Measuring Kerrness in Binary Black Hole Simulation Ringdowns

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Acknowledgments

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Kerr

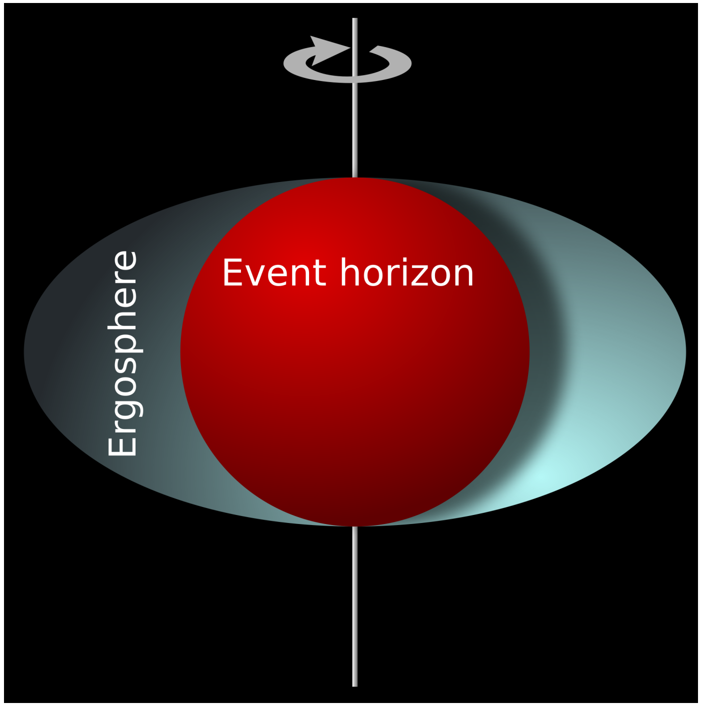
- Kerr is the spacetime of a single black hole which is

  Axially symmetric

  Uncharged

  Asymptotically flat

  Spinning



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“Kerrness”

- **Local** scalar representing closeness to Kerr

  Local quantity: At a given (4D) point in spacetime, we may compute the similarity to Kerr

- These quantities are **invariant of choice of coordinates**

  Important for implementing in a numerical code

Why measure Kerrness?

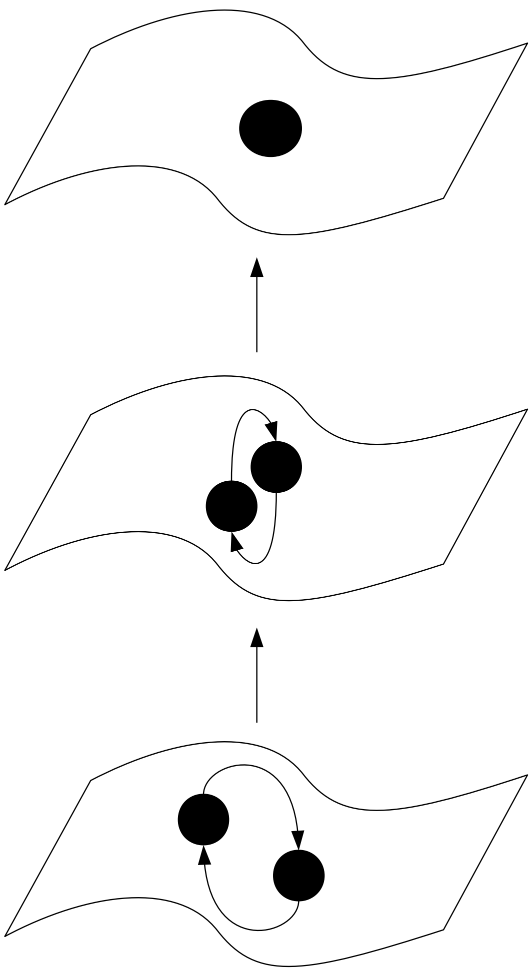
- Analyses of binary black hole mergers assume a Kerr remnant (or a perturbation) [4, 5]
- LIGO data analyses may assume that the resulting spacetime is Kerr (or a perturbation) [4, 5]
- Measures of Kerrness applied to binary black hole merger simulations may help to **quantify** when it is valid to make these assumptions

SpEC

- The simulations were run using the **Spectral Einstein Code (SpEC)**

    Spectral methods: compute coefficients of basis functions

- Codebase in **C++** (with Perl for parsing input files)
- Simulations consist of **slices** (three dimensional spacelike hypersurfaces) evolved in time
- **3 + 1** formalism



Speciality Index [1]

$$\mathcal{S} = \frac{27J^2}{I^3}, \quad I = \tilde{C}_{abcd}\tilde{C}^{abcd}, \quad J = \tilde{C}_{abcd}\tilde{C}^{cd}{}_{mn}\tilde{C}^{mnab} \tag{1}$$

$$\tilde{C}_{abcd} = C_{abcd} + (i/2)\epsilon_{abmn}C^{mn}{}_{cd} \tag{2}$$

- Computed from contractions of the self-dual **Weyl Tensor**
- **Complex** quantity
- Re[S] → 1, Im[S] → 0
- **Necessary** but **not sufficient** condition

  This quantity is 1 for any algebraically special spacetime

  Kerr ⊂ Algebraically special [1]

García-Parrado 2015 [2]

$$\mathcal{L} \equiv F_1 + F_2 + F_3 + F_4 + F_5 + F_6 \quad (1)$$

- F_1 – F_6 are expressions involving contractions and covariant derivatives
- **Directly** measures similarity to Kerr
- **Real, non-negative** quantity which vanishes for Kerr spacetime

  Each term **independently** vanishes

  It is possible to consider each term independently of the others during debugging and analysis

García-Parrado 2015

$$\mathcal{L} \equiv \frac{(\mathfrak{r}(A)+\mathfrak{r}(B))^2 + (\mathfrak{j}(A)_i+\mathfrak{j}(B)_i)(\mathfrak{j}(A)^i+\mathfrak{j}(B)^i) + (\mathfrak{t}(A)_{ij}+\mathfrak{t}(B)_{ij})(\mathfrak{t}(A)^{ij}+\mathfrak{t}(B)^{ij})}{\sigma^{14}} +$$

$$\frac{\mathfrak{a}_{ij}\mathfrak{a}^{ij}+\mathfrak{b}_{ij}\mathfrak{b}^{ij}}{\sigma^4} + \frac{\big((1-3\lambda^2)\beta+\lambda(3-\lambda^2)\alpha\big)^2}{\sigma^2} + \frac{(\mathfrak{B}_{ij}\mathfrak{B}^{ij})^3}{\sigma^4} + \frac{(\mathfrak{C}_{ij}\mathfrak{C}^{ij})^3}{\sigma^7} + \frac{\Omega}{\sigma^2}\ , \tag{1}$$

$$\mathfrak{r}(A) \equiv A_{\nu\mu}n^\nu n^\mu = \mathcal{E}(Q)_{\mu\nu}D^\mu A D^\nu A\ ,$$

$$\mathfrak{j}(A)_\mu \equiv A_{\nu\rho}n^\nu h^\rho{}_\mu = -\varepsilon_{\mu\kappa\pi}\mathcal{B}(Q)_\lambda{}^\pi D^\lambda A D^\kappa A - \mathcal{E}(Q)_{\mu\lambda}\mathcal{A}(A)D^\lambda A,$$

$$\mathfrak{t}(A)_{\mu\nu} \equiv A_{\kappa\pi}h^\kappa{}_\mu h^\pi{}_\nu =$$

$$D^\kappa A(-2D_{(\nu}A\mathcal{E}(Q)_{\mu)\kappa} + h_{\mu\nu}\mathcal{E}(Q)_{\kappa\pi}D^\pi A + 2\mathcal{A}(A)\mathcal{B}(Q)_{(\mu}{}^\pi\varepsilon_{\nu)\kappa\pi}) +$$

$$+\mathcal{E}(Q)_{\mu\nu}(D_\kappa A D^\kappa A + (\mathcal{A}(A))^2).$$

$$\mathfrak{a}_{\mu\nu} \equiv -B_\mu{}^\lambda B_{\nu\lambda} + E_\mu{}^\lambda E_{\nu\lambda} - \frac{1}{3}Ah_{\mu\nu} - E_{\mu\nu}\alpha - B_{\mu\nu}\beta = 0\ ,$$

$$\mathfrak{b}_{\mu\nu} \equiv B_\mu{}^\lambda E_{\nu\lambda} + B_\nu{}^\lambda E_{\mu\lambda} - \frac{1}{3}Bh_{\mu\nu} - B_{\mu\nu}\alpha + E_{\mu\nu}\beta = 0.$$

$$A \equiv \frac{1}{8}C^{\mu\nu\lambda\rho}C_{\mu\nu\lambda\rho},$$

$$B \equiv \frac{1}{8}C^{\mu\nu\lambda\rho}C^*_{\mu\nu\lambda\rho},$$

$$D \equiv \frac{1}{16}C\mu\nu\lambda\rho C\sigma\pi\mu\nu C\lambda\rho\sigma\pi,$$

$$E \equiv \frac{1}{16}C\mu\nu\lambda\rho C\sigma\pi\mu\nu C{*}\lambda\rho\sigma\pi.$$

$$T \equiv \frac{\beta}{\alpha} = \frac{BD-AE}{AD+BE}\ ,$$

$$\lambda \equiv \frac{K(KT+1)}{K^2-3KT-2}\ ,$$

$$K = \frac{2R^\parallel\Theta^\parallel - 2R^{\perp\mu}\Theta^\perp_\mu}{R^{\parallel 2} - \Theta^{\parallel 2} - R^\perp_\mu R^{\perp\mu} + \Theta^\perp_\mu\Theta^{\perp\mu}}\ ,$$

$$\sigma = \tfrac{1}{4}(-2+K^2-3KT)^2 \times$$

$$\left(\frac{R^{\parallel 2} - \Theta^{\parallel 2} - R^\perp_\mu R^{\perp\mu} + \Theta^\perp_\mu\Theta^{\perp\mu}}{K^2(1+KT)^2 - (K^2-3KT-2)^2} - \frac{8\alpha}{(K^2-3KT-2)^2 - 3K^2(1+KT}\right.$$

$$\alpha = -\frac{AD+BE}{A^2+B^2}\ ,$$

$$\beta = \frac{AE-BD}{A^2+B^2}.$$

García-Parrado 2015 [2]

$$\mathcal{L} \equiv F_1 + F_2 + F_3 + F_4 + F_5 + F_6 \quad (1)$$

- Can be computed on an individual slice

  The computations involving the pulled-back tensors **do not involve time derivatives**

  Can be computed using **3D quantities** implemented in SpEC

García-Parrado 2015 (continued)

$$\mathcal{L} \equiv F_1 + F_2 + F_3 + F_4 + F_5 + F_6 \quad (1)$$

- The first three terms have been implemented and evaluated on single and binary black hole simulations

  Fourth and fifth terms: Equivalent terms from the 2016 paper [3] have been implemented

Simulations

- New code was written for the García-Parrado 2015 quantity
- Single black hole simulations

  Kerr black hole with mass 1, spin vector (0., 0., 0.4)

  Sanity check that the quantities behave

  Allows checking the **convergence** of quantities with respect to resolution

- Binary black hole ringdown

  The quantities were computed on the ringdown phase of a simulation of the GW150914 event at a single resolution (obtained from California State University, Fullerton)

Evaluation of Kerrness Quantities

Analysis of Convergence

- "Error" measured by L2 norm of deviation from theoretical values (Re[S] = 1, Im[S] = 0)

$$\sqrt{\sum_{i}^{N} (x_i - x)^2 / N}$$

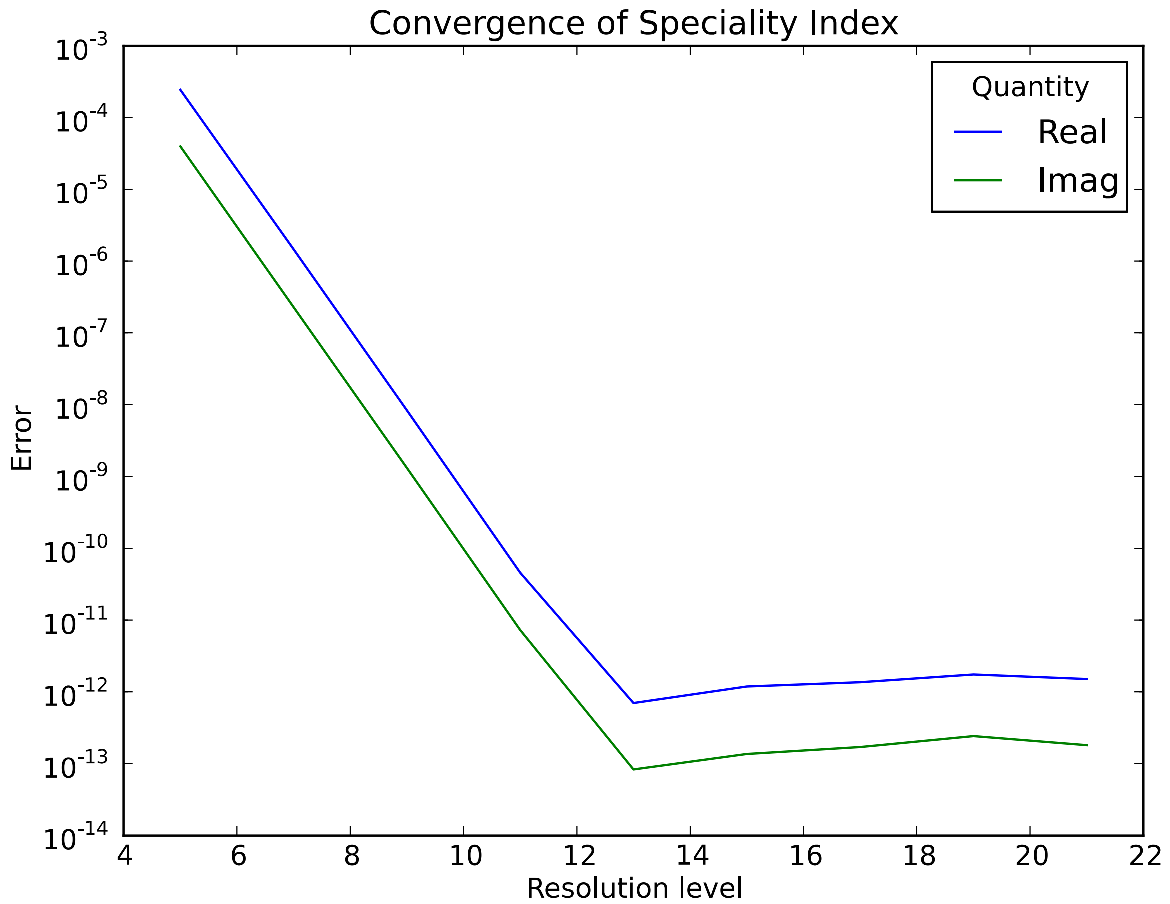
- "Resolution" refers to simulation angular resolution

  Spherical harmonics: Y_{lm}

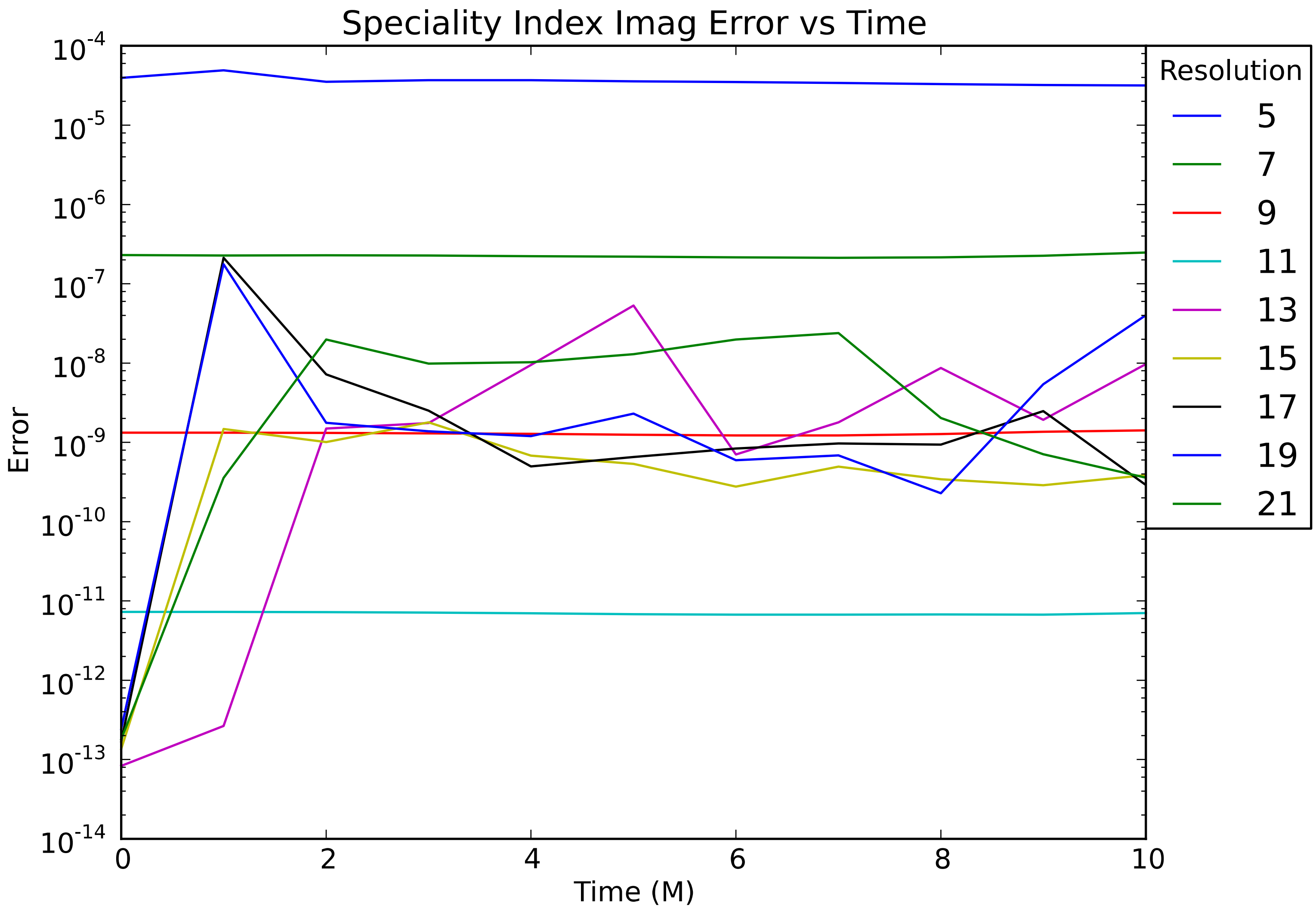
Analysis of Convergence

- The error was computed for points on a spherical shell located 12M from the origin.
- Other spheres (other than the innermost) exhibit similar convergence patterns.

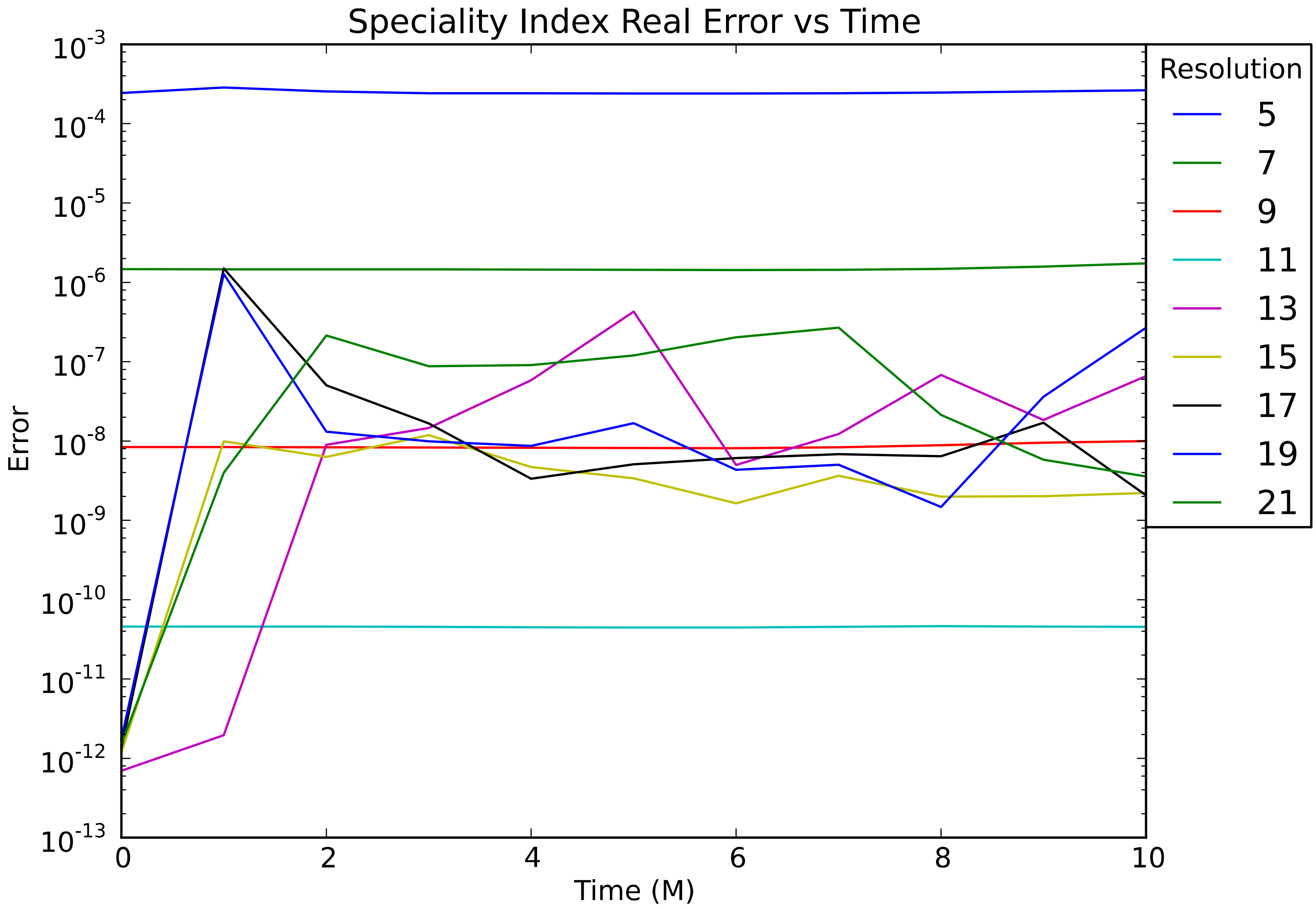
Speciality Index: Single Black Hole



Speciality Index: Single Black Hole



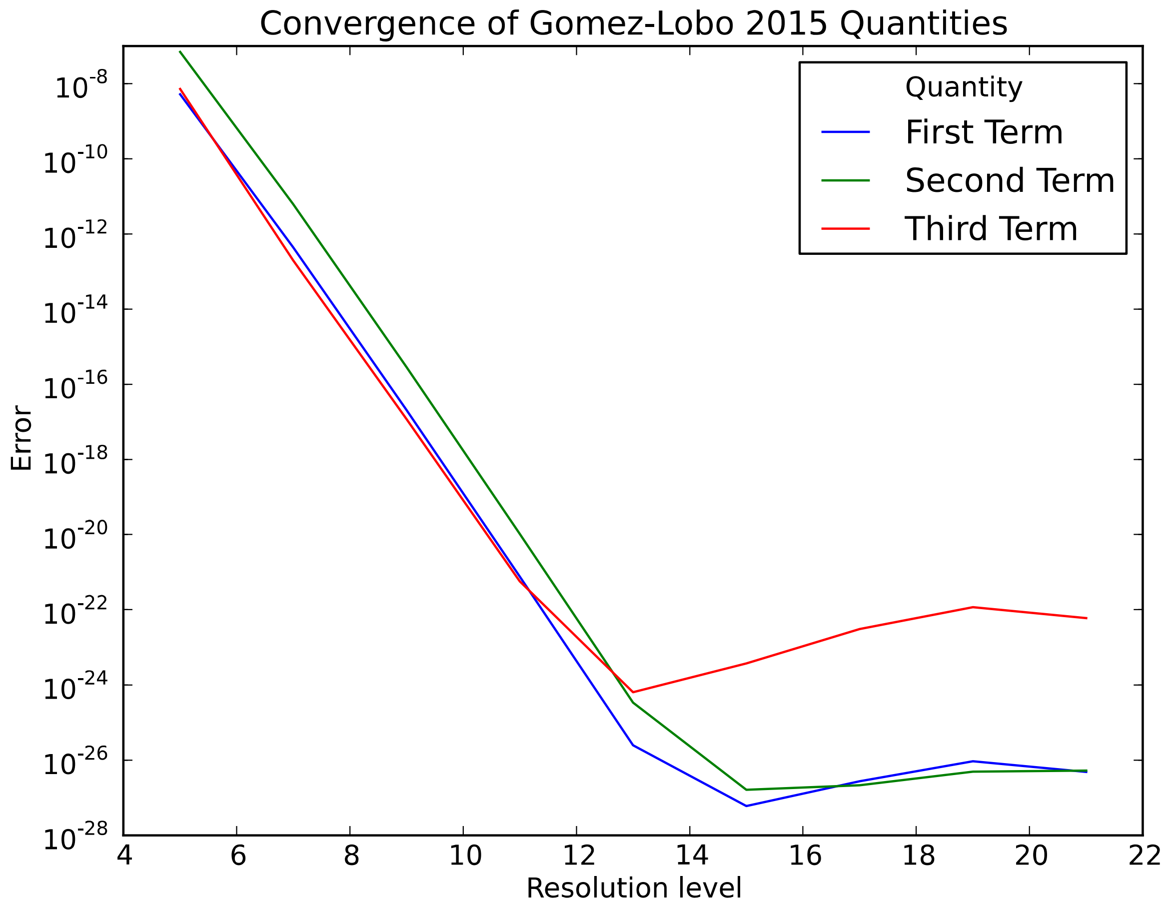
Speciality Index: Single Black Hole



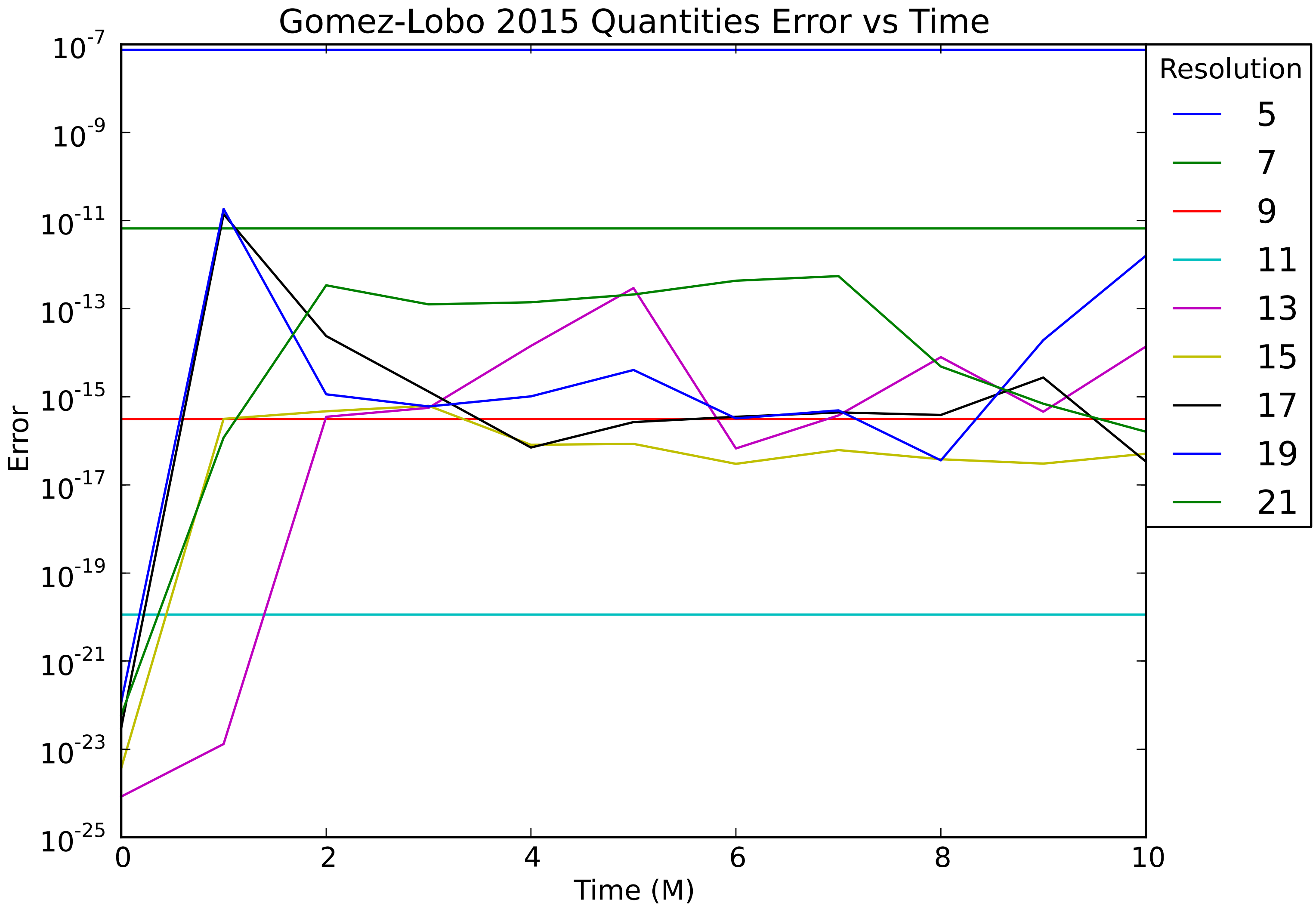
Speciality Index: Binary Black Hole

- Qualitatively, Re[S] $\rightarrow$ 1 and Im[S] $\rightarrow$ 0 as the ringdown progresses
- Computations of the quantities at time resolutions to generate plots are ongoing

García-Parrado 2015: Single Black Hole



García-Parrado 2015: Single Black Hole



García-Parrado 2015: Binary Black Hole

- Qualitatively, $F_{1\text{-}3}[S] \rightarrow 0$ as the ringdown progresses
- Computations of the quantities at time resolutions to generate plots are ongoing

Further Work

- Short term:

  Finish implementation of remaining terms and compare with corresponding terms from the 2016 paper [3]

- Long term:

  Apply Kerrness measures to quantify similarity to Kerr during ringdowns

References

[1] J. Baker and M. Campanelli. Making use of geometrical invariants in black hole collisions. *Physical Review D*, 62(12):127501, December 2000.

[2] A. García-Parrado Gómez-Lobo. Local non-negative initial data scalar characterization of the Kerr solution. *Physical Review D*, 92(12):124053, December 2015.

[3] A. García-Parrado Gómez-Lobo. Vacuum type d initial data. *ArXiv e-prints*, February 2016.

[4] B. P. Abbott et al. Properties of the binary black hole merger GW150914. 2016.

[5] B. P. Abbott et al. Tests of general relativity with GW150914. 2016.