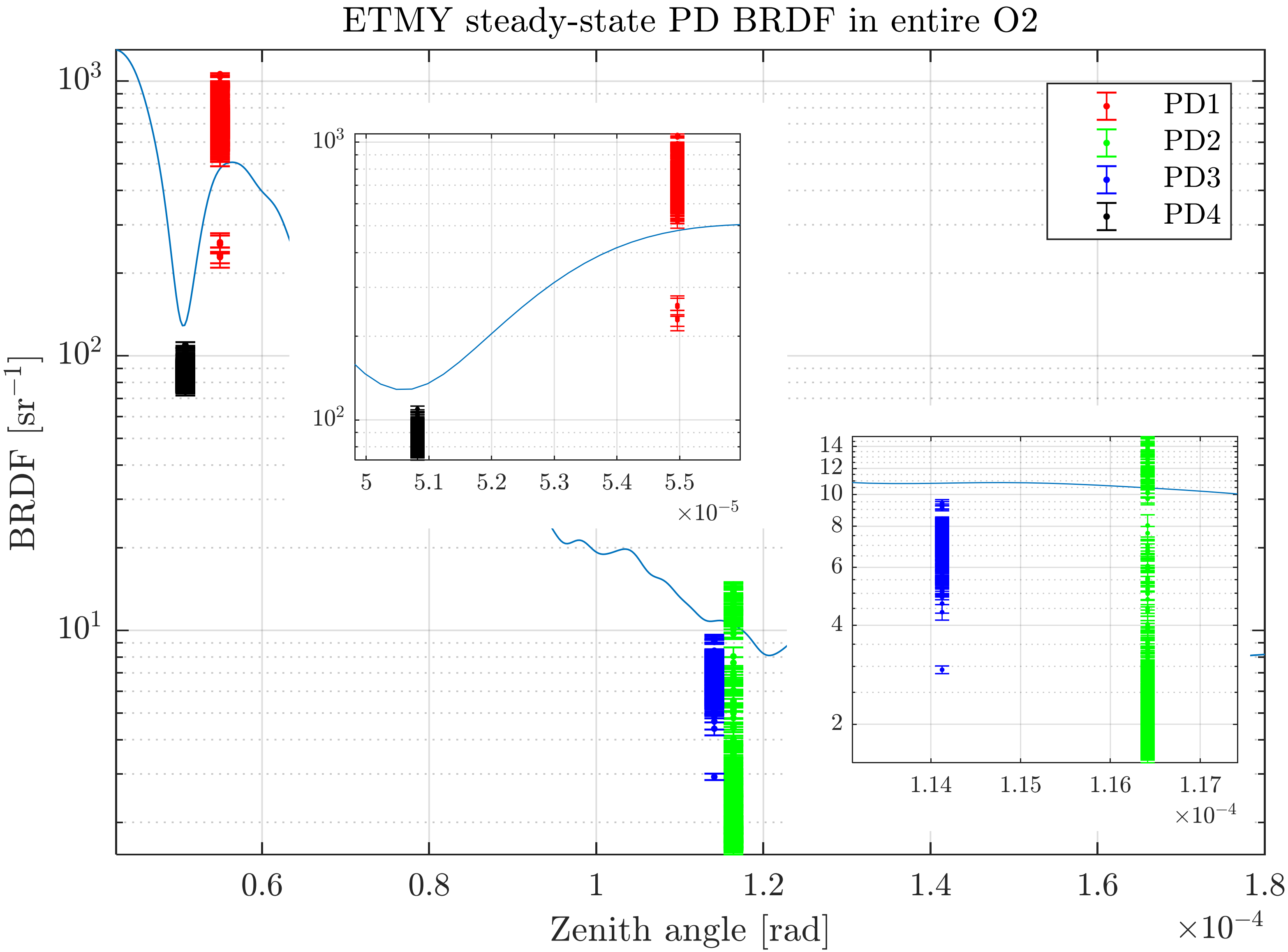
Steady-state BRDF



Steady-state BRDF



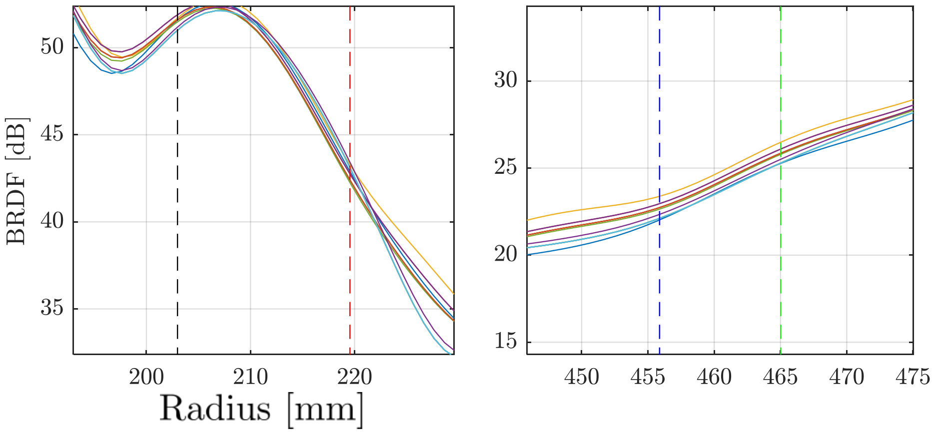
Steady-state BRDF



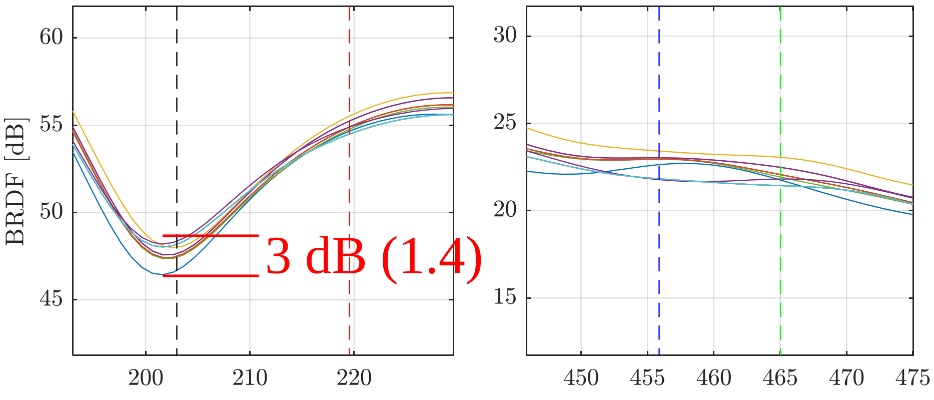
Scatter with Beam Positions in O2

Scatter simulation with all lock beam positions (mirror tilting)

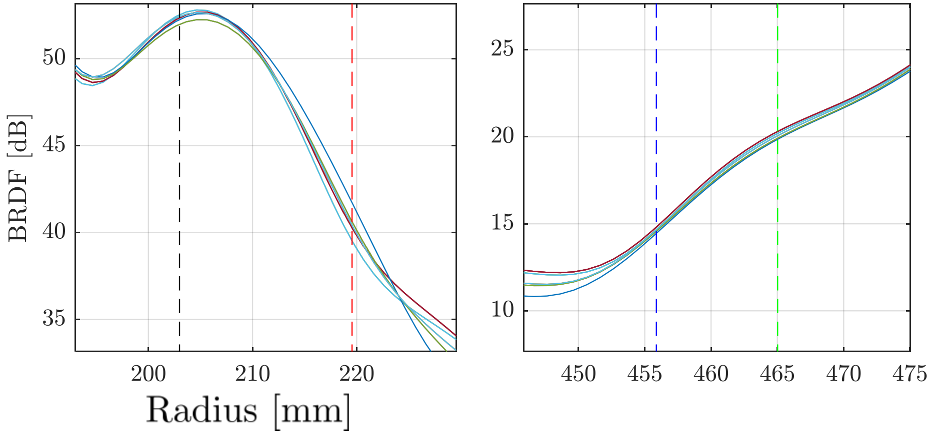
ITMX PD



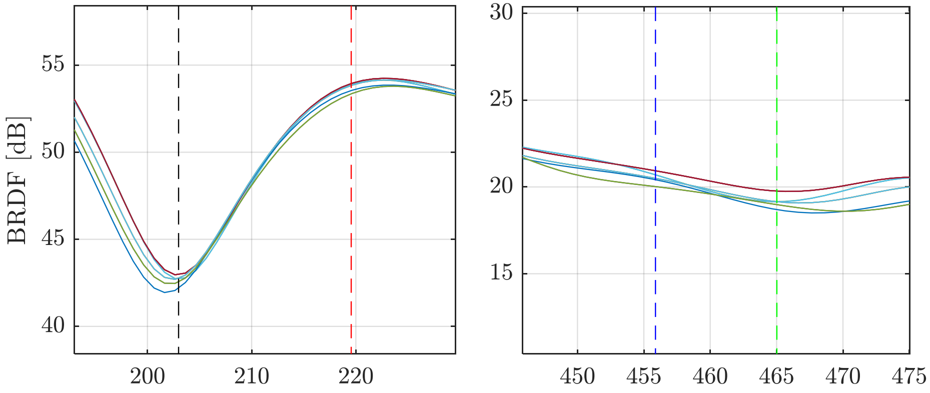
ETMX PD



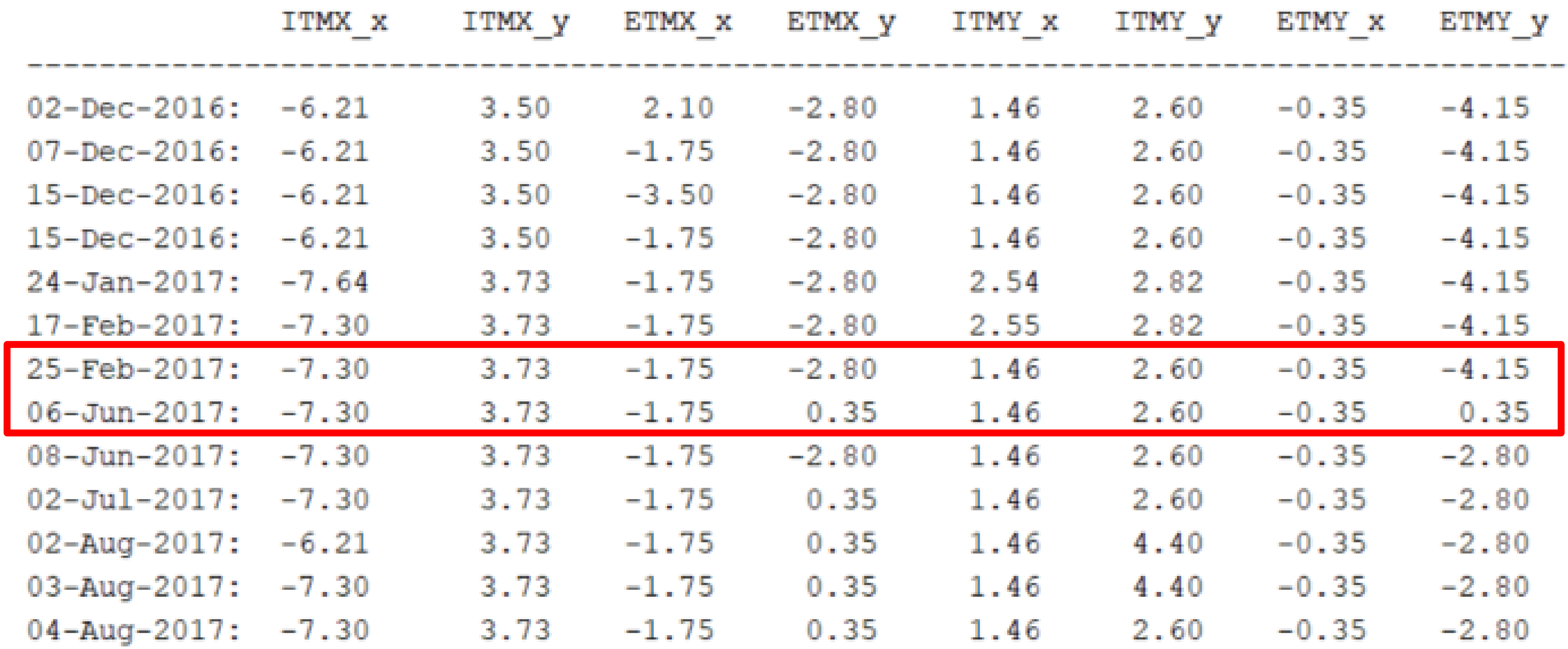
ITMY PD



ETMY PD



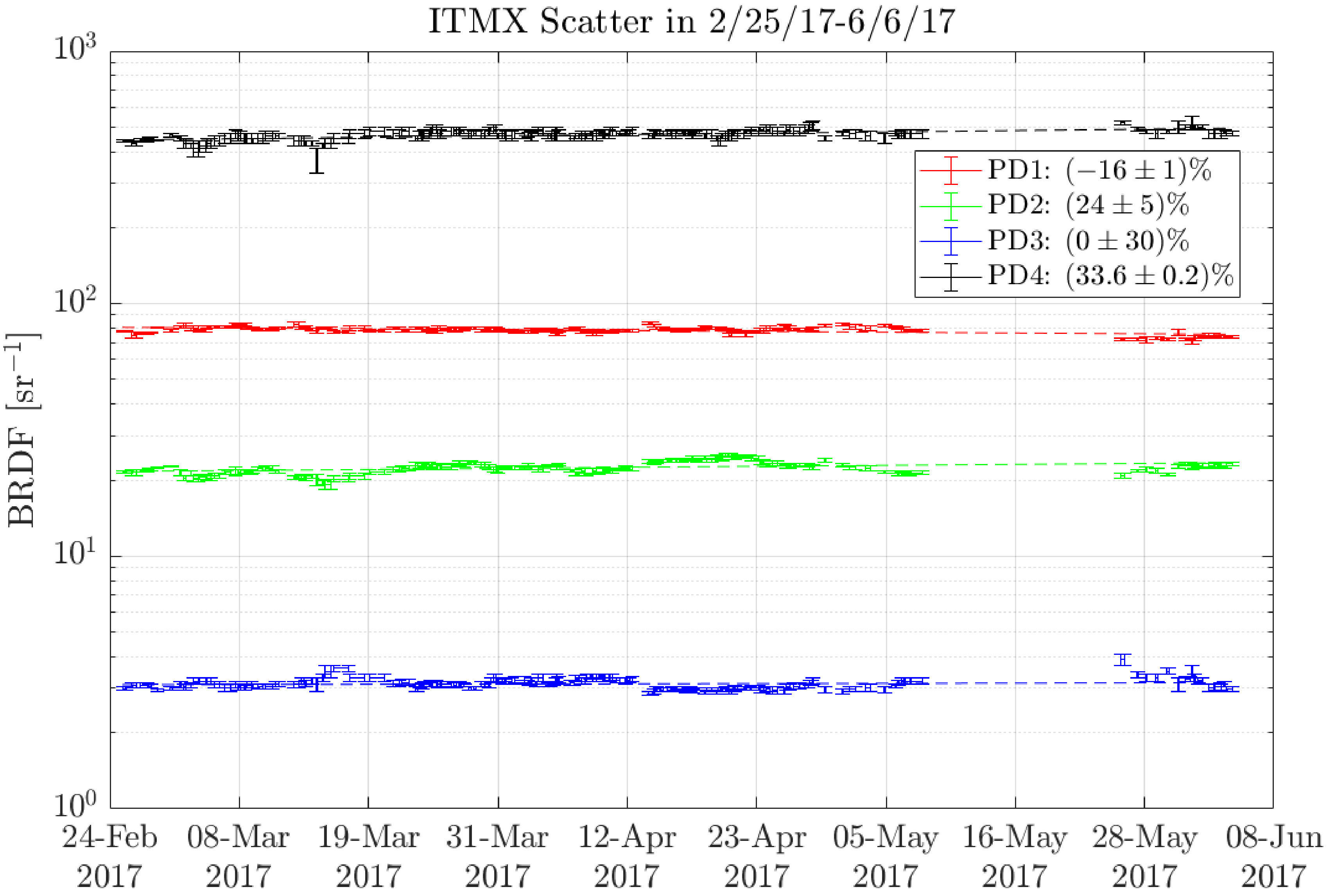
Longest Stable Duration in O2



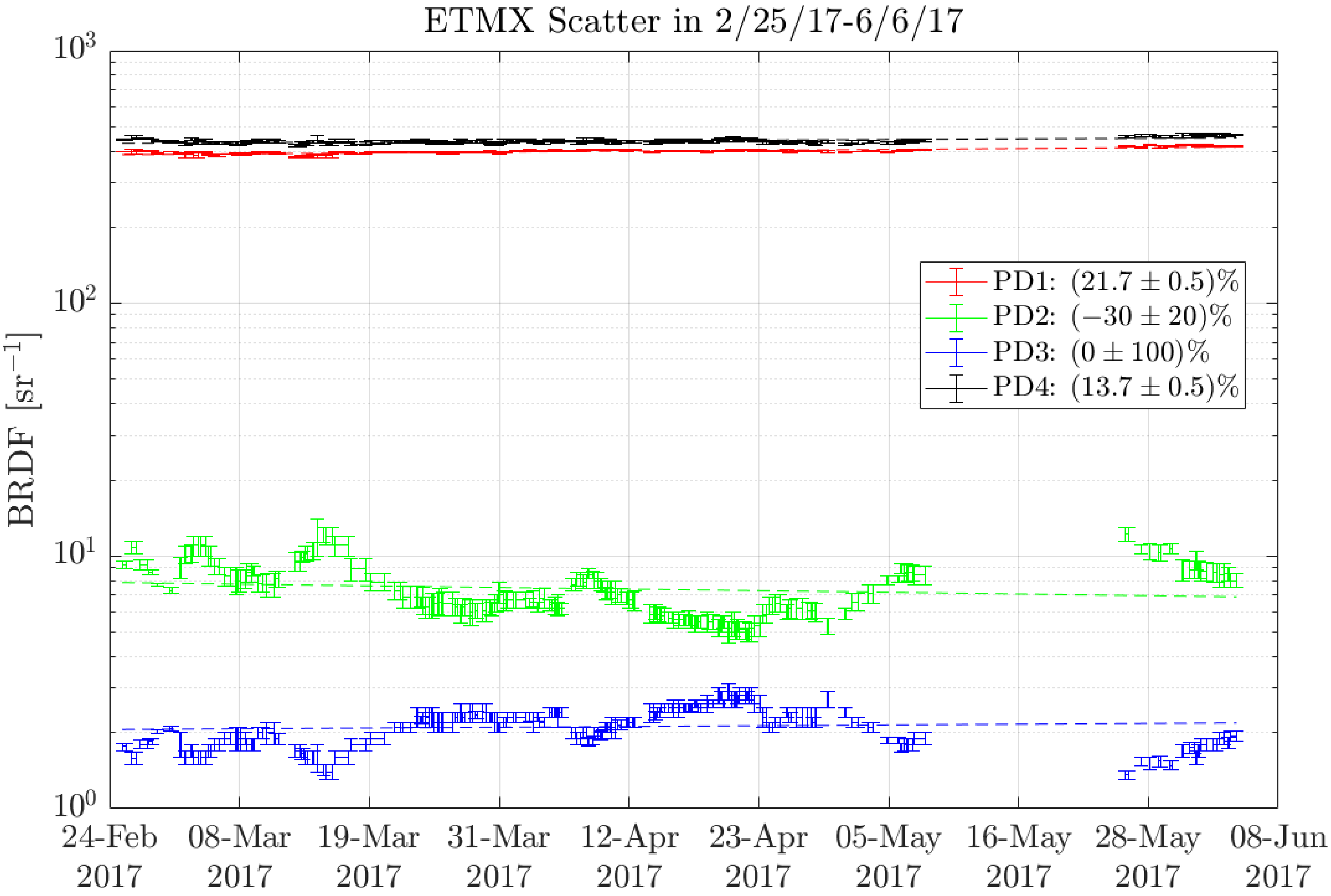
```
              ITMX_x    ITMX_y    ETMX_x    ETMX_y    ITMY_x    ITMY_y    ETMY_x    ETMY_y
--------------------------------------------------------------------------------------------
02-Dec-2016:  -6.21      3.50      2.10     -2.80      1.46      2.60     -0.35     -4.15
07-Dec-2016:  -6.21      3.50     -1.75     -2.80      1.46      2.60     -0.35     -4.15
15-Dec-2016:  -6.21      3.50     -3.50     -2.80      1.46      2.60     -0.35     -4.15
15-Dec-2016:  -6.21      3.50     -1.75     -2.80      1.46      2.60     -0.35     -4.15
24-Jan-2017:  -7.64      3.73     -1.75     -2.80      2.54      2.82     -0.35     -4.15
17-Feb-2017:  -7.30      3.73     -1.75     -2.80      2.55      2.82     -0.35     -4.15
25-Feb-2017:  -7.30      3.73     -1.75     -2.80      1.46      2.60     -0.35     -4.15
06-Jun-2017:  -7.30      3.73     -1.75      0.35      1.46      2.60     -0.35      0.35
08-Jun-2017:  -7.30      3.73     -1.75     -2.80      1.46      2.60     -0.35     -2.80
02-Jul-2017:  -7.30      3.73     -1.75      0.35      1.46      2.60     -0.35     -2.80
02-Aug-2017:  -6.21      3.73     -1.75      0.35      1.46      4.40     -0.35     -2.80
03-Aug-2017:  -7.30      3.73     -1.75      0.35      1.46      4.40     -0.35     -2.80
04-Aug-2017:  -7.30      3.73     -1.75      0.35      1.46      2.60     -0.35     -2.80
```

- In the longest stable duration (LSD), the beam positions are constant in all locks.
- It would be interesting to see how scatter varied during LSD

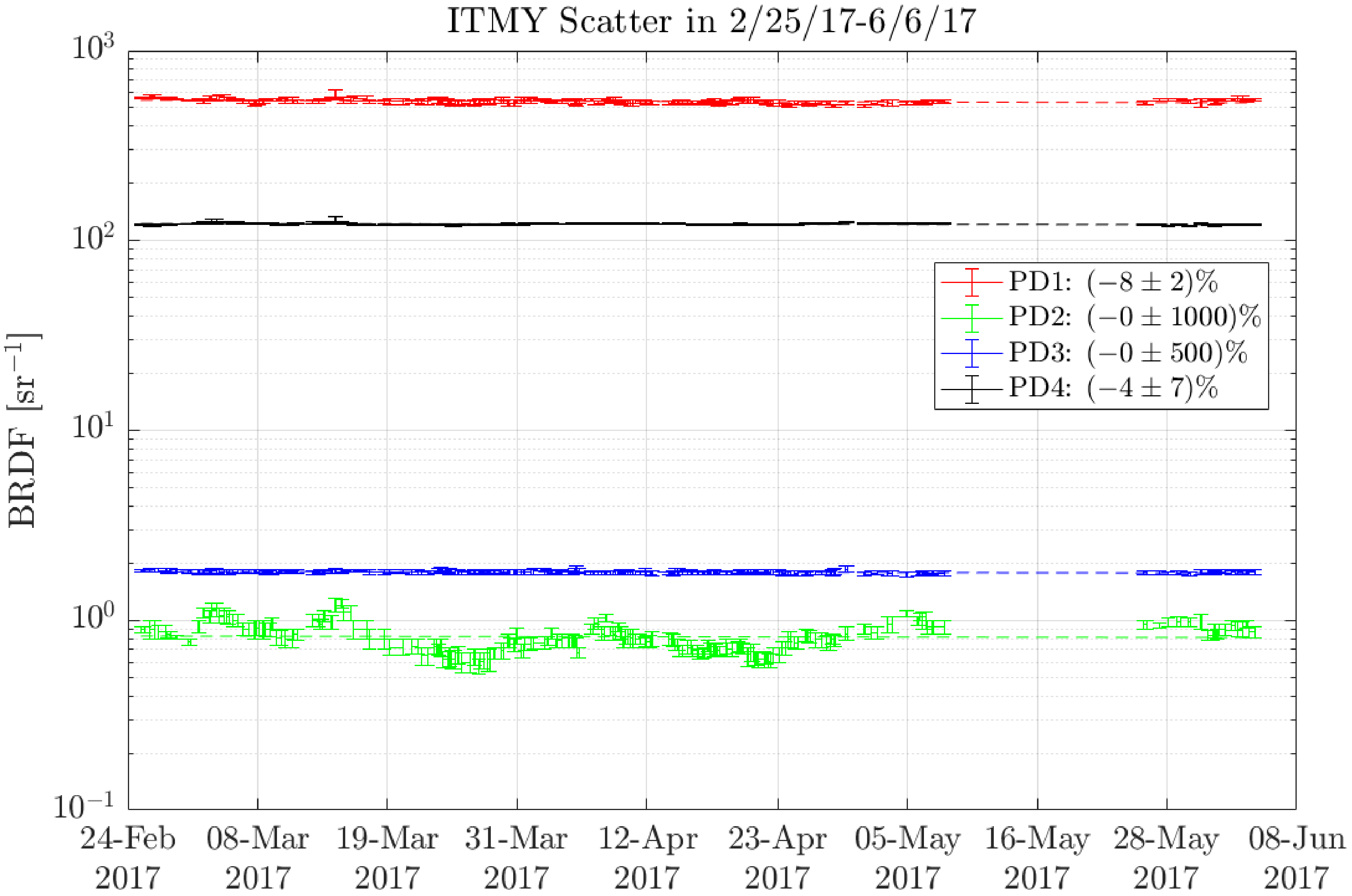
Scatter during LSD



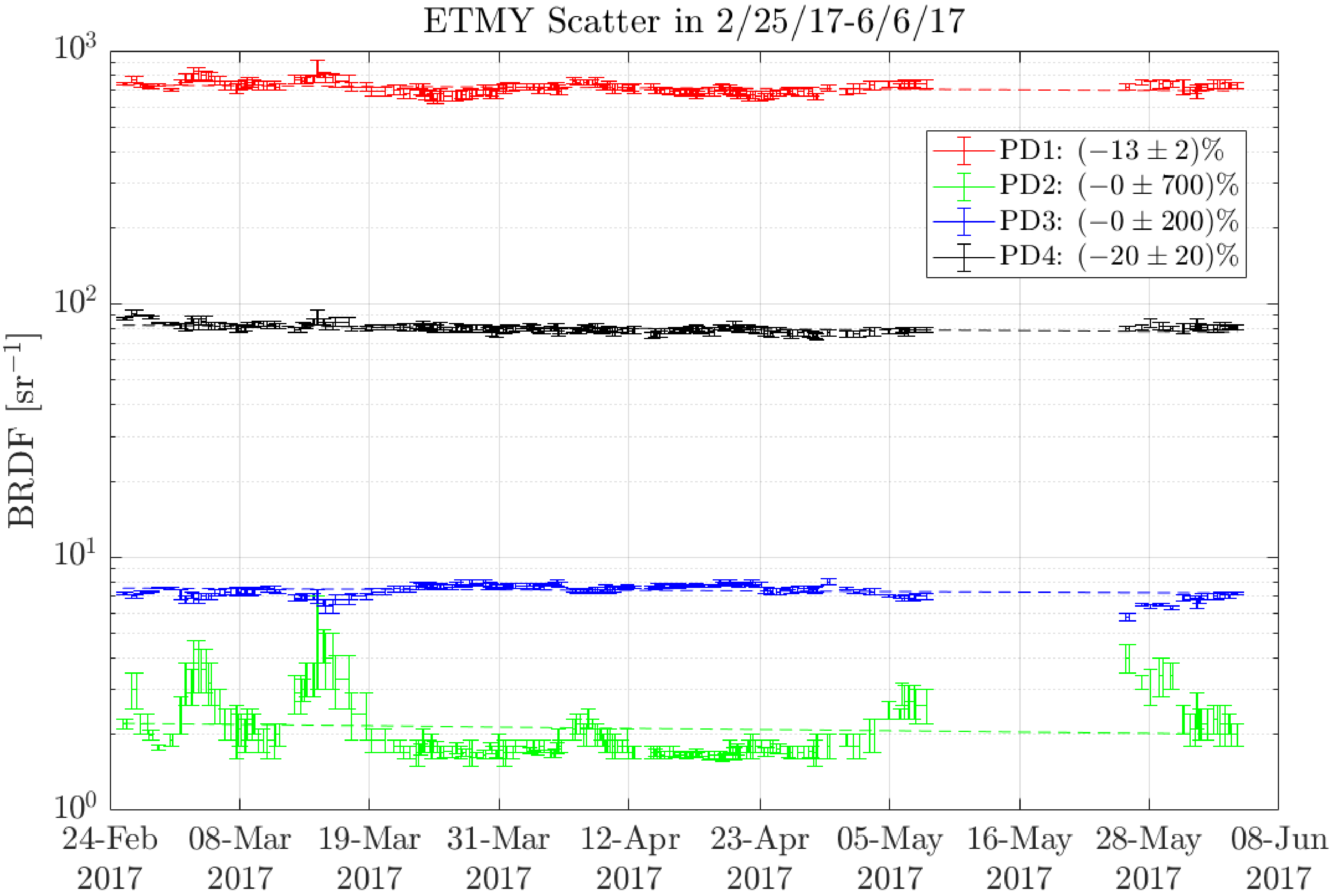
Scatter during LSD



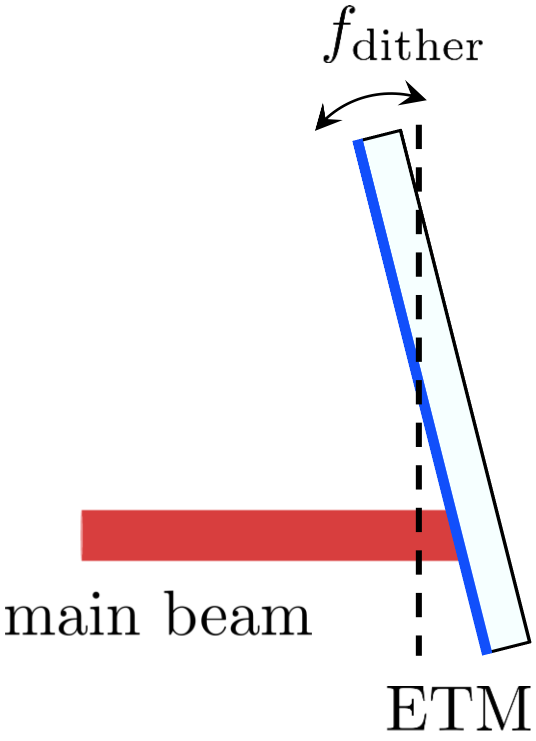
Scatter during LSD



Scatter during LSD



Alignment Dither System



High energy at f_{dither} in PSD of DARM

Demodulate & calibrate

Beam position on ETM

- Demodulated amplitude $\propto$ dither amplitude $\propto$ beam offset

Alignment Dither System

L1:ASC-ADS GAIN

Scatter Data Processing

Factors	**Channel Names**
General feedback gain	*L1:ASC-ADS GAIN*
ETMX(Y/PR2) Pitch dither magnitude	*L1:ASC-ADS PIT3(4/5) OSC CLKGAIN*
ETMX(Y/PR2) Pitch error	*L1:ASC-ADS PIT3(4/5) DOF INMON*
ETMX(Y/PR2) Yaw dither magnitude	*L1:ASC-ADS YAW3(4/5) OSC CLKGAIN*
ETMX(Y/PR2) Yaw error	*L1:ASC-ADS YAW3(4/5) DOF INMON*

- 25 W-laser output alignment starts when *LOCK_STATE_N* = 1731
- Select locks with duration longer than 1 hour.

$$\text{X/Y offset [m]} = \frac{INMON}{(CLKGAIN)(ADSGAIN)}k, \quad k = \begin{cases} 0.98, & \text{ETMX pitch} \\ -0.72, & \text{ETMX yaw} \\ 0.95, & \text{ETMY pitch} \\ -0.76, & \text{ETMY yaw} \end{cases}$$

Regression Model

- Nonparametric regression model
  - High-dimensional predictors
  - Complex
- Parametric regression model
  - Physically interpretable
  - Computationally efficient
  - Easy diagnostics
  - Most common one: linear model

General Linear Regression

- General linear regression model
  - Response: y
  - *p*-dimensional predictors: $x_i, \;\; i = 1, 2, ..., p$
  - With n-observation: $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, $n \sim 10^5$ in LSD

the design matrix (without interaction):

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{11}^2 & \dots & x_{11}^k & x_{21} & \dots & x_{21}^k & \dots & x_{p1}^k \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ 1 & x_{1j} & x_{1j}^2 & \dots & x_{1j}^k & x_{2j} & \dots & x_{2j}^k & \dots & x_{pj}^k \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ 1 & x_{1n} & x_{1n}^2 & \dots & x_{1n}^k & x_{2n} & \dots & x_{2n}^k & \dots & x_{pn}^k \end{bmatrix}$$

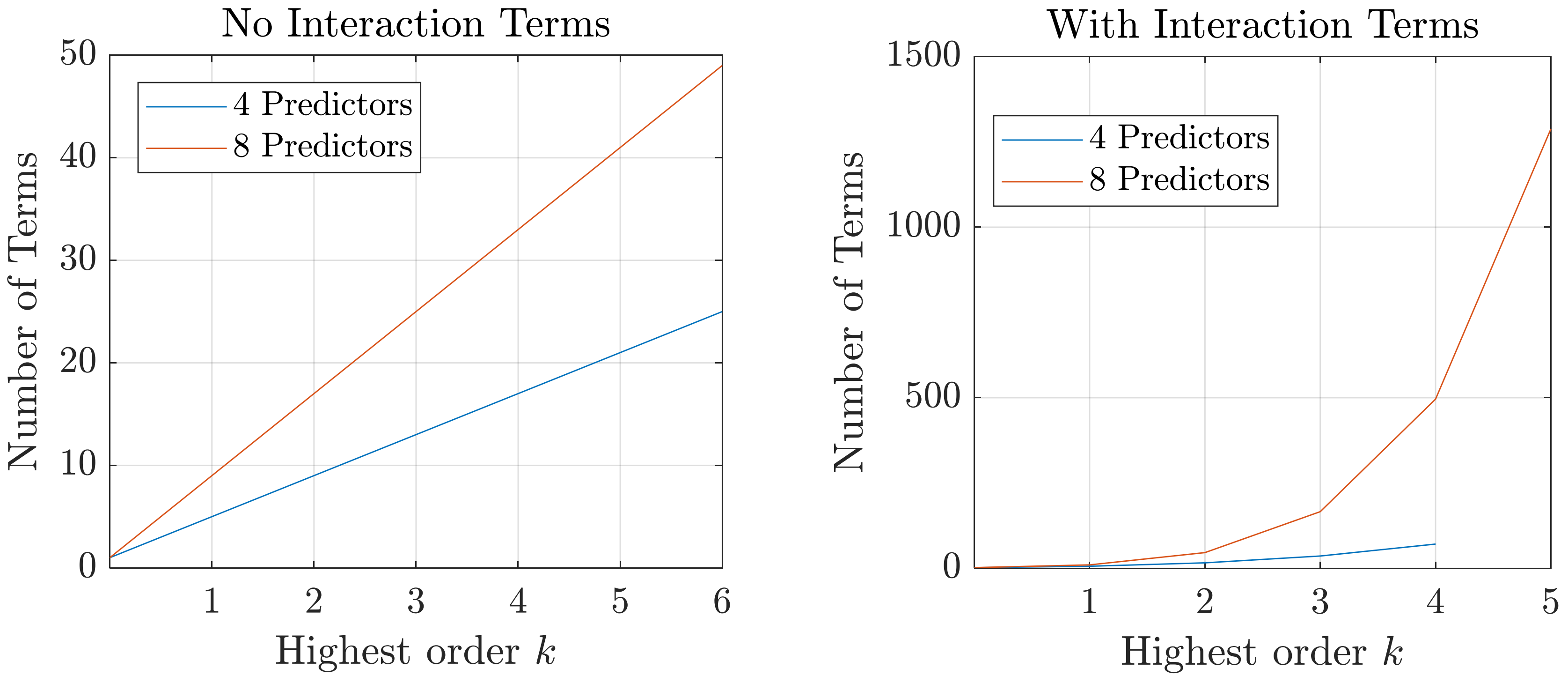
with interaction:

$$\mathbf{X}_j = \begin{bmatrix} 1 & x_{1j} & \dots & x_{1j}^{k_1} x_{2j}^{k_2} ... x_{pj}^{k_p} & \dots & x_{pj}^k \end{bmatrix}$$

where $k_1, k_2 ... k_p \in \{0, 1 ... k\}$ and $\sum_{i=1}^{p} k_i \leqslant k$

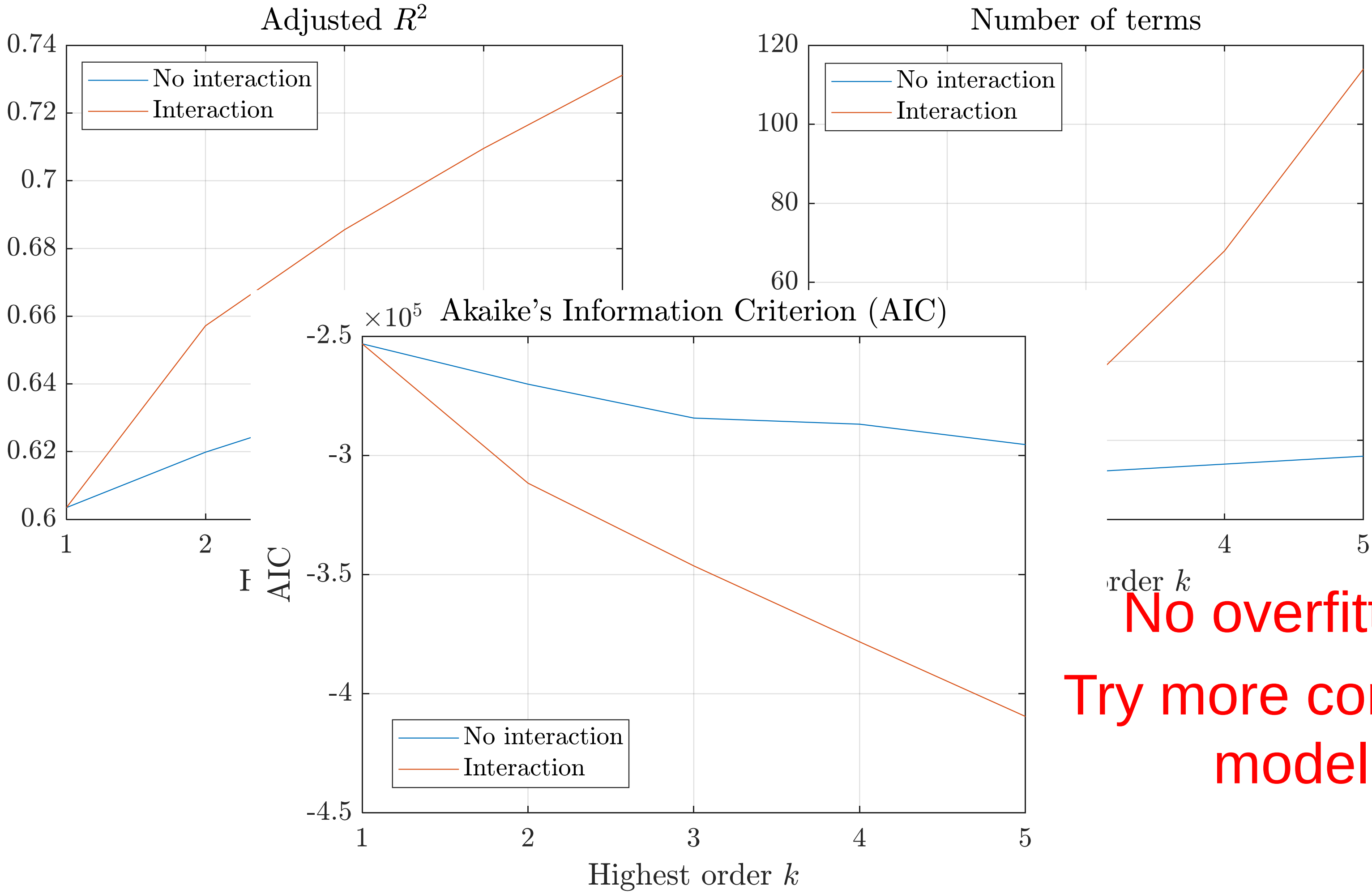
Stepwise Linear Regression

- Backward stepwise regression
  - Dimensional reduction
  - Trim model parameters to avoid overfitting and complexity
  - 1 response: x/y beam position
  - Use 4 predictors (4 PD) to avoid blowing up terms



Stepwise Linear Regression

Use measured beam position (*x*) on ETMX from 4 ETMX PD:



No overfitting

Try more complex model