

LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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Technical Note	LIGO-T1800201-v2	2018/06/02
Developing Phase Map of Cavity Mirrors using Laser Mode Spectroscopy Intermediate Report - SURF 2018		
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1 Introduction

The existence of gravitational wave was predicted by Albert Einstein in 1916 with the help of his general theory of relativity. According to Einsteins relativity presence of mass or energy bends the spacetime and accelerating massive objects like neutron star or blackholes produce time varying ripples in the spacetime. These ripples are known as the Gravitational Waves[1]. These waves would travel at the speed of light through the Universe. Study of these waves will give us information about the origin of these waves, nature of gravity and also a better picture of the universe.

The Laser Interferometer Gravitational Wave Observatory (LIGO) is a large-scale physics experiment and observatory to detect cosmic gravitational waves. It works with the help of two detectors located in Livingston, Louisiana and in Hanford, Washington. The first direct detection of Gravitational Waves was done by LIGO on September 14, 2015 [2]. This work earned three scientist the 2017 Nobel Prize in Physics.

The interferometers work on the principle of merging of two or more EM waves by a laser to create an interference pattern, which contain information about the object or phenomenon influencing it. This technique can be used to make extremely small measurements that are not achievable using other techniques. The detector used in LIGO is a modified Michelson interferometer with Fabry-Perot cavities introduced to it. This modified Michelson interferometer has a L shape, in which each arms are of 4 km length. Each arm of the detector forms a Fabry-Perot cavity capped by a semi-transparent Input Test Mass (ITM) and an End Test Mass (ETM) [3].

Assuming that the suspended mirrors of the interferometer are still, the returning beam from each arm has a constant phase in the absence of gravitational waves. Thus, the interference condition of the Michelson interferometer is stationary. When a gravitational wave passes through the detector it changes the differential of the arms by making it contract or expand. Now, the intensity of the TEM_{00} mode obtained at the output port will be different from the one we obtained when both the arms were of same length. Thus, these interference patterns can be used to make the precise measurement of the change in length occurred and also the nature of gravitational waves causing this change.

The change in length of the interferometer arm caused by the gravitational wave is about 10^{-19} m. Measurements of this scale is done by increasing the interaction time of Gravitational Waves with each arm. This is done by introducing a Fabry-Perot cavity inside each arm in which, laser beam effectively undergoes multiple reflections. But this increases the chance of power loss due to unwanted scattering from the mirror surfaces (ITM and ETM). The scattering can be due to point defects, scratches, sleeks, contaminations and Figure error of the mirrors [5]. The aim of this project is to precisely characterize the figure error of an optical cavity using an interferometric measurement. If we can reduce the power loss efficiently, it will help us to operate the interferometers with increased power efficiency and reduced scattered light problem.

2 Objectives

The objectives of this project are listed below.

1. Using Finesse software for obtaining the ideal cavity scan data and estimating the free spectral range and transverse mode spacing corresponding to a particular frequency scan.
2. Obtaining the cavity scan data from the experiment and comparing these experimental deviations with theoretical mode spacings.
3. By analysing this data, precisely characterizing the figure error of an optical cavity by plotting Phase Maps.

Mirror figure errors are the low frequency surface defects present on the test masses which results in the low angle scattering of the light. This causes power loss in the optical cavities, which ultimately leads to destruction of squeezed state of light. Through this project we are trying to develop a in-situ technique to precisely characterizing the figure error of an optical cavity.

Mirror figure error can be characterised by using Phase Maps. Phase Maps are those diagrams that show us the imperfections in the surface of mirror which contribute towards the power loss through scattering. The conventional technique to create these mirror map is Fizeau interferometry. But this cannot be done inside the original LIGO interferometer without disturbing the output. Thus, we are interested in developing a technique which can be used in the original LIGO interferometer. Such techniques are classified as In-situ measurement techniques.

3 Approach

3.1 Optical Resonator

An optical resonator is a system containing two mirrors (ITM and ETM) separated by a distance. The Fabry-Perot cavity introduced in the Michelson interferometer act as an optical resonator. The laser beam passing through each arm bounces back and forth between the mirrors, which leads to power recycling. Optical resonators are characterised by the radius of curvature of the mirrors and the absolute distance between them. An optical resonator

has a certain set of modes which are allowed to resonate inside it. All the modes obtained in the transmission spectrum of the Fabry-Perot Michelson Interferometer can be expressed as a linear combination of these cavity eigen modes. The resonance happens at different laser frequencies for different cavity eigen modes. For perfect spherical mirrors, eigen modes become Gaussian Modes.

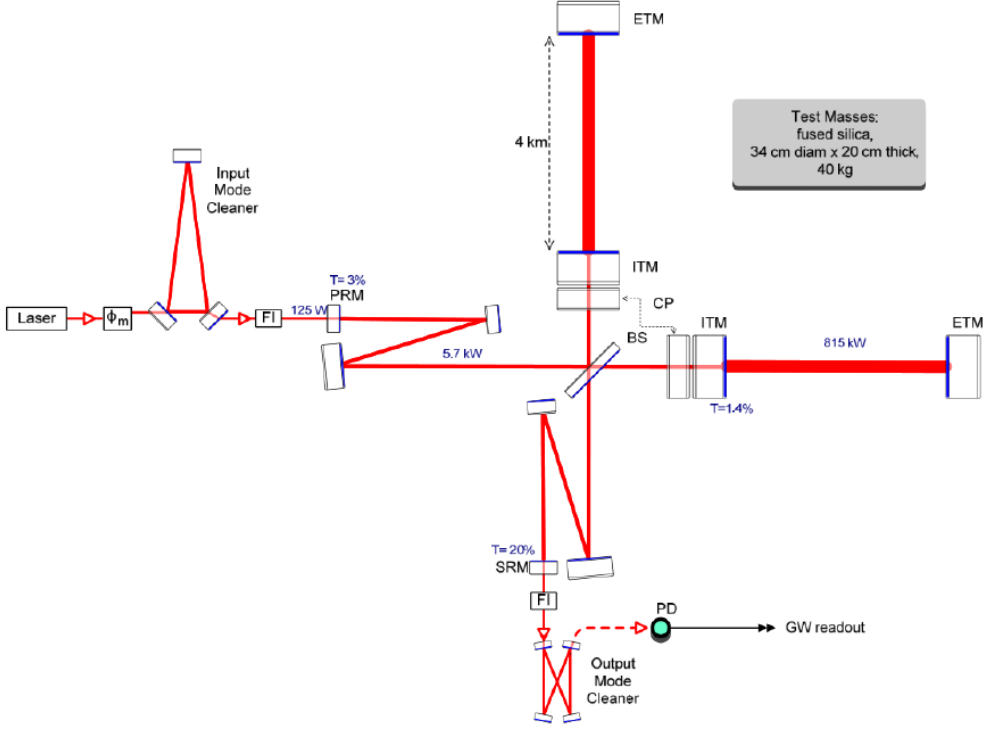


Figure 1: Schematic diagram of the LIGO 40m interferometer [4].

The cavity has repetitive resonances with the laser frequency. The spacing between two adjacent such modes are known as the Free Spectral Range (FSR) [8]. The free spectral range frequency (ν_{FSR}) is defined as,

$$\nu_{FSR} = \frac{c}{2L} = 3.7474 \times 10^6 \text{ Hz}$$

Here, c is the speed of light and L is the length of the cavity which is equal to 40 m. These resonances are known as the fundamental mode resonance. Apart from these fundamental mode resonances in the transmission spectrum, there are higher order Transverse mode resonances. The distance between fundamental mode resonance and higher order Transverse mode resonance is known as the Transverse mode spacing (ν_{TMS}). For TEM_{nm} mode, transverse mode spacing will be equal to,

$$\nu_{TMS} = \nu_{FSR} \times \left(\frac{n+m}{\pi}\right) \cos^{-1} \sqrt{g_1 g_2}$$

The g-parameter for this cavity are given by,

$$g_1 = 1 - \frac{L}{R_1} \approx 1$$

$$g_2 = 1 - \frac{L}{R_2} \approx 0.298$$

Here $R_1 = \infty$ and $R_2 = 60$ m are the radius of curvature of the ITM and ETM mirrors. Then for TEM_{10} and TEM_{01} modes, the ν_{TMS} will be equal to

$$\nu_{TMS} = 1.14 \times 10^6 \text{ Hz}$$

3.2 Experiential Technique

A schematic diagram of the interferometer we will be working on in this project is shown in Figure 1. We will be using scan data from only one arm of the interferometer. A simplified diagram of one arm of the interferometer is provided in Figure 2. Later this procedure can be done to the other arm in order to find out the Phase Map of the mirrors used in that arm. We are expecting the experimental scan data to look similar to that we predicted with the help of Finesse software. By analytical calculation we know the mode spacing corresponding to each mode. But the experimental scan data will show some shifts in frequency from the predicted frequencies. This shift is due to the presence of figure error in the mirrors used in the cavity.

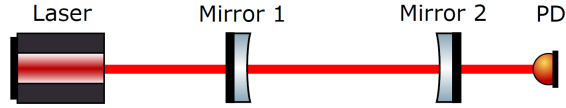


Figure 2: A simplified diagram of one arm of the interferometer [6].

3.2.1 Frequency Scan

Neither the laser frequency nor the cavity length of the arms is stable enough to sustain the resonance of the cavity. Thus, we need to control either the laser frequency or the arm length to maintain the optical resonance. Thus, the main laser used in the interferometer is a pre-stabilised laser (PSL) whose frequency is locked to a particular value with the help of Pound Drever Hall Locking technique. For giving a scan to the frequency we cannot do any tuning to the main laser as it will make the cavity unstable. In order to give a scan to the frequency, we need to use some other techniques. Few of the techniques used by the previous students include simple cavity scan, cavity scan using Electric Optical Modulator (EOM), cavity scan using Additional Auxiliary Laser and Arm length stabilisation scheme. The method we are going to follow in this project to obtain the cavity scan is by using an Auxiliary laser. This process is known as the Phase locking. A schematic diagram of the Phase locking loop including the AUX laser is shown in Figure3. The auxiliary laser beam is injected from an optical port, while the main laser is locked to one of the arm.

The electric field (E) of both PSL and AUX lasers are given below,

$$E_{PSL}(t) = \sqrt{P_{PSL}} e^{i\phi_{PSL}(t)}$$

$$E_{AUX}(t) = \sqrt{P_{AUX}} e^{i\phi_{AUX}(t)}$$

AUX-PSL Phase-Locked Loop

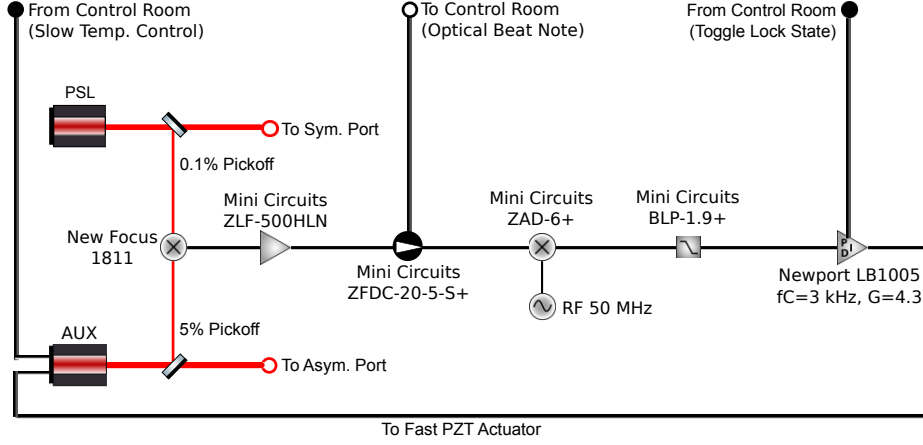


Figure 3: Schematic diagram of the AUX-PSL Phase locked loop

where P is the power of the laser and ϕ is the phase of the laser. These two fields undergo optical mixing to produce the beat note at the photo-detector (New Focus 1811).

$$P_{beat}(t) = | E_{PSL}(t) + E_{AUX}(t) |^2$$

$$P_{beat}(t) = P_{PSL} + P_{AUX} + 2\sqrt{P_{PSL}P_{AUX}} \cos [\phi_{PSL}(t) - \phi_{AUX}(t)]$$

The high-passed NF1811 beat readout signal from the photo-detector,

$$V_{PD}(t) = 2K_{PD} \sqrt{P_{PSL}P_{AUX}} \cos [\phi_{PSL}(t) - \phi_{AUX}(t)]$$

The RF-amplified readout signal is given by,

$$V_{amp}(t) = 2K_{amp} K_{PD} \sqrt{P_{PSL}P_{AUX}} \cos [\phi_{PSL}(t) - \phi_{AUX}(t)]$$

A part of this beat is taken to the control room with the help of coupler. This is connected to the frequency counter for getting the frequency display at each point of time. This display helps us to monitor the beat frequency when we want to engage the PLL to acquire the lock. If the beat frequency is far away from the oscillatory frequency, we will not be able to acquire the lock of the PLL. In such cases, we need to observe the beat frequency and tune it by slowly adjusting the laser temperature. Once the beat frequency reach near the target frequency, the loop can be turned on and the lock can be acquired. We are providing an oscillatory frequency through the RF generator. The system will lock the two lasers in such

a way that the frequency difference between PSL and AUX lasers remains a constant and this will be equal to the frequency provided by the RF frequency generator.

The RF generator signal is given as,

$$V_{RF}(t) = V_{amp} \sin[\phi_{RF}(t)]$$

The beat signal and the RF signal passes through the mixer where it down convert beat signal to DC. Here the upper and the lower side bands are formed.

$$\begin{aligned} V_{mix}(t) &= K_{mix} V_{amp}(t) V_{RF}(t) \\ &= K_{mix} K_{amp} K_{PD} \sqrt{P_{PSL}P_{AUX}} V_{amp} \sin [\phi_{PSL}(t) - \phi_{AUX}(t) + \phi_{RF}(t)] \text{ (upper side band)} \\ &\quad - K_{mix} K_{amp} K_{PD} \sqrt{P_{PSL}P_{AUX}} V_{amp} \sin [\phi_{PSL}(t) - \phi_{AUX}(t) - \phi_{RF}(t)] \text{ (lower side band)} \end{aligned}$$

The Low-Pass filter only allow the low frequency component to pass through and it eliminates the high frequency component. Thus the filter output is given as,

$$V_d(t) = -K_{mix} K_{amp} K_{PD} \sqrt{P_{PSL}P_{AUX}} V_{amp} \sin [\phi_{PSL}(t) - \phi_{AUX}(t) - \phi_{RF}(t)]$$

At lock point,

$$\phi_{AUX}(t) = \phi_{PSL}(t) - \phi_{RF}(t)$$

This signal pass through the PI control system (Proportional and Integration) and as a result feed back is given back to the fast frequency tuning of the AUX laser for adjusting its frequency in order keep the difference in frequency between PSL and AUX a constant.

4 Project Schedule and Updates

Different stages involved in this project are shown as a flow chart in Figure 4. For completing this project efficiently, the project is divided into different stages as listed below.

1. Developing the experimental setup needed to obtain the cavity scan data. The control systems need to be connected to the computers. We need to write codes to adjust the parameters using computer itself. I have developed the codes for controlling the scan frequency by the frequency oscillator. A picture of the MEDM control screen for this experiment is shown below in Figure 5.

2. Developing Ideal cavity scan of the system by using the same system parameters as that of the experimental setup with the help of Finesse software. A flow chart for using Finesse software is shown in Figure 6. The parameters of the cavity which are needed for producing the ideal cavity scan is given below in the Table 1. The cavity is scanned over range of frequencies. The transmitted power of the cavity is plotted as a function of frequency. From

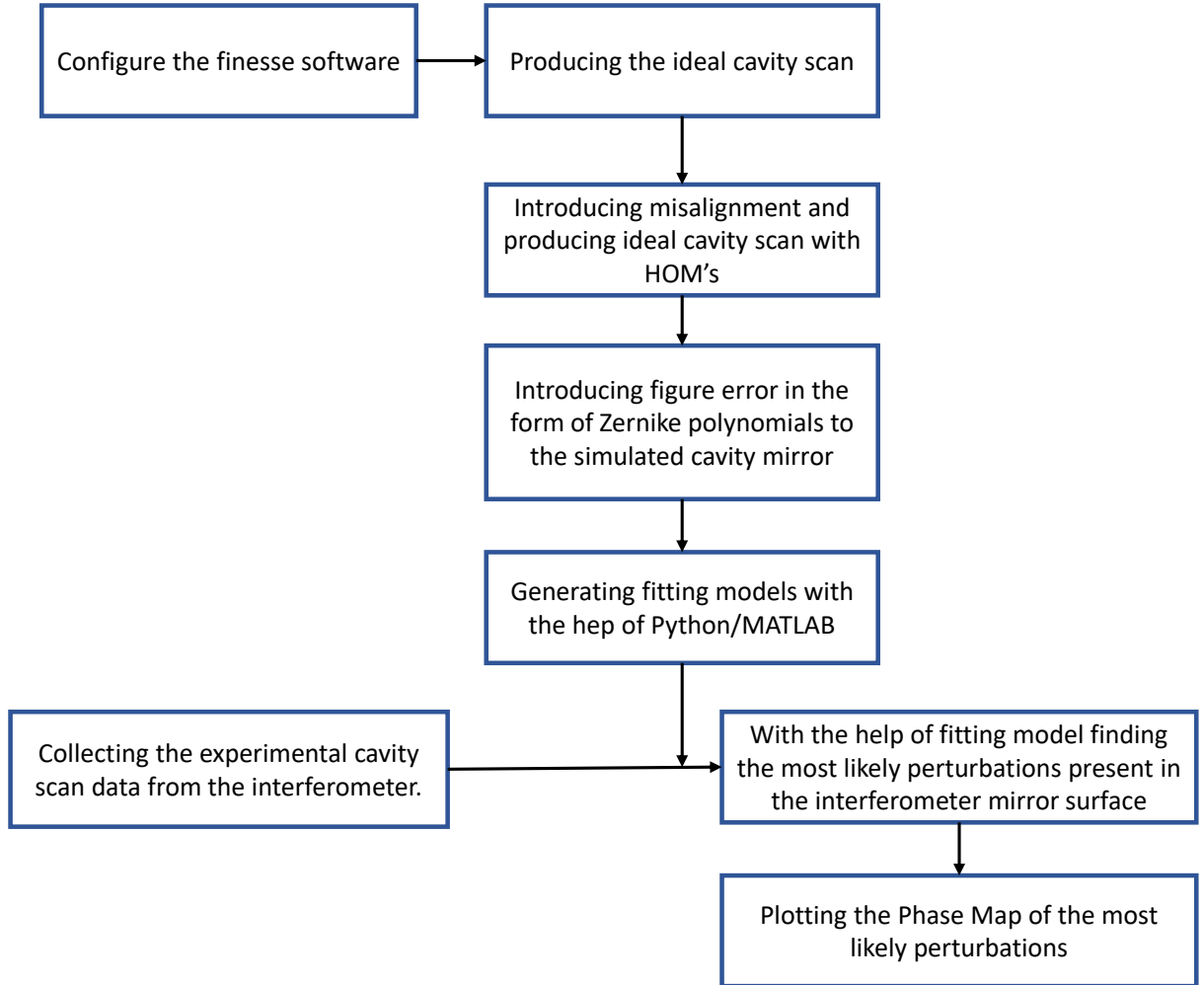


Figure 4: Flow chart showing various stages of the project.

Parameter	Value
Cavity length (L)	40 m
ITM Reflectivity (R)	0.98616
ITM Transmissivity (T)	0.01384
ETM Reflectivity (R)	0.9999863
ETM Transmissivity (T)	1.37×10^{-6}
ITM Radius of curvature (R_1)	∞
ETM Radius of curvature (R_2)	60 m

Table 1: Parameters of the 40m LIGO interferometer used for the Finesse simulation.

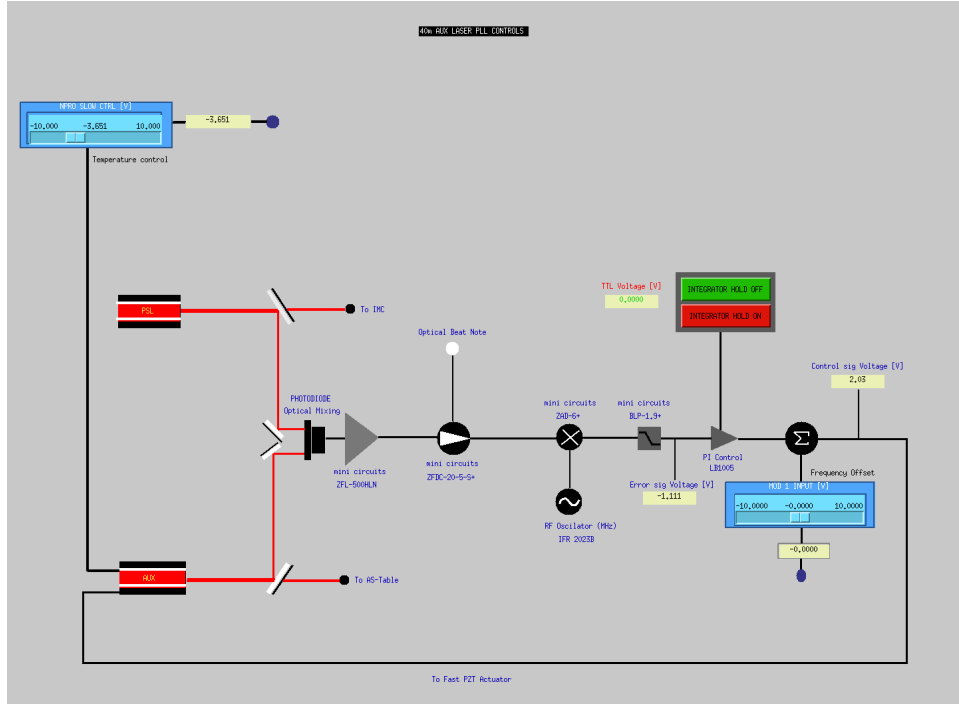


Figure 5: A picture of the MEDM screen for controlling the system parameters.

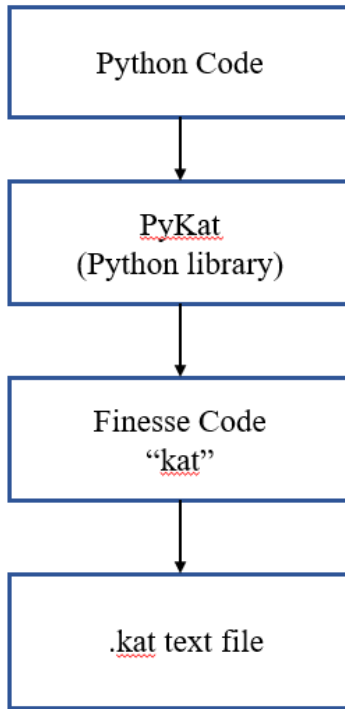


Figure 6: Flow chart showing the various stages involved in producing a Finesse simulation.

this the resonant transverse modes of the cavity are observed. Higher order modes (HOM's) can be introduced by providing a misalignment to the simulated cavity.

3. Introducing random figure error in the form of Zernike polynomials to one of the cavity mirrors in the simulation. Obtaining the plot of transmitted power as a function of frequency. Observing the effect of this figure error on the higher order mode spacing of the cavity.
4. Generating thousands of potential perturbations in the cavity mirror surfaces and studying their effect on the higher order mode spacing.
5. Running the experiment and obtaining the experimental cavity scan data. Using Monte Carlo method to try to fit actual cavity scan transmission data to a linear combination of these potential mirror surface perturbations. Using Bayesian inference method to predict the probability for a particular perturbation being present in the cavity mirror surface.
6. The set of perturbations which gives us the cavity scan similar to that we obtained in the experiment is noted and it is plotted as a Phase Map diagram.

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