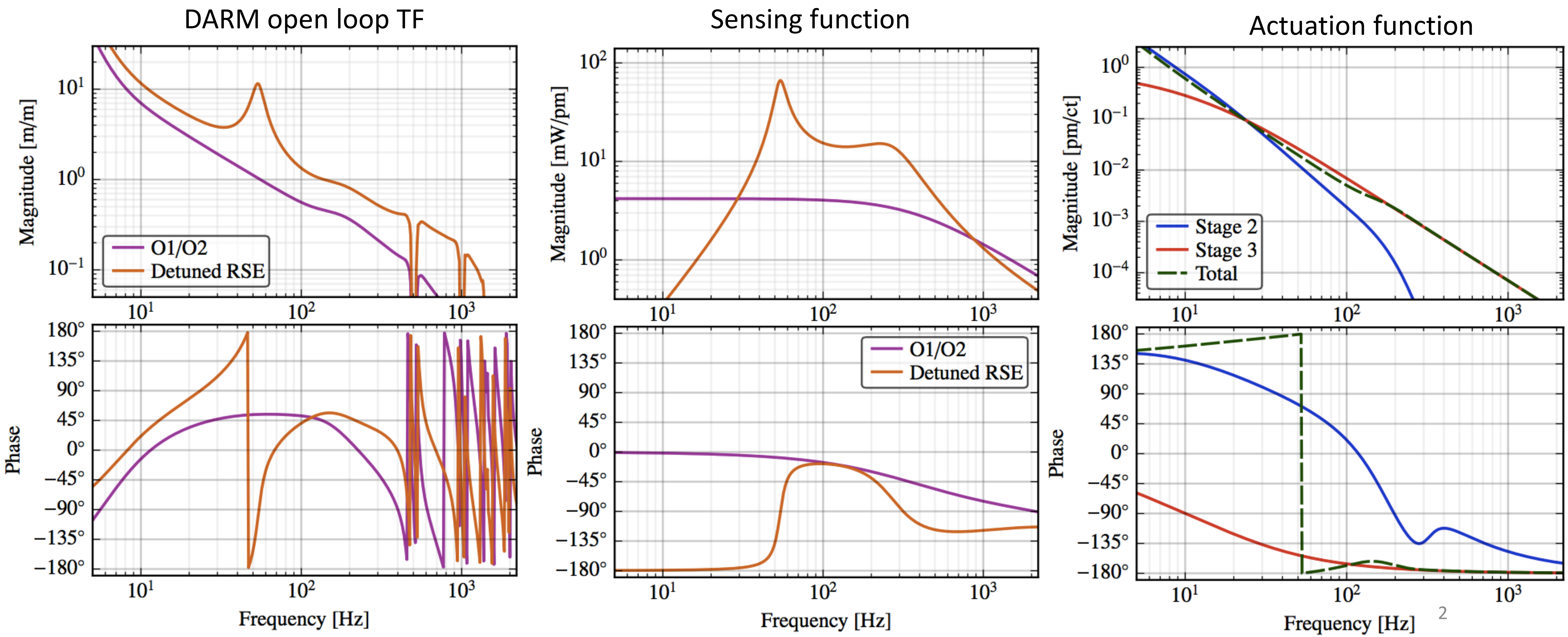
Paper summary

- Question: what should the requirements be on systematic calibration errors? (optical gain, DARM pole, etc.)
- Answer: they should induce negligible* systematic error in the astrophysical parameter estimation (PE) of a CBC signal. (masses, distance, etc.)
- This paper uses some calculus to form a **semi-analytical approach to setting calibration requirements for a signal like GW150914**
- Similar to semi-analytical approaches of Cutler and Vallisneri for systematic waveform errors

*Quantified later

What is the calibration model? [1/2]

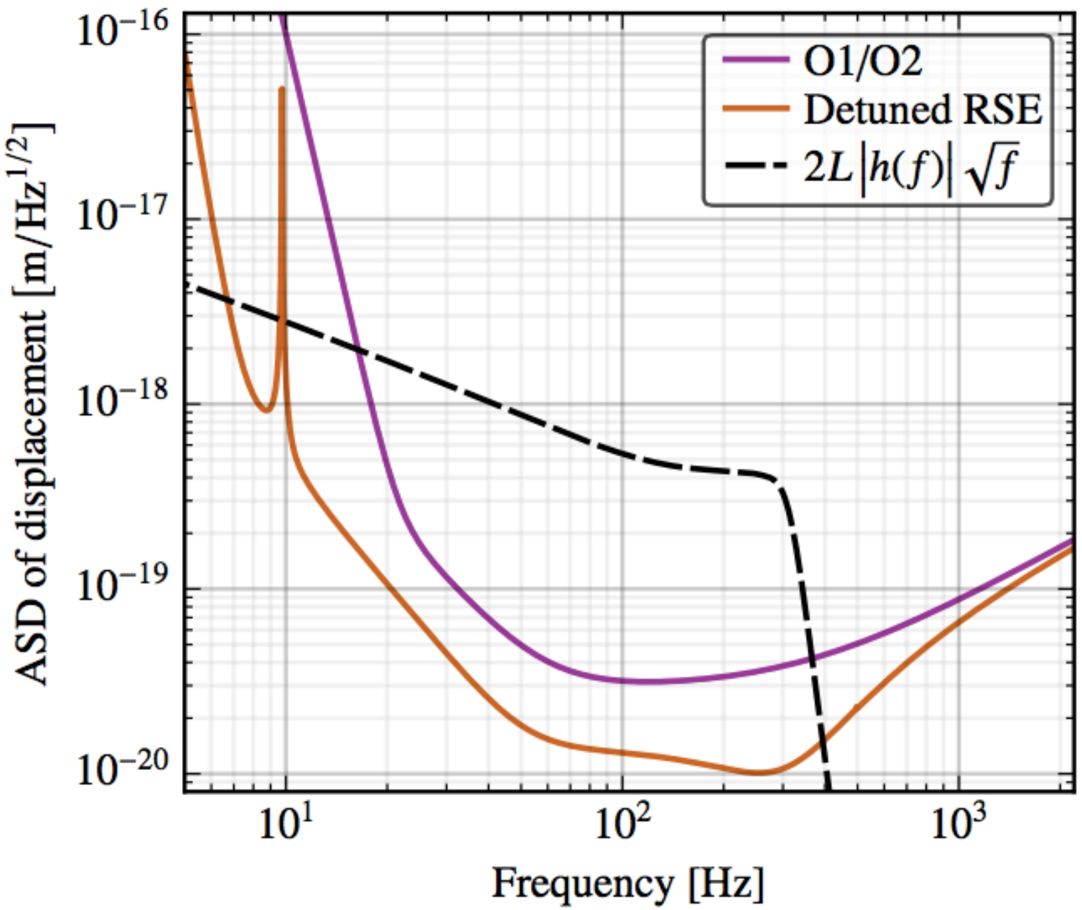


What is the calibration model? [2/2]

$$C(f) = \frac{g\,\mathrm{e}^{-2\pi \mathrm{i} f L/c} \times (1 + \mathrm{i}f/z)}{1 + \mathrm{i}f/|p|Q_p - f^2/|p|^2 - \xi^2/f^2},$$

Quantity	O1/O2	Detuned RSE	Unit
g	4.2	9.8	mW/pm
z	365	749	Hz
$\|p\|$	365	276	Hz
Q_p	0.50	1.33	—
ξ^2	0.0	$+53.4^2$	Hz^2
a	0.11	0.11	μN/V

What is the noise model? and signal model?



Using IMRPhenomD with spins set to zero

Masses and effective distance similar to GW150914

We also consider an analysis with a massive graviton

CBC parameter estimation in one slide

PE basically* boils down to maximizing the following log-likelihood:

$$\ell \equiv \ln \mathcal{L} = -\frac{1}{2} \int_0^\infty \mathrm{d}f \, \frac{\left| d(f) - h(f;\boldsymbol{\theta}) \right|^2}{S_{nn}(f)}, \qquad (12)$$

where $d(f)$ is the frequency-domain detector data, $h(f;\boldsymbol{\theta})$ is the frequency-domain waveform model (which is a function of the parameters $\boldsymbol{\theta}$), and $S_{nn}(f)$ is the power spectral density (PSD)

* We assume flat priors.

The effect of calibration systematics on PE

After some calculus, we arrive at the following:

$$-\underbrace{\begin{bmatrix} \ell_{\theta_1\theta_1} & \ell_{\theta_1\theta_2} & \cdots & \ell_{\theta_1\theta_M} \\ \ell_{\theta_2\theta_1} & \ell_{\theta_2\theta_2} & \cdots & \ell_{\theta_2\theta_M} \\ \vdots & \vdots & \ddots & \vdots \\ \ell_{\theta_M\theta_1} & \ell_{\theta_M\theta_2} & \cdots & \ell_{\theta_M\theta_M} \end{bmatrix}}_{\equiv\mathcal{H}} \begin{bmatrix} \Delta\hat{\theta}_1 \\ \Delta\hat{\theta}_2 \\ \vdots \\ \Delta\hat{\theta}_M \end{bmatrix} = \underbrace{\begin{bmatrix} \ell_{\theta_1\lambda_1} & \ell_{\theta_1\lambda_2} & \cdots & \ell_{\theta_1\lambda_N} \\ \ell_{\theta_2\lambda_1} & \ell_{\theta_2\lambda_2} & \cdots & \ell_{\theta_2\lambda_N} \\ \vdots & \vdots & \ddots & \vdots \\ \ell_{\theta_M\lambda_1} & \ell_{\theta_M\lambda_2} & \cdots & \ell_{\theta_M\lambda_N} \end{bmatrix}}_{\equiv\mathcal{M}} \begin{bmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \\ \vdots \\ \Delta\lambda_N \end{bmatrix}$$

Systematic PE error

Systematic calibration error

Results

- E.g., for an O1/O2 configuration, and a massless graviton:

Applying Eq. 26 to this scenario yields the following relationship between $\Delta\hat{\boldsymbol{\theta}}$ and $\Delta\boldsymbol{\lambda}$:

$$\begin{bmatrix} \Delta\mathcal{M}/\mathcal{M} \\ \Delta\eta/\eta \\ \Delta t_c/(1\,\mathrm{ms}) \\ \Delta\phi_c/(1\,\mathrm{rad}) \\ \Delta D/D \end{bmatrix} = \frac{1}{10^3}\times\begin{bmatrix} -12 & -21 & 1 \\ -93 & -80 & -181 \\ -1745 & -121 & -2827 \\ -5058 & -10497 & 7770 \\ 736 & -90 & -537 \end{bmatrix}\begin{bmatrix} \Delta g/g \\ \Delta|p|/|p| \\ \Delta a/a \end{bmatrix} \tag{28}$$

“Negligible” systematic calibration error?

Sensing function

- **If the systematic calibration error is made small enough, the astrophysical parameter estimate will instead be dominated by systematic error from detector noise**
- Given a few assumptions (Gaussian noise, loud SNR), the distribution of detector noise systematic errors can be computed from the Fisher matrix of the signal:

$$\Gamma_{ij} = \left\langle \frac{\partial h}{\partial \theta_i} \middle| \frac{\partial h}{\partial \theta_j} \right\rangle$$

- Invert the Fisher matrix to get a bound on the covariance matrix:

$$\Sigma_{ij} \geq (\boldsymbol{\Gamma}^{-1})_{ij}.$$

Results

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$$\begin{aligned} \Sigma^{1/2}_{\mathcal{M}\mathcal{M}}/\mathcal{M} &= 0.015 \\ \Sigma^{1/2}_{\eta\eta}/\eta &= 0.051 \\ \Sigma^{1/2}_{t_c t_c} &= 0.45\,\mathrm{ms} \\ \Sigma^{1/2}_{\phi_c\phi_c} &= 8.3\,\mathrm{rad} \\ \Sigma^{1/2}_{DD}/D &= 0.054. \end{aligned}$$

Calibration results summary

TABLE II: Requirements on systematic calibration errors. These requirements are set such that the systematic calibration errors induce an error in the astrophysical parameter estimation that is less than one tenth of the error introduced by detector noise (as determined from the Cramér–Rao bound as described in the text).

Quantity	**O1/O2, MG**	**no O1/O2, MG**	**Detuned RSE, no MG**	**Detuned RSE, MG**
$\Delta g/g$	0.7%	0.8%	1.2%	1.2%
$\Delta z/z$	—	—	2%	2%
$\Delta\lvert p\rvert/\lvert p\rvert$	6%	5%	0.3%	0.3%
$\Delta Q_p/Q_p$	—	—	1.3%	0.9%
$\Delta\xi^2/\xi^2$	—	—	0.9%	1.0%
$\Delta a/a$	1.0%	0.9%	0.17%	0.17%

Possible future work

- Network of multiple detectors
- Consideration of spin
- Numerical analysis using actual PE codes