LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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Simulating Beam Jitter Coupling for Advanced LIGO

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1 Overview

1.1 Scope

Beam pointing jitter of the input beam can be a limiting factor for achieving the goal sensitivity in Advanced LIGO. This document describes simulation study which quantitatively assesses such couplings in frequency domain using FINESSE [1]. For now, the simulation has been done for horizontal spatial jitter only (jitter along the interferometer plane).

A similar work had been previously done by G. Mueller in 2009 [2]. The study presented in this document is an update of his work with more realistic interferometer parameters.

Those who are not interested in the details of the simulation should skip section 2.

2 Simulation Setup

The actual simulation codes can be found in LIGO git [3].

2.1 Input optics

Figure 1 shows a schematic layout of the input optics that has been used for this simulation study. The spatial mode of the laser field is virtually mode matched to the interferometer. The laser power is set to the maximum of 125 W.

Two flat mirrors are placed between the laser and PRM in order to inject two quadratures of spatial beam jitter. The first mirror is named *JM* and the other is *IM4*, see the figure. The distance between these two mirrors is adjusted such that the Gouy phase shift between them is exactly at 90 deg. While IM4 in this simulation corresponds to the actual IM4 of Advanced LIGO which is 0.4 meters away from PRM, JM doesn't have a corresponding mirror in reality. JM is merely meant for injecting the other jitter quadrature. We purposely omit the input mode cleaner and other relay mirrors in the simulation in order to simplify the simulation setup and to accommodate JM.

Figure 2 shows the evolution of the beam size and Gouy phase as a function of the travel distance for the field propagating around the jitter injection mirrors. The propagation co-

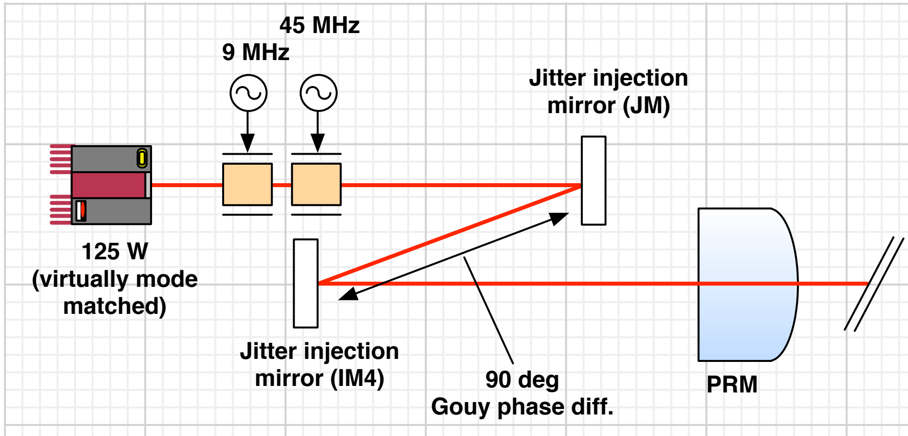

Figure 1: A schematic of the simulation setup for the input light injection.

ordinate is set such that IM4 is located at $z = 0$. The location of JM is indicated by a dashed vertical line in the plots. JM is placed at $z = -7.2$ meters where the Gouy phase with respect to that for IM4 is 90 deg.

After the virtual mode matching is completed, the complex curvature is found to be $q = 5.99 + i3.02$ for the field right before entering the AR surface of PRM propagating from IM4. This means that the beam has already passed the beam waist by 5.99 meters (see figure 2). Going through the substrate and curved HR surface of PRM, the complex curvature becomes $q = 7.17 + i5.21$ at the HR surface, which gives us a radius of curvature of 10.95 m and a beam size of 2.3 mm. These values are consistent with the design values.

2.2 Main interferometer

We use a set of the interferometer parameters (reflectivity, distances and etc.) specific for H1 by loading version 7 of the LHO FINESSE input file [4]. These parameters are based on the galaxy web page [5] and some measurements.

Below, we list some of the important parameters and settings.

- $T_{\mathrm{SRM}} = 37\%$ (SRM-w-14 [5]).
- Broadband RSE
- $P_{\mathrm{in}} = 125$ W.

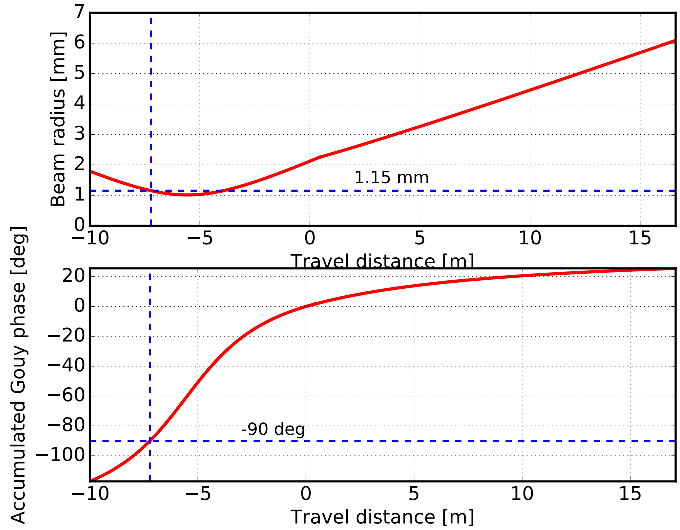

Figure 2: The evolution of the beam size and Gouy phase at around the jitter injection mirrors. IM4 and JM are located at $z = 0$ and $z = -7.2$ m, respectively.

- OMC transmitted light: $P_{\rm omc} = 20$ mW.
- Inferred DARM offset: $L_{\rm x} - L_{\rm y} = 1.3\,$pm.

Note that the DARM offset at LHO during O2 was adjusted to provide a summed DC current of 20 mA from the OMC DCPDs, corresponding to a transmitted light of 25 mW[1].

2.3 Jitter injection

As described in section 2.1, two mirrors are used to inject two quadrature components of spatial jitter in frequency domain by modulating the angle of these mirrors. A small change in the angle of IM4, $\Delta\alpha$, causes a first order field component [7] as

$$E_{10} = -2i\frac{\pi w(z)}{\lambda}E_{00}\Delta\alpha, \tag{1}$$

where $w(z)$ is the size of the beam at IM4, λ is the laser wavelength, and E_{00} is the field amplitude of the fundamental mode or the 00 mode. This excited field comes with an imaginary i, indicating that this field represents an *angular jitter* of the beam at IM4.

[1] assuming a responsivity of 0.8 A/W

In contrast, first order components excited by JM (at location $z = z_{\rm IM}$) turn into *translation jitter* when they arrive at IM4 as

$$
\begin{aligned}
E_{10} &= -2i\frac{\pi w(z_{\rm JM})}{\lambda}E_{00}e^{i\pi/2}\Delta\alpha, \\
&= 2\frac{\pi w(z_{\rm JM})}{\lambda}E_{00}\Delta\alpha,
\end{aligned}
\tag{2}
$$

where we have omitted the propagation phase $2\pi z/\lambda$ because it is irrelevant. As opposed to the angular jitter, this component is a pure real number at IM4.

In the subsequent sections, we often use the normalized amplitude for the first order higher order modes as

$$
A_{10} = \left|\frac{E_{10}}{E_{00}}\right| = \frac{|E_{10}|}{\sqrt{P_{\rm in}}}. \tag{3}
$$

2.4 Other important points

Longitudinal control-. The main interferometer is "locked" in the simulation prior to the jitter coupling simulation. This operation is done by the *lock* function of FINESSE [1]. The lock function, very similarly to the actual experiment, looks at a specific combination of error signals (e.g. REFL9, OMC DC and etc) and servos mirrors' longitudinal position until the residual of the error signals satisfy user-defined precision. Once the lock operation finishes, the control servos are switched off. Therefore the mirrors will numerically stay at the positions provided by the lock operation for the rest of the simulations. This means that the simulation results don't include any servo effects such as noise coupling through frequency servo.

Angular control-. Unlike the longitudinal control, we don't actively control the angle of any mirrors all through the simulation. The only exceptions are (1) angular dither on JM or IM4 for injection of the first order higher order fields and (2) intentional misalignment of an interferometer optic (section 3).

OMC alignment-. Even though we intentionally misalign some of the interferometer mirrors, we leave the OMC alignment untouched all the time. As a consequence, the OMC cavity will keep attenuating the first order higher order modes that are generated by the misaligned interferometer optics and/or the jitter injection mirrors. So the OMC DCPDs

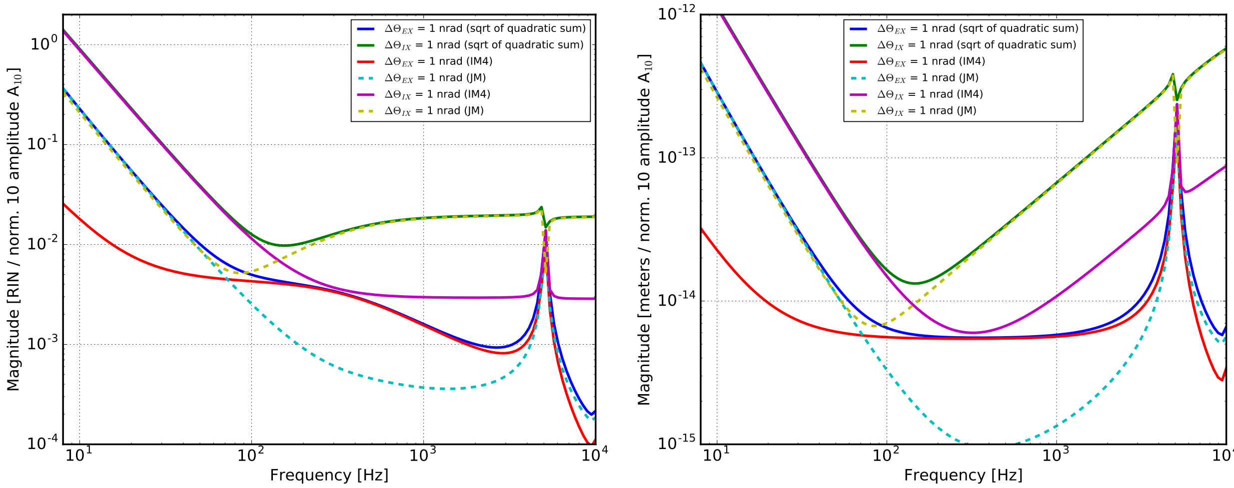

Figure 3: (Left) Beam jitter couplings to OMC RIN. (Right) Beam jitter couplings to DARM displacement.

are mostly sensitive to the fundamental mode which is contaminated by pointing jitter noise via scattering matrices [2].

3 Jitter coupling

As described in [6], input jitter couples to gravitational wave channels when the interferometer is misaligned from the optimum alignment. For this reason, we simulate the couplings by intentionally misaligning one interferometer optic at a time. We misalign PRM, SRM, ITMs and ETMs. Each test mass is horizontally tilted by 1×10^{-9}rad. On the other hand, PRM and SRM are tilted by a larger amount of 30×10^{-9}rad in order to compensate for their smaller beam sizes ($w_{\mathrm{PRM}} \sim w_{\mathrm{SRM}} \sim 2\,\mathrm{mm}$ whereas $w_{\mathrm{TM}} \sim 50\,\mathrm{mm}$).

As described in [2], the highest couplings are obtained when test masses are misaligned. For the reason, we are not going to show the results for the ones with PRM or SRM misaligned in this section. A complete set of plots including those with PRM and SRM misaligned can be found in section A.

3.1 Simulation with tilted test masses

Figure 3 shows the results of angular and translation jitter couplings obtained by modulating the angle of either IM4 or JM. Since there were little difference between the results with misaligned ETMX(ITMX) and those with misaligned ETMY(ITMY), we show the results with either ETMX or ITMX misaligned. All the coupling transfer functions show higher coupling at low frequencies presumably due to photon radiation pressure effects.

As shown in the figure, the highest coupling at 200 Hz is given by translation jitter introduced by JM with an ITM misaligned. Using the quadratic sum of angular and translation jitter couplings as a worst estimate, we can write down the couplings as

$$L_{-}\Big|_{f=200\,\mathrm{Hz}} = 1.5 \times 10^{-14} \times \left(\frac{\Delta\Theta_{\mathrm{IX}}}{10^{-9}\,\mathrm{rad}} \right) \times (A_{10}) \text{ meters,} \tag{4}$$

$$L_{-}\Big|_{f=200\,\mathrm{Hz}} = 5.6 \times 10^{-15} \times \left(\frac{\Delta\Theta_{\mathrm{EX}}}{10^{-9}\,\mathrm{rad}} \right) . \times (A_{10}) \text{ meters} \tag{5}$$

Also, as described in [2, 6], differential misalignment of the test masses is the predominant coupling path. This is also confirmed in out simulation and the results are shown in appendix A.3.

3.2 Notes on unit conversion

If one wants to convert OMC RIN into raw OMC trans in watts, the transfer functions shown in the left panel of figure 3 should be read with an extra multiplication factor of $P_{\mathrm{omc}} = 20 \times 10^{-3}$ W. Similarly, multiplying an extra factor of $1/\sqrt{P_{\mathrm{in}}} = 0.09$ to a transfer function is equivalent to converting its denominator from A_{10} to the raw amplitude of the excited first order higher order field or E_{10}.

4 Conclusions

We have simulated beam pointing jitter couplings with a set of realistic interferometer parameters. Differential ITM misalignment gives the largest coupling at almost all frequencies. The coupling coefficient at 200 Hz was estimated to be 1.5×10^{-5} meters per misalignment of an ITM in radians per normalized first order higher-order-mode amplitude.

A Other plots

A.1 Angular jitter

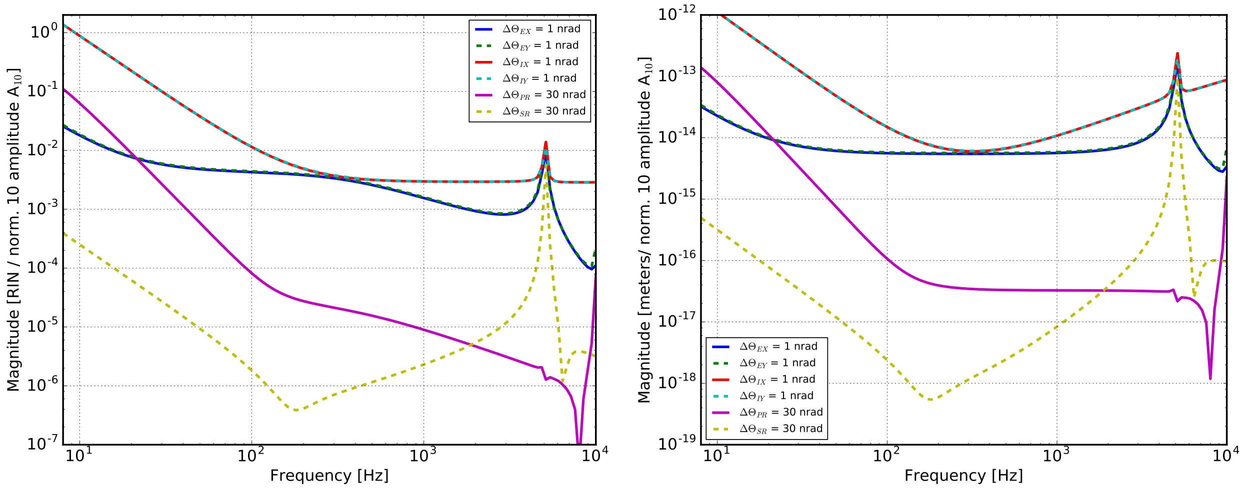

Figure 4: (Left) Angular jitter coupling into OMC RIN. (Right) Angular jitter coupling to DARM displacement. They are obtained by shaking the angle of IM4.

A.2 Translation jitter

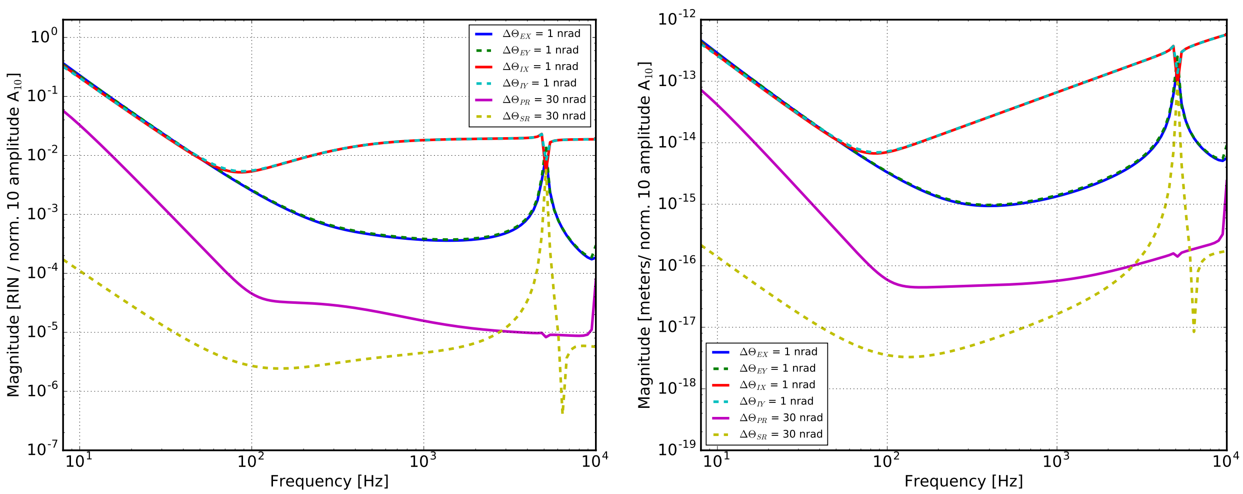

Figure 5: (Left) Translation jitter coupling into OMC RIN. (Right) Translation jitter coupling to DARM displacement. They are obtained by shaking the angle of JM.

A.3 Differential/Common basis

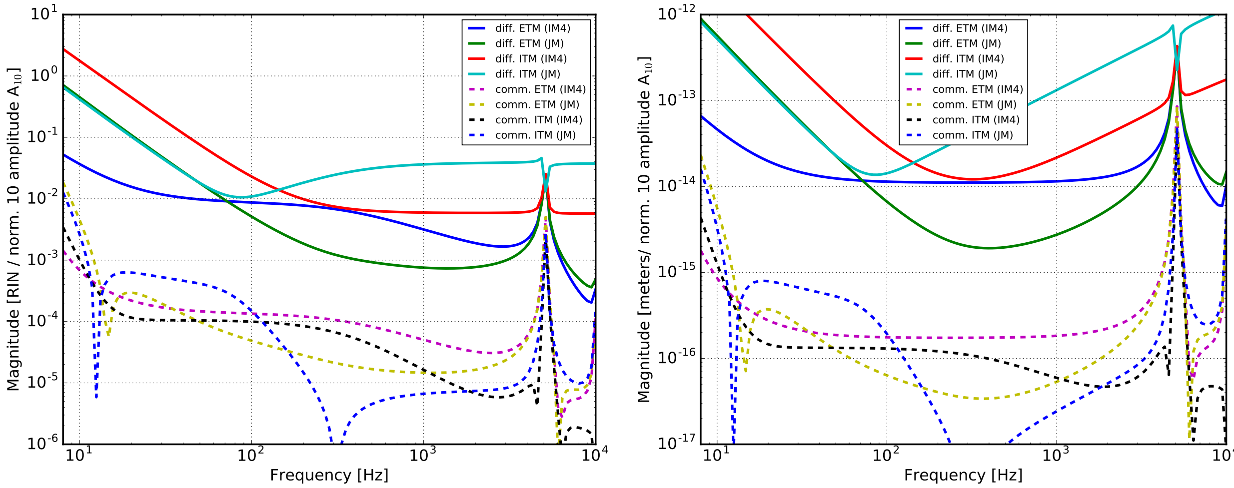

Figure 6: (Left)Jitter couplings into OMC RIN in the differential/common basis. (Right) Jitter couplings to DARM displacement in the differential/common basis.

B Code verification, back of envelop and etc

To be filled.

C Comparison with previous requirement

In order to directly compare our results with the previous one [2], we derive new requirement curves for input pointing jitter in the same fashion.

We use the same target sensitivity as the previous one,

$$h = 3 \times 10^{-24} \sqrt{1 + \left(\frac{40\,\mathrm{Hz}}{f}\right)^4} \frac{1}{\sqrt{\mathrm{Hz}}}. \tag{6}$$

Using our simulated coupling transfer functions T [meters/normalized amplitude of 10 mode] which are shown in the right panel of figure 6, we compute the requirement as

$$A_{10}^{(\mathrm{requirement})} = \frac{1}{10} \times \frac{hL_{\mathrm{arm}}}{T}, \tag{7}$$

where L_{arm} is the arm length and set to 4 km. An extra factor of 10 upfront is the safety factor as was done for the derivation of the previous requirement.

The results are shown in figure 7.

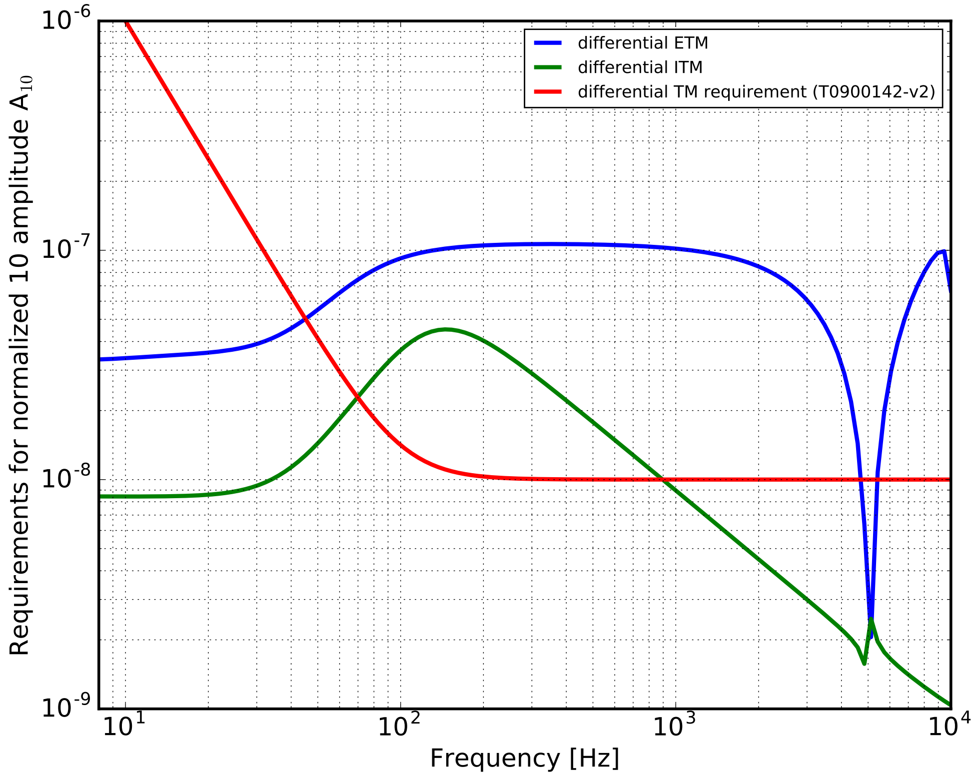

Figure 7: Requirements for A_{10}

References

[1] http://www.gwoptics.org/finesse/

[2] G. Mueller, “Pointing Requirements for Advanced LIGO,” LIGO-T090142-v2 (2009) https://dcc.ligo.org/LIGO-T0900142-v2

[3] Codes can be found in git.ligo.org https://git.ligo.org/IFOsim/Finesse_playground/tree/master/kiwamu/JitterCoup

[4] C. Bond, et.al., “Finesse input files for the H1 interferometer,” LIGO-T1300904-v7 (2015) https://dcc.ligo.org/LIGO-T1300904-v7

[5] Galaxy web page https://galaxy.ligo.caltech.edu/optics/

[6] P. Fritschel, “Misalignment-Beam Jitter Coupling in LIGO,” LIGO-T960-120-00-D (1996) https://dcc.ligo.org/LIGO-T960120

[7] T. Hefetz, et.al., “Principles of calculating alignment signals in complex resonant optical interferometers.” J. Opt. Sci. Am. B, 14(7), 15971606 http://doi.org/10.1364/JOSAB.14.001597