

# Absolute Calibration of A Gravitational Wave Detector Network

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LIGO Hanford Controls Engineer

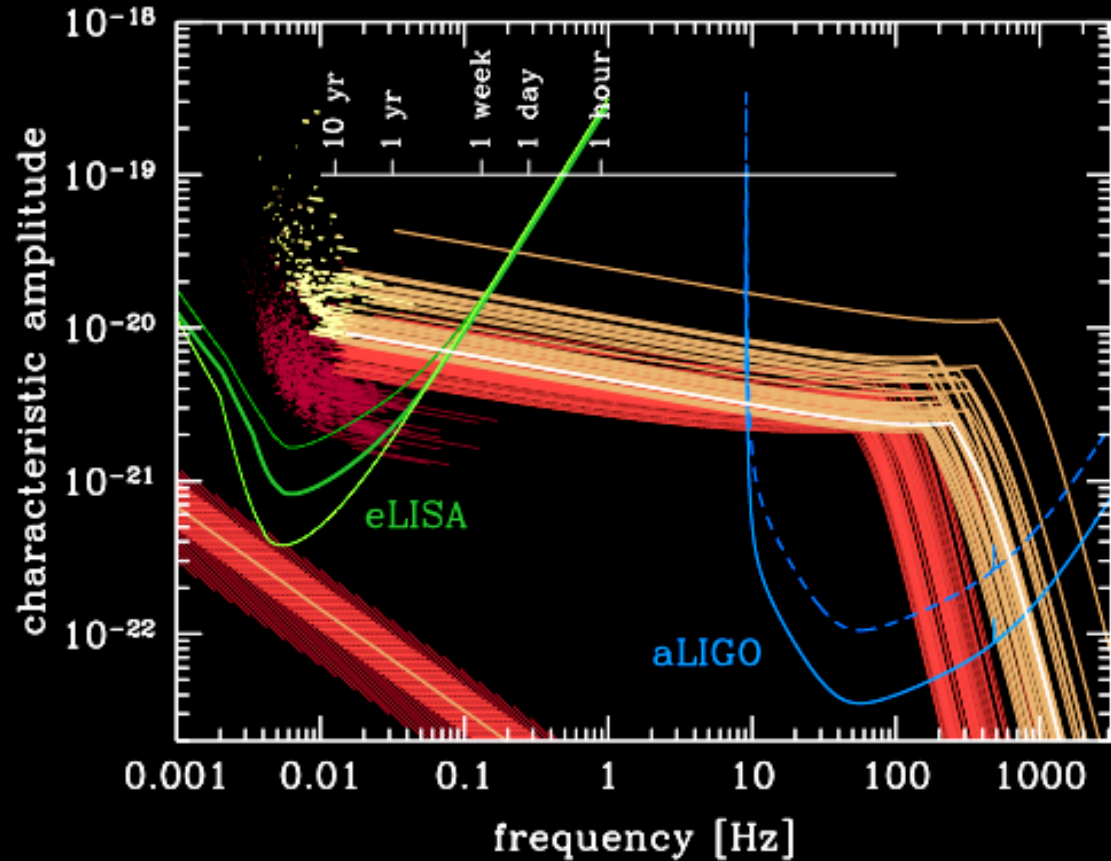
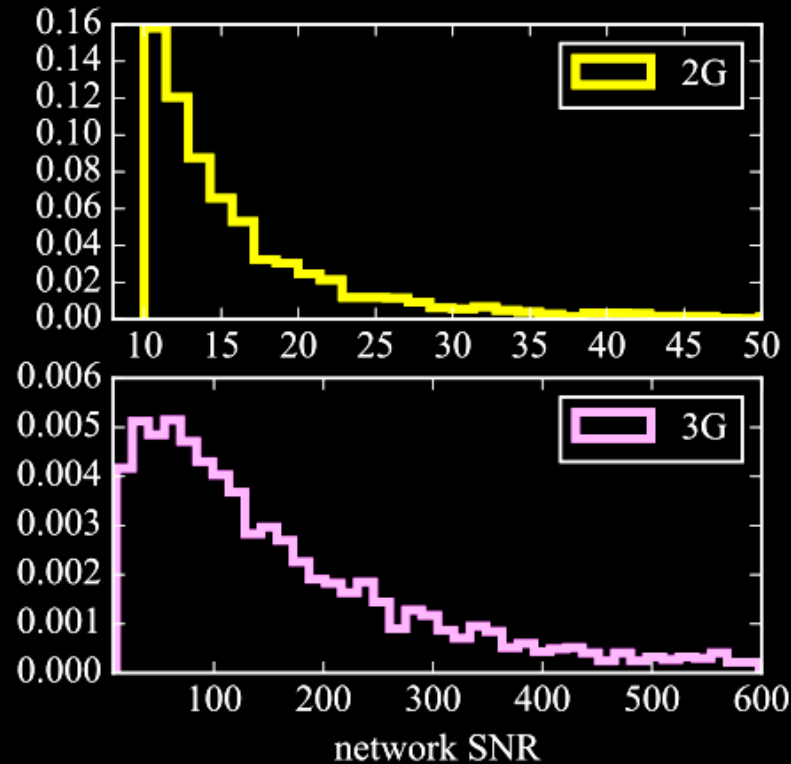
LIGO Calibration Group Co-Chair

# Inspiration



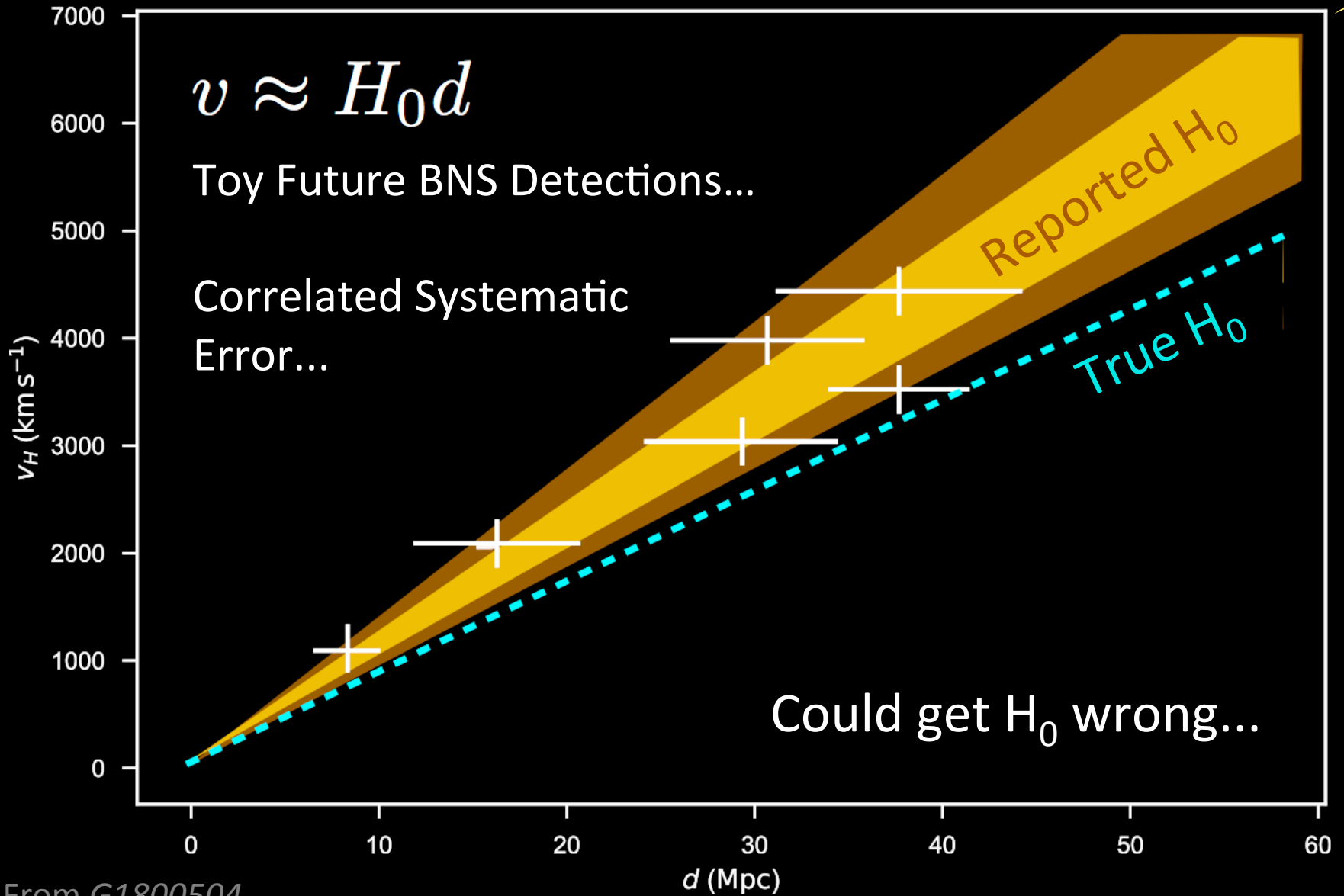
2G , 3G, Space, and PTAs  
will have events with SNRs  
of  $O(\sim 100)$ ...

...Detector calibration **must**  
get *better* than  $\sim 1\%$



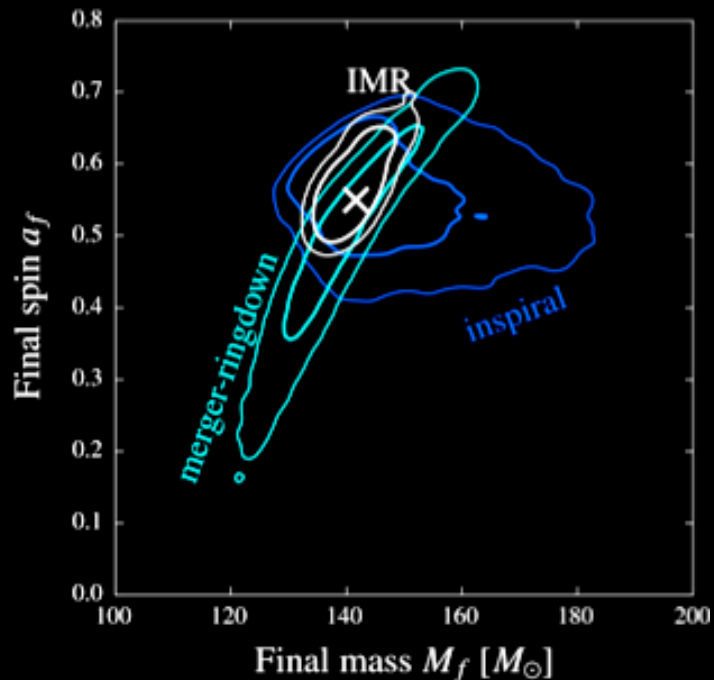
And if multi-band, we must understand each other's  
systematic errors.

# Cosmology and Correlated Systematic Errors



From G1800504

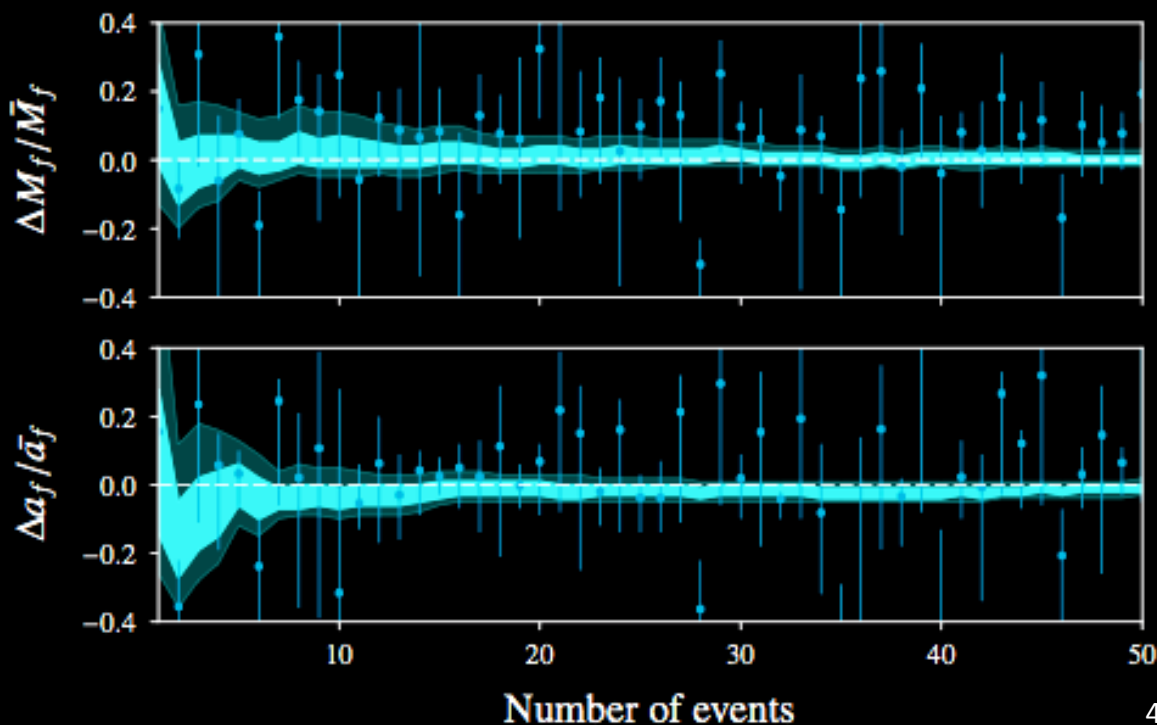
# Tests of GR and Correlated Systematic Errors



Similar systematic error across events may lead you astray in GR consistency checks

Will not get  $\text{SQRT}(N)$  improvement, because calibration's systematic errors (biases) are correlated b/w events.

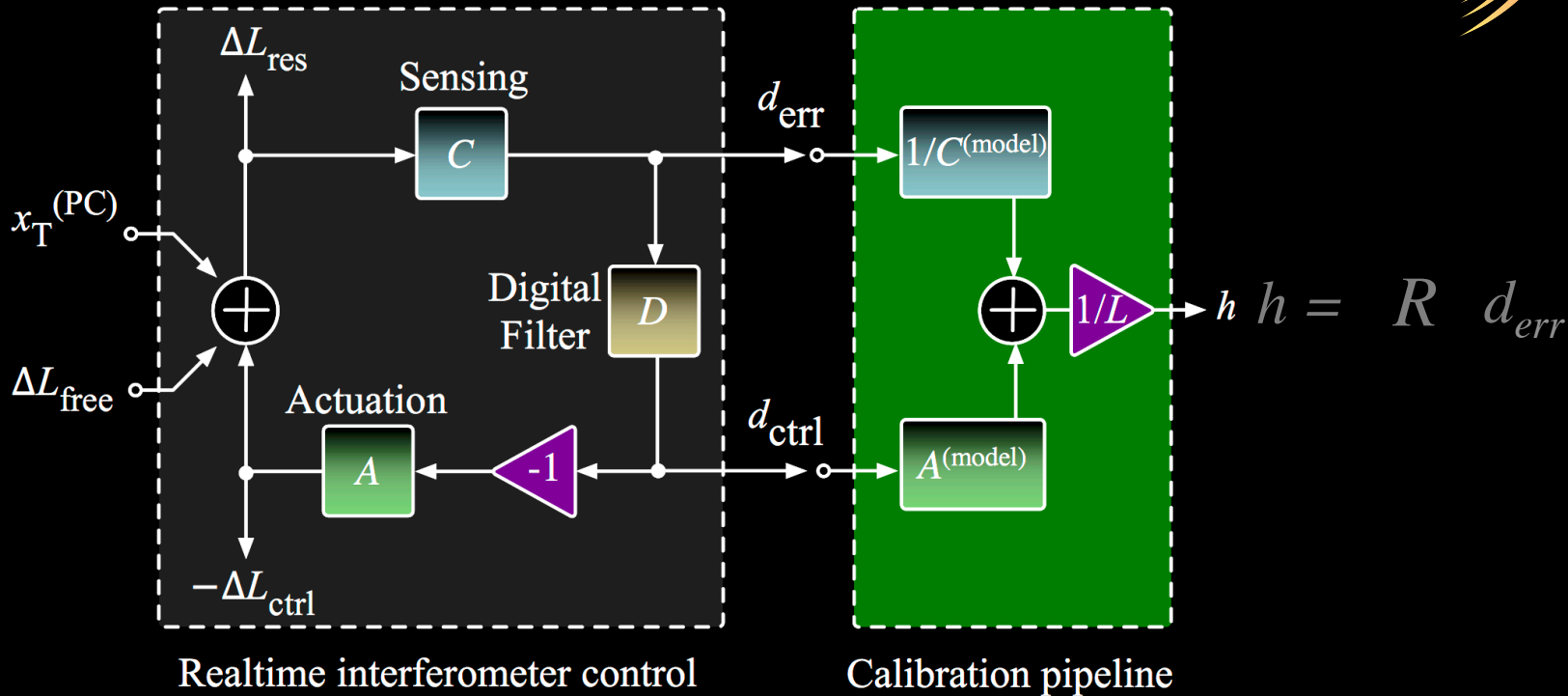
Will need to consider calibration uncertainty and systematic error over several events, several observation runs as the detector hardware changes



CQG 35.1 (2017): 014002.  
(and from yesterday; G1800976)



# State of the Art Example: LIGO 2G

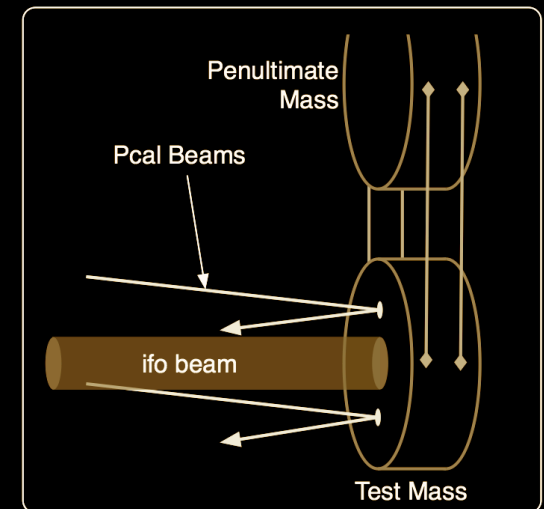
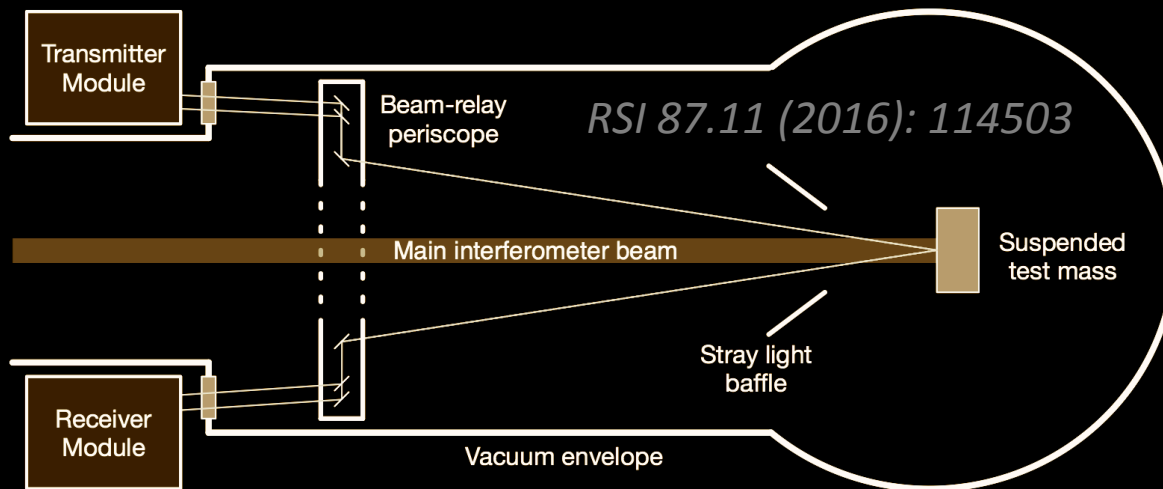
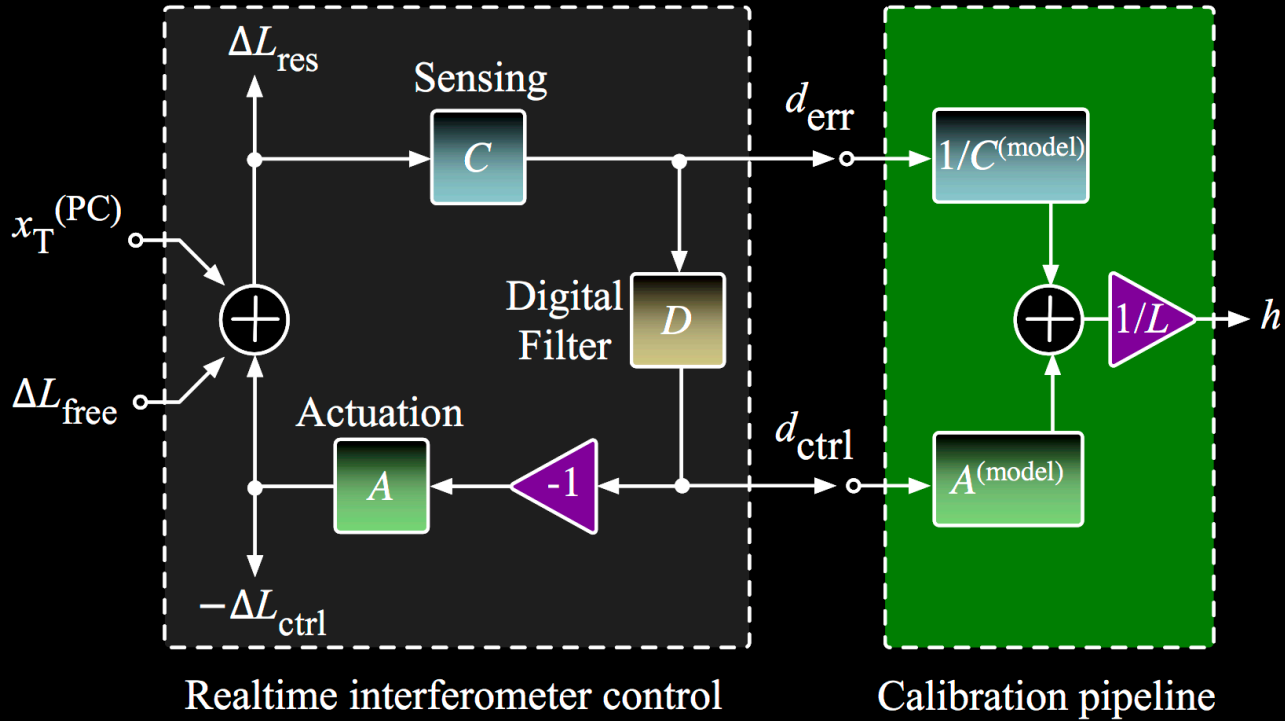


$$R = \frac{1}{L} \frac{1 + G}{C} \quad G = A D C \quad h L = \frac{1}{C} d_{err} + A d_{ctrl}$$

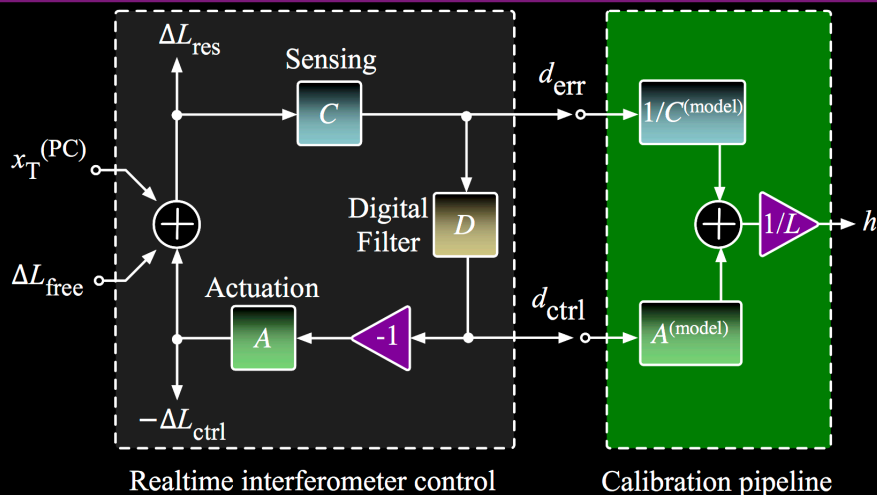
$$dR^2 = \left( \frac{1}{1 + G} \right)^2 \left( \frac{dC}{C} \right)^2 + \left( \frac{G}{1 + G} \right)^2 \left( \frac{dA}{A} \right)^2$$

**+ Systematic Error**

# Absolute Reference: Radiation Pressure



# How does LIGO Determine dC and dA

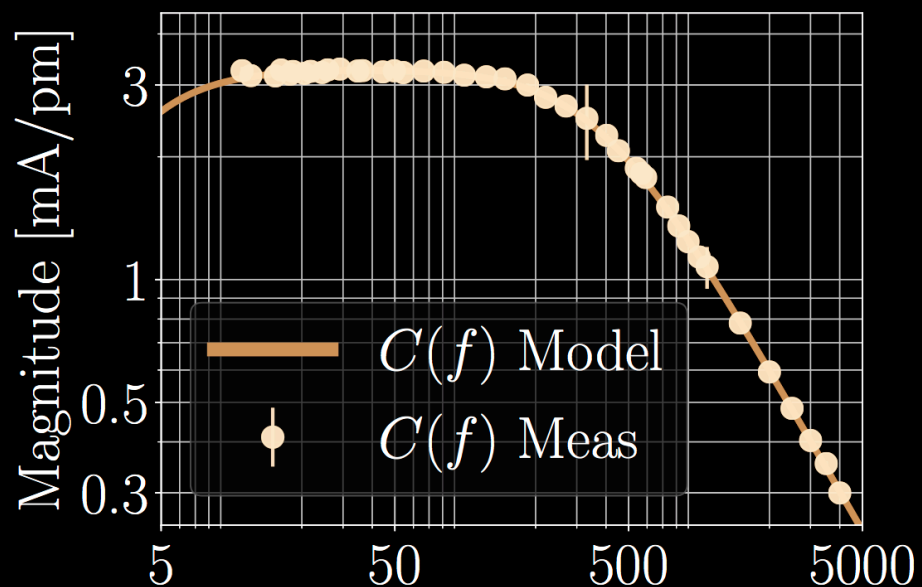


$$C = \frac{d_{exc}}{d_{IN2}} \times \frac{d_{err}}{x_{pcal}} \times \frac{x_{pcal}}{pd_{pcal}}$$

Complex TFs w/ Full IFO

$$A = \frac{d_{err}}{d_{exc}} \times \frac{x_{pcal}}{d_{err}} \times \frac{pd_{pcal}}{x_{pcal}}$$

Absolute Scale Factor

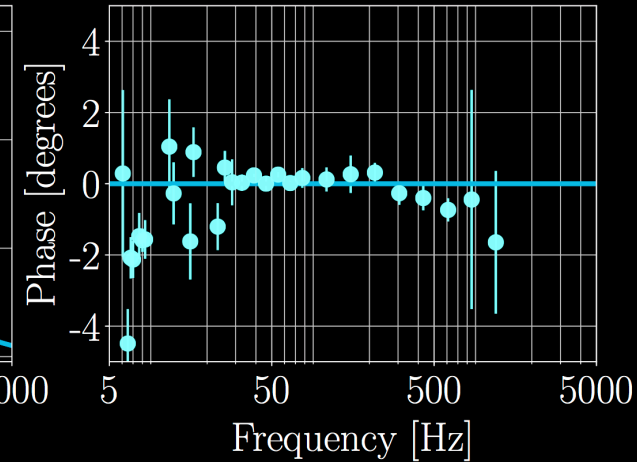
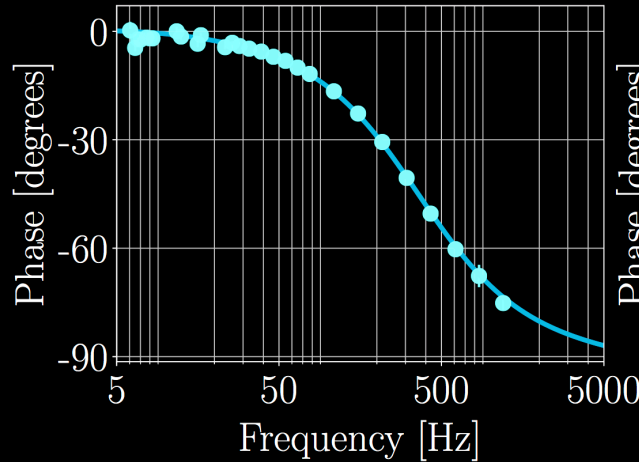
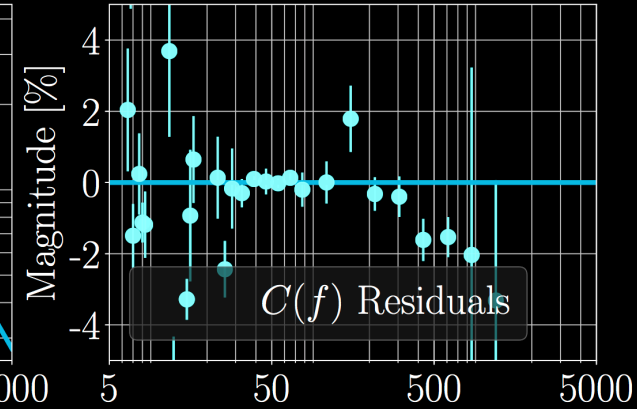
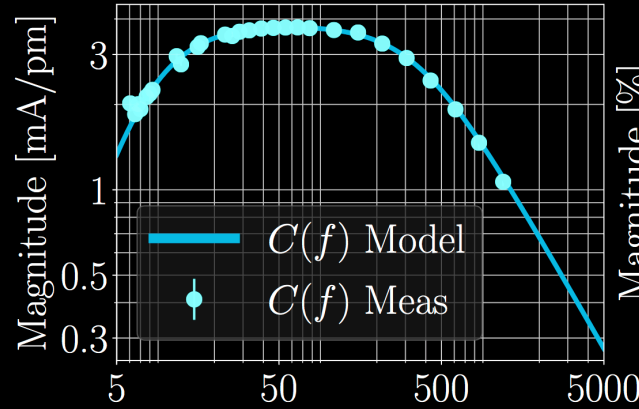
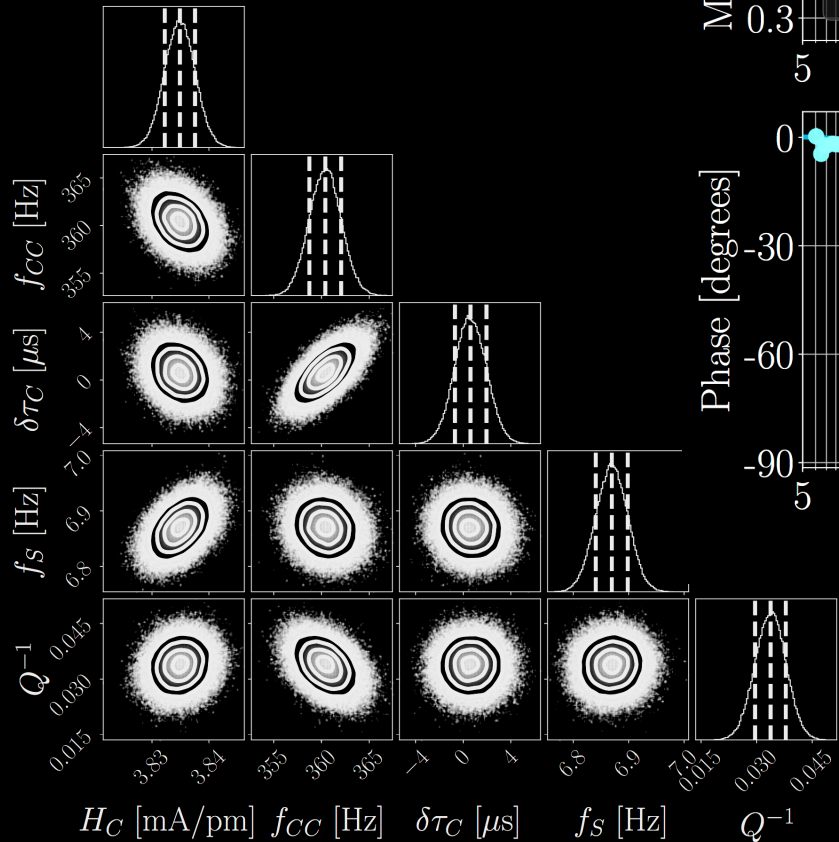


**Phase uncertainty is entirely from Full IFO transfer functions, not set by absolute scale.**

# Systematic Error vs. Statistical Uncertainty



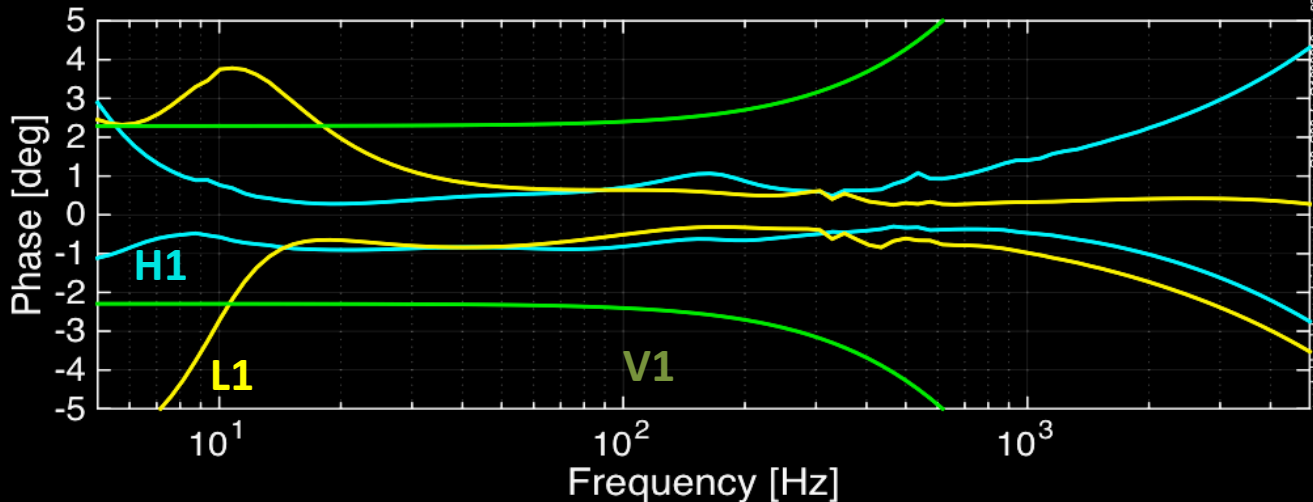
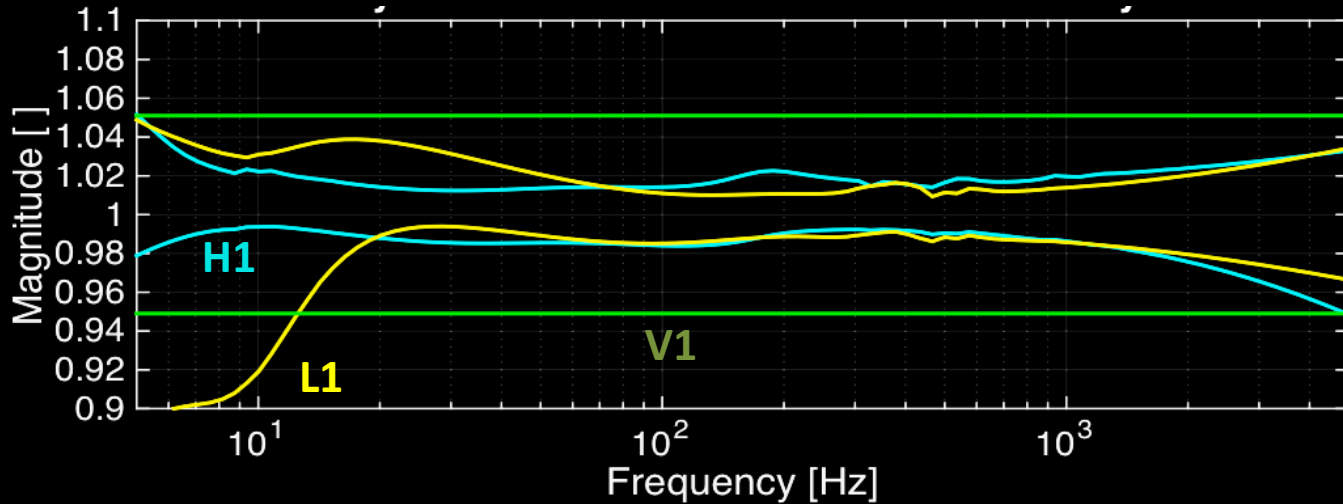
**Statistical uncertainty:**  
MCMC posteriors to the model parameters sampled to form a sample response function.



## Systematic Error:

- Any residual, statistically significant, frequency dependence
- Fundamental reference error

# 2G Detector (Known) Error and Uncertainty



created by plotuncertaintyspectrograms\_O2\_C02\_forG1800393 on 20-Mar-2018

$$dR^2 \approx \left( \frac{1}{1+G} \right)^2 \left( \frac{dC}{C} \right)^2 + \left( \frac{G}{1+G} \right)^2 \left( \frac{dA}{A} \right)^2 + \textit{Systematic Error}$$

# What are the 2G limits?



- Yes... Propagation of NIST factor
  - Years of practice has this nailed down

## BUT ALSO:

- PCAL Actuator Alignment
  - Will be improved for O3!
  - But can't track anymore...
- IFO Actuator Strength
  - To measure open loop gain and DARM actuator drive
- Detector Performance
  - If noise improves, SNR improves for fixed actuator
  - For detector upgrades, actuation strength will decrease

$$C = \frac{d_{exc}}{d_{IN2}} \times \frac{d_{err}}{x_{pcal}} \times \frac{x_{pcal}}{pd_{pcal}}$$

↑
↑
↑

Complex TFs  
 w/ Full IFO

$$A = \frac{d_{err}}{d_{exc}} \times \frac{x_{pcal}}{d_{err}} \times \frac{pd_{pcal}}{x_{pcal}}$$

↓
↓
↓

Absolute Scale Factor

## AND OF COURSE,

- ... patience
- ... person power
- ... interferometer time  
(see last year's talk, G1700810)

# The International Reference Exchange



# NIST

“GOLD”  
Standard

“Working”  
Standards

ETMY

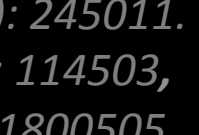
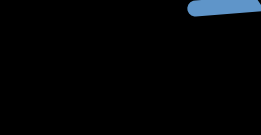
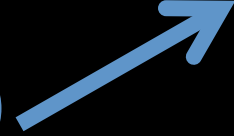
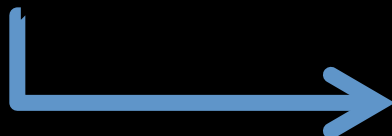
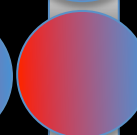
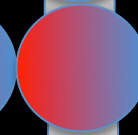
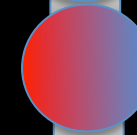
RX

TX

ETMX

RX

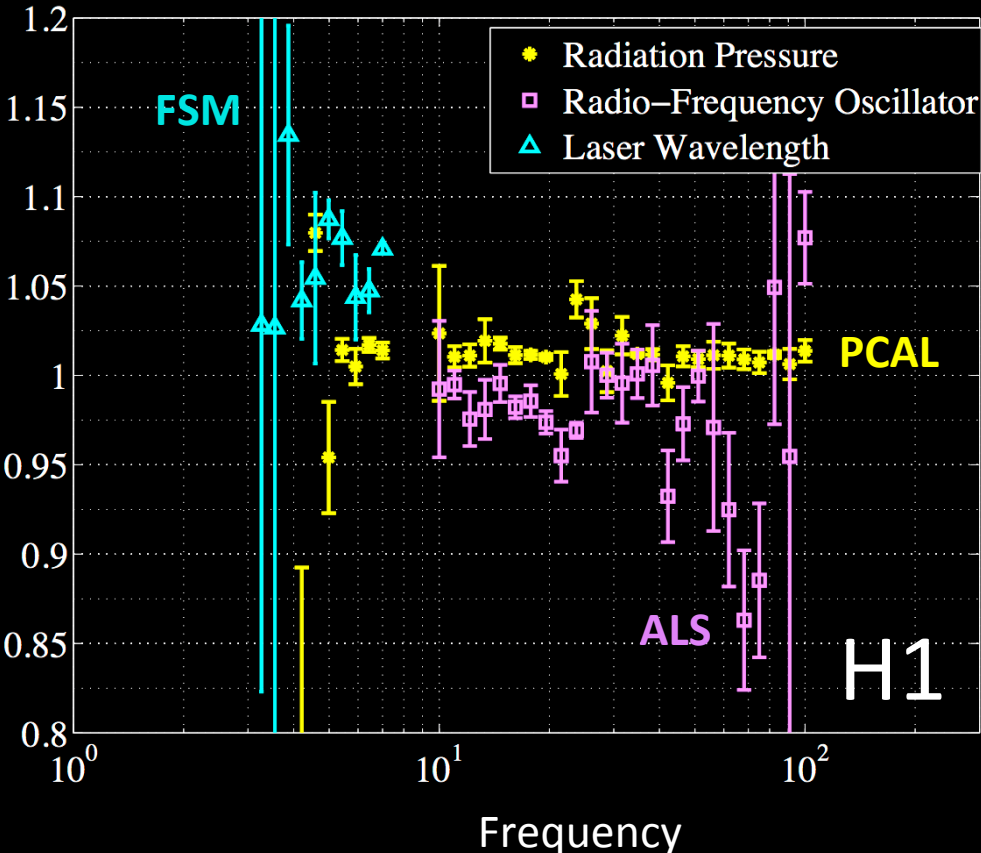
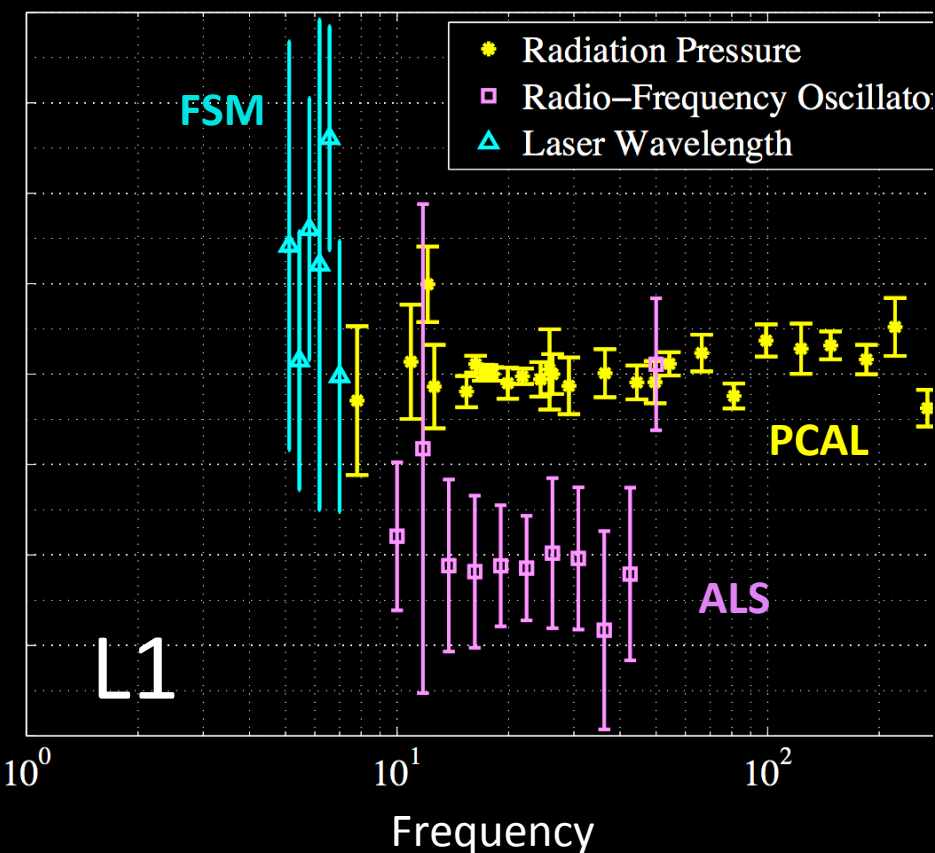
TX



- “Propagation of reference” program, with over ~10 years of experience, led by Rick Savage at LHO.
- KAGRA has already joined the reference comparison program. VIRGO has begun discussions of doing so.
- Mitigates differential scale error between detectors (single event: Sky Position)
- Still leaves vulnerability to common scale error (single event: Distance)

iLIGO: CQG 26.24 (2009): 245011.  
aLIGO: RSI 87.11 (2016): 114503,  
T1800046, T1800207, G1800505

# In-House PCAL Reference Check



- **Free-swinging Michelson** doesn't work well for LIGO. ITM must drive at PUM, too weak a drive.
- **Arm Length Stabilization** is too noisy, and electronics weren't well understood at the time.

Checks are not (yet?) precise.

Plans are to revisit this for O3.



# Terrestrial Detectors: The Next “New” Idea



- Newtonian Gravitational Calibrators (NCAL, GCAL) have recently (re)-picked up steam
- In-theory a ~0.1% level reference
  - PCAL starts at ~0.5%
- May be ready to *corroborate* PCAL by O3

Mirror



d

m

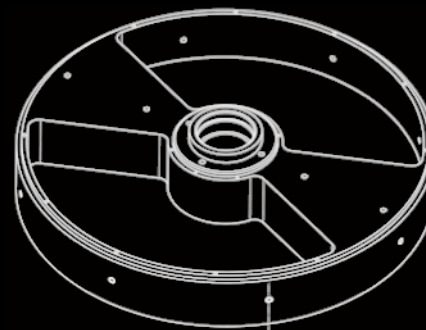
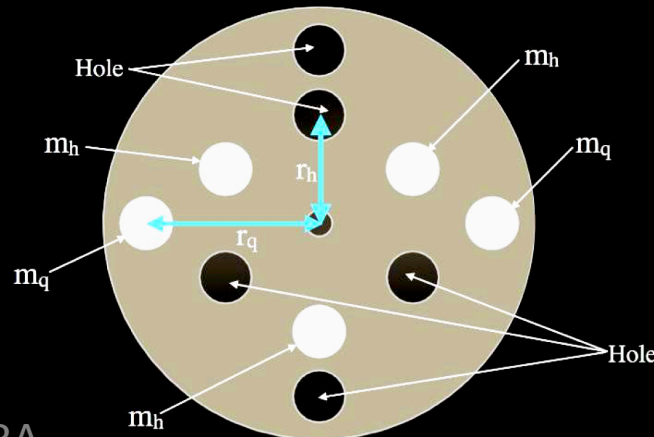
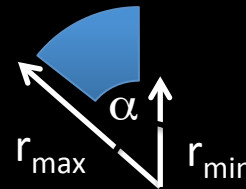
2r

m

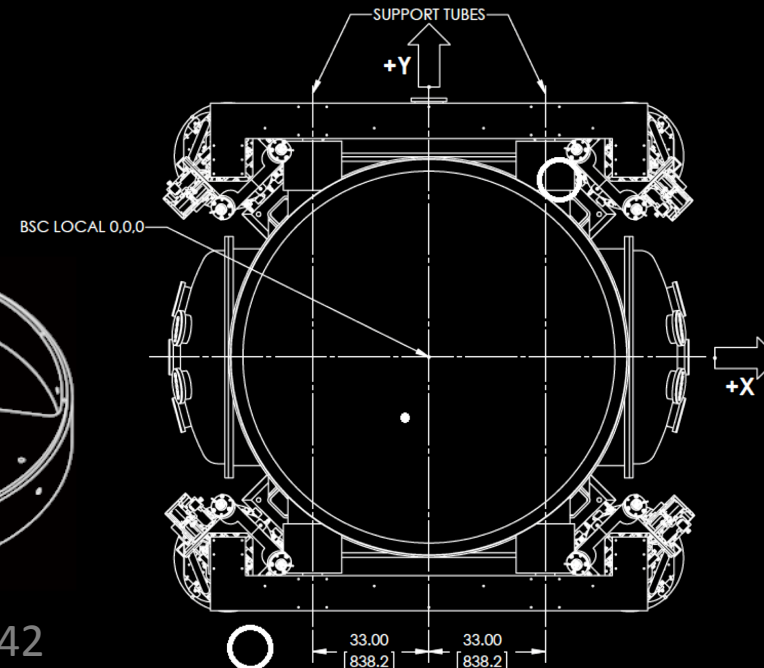
Basic Principle

$$x(t) = C \cos(4\pi f_{\text{rotor}} t) / (d^4 f_{\text{rotor}}^2)$$

$$\text{where } C = (9G/32\pi^2)(z \rho \sin(\alpha)(r_{\text{max}}^4 - r_{\text{min}}^4)/4)$$



VIRGO - G1800442



LIGO – “in prep.”

# What about Space and Galactic Detectors?

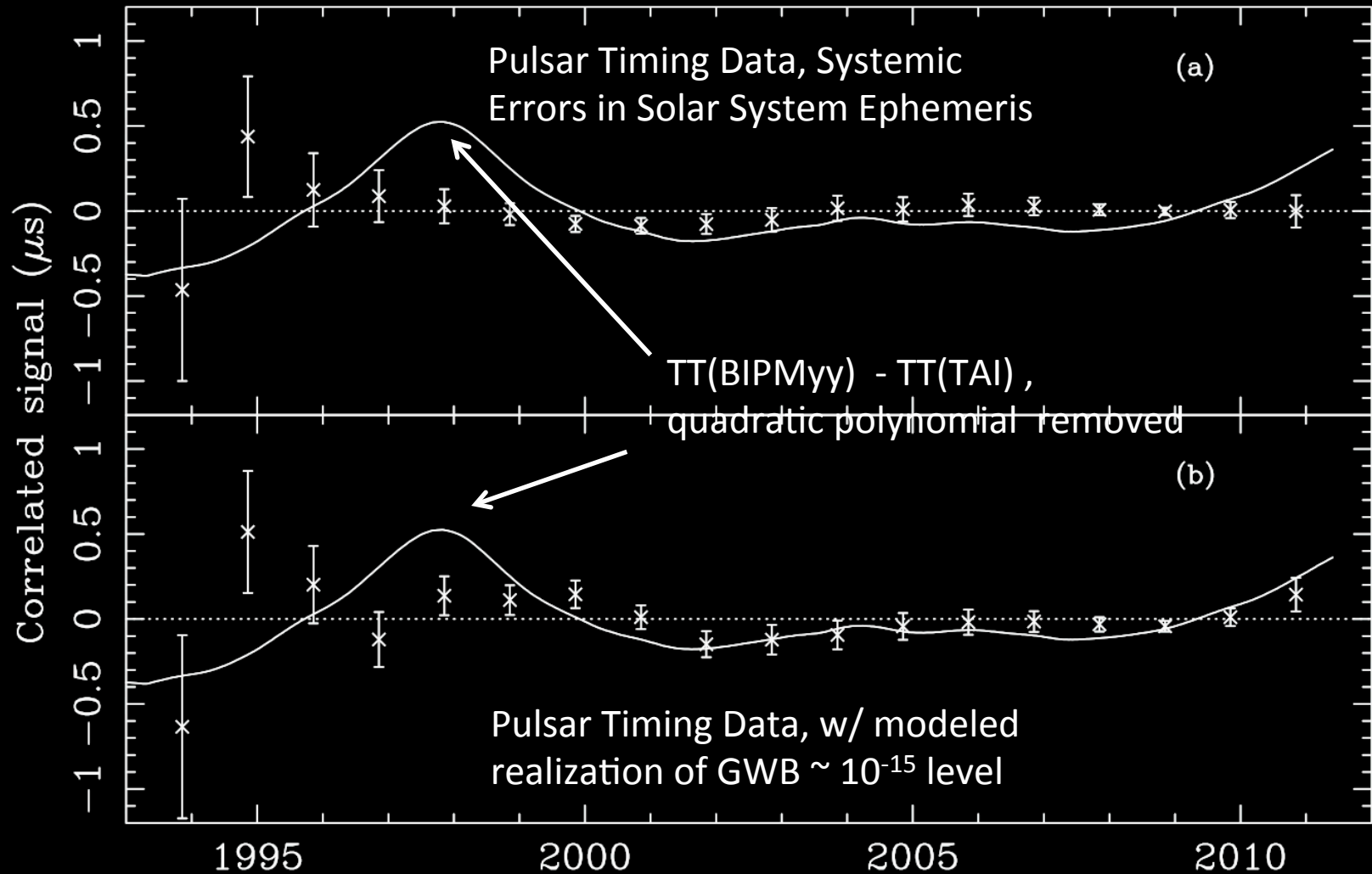


- Pulsar Timing Arrays (timing systems)
  - Limited by diverse Timing Systems in Array, over Decades?
  - Everyone uses TEMPO2... (*Hobbs et. al 2006*)



- LISA (phase meters and microthrusters)
  - Time Delay Interferometry – More clocks and Frequency Refs.
    - needs to measure individual,  $\sim 1e6$  m arm lengths to  $\sim 10$  m precision
    - needs clocks synchronized at the  $\sim 50$  nsec level

# Example Timing System Systematics



TT = Terrestrial Time

BIPM = Bureau International des Poids et Mesures

TT(TAI) = International Atomic Time -> *Basis for UTC*

TT(BIPMyy) -- updates / corrections, released over time

# Discussion & Questions

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- Calibration is ~~service work~~ **precision engineering**
- Precision/accuracy will not linearly increase with time and costs real money, not “just” time and person power.
- We’re actively working with NIST to understand the EUROMET study – Rick @ NIST this week!
  - *(+ soon new post-doc John Cripe!)*
- Already working to make ground-based detectors internally consistent via Photon Calibrator, starting to think how to get past its limits
- Need to begin dialogue with fundamentally different GW detectors

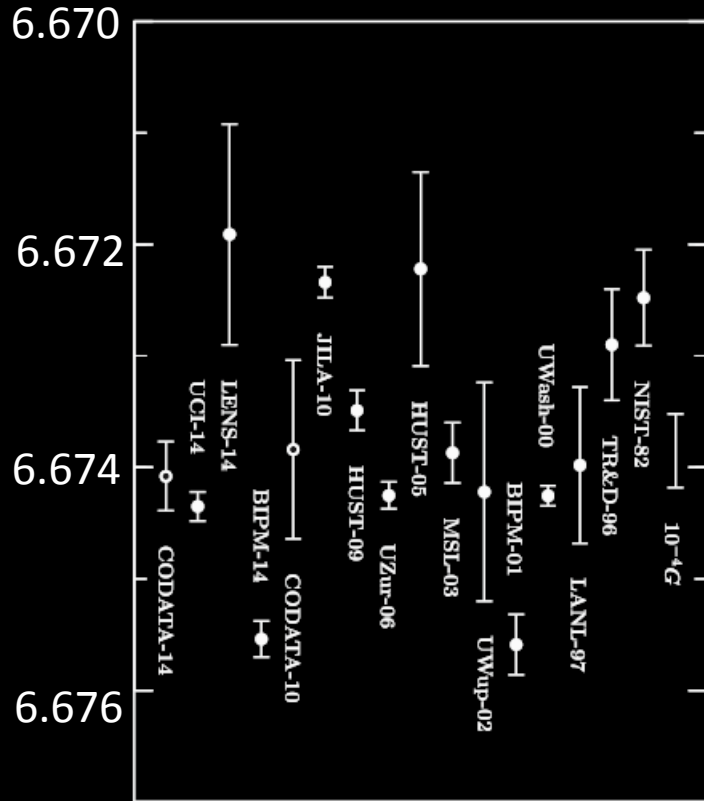




Thank You!

# Bonus Slides

## Big G / ( $1e-11 \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ )



## Response of a Photodetector

### Percent Difference from Weighted Mean Response of a Reference Photodetector

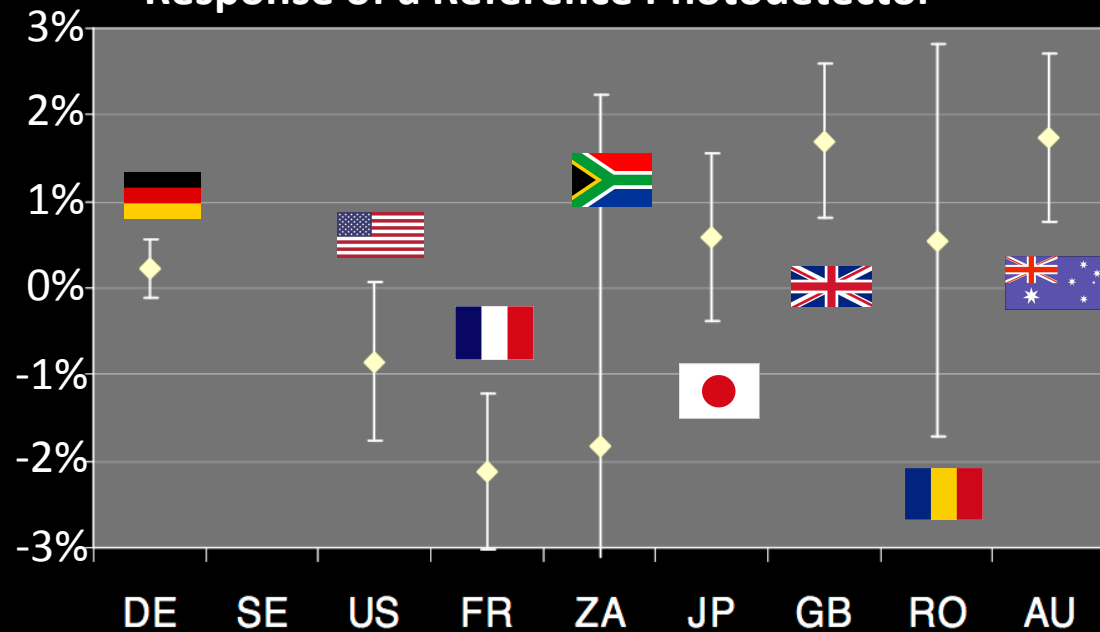
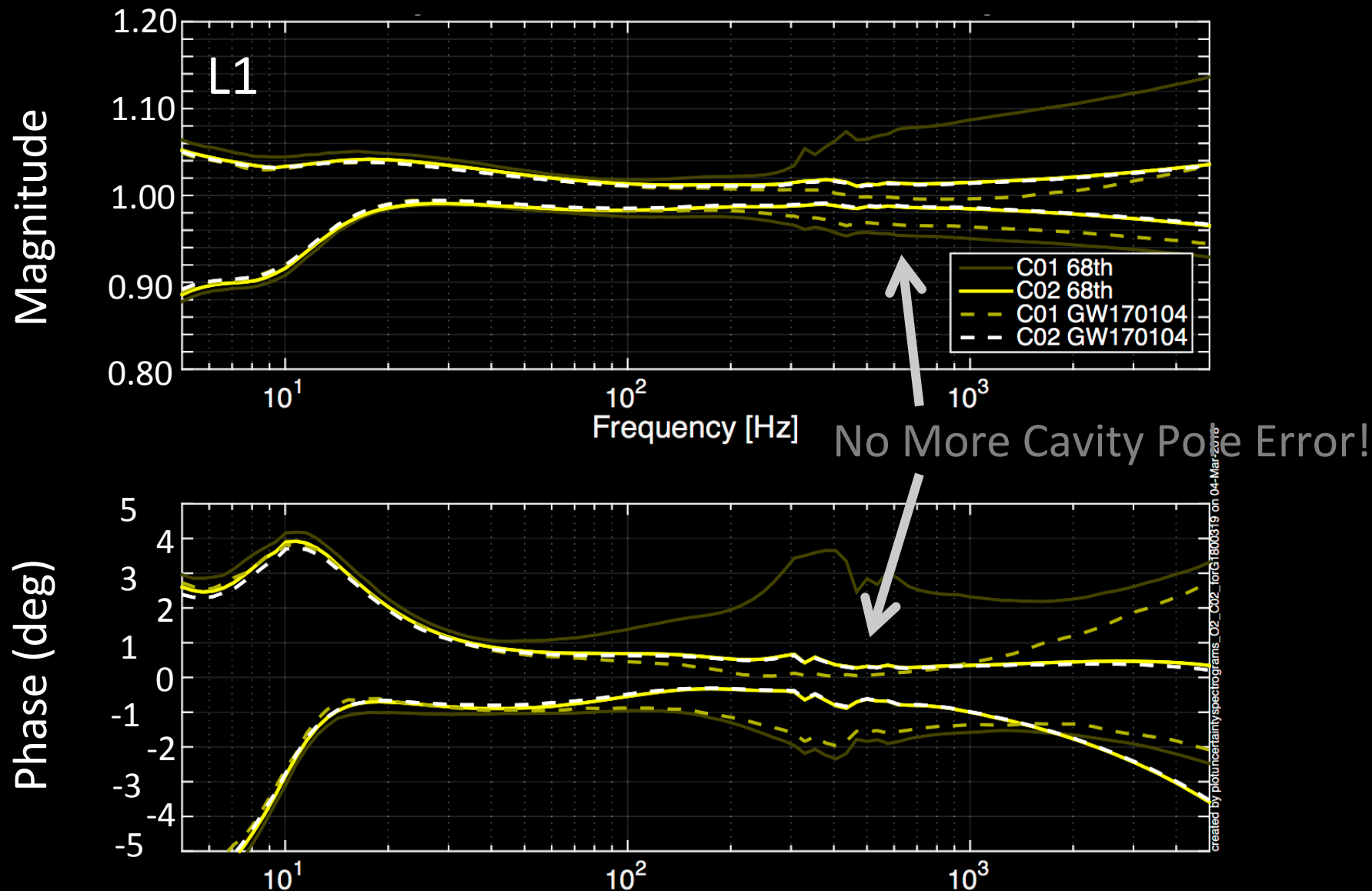


Figure 6  
CODATA 2014: Mohr, Newell, and Taylor. Phys. Chem. Ref. Data 45 (2016): 043102.

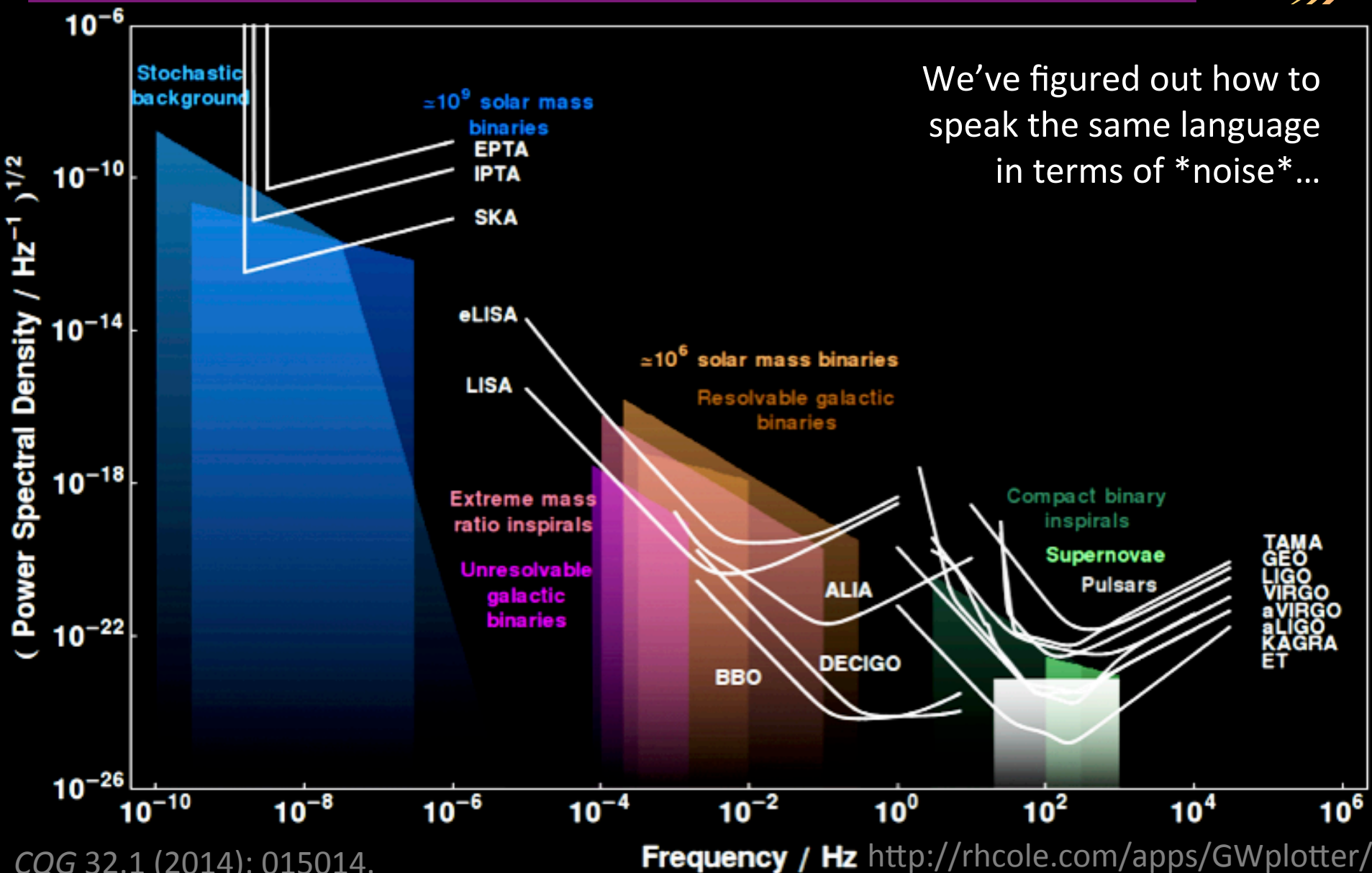
Figure 9  
Kück *Metrologia* 47.1A (2010): 02003.

# Bonus Slides: Removal of Systematic Error





# Common Sensitivity Plot



We've figured out how to speak the same language in terms of \*noise\* ...

CQG 32.1 (2014): 015014.

Frequency / Hz <http://rhcole.com/apps/GWplotter/>

- Current IPTA is at 49 Pulsars
  - *MNRAS* 458.2 (2016): 1267-1288
- 7 parameters – 3 for Sky Location, Amplitude, Source Inclination, Polarization Angle, Rotation Frequency
- ( Distance and Mass are degenerate ☹ )
- For SNR = 10, sky location uncertainty typically simulate (Fisher Matrix) to  $\sim 40 \text{ deg}^2$  and 30% in amplitude
  - *PRD* 81.10 (2010): 104008.
- Amplitude uncertainty scales with SNR, sky location scales with  $\text{SNR}^2$

# Pulsar Timing Array Papers

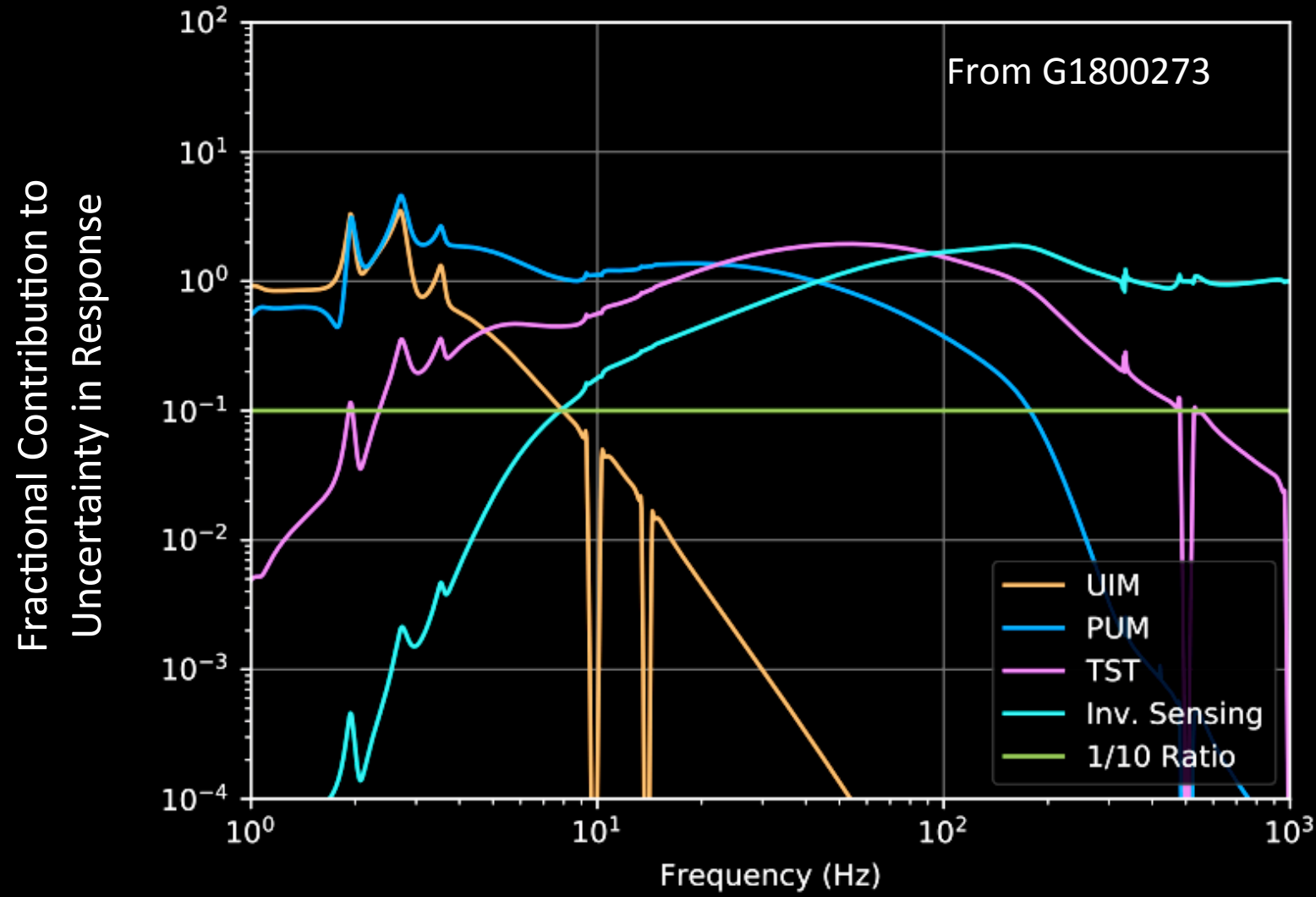
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- Verbiest, J. P. W., et al. "Status update of the Parkes pulsar timing array." *Classical and Quantum Gravity* 27.8 (2010): 084015.
- Sesana, Alberto, and Alberto Vecchio. "Measuring the parameters of massive black hole binary systems with pulsar timing array observations of gravitational waves." *Physical Review D* 81.10 (2010): 104008.
- Arzoumanian, Z., et al. "Gravitational waves from individual supermassive black hole binaries in circular orbits: Limits from the North American Nanohertz Observatory for Gravitational Waves." *The Astrophysical Journal* 794.2 (2014): 141.
- Verbiest, J. P. W., et al. "The international pulsar timing array: first data release." *Monthly Notices of the Royal Astronomical Society* 458.2 (2016): 1267-1288.
- Hobbs, G., et al. "Development of a pulsar-based time-scale." *Monthly Notices of the Royal Astronomical Society* 427.4 (2012): 2780-2787.
- Becker, Werner, Michael Kramer, and Alberto Sesana. "Pulsar Timing and Its Application for Navigation and Gravitational Wave Detection." *Space Science Reviews* 214.1 (2018): 30.

- Tinto, Massimo, and Sanjeev V. Dhurandhar. "Time-delay interferometry." *Living reviews in relativity* 17.1 (2014): 6.
- Hellings, Ronald W. "Elimination of clock jitter noise in spaceborne laser interferometers." *Physical Review D* 64.2 (2001): 022002.
- Armstrong, J. W., F. B. Estabrook, and Massimo Tinto. "Time delay interferometry." *Classical and Quantum Gravity* 20.10 (2003): S283.
- Tinto, Massimo, Frank B. Estabrook, and J. W. Armstrong. "Time-delay interferometry for LISA." *Physical Review D* 65.8 (2002): 082003.

# What Contributes Where?



$$dR^2 = \left( \frac{1}{1+G} \right)^2 \left( \frac{dC}{C} \right)^2 + \left( \frac{G}{1+G} \right)^2 \left( \frac{dA}{A} \right)^2$$

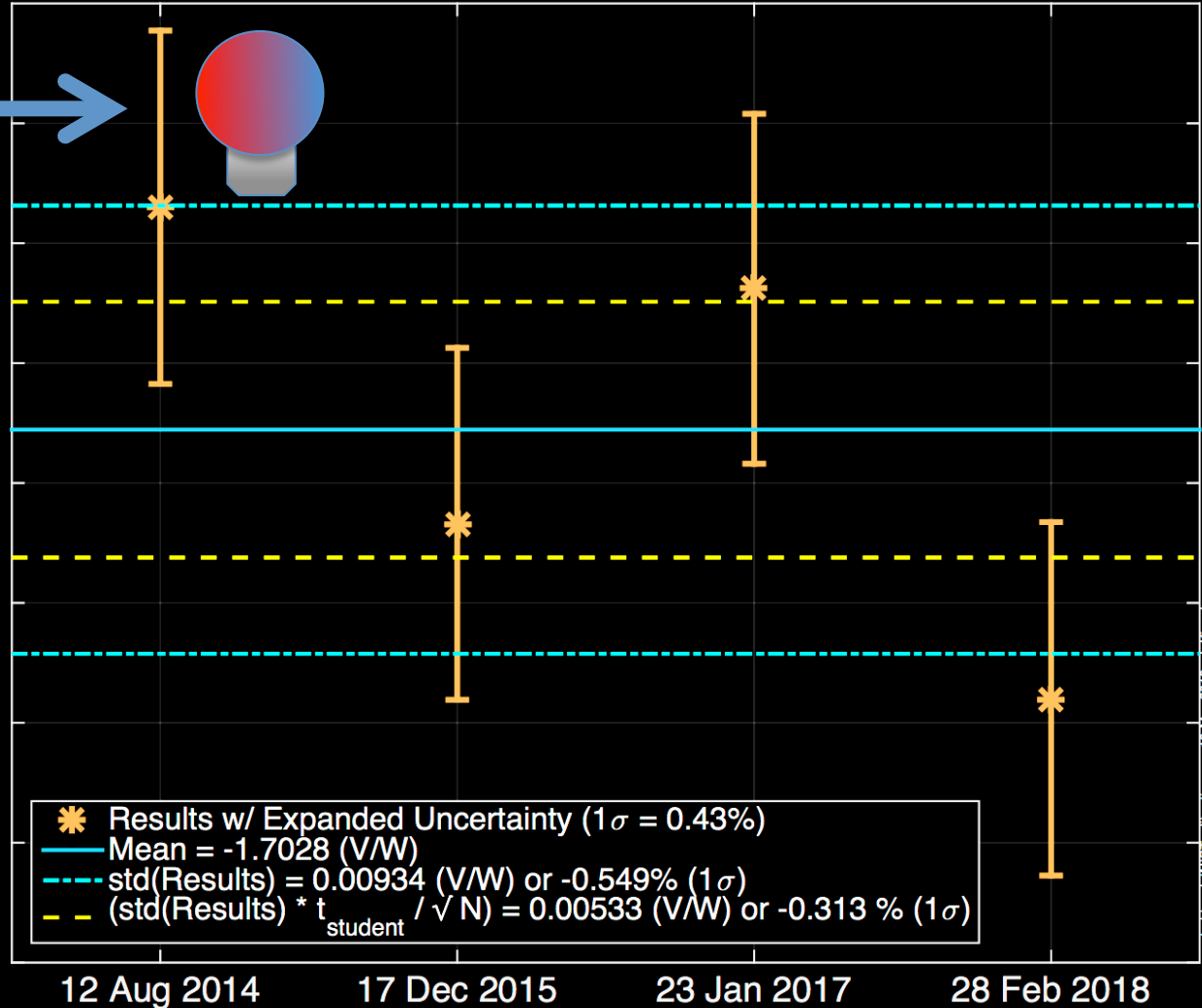
# 4 Years of aLIGO NIST Data



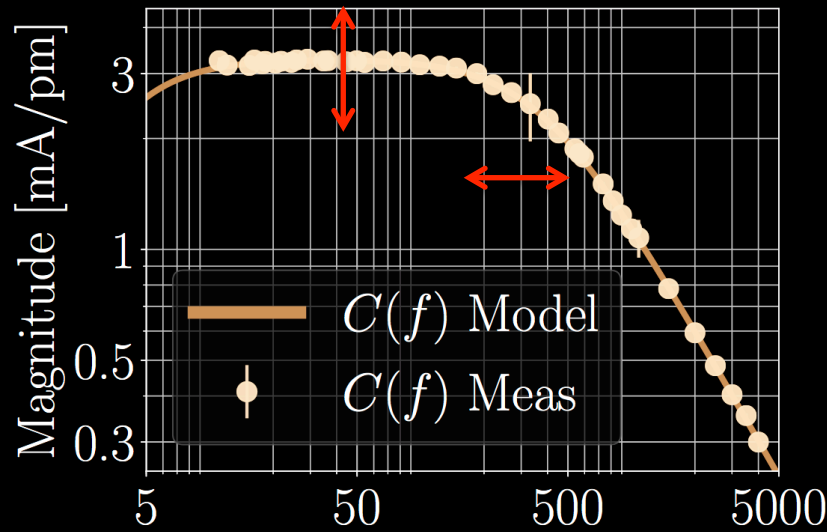
# NIST

“GOLD”  
Standard

~\$8k per  
calibration  
T1100068



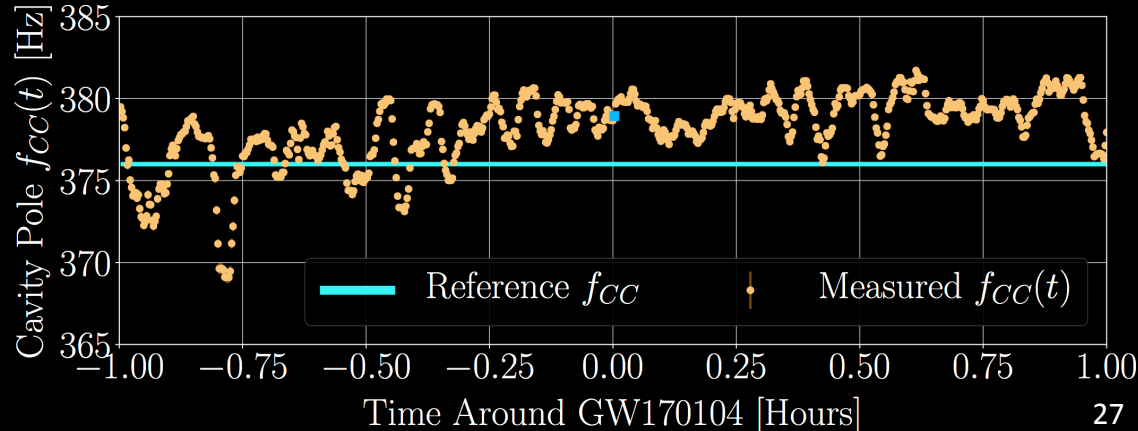
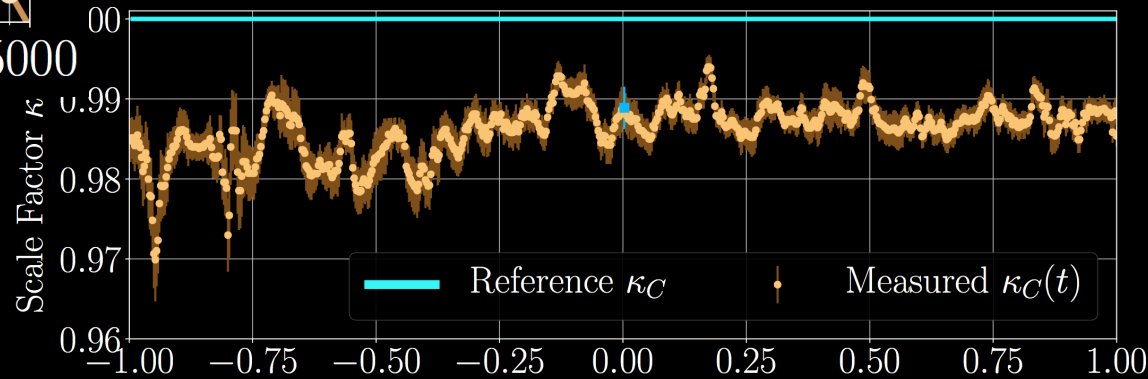
# Systematic Error vs. Statistical Uncertainty



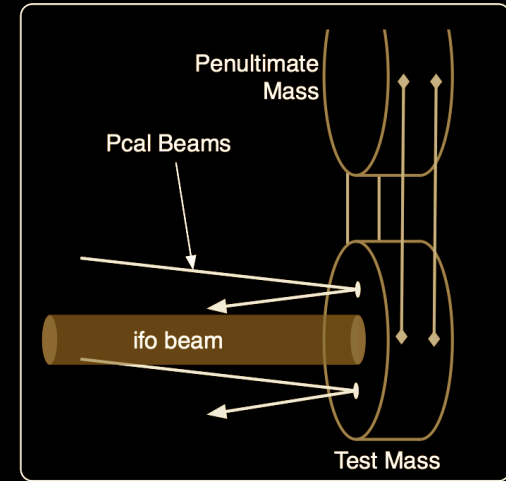
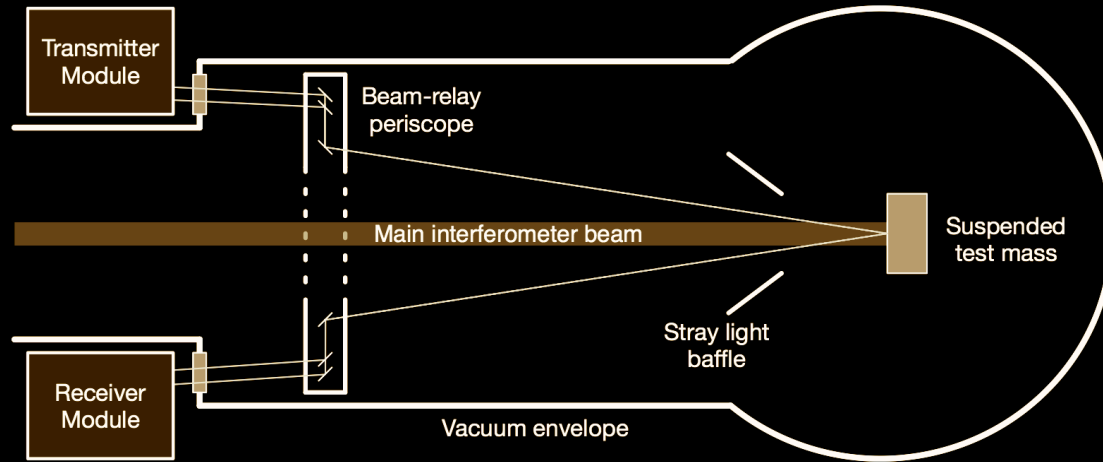
Interferometer parameters vary as a function of time.

Calibration Lines > Constantly Measuring C and A at single frequencies

- Alignment Drifts
- Thermal Equilibrium
- Charge on the Test Mass
- Distance between Reaction Chain and Test Mass Chain

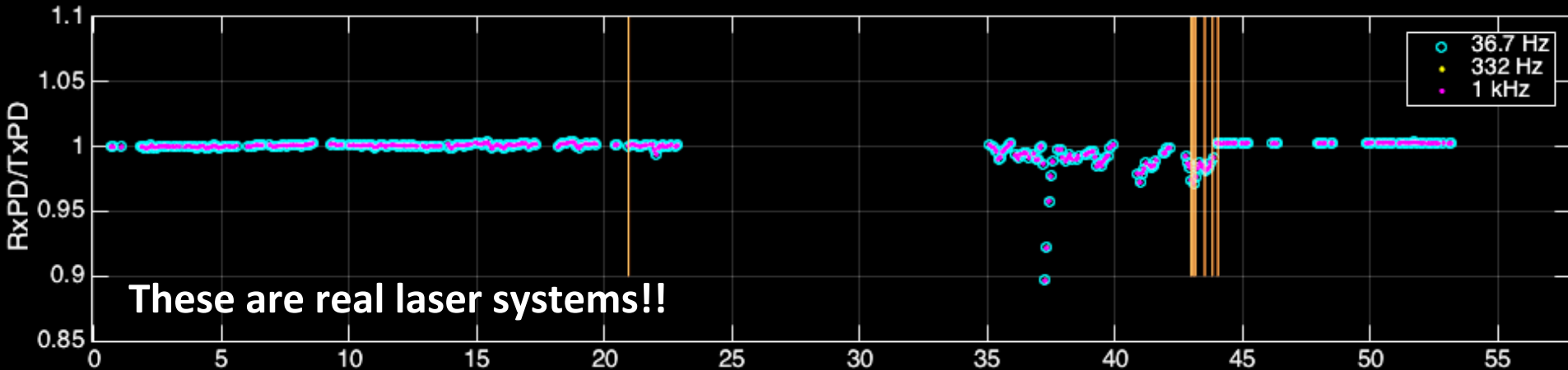


# Systematic Error vs. Statistical Uncertainty



During O2 – the response of the receiver reference dropped compared the the transmitter reference

**Temperature dependence of Beam Relay Periscope. Systematic error of reference!**





# In summary (From G1700810 )



Any one can build a calibration to within a factor of 2 once.  
But can you build a 1% / 1 deg calibration over  $\sim 1$  year?

## Seven Commandments of Calibration

- 1) Your **GW response** will be more complicated than you want it to be
- 2) You'll need to invert **it**
- 3) **It** will be time dependent
- 4) You'll be fighting your awesome isolators
- 5) Your reference will not be perfect
- 6) Your calibration will change between runs, even with the same detector
- 7) 1%/1 deg will be a bookkeeping nightmare