

A Carillon of Black Holes

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Abstract

Scientists collaborating internationally have developed a new way to learn about our universe through gravitational waves, which are ripples in space-time caused by the motion and vibration of celestial bodies. By analogy, gravitational waves are akin to the vibrations carried through the air as sound. Quite remarkably, black holes, which are the densest objects in the universe formed from dead stars can vibrate and emit gravitational waves at frequencies that are within the range of human hearing once the gravitational waves are detected and amplified by instruments such as the LIGO and Virgo gravitational wave detectors. In this work, we explore how to make musical instruments based on gravitational waves by mapping a different gravitational wave pattern to each of the 88 keys of a piano, much like a carillon, which has its bells mapped to the batons of a carillon-keyboard. We rely on theoretical calculations for black hole vibrations to construct our digital black hole instruments. Our software and music samples are freely available to those who want to explore the music of gravitational waves.

Introduction

The universe contains a plethora of exotic objects that humans have been studying through the medium of electromagnetic (EM) radiation, e.g., visible light, for thousands of years. However, light is only one medium through which we are able to sense the universe.

In recent years, sound has been regarded as another way of sensing cosmological phenomena. Cosmic microwave background radiation was first detected aurally from radio telescope signals, and it has been used in artistic outreach projects such as "Rhythms of the Universe," a short film by Smoot and Hart (2013). Cosmologist Janna Levin (2011) describes listening for gravitational waves (GWs) in her TED talk, which has the theme of "space as a drum." Organizations such as the International Community for Auditory Display (www.icad.org) and Interactive Sonification (interactive-sonification.org) are dedicated to exploring the emerging use of sound in informatics, and researchers in a variety of fields are taking an interest in the potential of musically-based sound for new forms of data exploration and education/outreach.

Gravitational waves were first theorized in 1916 by Albert Einstein as a consequence of his theory of gravitation known as General Relativity (GR): he predicted that gravitational interactions were encoded as waves in the fabric of space-time. Although his theory has since been acknowledged and accepted, it took 100 years to prove his final prediction because the technology required to detect GWs involves measuring changes in distances much, much smaller than the diameter of a proton. The ability to detect changes on this scale has only become feasible over the last decade (Harry et al. 2010; Aasi et al. 2015; Acernese et al. 2014).

On September 14, 2015, at 09: 50: 45 UTC, the two Laser Interferometer Gravitational- Wave Observatory (LIGO) detectors (Aasi et al. 2015), located at

Livingston, LA, and Hanford, WA, USA, made the first direct detection of GWs from two colliding black holes (BHs) (Abbott et al. 2016c) that occurred some 1.3 billion light years away. With this detection, along with several more in the years since (Abbott et al. 2016b, 2017b,c,d,a), a new era of gravitational wave astronomy as the Universe's soundtrack has expanded.

We are now able to literally 'listen' to the GWs emitted from cataclysmic astrophysical events in the Universe. The present generation of gravitational-wave detectors have a frequency response that lies within the human hearing range of 20 – 20,000¹ Hz. A natural question to ask is, what do GW sources sound like? There are many GW signals that we could expect to 'hear'; chirping signals from colliding compact objects, burst signals from a supernova, or the background 'hum' from the birth of the Universe itself. Already, we have 'chirp' signal sounds from GWs emitted from the collision of binary-BH systems as BHs pick up speed right before merging. In this paper we consider the GWs emitted by the perturbation of a BH and compare and contrast these sounds to those of traditional musical instruments such as drums and bells. We present a method to construct a digital instrument from a collection of BH spectra, using results from a branch of physics known as BH perturbation theory. We provide software for readers to construct their own BH instrument mapped to the chromatic pitches found on a piano and provide soundfonts of our favorite examples.

Gravitational waves and the sounds of a black hole

Gravitational waves stretch and squeeze space-time in a pattern not dissimilar to ripples in water. As they pass a given area, the relative distance between two points within it changes as a function of time. The stretching and squeezing is characterized by

¹LIGO can listen to sources within this range, but loses sensitivity after 8,000 Hz

the GW strain, $h(t)$, defined as

$$h(t) = \frac{\Delta L(t)}{L}, \quad (1)$$

where $\Delta L(t)$ is the time-dependent change in distance between two points separated by distance L . Gravitational wave detectors such as LIGO and newly active detector Virgo measure $h(t)$ using laser interferometry and digitize the measured strain at a sample rate of 16,384 and 20,000 Hz respectively. The digitized strain time series can be normalized, filtered and encoded as a digital audio file. Thus, it is literally possible to listen to the ripples of the Universe.

Characterizing precisely what types of objects in the Universe produce GWs is an area of active research (Cutler and Thorne 2002), but here we will focus on GWs from perturbed BHs (Ruffini and Wheeler 1971). Black holes are of particular interest since they are the predominant source detected by the LIGO and Virgo GW detectors (Abbott et al. 2016a). Left undisturbed, BHs are quiet. They emit no GWs. However, when a BH is perturbed, say by a collision with another BH (Peters and Mathews 1963; Pretorius 2005), it will emit GWs at frequencies that can be predicted theoretically. A deformed BH quickly dissipates energy in the form of GWs until it again reaches a quiescent state. The GW emission of a perturbed BH depends on two factors: the BH's angular momentum and mass (Echeverria (1989)). The emission pattern consists of a spectrum of quasinormal modes, much like the modes of a bell or drum.

All musical instruments have a signature tone color dictated primarily by its shape, materials, and the mechanics of how it is played. These characteristics enable us to distinguish between different instruments by listening to them. Below we will explore the tone-color of a BH.

Tone-color of a black hole

Like bells, drums, and other various instruments, it is the available frequency partials that give an instrument its characteristic tone-color. With BHs, this too is the case. According to black hole perturbation theory, there are modes that most likely to be excited when a black hole is disturbed. In our analysis, we do not consider which modes may be difficult to excite or suppress. Instead, we choose to remain naive and assume that there should be astrophysical situations where it is physically possible to excite any mode. Therefore, we try to show that, based on the modes that are excited, BHs can emulate the sound of a bell, or of a drum. The frequencies and damping times of these modes are only dependent on its two physical parameters - its mass and spin. Roughly speaking, the mass determines the fundamental frequency of the BH ringing and the spin determines the quality factor, i.e., duration, of the modes or overtones. A higher spin allows for less damped, longer lasting modes. This is analogous to a bell, where the quality of the bell describes its ring-time. All BHs are equivalent, assuming they have the same mass and spin. Other than these two factors, the composition of all BHs are the same.

Through experience and intuition, we can often "hear the shape of the drum" (Kac 1966). In principle, the same intuition could be developed wherein we could hear differences among different perturbed BHs and potentially predict their characteristics - the mass and spin. Later, we show that a BHs truly sounds like unique musical instruments, precisely determined by which modes are excited, and by how fast it is spinning. Therefore, based on the manner of perturbation, a BH can be made to sound like a bell or a drum

A perturbed BH emits GWs according to a spectrum of possible quasinormal modes. The spectrum is defined by the indices of spheroidal harmonics l, m that describe the BH

perturbations along with an overtone factor n . Figure 1 shows a qualitative diagram of an $m = 2$ mode of a perturbed BH, defined by a vibration that stretches and squeezes the space around the BH in a time-dependent way. These waves have frequencies that depend only on the mass and spin of the BH, and a damping time determined by the “quality factor”, Q , which depends only on the spin parameter of the BH, j .

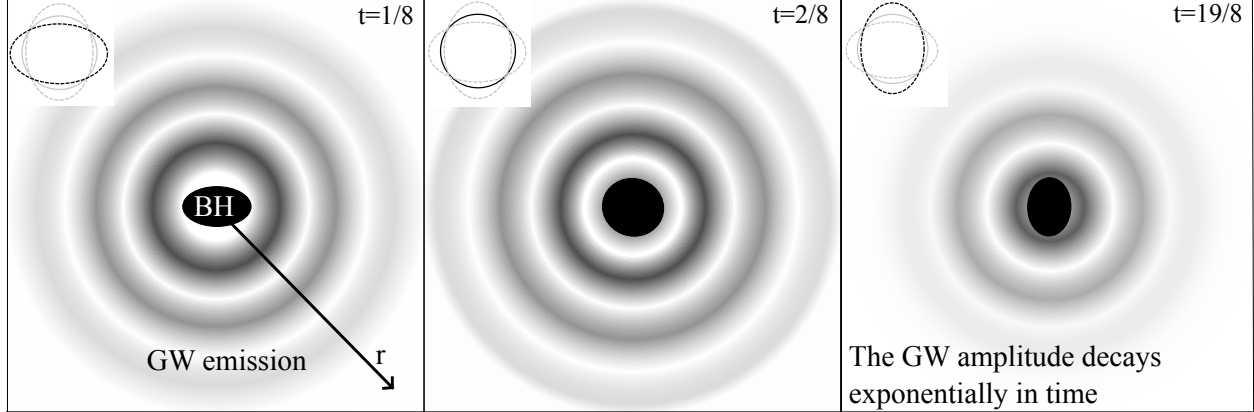


Figure 1. Artist’s rendition of GWs emitted from a perturbed BH. Shown is the equatorial plane of a BH that is “ringing” with an $m = 2$ mode where the concentric circles indicate the GW strain with darker values indicating regions of a higher strain. The three panels show the vibration of the BH at three distinct times. Note that the GW strain decreases with radial distance, r , from the source and with time, t . The top left insets show the relative phase of the $m = 2$ mode for each panel. LIGO and Virgo detect the passing crests and troughs of gravitational strain emanating from the BH as they reach our terrestrial detectors

Determining the precise spectrum of BH quasinormal modes has been an active research field for over sixty years. Berti et al. (2009) provides a thorough and modern review on the topic and we use their fitting formulas for the quasinormal mode frequencies and quality factors. The frequency f_{lmn} and quality factor Q_{lmn} of a given mode indexed by the two spheroidal harmonic numbers l and m , as well as an overtone number n , are given by

$$f_{lmn} \approx \frac{1}{2\pi GM/c^3} (f_{1lmn} + f_{2lmn}(1 - j)^{f_{3lmn}}), \quad (2)$$

$$Q_{lmn} \approx \pi f_{lmn} \tau_{lmn} \quad (3)$$

where $f_1, f_2, f_3, q_1, q_2, q_3$ are the given fitting parameters, M is the mass of the BH, j is the dimensionless spin parameter, which can take on any value between 0 and 1, G is the gravitational constant, c is the speed of light, and τ_{lmn} is the damping time.

Neglecting geometric factors, the detected GW strain from a perturbed BH at a detector on Earth is given by

$$h_{lmn}(t) = e^{-\pi f_{lmn} t / Q_{lmn}} [A_{lmn} \sin(2\pi f_{lmn} t) + B_{lmn} \cos(2\pi f_{lmn} t)], \quad (4)$$

where A_{lmn} and B_{lmn} correspond to complex mode amplitudes. In this work, we will only consider the sine phase to simplify the discussion and note that it has the benefit of being zero at time zero. Furthermore we will consider modes up to $l = m = 4$ and $n = 3$, for which the fitting formulas are defined in Berti et al. (2009). Therefore, the waveforms we consider are linear combinations of the sine phases of the above modes given by

$$h(t) = \sum_{n=1}^3 \sum_{l=2}^4 \sum_{m=-l}^l A_{lmn} e^{-\pi f_{lmn} t / Q_{lmn}} \sin(2\pi f_{lmn} t). \quad (5)$$

The amplitude of the mode, A_{lmn} encodes how the BH is perturbed and we leave it as a free parameter. We show that it is possible to obtain the A_4 frequency from a variety of black holes for the same mode in Figure 2 below

Black holes as bells or drums

A bell's ring typically lasts for many cycles of the fundamental frequency, i.e., a large Q , whereas a drum's duration could be very short, i.e., a small Q . From experience, we have seen that too short a quality factor, i.e., $Q \lesssim 5$ leads to more of a click, rather

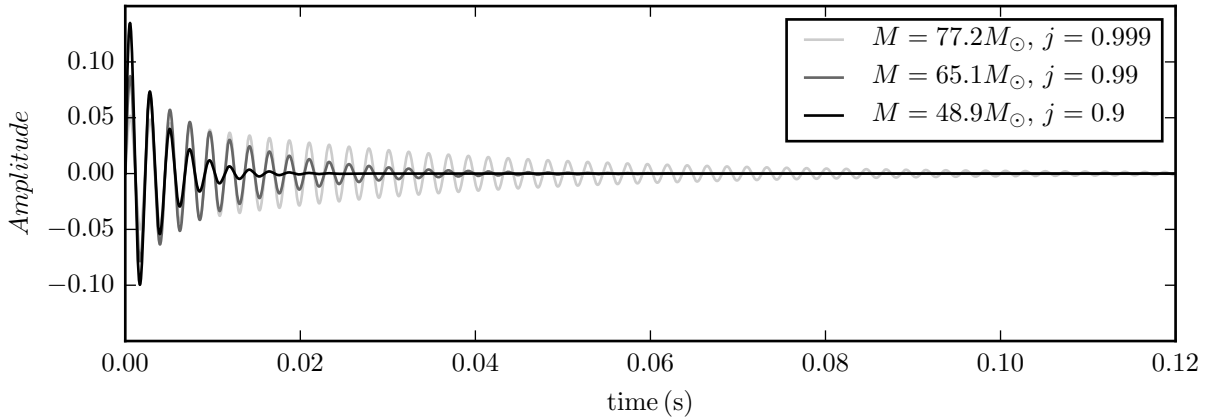


Figure 2. The $l = 2, m = 2, n = 0$ mode of three distinct BHs with different mass and spin. The masses and spins are chosen such that each produces an A_4 (440 Hz) as the central frequency for this mode. In general, additional modes for each BH may be excited according to (5).

than drum-thud type sound. We will classify BHs that produce long-lived gravitational-wave ringing as bell-like, and those with short ringing as drum-like/click-like. To distinguish the two classes, we will arbitrarily establish $Q = 100$ as the threshold between a drum/click and a bell. Subsequently, we will establish $5 \leq Q \leq 100$ as drum-like modes to avoid click-type tones. The fitting parameters q_1, q_2, q_3 range from $\sim [-20, 20]$. In order to have large Q , e.g., $Q > 100$, two conditions are required: 1) $q_3 < 0$ and 2) $j \sim 1$. These bell-like and drum-like modes are separated into two categories as seen in Figure 3.

Can a black hole make a good bell or drum?

Perrin et al. (1983) note that a “typical good quality church bell” in western music has its first five partials with the frequency ratios of $1 : 2 : 2^{15/12} : 2^{19/12} : 4$. The modes of black holes do not match these ratios, as the ratios fall between these values. Instead, we determine whether or not the ratios between *any* of the first 63 modes defined by Equation (5) for a given BH spin have similarities to the spectrum of a bell. Figure 4

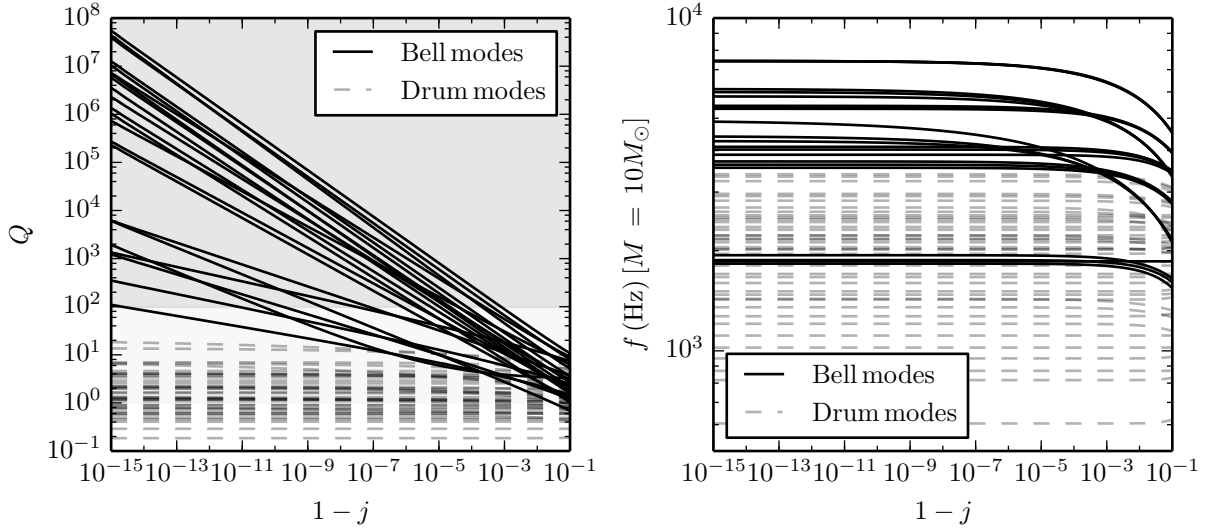


Figure 3. Quality factor, Q , (left) and frequency, f , (right) for the first 63 modes of a BH defined by (3). As $1 - j$ approaches zero, Q diverges for modes with $q_3 < 0$, but f asymptotes to a constant value. We denote modes with $q_3 < 0$ as “bell-like”, since as j approaches one the Q values exceed 100. The remaining modes we denote as “drum-like”. As j approaches zero, all the modes are drum-like.

shows the frequency spacing of the bell-like modes, $q_3 < 0$, as a function of $1 - j$, as well as the accuracy with which an ideal bell can be obtained. With a one percent accuracy, a BH with a spin greater than 0.9999999 can mimic an ideal bell in terms of frequency content. However, each mode has a substantially different quality factor, Q , meaning that the partials do not decay at a similar rate. So, although a perturbed BH may initially resemble the spectra of a typical good quality bell, the modes will not decay in the same way. We explore more of this particular example later.

Black holes may also be compared to a simple drum membrane. We repeat the same process, this time matching *any* black hole mode frequencies to have ratios representative of drums: $1 : 1.59 : 2.14 : 2.30 : 2.65$, relative to the first overtone (seen from Rossing (2007)). Figure 5 shows the frequency spacing of the drum-like modes, as a function of $1 - j$, as well as the accuracy with which an ideal drum can be obtained. We have placed constraints on the drum like modes to have $5 \leq Q \leq 100$, to a) exclude

bell-like modes and b) avoid click-like sounds. Under these constraints, we see that spins values of $10^{-5} \leq 1 - j \leq 10^{-2}$ are needed to obtain a typical, ideal drum. Later on, we explore an drum membrane example subject to these constraints.

For both the bell and drum cases, we also plot the accuracy of the mode with the largest deviation from the ideal frequency ratio. This estimation in accuracy does not take into account the errors associated with the fitting factors listed in Equation 2 (see Berti et al. (2009) for a complete list of tabulated errors), and is computed as follows:

$$Accuracy = \max_i \left[\min_{lmn} \left(\frac{f_{lmn}}{f_{\text{partial}_i}} \right) \right] \times 100. \quad (6)$$

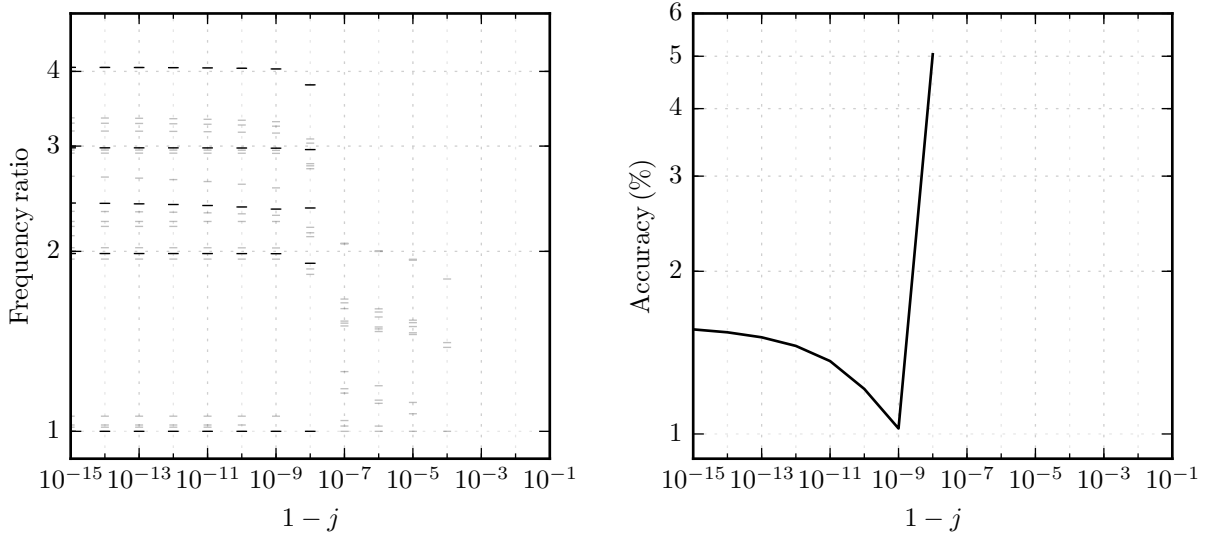


Figure 4. Frequency spacing of the bell-like modes of a BH (left) and accuracy to which a “typical good quality church bell” can be achieved (right) for the worst mode. We restricted the modes in this figure to be those with a quality factor, $5 \leq Q \leq 100$. Under these constraints, it can be seen that a quality bell is only achieved when $1 - j < 10^{-9}$. The damping timescales are considerably different for each mode as indicated by Figure 3. This implies that although a BH may start out sounding like a church bell, the modes will not decay as one would expect from a typical bell.

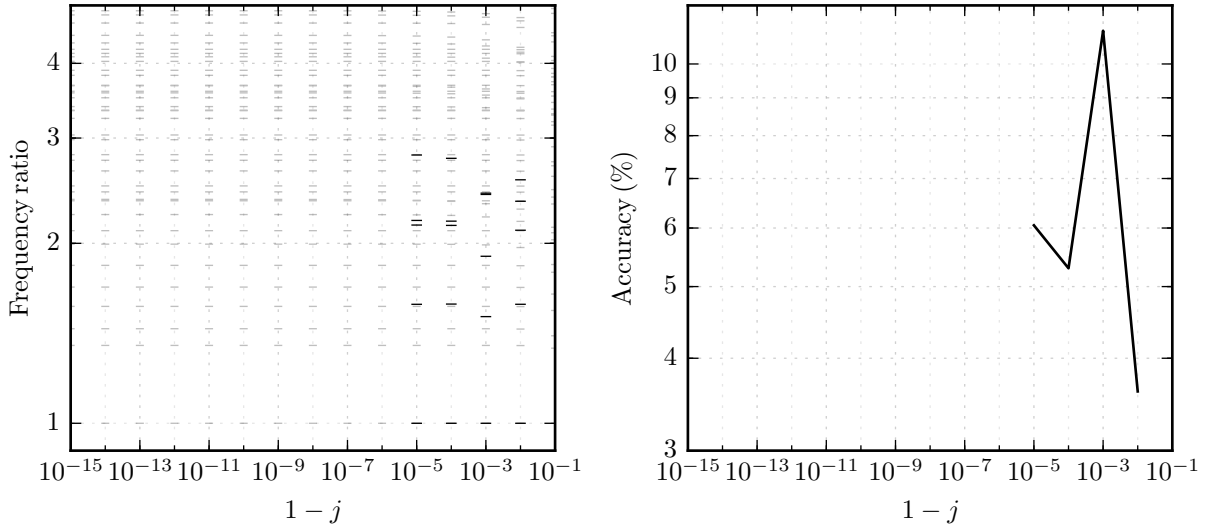


Figure 5. Frequency spacing of the drum-like modes of a BH (left) and accuracy to which a “typical good quality drum” can be achieved (right) for the worst mode. We restricted the modes in this figure to be those with a quality factor $5 \leq Q \leq 100$. Under these constraints, it can be seen that a quality drum is only achieved when $10^{-5} \leq 1-j \leq 10^{-2}$. The damping timescales are considerably different for each mode as indicated by Figure 3. This implies that although a BH may start out sounding like a drum, the modes will not decay as one would expect from a typical drum.

Carillons and drums from black holes

We have shown that for particular spins, it is possible to create black hole modes that approximate bell-like and drum-like timbres, derived from emitted GWs. While it may be possible to create instruments with more complicated waveforms, through more complicated astrophysical scenarios, we limit our study to these simpler instruments. Using the algorithm described above to find five accurate partials, we create an approximation to a church bell and a simple drumhead.

Our example instrument tones are tuned by setting the prime frequency to be the fundamental pitch. The 88 bells, each assigned to a chromatic pitch, create a kind of digital carillon: an instrument that contains an array of large church bells, played by striking a lever connected to the keys of a special type of piano. As there is much study done on the modal structure of a historic carillon in the city of Flanders, Belgium, we can compare our BH carillon to this historic carillon.

Typical church bell

As described above, it is possible to approximate a church bell-like sound based on a BH, although, the result differs from typical bell spectra due to 1) the ratios of mode frequencies not exactly matching those of a bell and 2) relative mode decay times sustaining longer than those of a bell. Table 1 shows a simple set of modes. It can be seen that the decay times increase with higher partials, which leads to a change in the waveform at later times (see Figure 6). This is in direct contrast to the behaviour of bell partials, which typically feature the longest sustain from the first partial (hum), and quicker decay times in the higher partials.

partial	mode	f[Hz]	Q
Hum	(2, 1, 2)	222	131
Fundamental	(3, 2, 1)	440	1660
Minor third	(2, 2, 1)	522	9840
Fifth	(4, 3, 0)	661	19600
Octave	(4, 4, 0)	896	68600

Table 1. A BH approximation to a good quality typical church bell tuned to A_4 . Here the mass is $M = 82.37M_\odot$ and the spin is $j = 0.999999999$. The tuning is fixed by setting the $l = 3, m = 2, n = 1$ mode to be the fundamental frequency, with the largest amplitude.

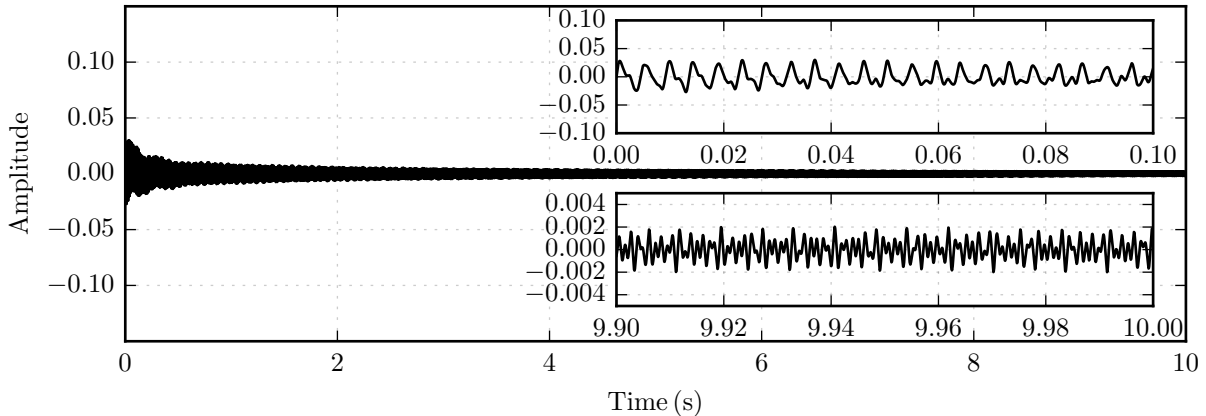


Figure 6. First 10 seconds of the waveform of good quality typical church bell tuned to A_4 made from a BH. As you can see, the waveform is very long-lived > 10 s and evolves over time as indicated by the difference between the top inset (first 0.10 seconds) and bottom inset (last 0.10 seconds).

Comparison to the actual carillon

This set of partials and decay times can be compared to the bells of a carillon cast by Joris Dumery circ. 1742 (Schneider and Leman (2016)). The modal structure of the Dumery bells has been studied extensively, particularly their frequency dependence on mass and diameter. Of particular interest are the first five partials, which we compare to the BH bell's modal dependence on mass and geometry.

As mentioned before, the modes of a BH depend on a 3-dimensional spheroidal

geometry. For bell-like structures, modes are described as nodal meridians m , and nodal circles n (see Strutt (2011)). Since there is no obvious translation between these two geometries, we list the first five Dumery bell partials in Table 2 which result from full meridians, seen in Schneider and Leman (2016), as a comparison.

partial	BH mode	carillon mode (m,n)
Hum	(2, 1, 2)	(2, 0)
Fundamental	(3, 2, 1)	(2, 1)
Minor third	(2, 2, 1)	(3, 1)
Fifth	(4, 3, 0)	(3, 1)
Octave	(4, 4, 0)	(4, 1)

Table 2. Comparison of the modal structure of first five partials of an ideal BH bell with the first five partials of a carillon bell. m and n for bells stand for nodal meridians, and nodal circles respectively.

We first investigate mode frequency dependence on mass for BHs and carillon bells, and then consider frequency dependence on geometry. By fixing the spin of an array of BHs with varying mass, we can utilize the methods of Equation (2) to compute BH mode frequency dependence on mass. In Figure 7 we present the results alongside carillon bell mode frequency dependence on bell mass, computed from Schneider and Leman (2016).

While it was relatively easy to compare mode frequency dependence on mass for bells and BHs, the problem becomes less well-defined when computing the mode frequency dependence on a BH's diameter. To quantify a diameter, we seek to assign the radius from which GWs are emitted. From GR, it follows that for spinning BHs, an axisymmetric surface is created from which GWs will propagate, without falling in. While this surface is spherical for a static BH, due to the spin it morphs into an axisymmetric surface with two radii that encompass a region termed the light curve. To simplify further discussion, we consider only one radius, r , of this light curve (Bardeen

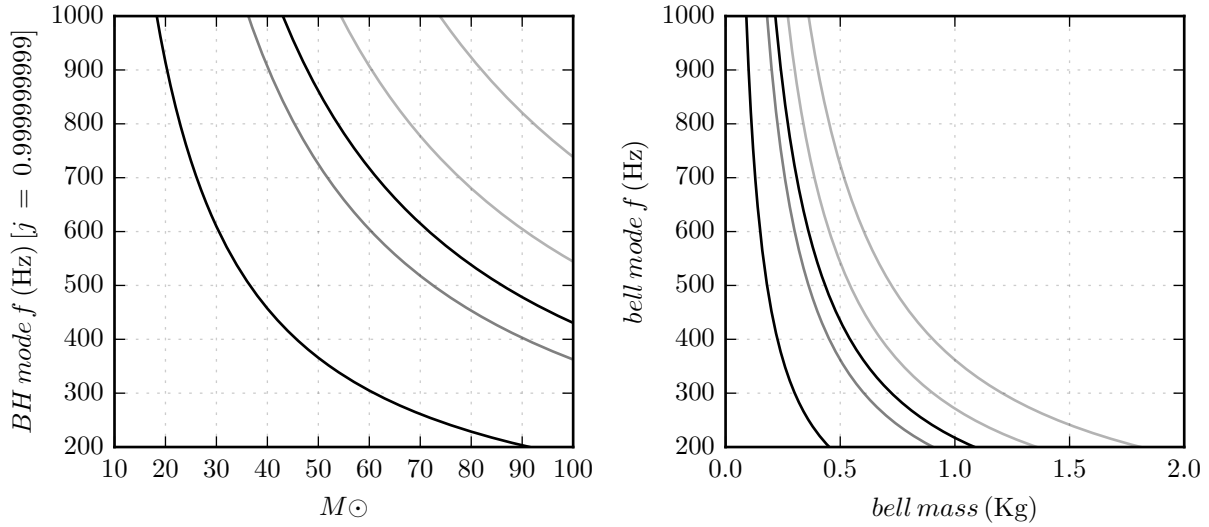


Figure 7. Left figure: Frequency dependence of quasinormal modes on BH mass (M_{\odot}), changed by varying spin. From left to right: BH partials are hum, minor third, fundamental, fifth, octave. Right figure: Frequency dependence of bell modes on bell mass Schneider and Leman (2016). From left to right: bell partials are hum, fundamental, minor third, fifth, octave.

et al. (1972))

$$r = \frac{2MG}{c^2} \left[1 + \cos \left(\frac{2}{3} \cos^{-1}(-j) \right) \right], \quad (7)$$

where M is the mass of the BH, and j is the dimensionless spin parameter. Given this BH 'diameter' ($D = 2r$), we can begin to investigate the frequency dependence of BH modes based on its diameter, and examine how it differs with the carillon bells' mode frequency dependence on bell diameter.

Only a change in the mass or spin of a BH can change the diameter of its light ring. By choosing a fixed mass, we can see how the radius changes with spin. We can reframe Equation (2) using Equation (7) to change the mode frequency dependence on spin j to its diameter D . Our results are plotted in Figure 8 and compared to Dumery bell mode frequency dependence on bell diameter (Schneider and Leman (2016))

$$f_{lmn} \approx \frac{1}{2\pi GM/c^3} \left[f_{1lmn} + f_{2lmn} \left[1 + \cos \left(\frac{3}{2} \cos^{-1} \left(\frac{rc^2}{2MG} - 1 \right) \right) \right]^{f_{3lmn}} \right]. \quad (8)$$

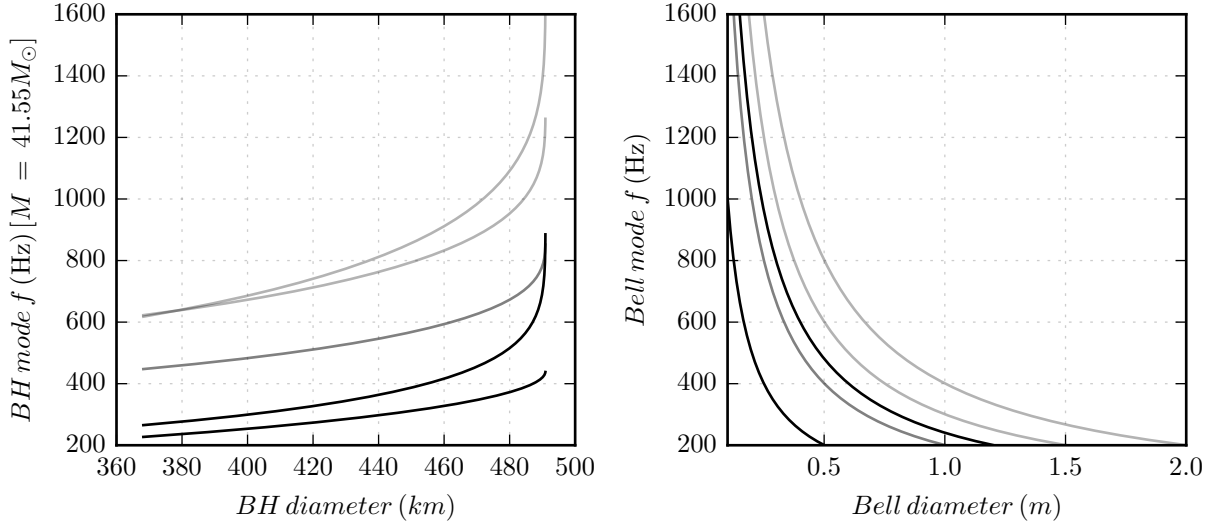


Figure 8. Left figure: Frequency dependence of quasinormal modes on BH diameter d , changed by varying spin. From top to bottom: BH partials are octave, fifth, fundamental, minor third, hum. Right figure: Frequency dependence of bell modes on diameter Schneider and Leman (2016). From left to right: bell partials are hum, fundamental, minor third, fifth, octave.

Typical drum membrane

For BHs with comparatively low spin, the quick decays of the modal frequencies emulate the thud of a typical drum membrane. Here, we present a very simple drum tuned to A_4 created by a BH spinning less rapidly than in our previous example ($j = 0.9999$), created from the overtones listed below. For quality factors $Q \lesssim 5$, we found that our approximation resembled a click, rather than the characteristic thud of a drum, while values of $Q > 100$ produced bell-type ringing modes. From Figure 5, it can be seen that there are fewer mode combinations that satisfy these constraints. We have listed the one with an accuracy of five percent in Table 3. As in the bell approximation,

the decay times increase with higher partials for our drum membrane approximation. Although the decay times are not as widely spread in this case, and the waveform is not long lived, there still occurs a change in the waveform at later times (see Figure 9).

mode	f[Hz]	Q
(2, 1, 1)	278	6
(4, 0, 0)	440	6
(4, 2, 1)	596	9
(4, 2, 0)	606	27
(4, 3, 2)	771	29

Table 3. A BH approximation to a good quality drum tuned to A_4 . Here the mass is $M = 67.03M_\odot$ and the spin is $j = 0.9999$. The tuning is fixed by setting the $l = 4, m = 0, n = 0$ mode to be the fundamental frequency, with the largest amplitude.

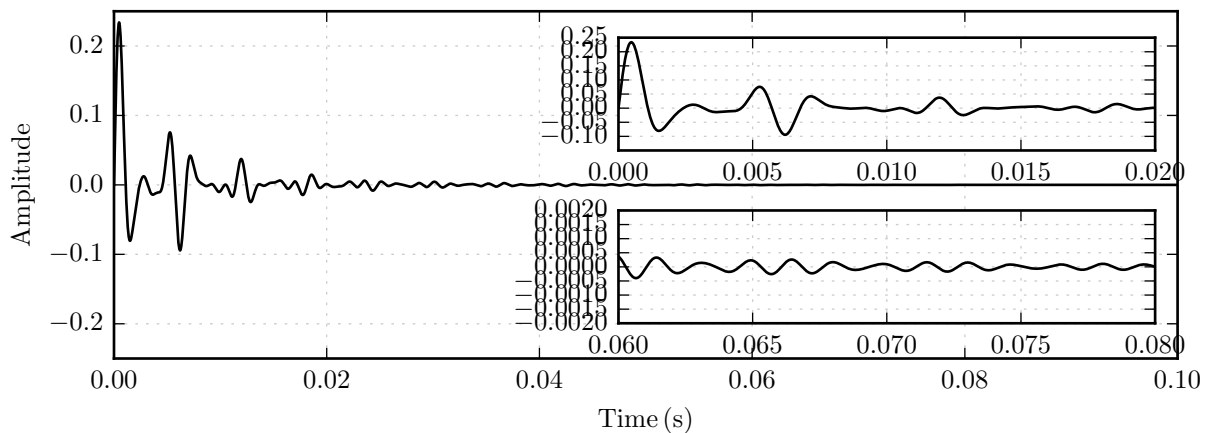


Figure 9. Waveform of a drumhead membrane simulated with five partials tuned to A_4 made from a BH. As you can see, the waveform is not long-lived (0.3 s) and evolves over time as indicated by the difference between the top inset (first 0.02 seconds) and bottom inset (later 0.02 seconds).

Constructing a black hole instrument

In order to construct a BH instrument, we create 88 bell-like timbres, save them as audio files, and assign each to a pitch that can be played by a MIDI keyboard. To start the process we construct our canonical BH by choosing a canonical mass M_{can} and a user-defined set of mode amplitudes, A_{lmn} . We denote the primary tone as the mode

frequency, f_{can} with the largest A_{lmn} . From our canonical BH we rescale the canonical mass to match the 88 keys of the piano through

$$\frac{M}{M_{\text{can}}} = \frac{f_{\text{can}}}{f_k}, \quad (9)$$

where f_k is given by

$$f_k = 2^{(k-49)/12} \times 440 \text{ Hz}, \quad (10)$$

where k denotes the keys of the keyboard (from 1 to 88).

Conclusion

We have showcased our method of creating digital BH instruments, and hope that this project inspires researchers and the public into creating their own types of instruments. The music that we create is from the perturbations of BHs, but understanding what kind of exact perturbation causes a BH to sound like a bell, drum or something else is out of the scope of this paper. We have seen that from the literature that the impact time of a force imparted to a bell directly contributes to which higher modes are excited, which in turn decides the tone-color of the sound emitted (see Fletcher et al. (2002)). We predict that, in turn, the manner and duration of a BH perturbation directly leads to the tone-color of the emitted GW.

One major limitation of our BH approximation of musical instruments is that the black hole instruments are only created from five partials. Therefore, at most one can approximate an ideal, generic bell. To be able to emulate a particular bell, more higher partials are needed to capture its true tone-color. Nevertheless, even if there were fitting factors that could derive frequencies of higher partials for any BH mass or spin, the

accuracy would undoubtedly plummet as there are only so many degrees of freedom available in choosing accurate partials. A possible solution to this is to have more overtones, i.e., $n > 3$ to have a larger pool of modes to choose from.

Another point to consider is the tuning of our BH bells and drums. We have simplified our analysis by setting the second partial (prime) to be the fundamental frequency, and having the largest amplitude, while this is not the case with all bells. We have also ignored issues of the perceived pitch at the beginning of perturbation. Moreover, we have found an inverse structure to the quality factor, Q , of our bell and drum approximations. In bells, the lower partials have a higher quality factor, with the hum frequency being longer lived than the higher partials. We find that this is not the case with BH bells. There seems to be a transfer of energy from lower to high partials, which results in the tuning of the bell approximation being fundamentally changed at later times.

Black hole perturbation theory is assisting GW astronomers to understand the parameters of observed BHs by analyzing the strain waveforms. We hope that this exploration into the similarities of modal structure between BHs and bells/drums, and the existing studies done on mode frequency dependence on the features of bells will assist the scientific community to obtain a different avenue in which to further explore the structure of BHs.

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instruments to re-create the Star Trek theme song.

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