Long-duration waveform models for millisecond magnetars born in binary neutron star mergers

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Abstract

We derive a model describing the gravitational-wave emission from a rapidly spinning-down millisecond magnetar born during a binary neutron star merger. Gravitational-wave emission and/or torques due to magnetic dipole radiation spin down the nascent neutron star. The waveform model described here allows for arbitrary braking indices, based on analytic work from Lasky et al. [1].

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I. INTRODUCTION

The merger of two neutron stars can result in the birth of a rapidly rotating, highly magnetised neutron star that spins down through some combination of gravitational-wave and electromagnetic emission. Rowlinson et al. [2] showed that simple spindown models [e.g., 3] can fit x-ray afterglow lightcurves following prompt short GRB emission. The model was generalised to include arbitrary braking indices n in Lasky et al. [1], who showed that a non-fiducial braking index — i.e., $n \neq 3$ — provides a better fit to the data for one of the two short GRBs analysed.

Here we show how the model of Lasky et al. [1] can be used to generate gravitational waveform models that can be utilised to benchmark searches for gravitational-wave transients following binary neutron star merger events.

II. GRAVITATIONAL-WAVE EMISSION MODEL

We assume the spindown of the nascent neutron star can be well-described by the general torque equation:

$$\dot{\Omega} = -k\Omega^n, \tag{1}$$

where Ω is the star's angular frequency, an overdot represents a derivative with respect to time, k is a constant of proportionality, and n is the braking index. A braking index of $n = 5$ describes a star being spun down through gravitational-wave emission only, while braking due to an unchanging, dipolar magnetic field in vacuo implies a theoretical braking index of $n = 3$. In practice, the braking index of isolated pulsars is almost always $n \lesssim 3$ [e.g., 4, 5], while the braking index has only been measured in two millisecond magnetars, and is $n = 2.9 \pm 0.1$ and $n = 2.6 \pm 0.1$ [1].

Rotating, non-axisymmetric stars emit gravitational waves at twice the star's spin frequency. For possible causes of such non-axisymmetries, see Refs. [1, 6] and references therein. Integrating Eqn. (1), and re-writing in terms of the gravitational-wave frequency implies

$$f_{\rm gw}(t) = f_{\rm gw,0}\left(1 + \frac{t}{\tau}\right)^{\frac{1}{1-n}}. \tag{2}$$

Here, $\tau = -\Omega_0^{1-n}/k(1-n)$ is the spindown timescale, and $f_{\rm gw,0}$ is the gravitational-wave frequency at $t = 0$.

In principle, $t = 0$ is the time following the binary neutron star merger at which the remnant has settled to rigid body rotation, and begins spinning down due solely to gravitational-wave and/or electromagnetic losses. This time is likely $\sim$ 100s of ms following the merger. In practice, this time is difficult to determine, but knowing the true value of $t = 0$ is not important as it is covariant with τ.

The gravitational-wave amplitude of a non-axisymmetric, rotating body is [7]

$$h_0(t) = \frac{4\pi^2 G}{c^4} \frac{I_{zz}\epsilon}{d} f_{\rm gw}^2(t), \tag{3}$$

where I_{zz} is the principal moment of inertia, ϵ is the ellipticity and d is the distance to the source. We assume these to be unchanging throughout the relevant evolution time. The strain amplitude is therefore

$$h_0(t) = \frac{4\pi^2 G}{c^4} \frac{I_{\rm zz}\epsilon f_{\rm gw,0}^2}{d} \left(1 + \frac{t}{\tau}\right)^{\frac{2}{1-n}}. \tag{4}$$

Both the ellipticity and principal moment of inertia are unknown for a given system. In principle we can therefore combine these two parameters, however in practice it is convenient to choose a fiducial value of I_{zz} and simply search over different values of ϵ.

The plus and cross polarisations of the strain are respectively:

$$h_+(t) = h_0(t) \frac{1 + \cos^2 \iota}{2} \cos \Phi(t), \tag{5}$$

$$h_\times(t) = h_0(t) \cos \iota \sin \Phi(t), \tag{6}$$

where ι is the inclination angle, and

$$\Phi(t) = \Phi_0 + 2\pi \int_0^t dt' f_{\rm gw}(t'), \tag{7}$$

is the phase with $\Phi_0 \equiv \Phi(0)$. Substituting the gravitational-wave frequency evolution, Eqn. (2), into the phase evolution implies

$$\Phi(t) = \Phi_0 + 2\pi\tau f_{\rm gw,0} \left(\frac{1-n}{2-n}\right) \left[\left(1 + \frac{t}{\tau}\right)^{\frac{2-n}{1-n}} - 1\right]. \tag{8}$$

In summary, Eqns. (4–6) and (8) constitute the full waveform model describing a rapidly rotating neutron star spinning down with arbitrary braking index.

A. Energy emitted

The total power emitted in gravitational waves is

$$\dot{E}_{\rm gw}(t) = -\frac{32G}{5c^5} I_{zz}^2 \epsilon^2 \Omega(t)^2. \tag{9}$$

Substituting in solution of Eqn. (1) for the evolution of the stars angular frequency and integrating to give the emitted gravitational-wave energy as a function of time gives

$$E_{\rm gw}(t) = -\frac{32\pi^6 G}{5c^5} I_{zz}^2 f_{\rm gw,0}^6 \epsilon^2 \tau \frac{n-1}{n-7} \left[\left(1 + \frac{t}{\tau}\right)^{\frac{7-n}{1-n}} - 1 \right]. \tag{10}$$

III. PARAMETER SPACE

In this section we highlight the contributions of various parameters in the above waveforms, and discuss likely parameter ranges for millisecond magnetars following binary neutron star mergers. The free parameters are $f_{\rm gw,0}$, τ, n, and ϵ (or alternatively ϵ/d).

A. Initial gravitational-wave frequency, $f_{\rm gw,0}$:

Conservation of angular momentum through the merger phase implies the post-merger remnant should be rotating at, or close to, the mass-shedding limit (e.g., see Ref. [8]). This spin period p_0 is a function of the equation of state, but in general the initial spin period is expected to be between $5 \gtrsim p_0/\text{ms} \gtrsim 0.7$, corresponding loosely to initial gravitational-wave frequencies between $500 \lesssim f_{\rm gw,0}/\text{Hz} \lesssim 3,000$. In principle, numerical relativity simulations of binary neutron star mergers can also inform the post-merger remnant's gravitational-wave frequency. In practice though, more needs to be done to understand the evolution of such systems from the end of numerical relativity simulations—typically tens of ms following the merger—to the point at which the body can adequately be described as a uniformly rotating body.

B. Damping timescale, τ:

It is instructive to consider the special case of $n = 3$, in which case the damping timescale is the electromagnetic dipole braking time

$$\tau_{\rm em} = \frac{3c^3 I}{B_p^2 R^6 \Omega_0^2}, \tag{11}$$

where B_p is the dipole, poloidal component of the external magnetic field, R is the stellar radius, and $\Omega_0 = 2\pi/p_0$ is the initial angular frequency of the star. To give an indication of reasonable values here, a 5×10^{15} G field, initial spin period of 1 ms and radius of 12 km (typical radii are between 10 and 14 km) implies $\tau_{\rm em} \approx 350$ s, while a 5×10^{14} G field with the same parameters implies $\tau_{\rm em} \approx 35,000$ s. These two numbers do a reasonable job of covering the extremes of this parameter space.

A complementary approach to understanding this parameter space is by looking at fits of the milliesecond magnetar model to x-ray light-curves following short GRBs [1, 2, 9]. Rowlinson et al. [2] fit many tens of GRB light curves and extracted, among other things, values for $\tau_{\rm em}$ under the fiducial assumption of $n = 3$. The range of $\tau_{\rm em}$ values ranged from many tens of seconds to tens of thousands of seconds, confirming our above estimates.

C. Braking index, n:

There are only two measurements of braking indices of millisecond magnetars [1], and the physics dictating those values is ill-understood. However it is also prudent to explore the $n = 5$ parameter space as this describes gravitational-wave dominated emission.

D. Ellipticity, ϵ:

Non-zero ellipticities can be caused by a number of factors. These include wound-up internal magnetic fields coupled to the Mestell-Jones-Cutler spin-flip instability [e.g., 10] and secular bar modes [e.g., 11, 12]. For a discussion of many relevant mechanisms, their associated timescales and expected stellar deformations, see Ref. [13]. In general, it is difficult to have stellar ellipticities larger than $\sim 10^{-3}$ [13]. However ellipticities much smaller than this will unlikely emit gravitational waves of sufficient amplitude to be detected with second-generation interferometers. It is therefore reasonable to assume $10^{-4} \lesssim \epsilon \lesssim 10^{-2}$.

$f_{\rm gw,0}$	τ	n	ϵ
1.0 kHz	10^2 s	2.5	10^{-2}
2.0 kHz	10^3 s	3.0	10^{-3}
3.0 kHz	10^4 s	5.0	

TABLE I: Waveform model parameters. We show 21 of these waveform models with combinations of these parameters in Fig. 1

IV. WAVEFORMS

We illustrate the effect each parameter has on the waveform model by creating 54 waveforms with every combination of parameters in Table I. We show the evolution of h_0 and $f_{\rm gw}$ for each of these waveforms in Fig. 1. In each model we show a fiducial waveform in black, with parameters: $f_{\rm gw,0} = 2$ kHz, $\tau = 10^3$ s, $n = 3.0$ and $\epsilon = 10^{-2}$. The top two rows of Fig. 1 show the evolution of h_0 (top row) and $f_{\rm gw}$ (second row) where the axes are linear, and therefore highlight the early-time rate of change of these parameters. The bottom two rows show the same, however on log-log axes to show the initial, almost monochromatic plateau phase for $t \lesssim \tau$, and the power-law decay for $t \gtrsim \tau$.

Figure 1 is divided into four columns where we vary $f_{\rm gw,0}$, τ, n, and ϵ in each column (from left to right, respectively) as indicated in the legends. The effect of each parameter can clearly been seen in each plot.

V. CONCLUSION

We derive and show gravitational-waveform models for the evolution of a nascent neutron star spinning down with an arbitrary braking index. This model includes as subsets the standard $n = 3$ spindown where the torques are due to an unchanging, dipolar magnetic field in vacuo, as well as a star being spun down through gravitational-wave emission only ($n = 5$). These models are designed to be used in long transient gravitational-wave signals from newly formed neutron stars.

These models do *not* take into account a number of things, including additional torques from fallback accretion or alternative gravitational-wave emission mechanisms such as stellar modes unstable to the Chandrasekhar-Friedmann-Schutz (CFS) instability. While fallback

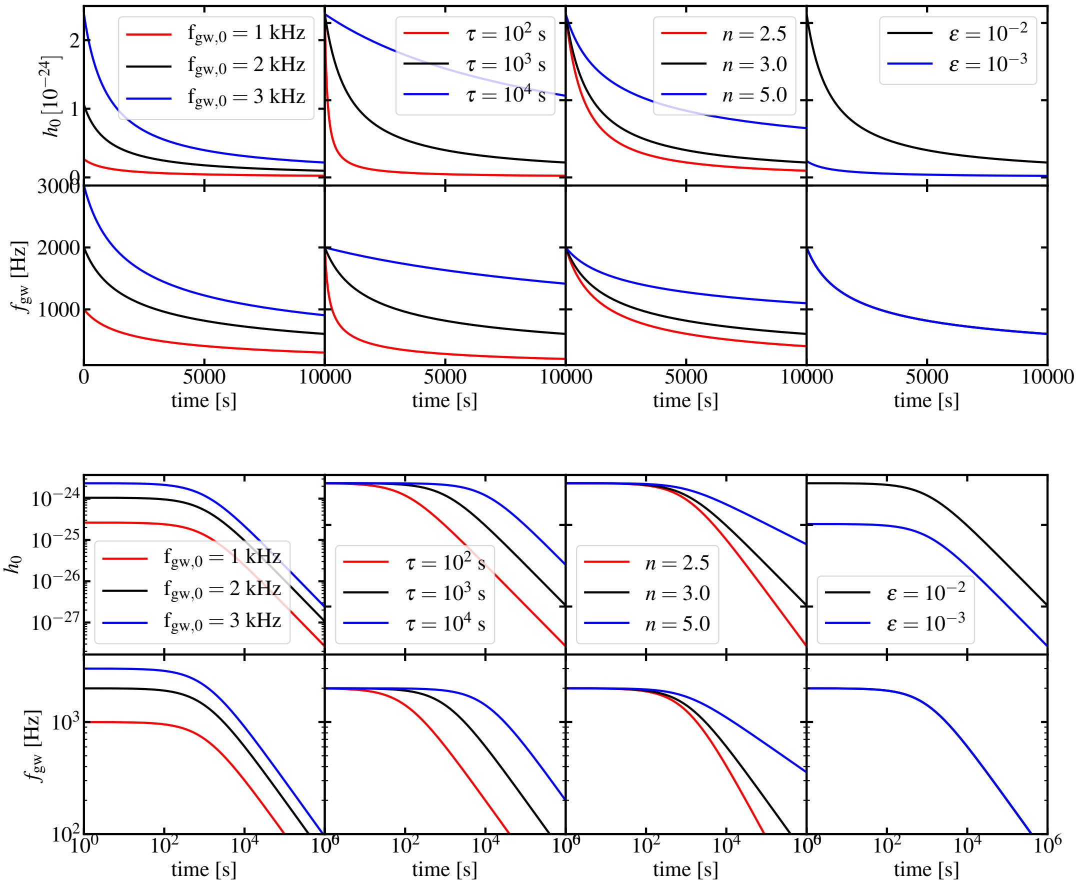

FIG. 1: Strain amplitude h_0 and gravitational-wave frequency $f_{\rm gw}$ evolution for our 54 different waveform models. The top two rows show the evolution of h_0 (top row) and $f_{\rm gw}$ (second row) where the axes are linear, and therefore highlight the early-time rate of change of these parameters. The bottom two rows show the same, however on log-log axes to show the initial, almost monochromatic plateau phase for $t \lesssim \tau$, and the power-law decay for $t \gtrsim \tau$. Each panel shows our fiducial model in black (see text for a description), and varies one parameter in each column: $f_{\rm gw,0}$ (first column), τ (second column), n (third column), ϵ (fourth column).

accretion is expected to be minimal for binary neutron star remnants, CFS instabilities may play an active role—see e.g., Refs. [11, 12] and references therein—although parameters associated with those models are highly uncertain.

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