Mirror Metrology using Mode Spectroscopy

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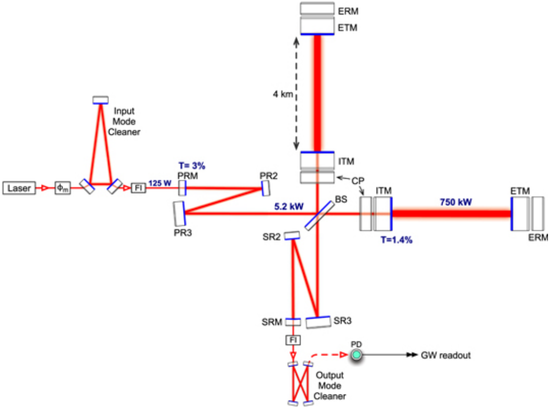
LIGO-IndIGO SURF 2017

July 26, 2017

LIGO as an FPMI

LIGO is an FPMI $\rightarrow$ multiple cavities $\Rightarrow$ multiple mirrors.



Optical losses

↙ ↘

reduced circulating power

destruction of squeezed state

↘ ↙

Increase in shot noise in GW output.

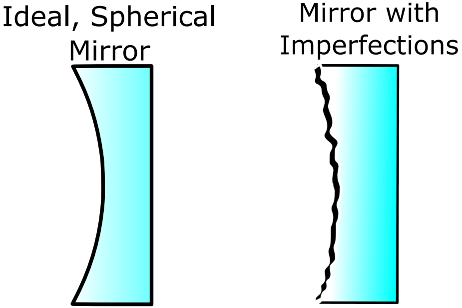
Optical Losses and Mirror Figure Error

Many Optical Losses: Point defects, scratches, contamination, absorption and transmission, coating loss, etc.[1]

This Project's Focus:

Mirror Figure Error

→ low spatial frequency
→ low angle scattering

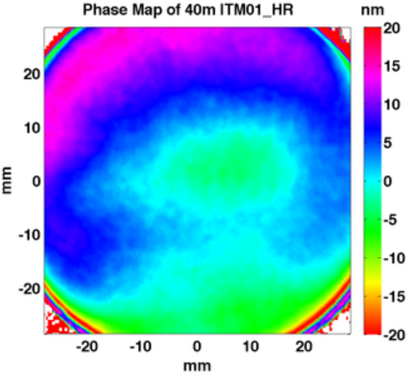


GariLynn Billingsley, *Characterization of Advanced LIGO Core Optics*, LIGO DCC - P1700029

In-situ Measurement of the Mirror Figure Error

Characterize mirror surface defects with phase maps(conventionally via Fizeau interferometry).

But we need in-situ measurement required:
$\longrightarrow$ Use the actual beam used in the interferometer $\rightarrow$ specify the region of the phase map contributing towards losses.
$\longrightarrow$ Quantify the loss using a cavity interferometer with high sensitivity.

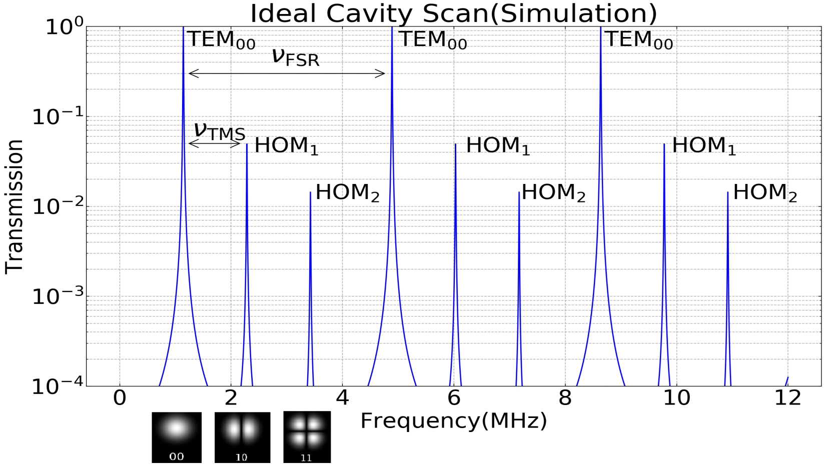


$\Rightarrow$ Mode Spectroscopy

Ideal FP Cavity

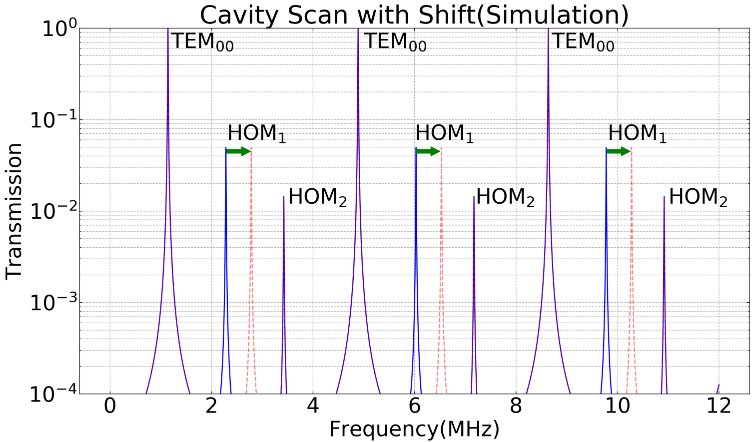
FSR and TMS $\rightarrow$ characterize *ideal* cavity parameters. Notice periodicity.

$$\nu_{FSR} = \frac{c}{2L} \qquad \nu_{TMS} = \frac{\nu_{FSR}}{\pi} \times \arccos(\sqrt{g_1 g_2})$$



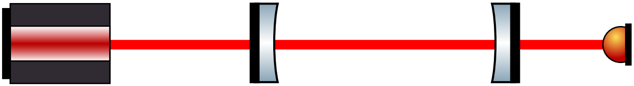
Frequency Shift for HOMs

Real mirrors $\Rightarrow$ Mirror figure error $\longrightarrow$ Shift in HOM resonant frequencies
Analogous to harmonic oscillator: *scattering* of 'energy' in eigenstates when potential has non-harmonic component.



Cavity Scan

Cavity Scan: 'sweeping' the laser freqeuncy for a few MHz.



Scan a cavity to collect the transmission data values

↓

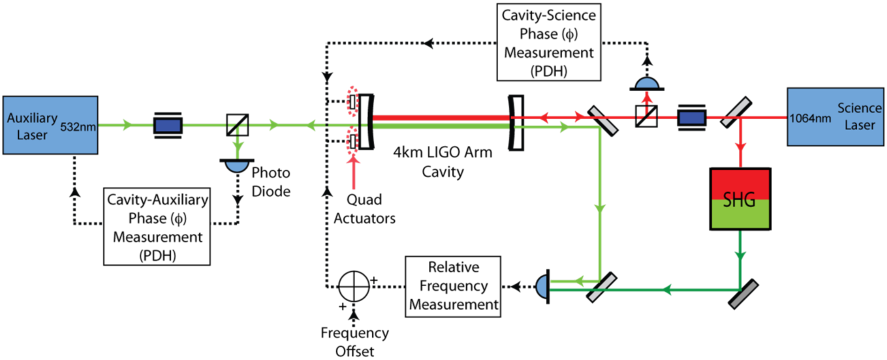
Identify HOMs

↓

Fit data to find shift in frequency from ideal equal spacing

Cavity Scan Setup(ALS)

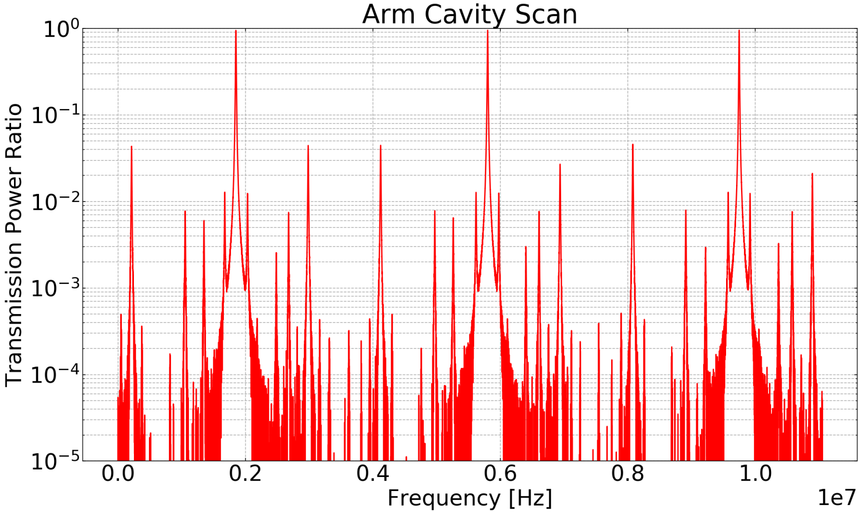
Arm cavity stabilised using *beat* note. Feedback loops to the lasers. Beat frequency swept by slowly varying the stabilized cavity length(ALS [1]).



B. Slagmolen *et al.*, *Advanced LIGO Arm Length Stabilisation System Design*, LIGO Document T0900144, (2010)

Fitting a Cavity Scan

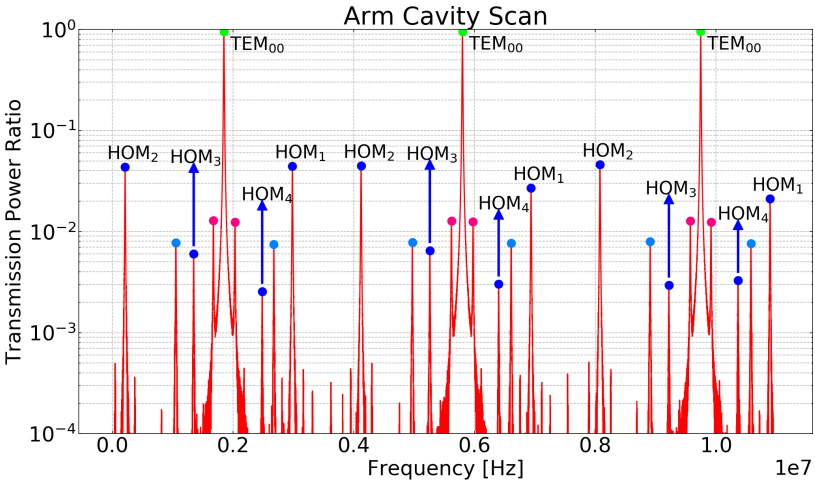
Past cavity scan data.



Fitting a Cavity Scan

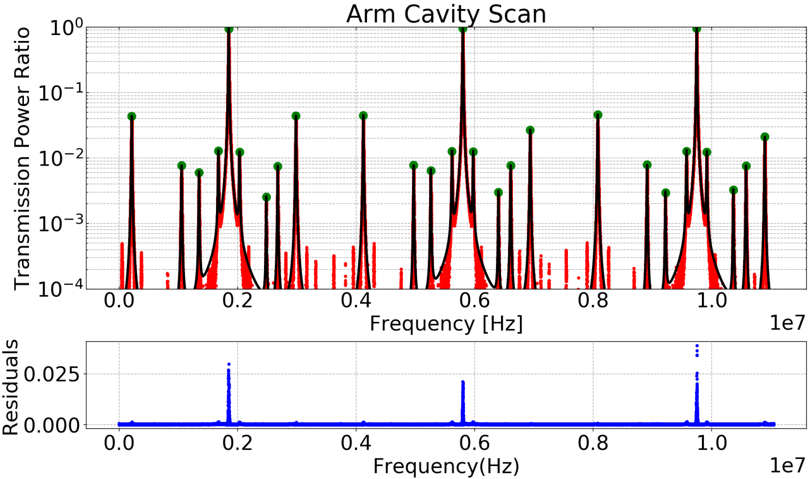
Identify peaks using *'peakutils'* function in python.

- ● → Fundamental Resonances
- ● → 11MHz Resonances
- ● → HOM Resonances
- ● → 55MHz Resonances



Fitting a Cavity Scan

$$T = \frac{a}{1 + (\frac{\nu - \nu_0}{b})^2} \tag{1}$$



Evaluating the Fit

$$a = \left(\frac{t_1 t_2}{1 - r_1 r_2}\right)^2 \qquad b = \frac{\nu_{FSR}}{2\mathcal{F}}$$

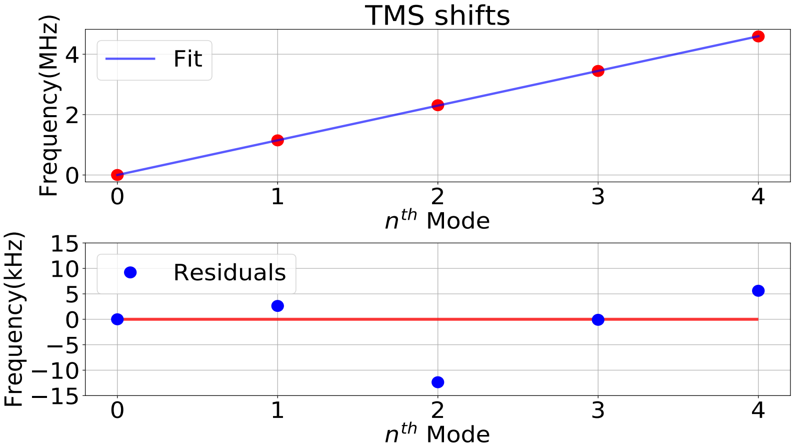
$\nu_0 \longrightarrow$ resonant frequency

Fitting parameters($'a'$,$'b'$ and $'\nu_0'$) $\Longrightarrow$ FSR/cavity length, Finesse and frequency shifts.

$$\text{FSR, } \nu_{FSR} = 3.9703 \pm 0.00022 \text{ MHz}$$
$$\text{Cavity Length, } L = 37.754 \pm 0.00207 \text{ m}$$
$$\text{Finesse, } \mathcal{F} = 402 \pm 21$$

Note: The actual error in FSR frequency will be much higher due to the non-linearity in the delay-line frequency discriminator.

TMS shifts



Note: FSR, TMS defined only for ideal case. Shift taken from average values.

Discussion and Future Prospects

Now we ask ourselves:

1. How accurate our results are?
2. Is it just a statistical error?

What we can do:

$\rightarrow$ Use *'frequency counter'* for accurate measurement.
$\rightarrow$ *'Finesse'* $\rightarrow$ simulate mirror defects iteratively $\rightarrow$ recreate identified frequency shifts(**Monte Carlo** method)

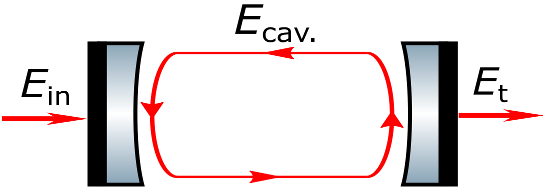
SUMMARY

We discussed the following:

1. LIGO $\rightarrow$ Real mirrors $\rightarrow$ Optical Losses $\rightarrow$ Increase in shot noise.
2. Mirror Figure Error $\rightarrow$ In-situ technique required $\Rightarrow$ Mode Spectroscopy.
3. Figure Error $\rightarrow$ Shift in HOM resonances(akin to harmonic oscillator).
4. Cavity Scan $\rightarrow$ Identify HOMs $\rightarrow$ Fit data and find shifts. Question the authenticity of result $\rightarrow$ Use other tools.

Fabry-Perot Cavity(Extra)

In a simple **FP cavity**, the *'cavity' equation*(Eq. 2) gives us the physical parameters(Eq. 4 - 6)



$$\text{Cavity Equation: } T = \frac{P_t}{P_{in}} = \left|\frac{E_t}{E_{in}}\right|^2 = \left|\frac{t_1 t_2}{1 - r_1 r_2 e^{-\mathbf{i}\phi_{RT}}}\right|^2 \quad (2)$$

The round trip phase change for a TEM_{nm} mode is:

$$\phi_{RT} = \frac{4\pi\nu L}{c} - 2(n + m + 1)\phi_G \quad (3)$$

Fabry-Perot Cavity(Extra)

Using Eq. 2 and 3 the following cavity parameters are what describe and characterize the cavity:

$$\text{Finesse, } \mathcal{F} = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2} \quad (4)$$

$$\text{Free Spectral Range, } \nu_{FSR} = \frac{c}{2L} \quad (5)$$

$$\text{Transverse Mode Spacing, } \nu_{TMS} = \frac{\nu_{FSR}}{\pi} \times \arccos(\sqrt{g_1 g_2}) \quad (6)$$

Also, Eq 2 can be reduced to a Lorentzian(Eq. 7) distribution in relatively *small* frequency intervals around the *'peak'* resonant frequencies(ν_0).

$$T = \frac{a}{1 + (\frac{\nu - \nu_0}{b})^2} \quad (7)$$

Note: Eq. 7, the Lorentzian distribution is what we will be using as our fitting model for the peak resonances we identify in a *'cavity scan'*.