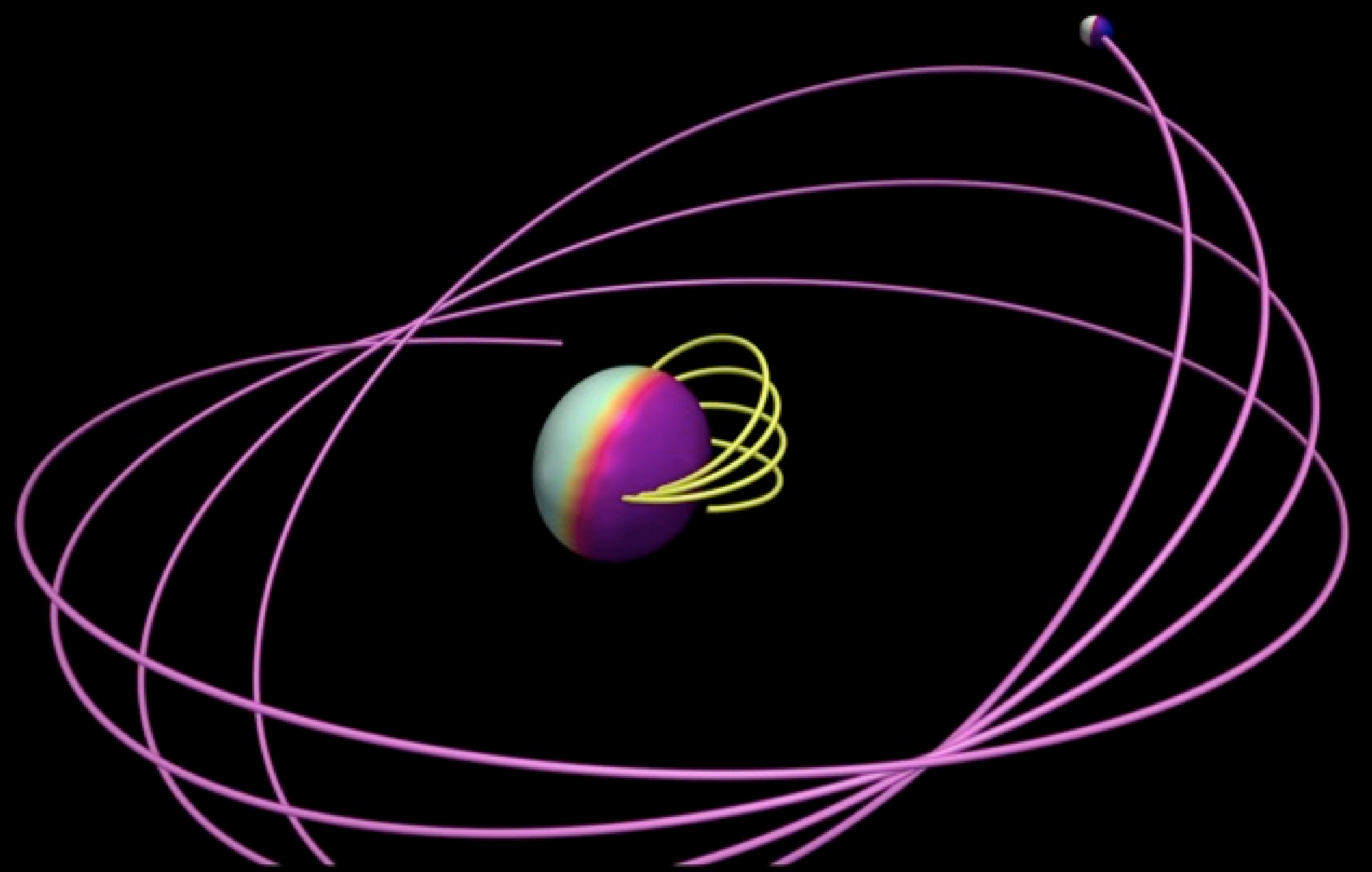
The Effect Of Orbital Eccentricity In Binary Black Hole Simulations On Gravitational Waveforms



Halston Lim

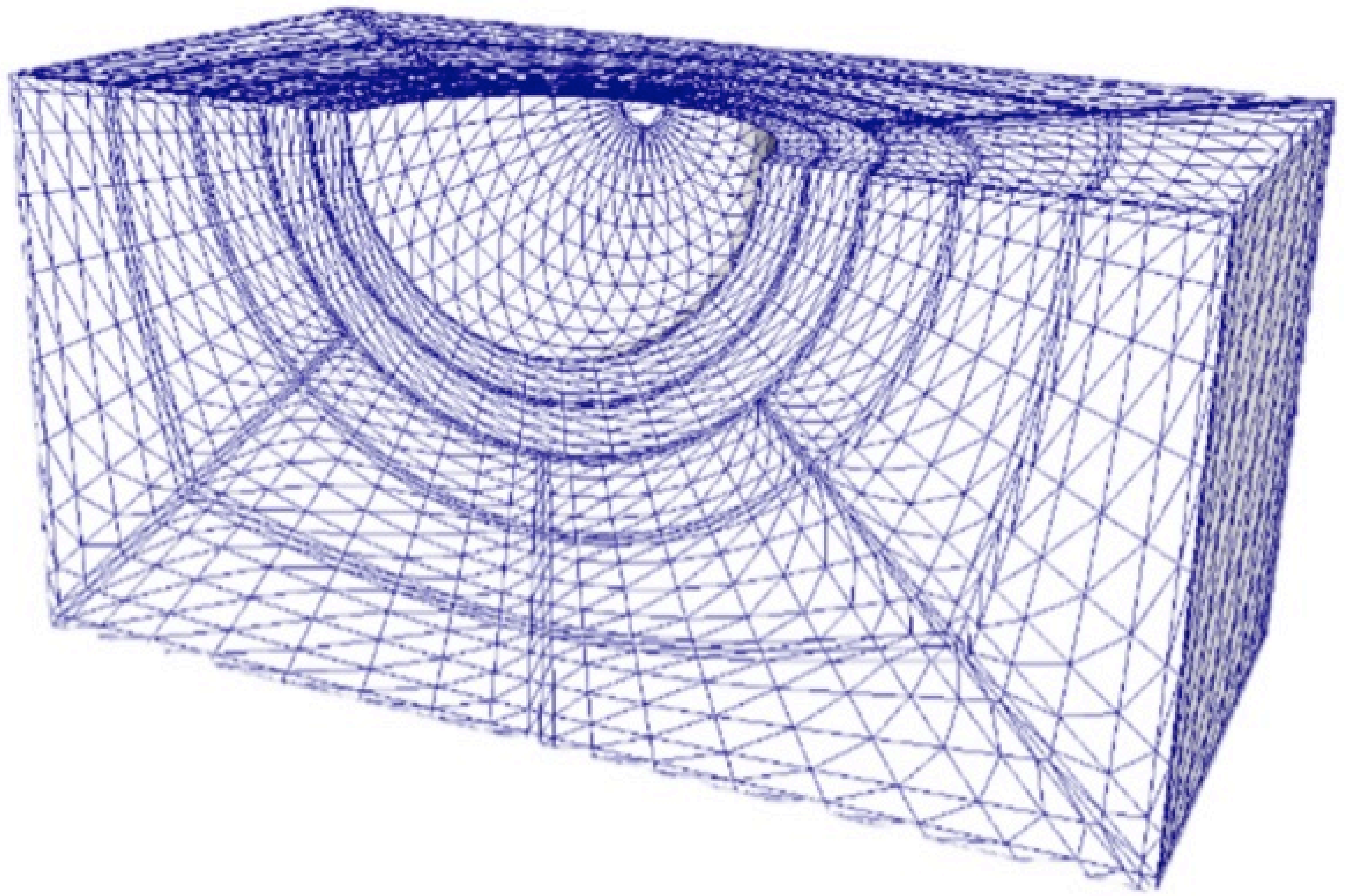
Mark Scheel, Mentor

LIGO SURF 2016

Research Funded by the Mellon Mays Undergraduate Fellowship

Introduction to Numerical Relativity

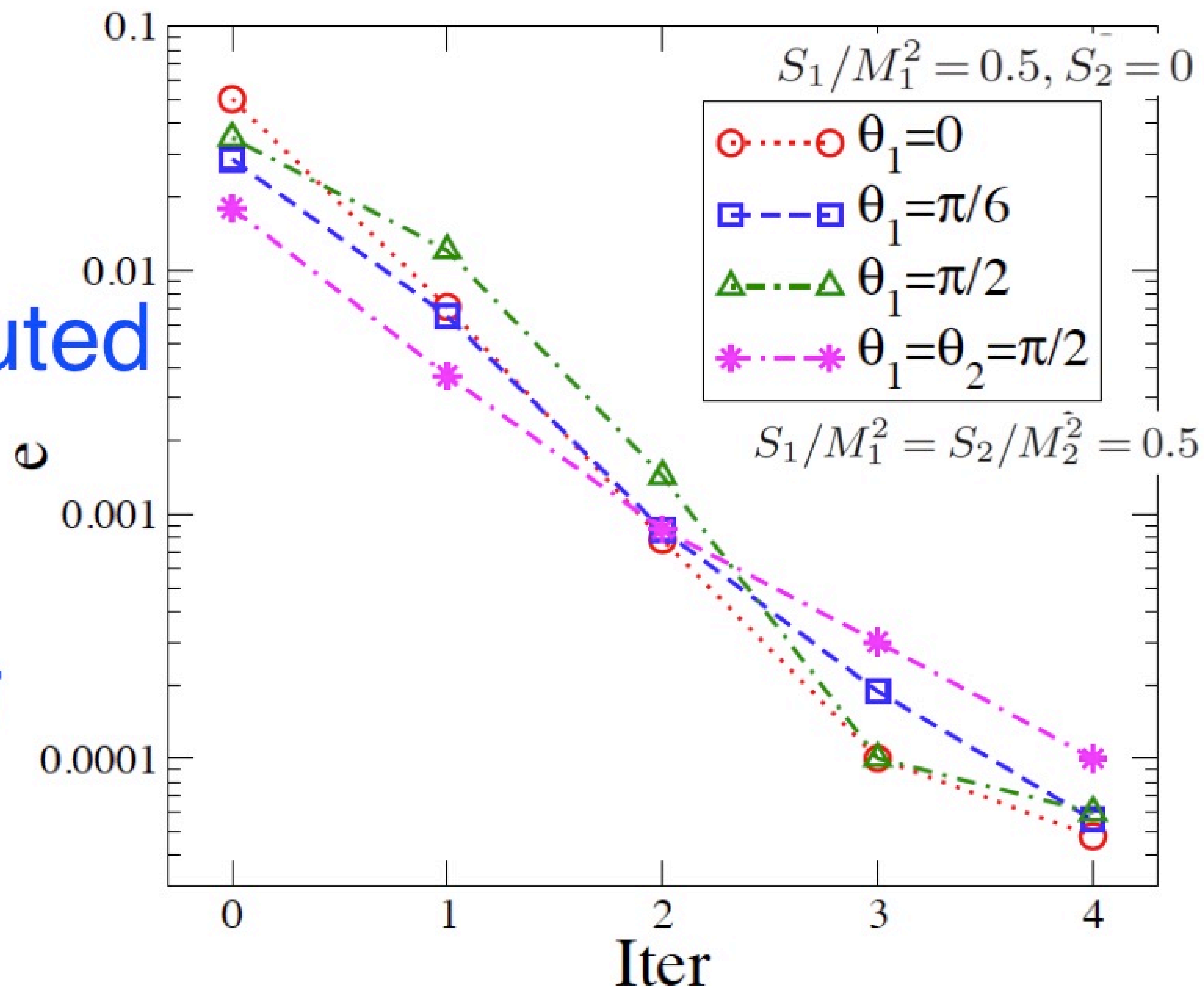
- Only analytic, circular waveforms (e.g. PN, EOB) are utilized by LIGO
- To get the most accurate waveforms, NR is required
  - » Spectral Einstein Code (SpEC)



Béla Szilágyi, Lee Lindblom, and Mark Scheel Phys Rev D **80**, 124010 (2009).

Orbital Eccentricity in BBHs

- CBCs detected by LIGO are expected to be circular
- Unlike PN, initial data for a quasi-circular orbit in NR can't be computed in closed form
- GWs from eccentric binaries differ from that of circular binaries



Alessandra Buonanno et al., Phys Rev D **80**, 104034 (2011).

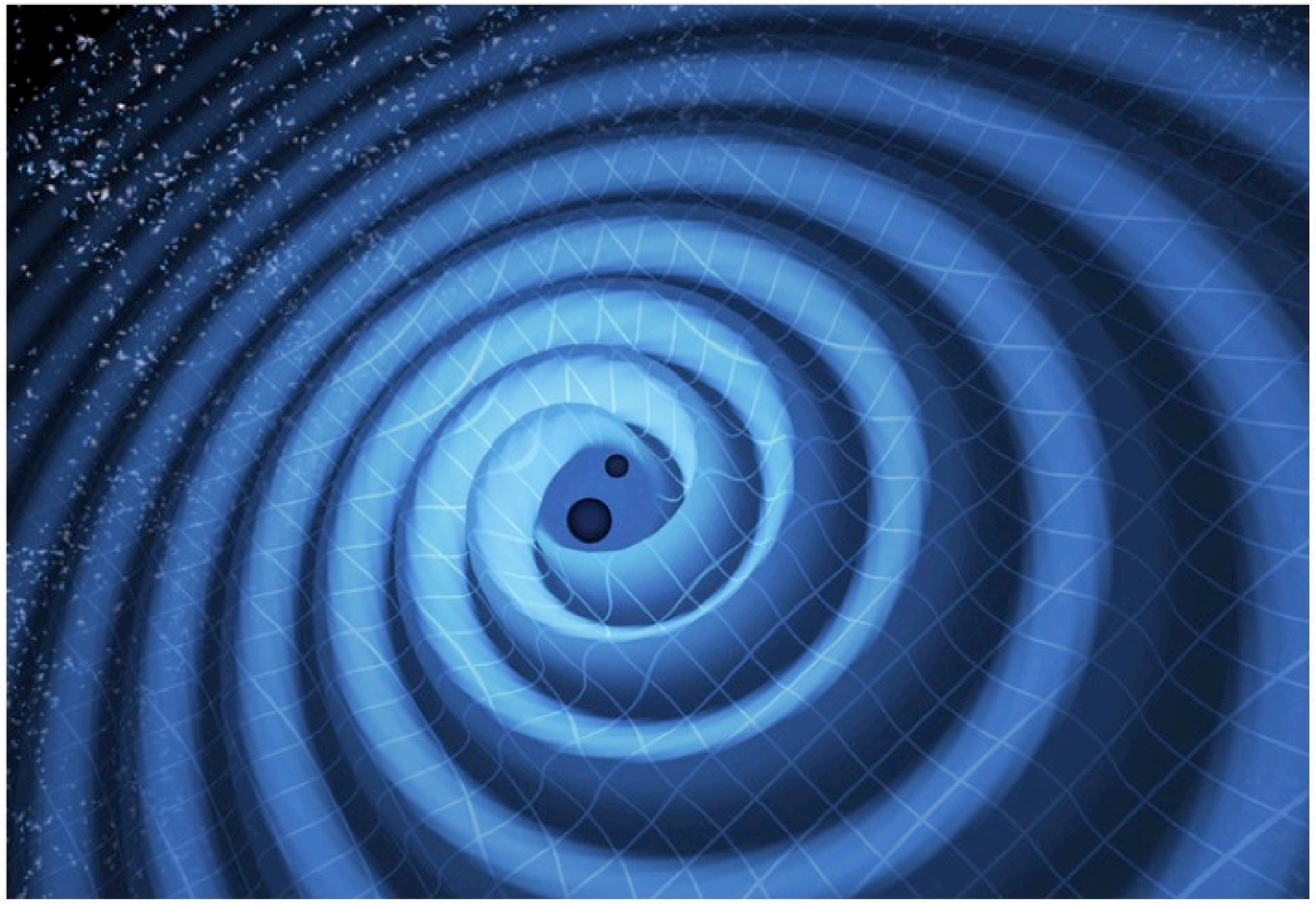
Research Goals

To determine:

- » The effect of eccentricity on BBH GW emission by comparing waveforms
- » The smallest orbital eccentricity required in order to be experimentally indistinguishable from zero eccentricity

To investigate:

- » The experimental implications of using eccentric waveforms on detection and parameter estimation



To learn:

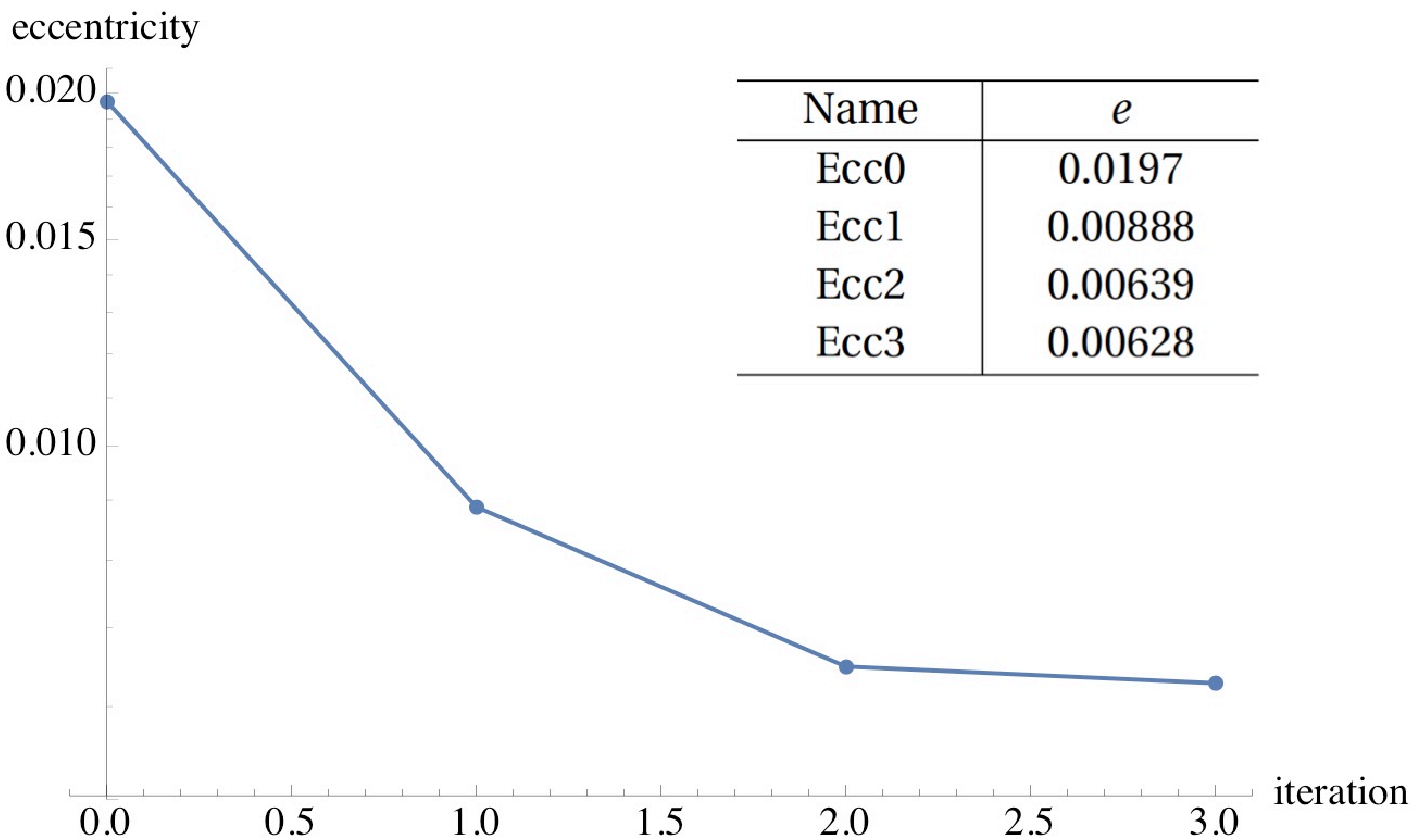
- » About computational aspects of NR by using SpEC

SpEC Simulations

- Need to compare BBH orbits with same masses/spins, but different eccentricities
  - » Two options: SXS Waveform Catalog or run new simulations

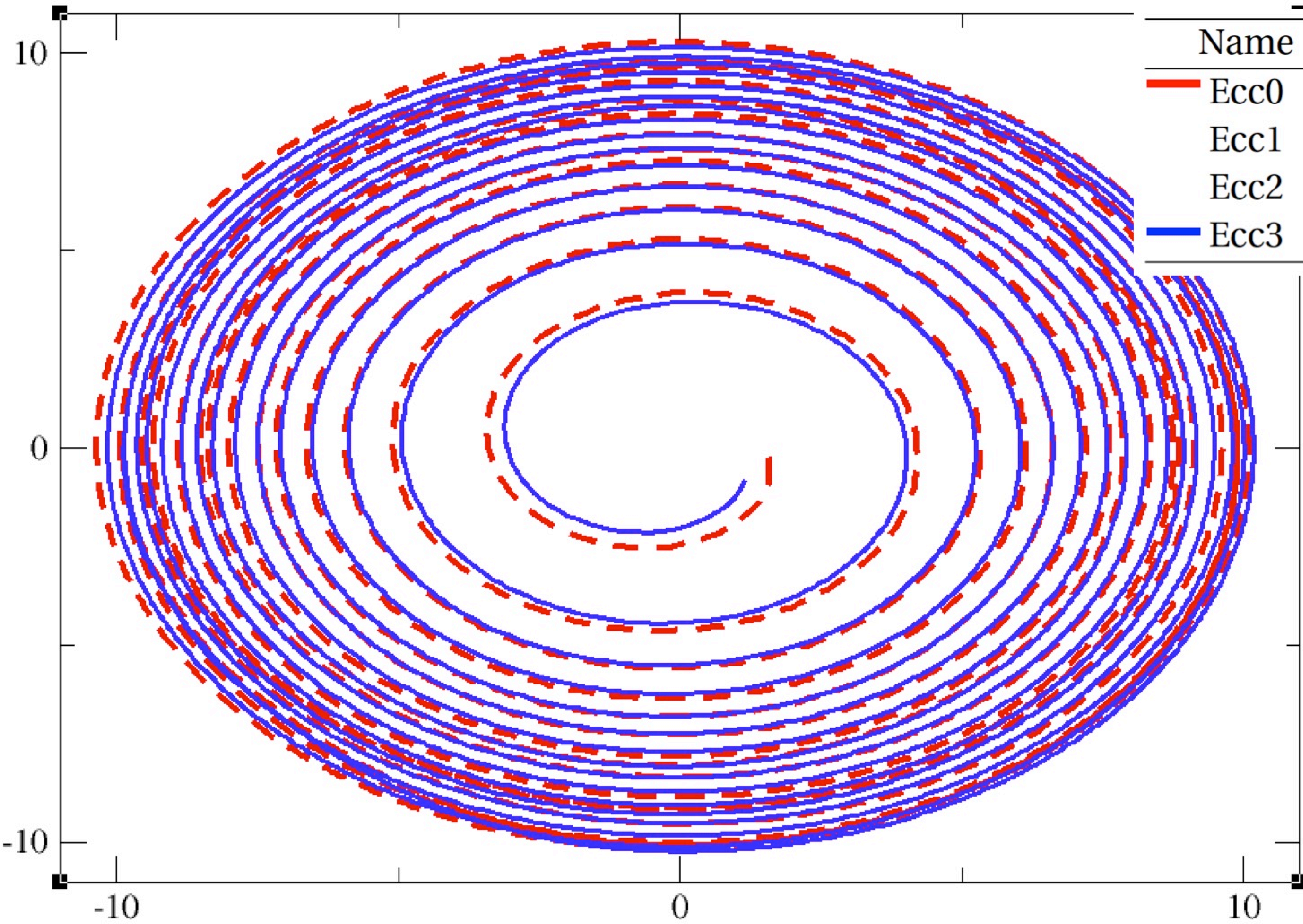
Name	q	$\|\chi^A\|$	χ^A_θ	χ^A_ϕ	$\|\chi^B\|$	χ^B_θ	χ^B_ϕ
7.1	3.0	0.7	0	0	0.6	0	0
7.2	3.0	0.7	3.14159	0	0.6	3.14159	0
7.3	3.0	0.7	1.0	0.5	0.6	0.5	1.0
7.4	3.0	0.7	1.0	0.5	0.6	2.0	4.0
7.5	2.0	0.0	0.0	0.0	0.0	0.0	0.0
7.6	1.0	0.0	0.0	0.0	0.0	0.0	0.0

Existing Data from “EccSeries_q3”



Name	e
Ecc0	0.0197
Ecc1	0.00888
Ecc2	0.00639
Ecc3	0.00628

BBH Trajectories (of the more massive BH) for q = 3 no spin



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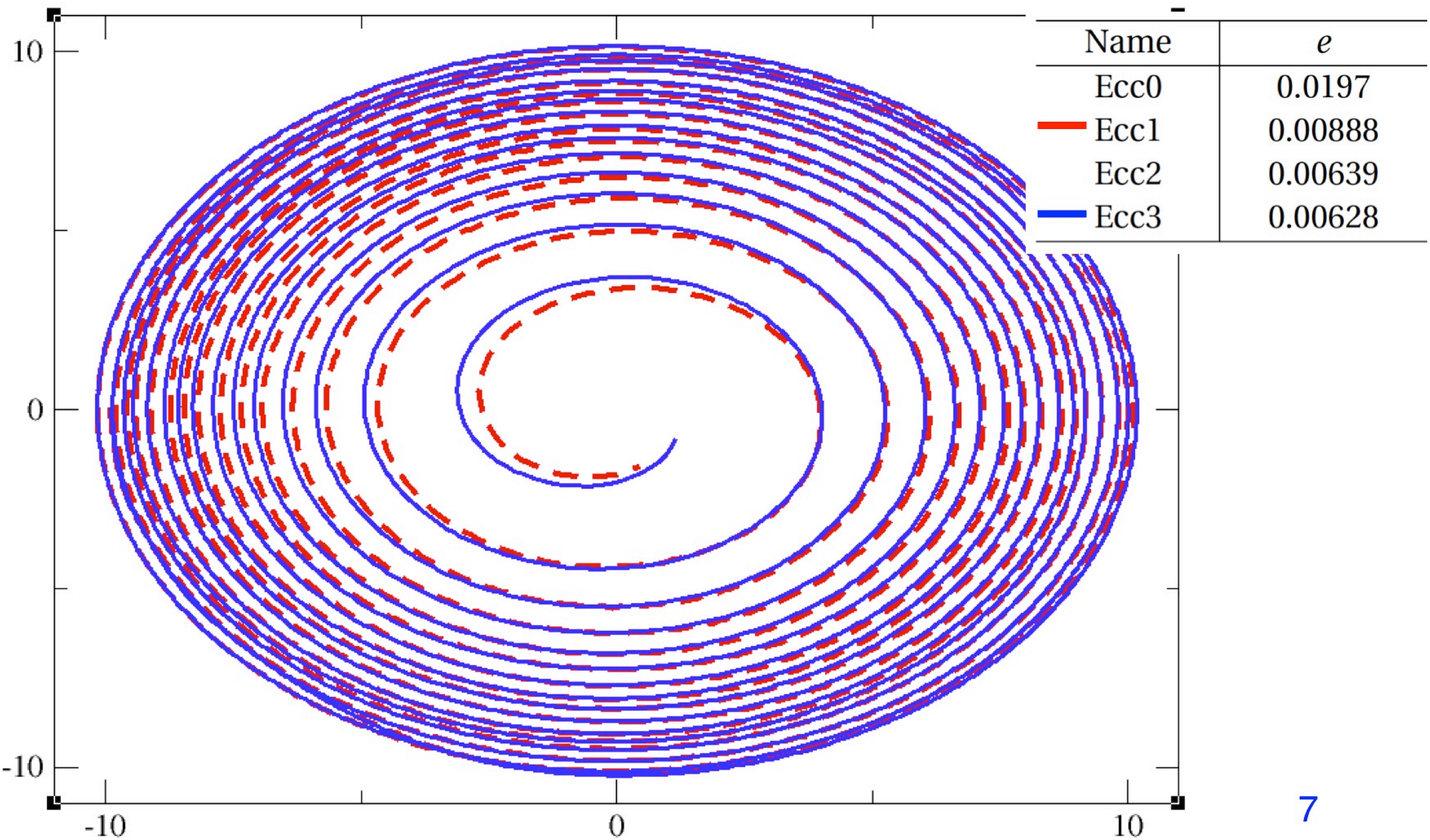
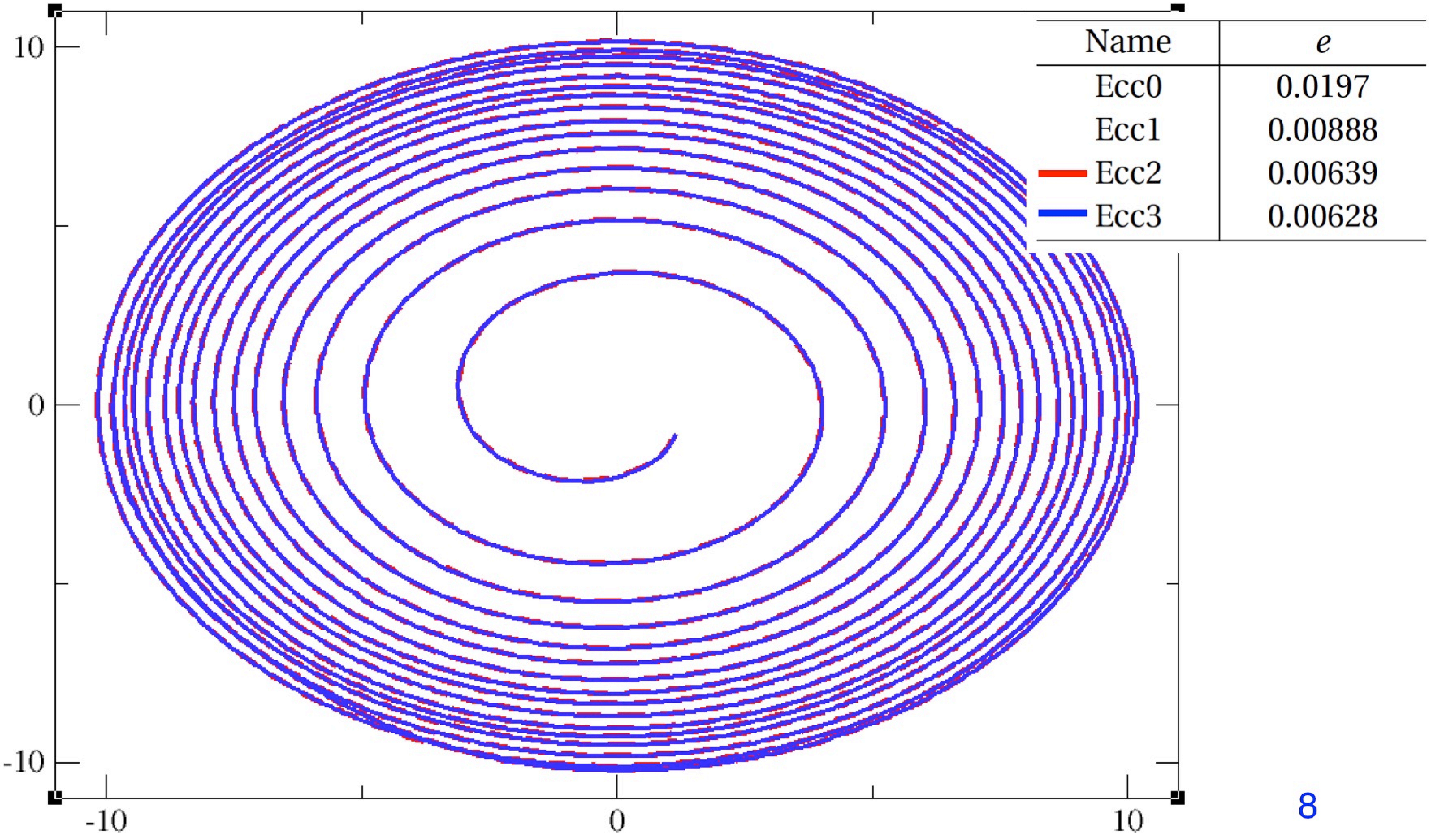
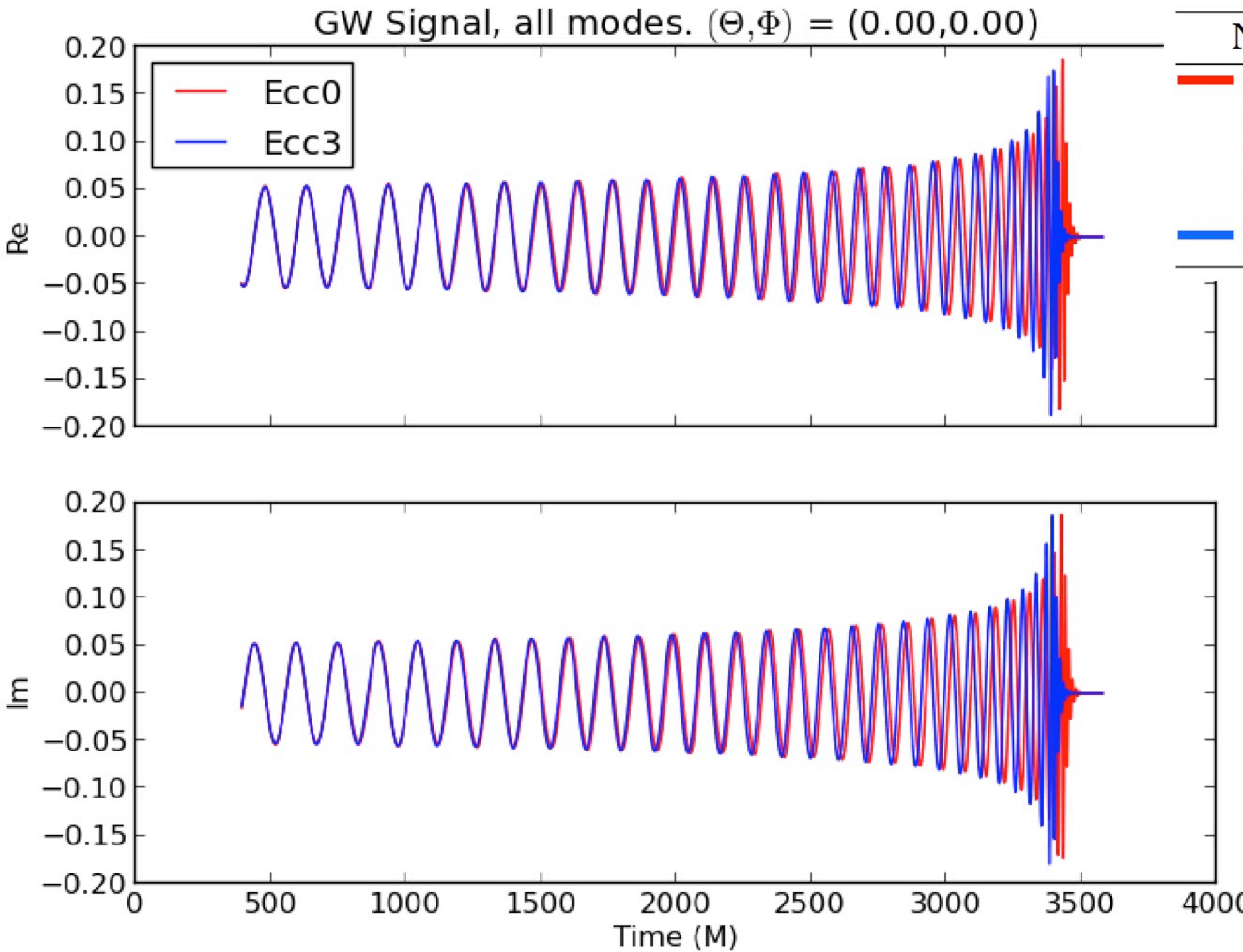


BBH Trajectories (of the more massive BH) for q = 3 no spin



Unaligned Time Series Waveform



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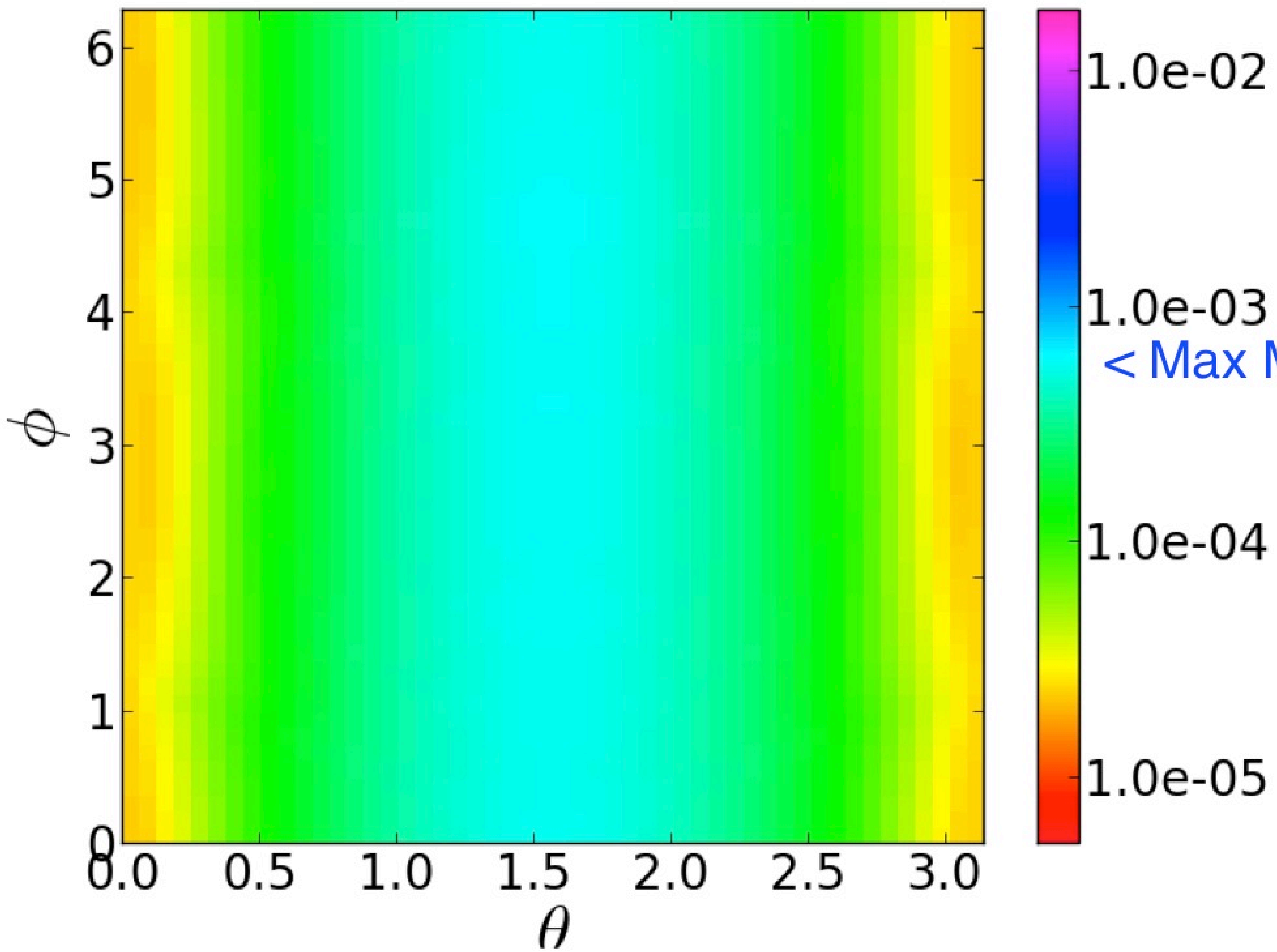
Competing Numerical Errors

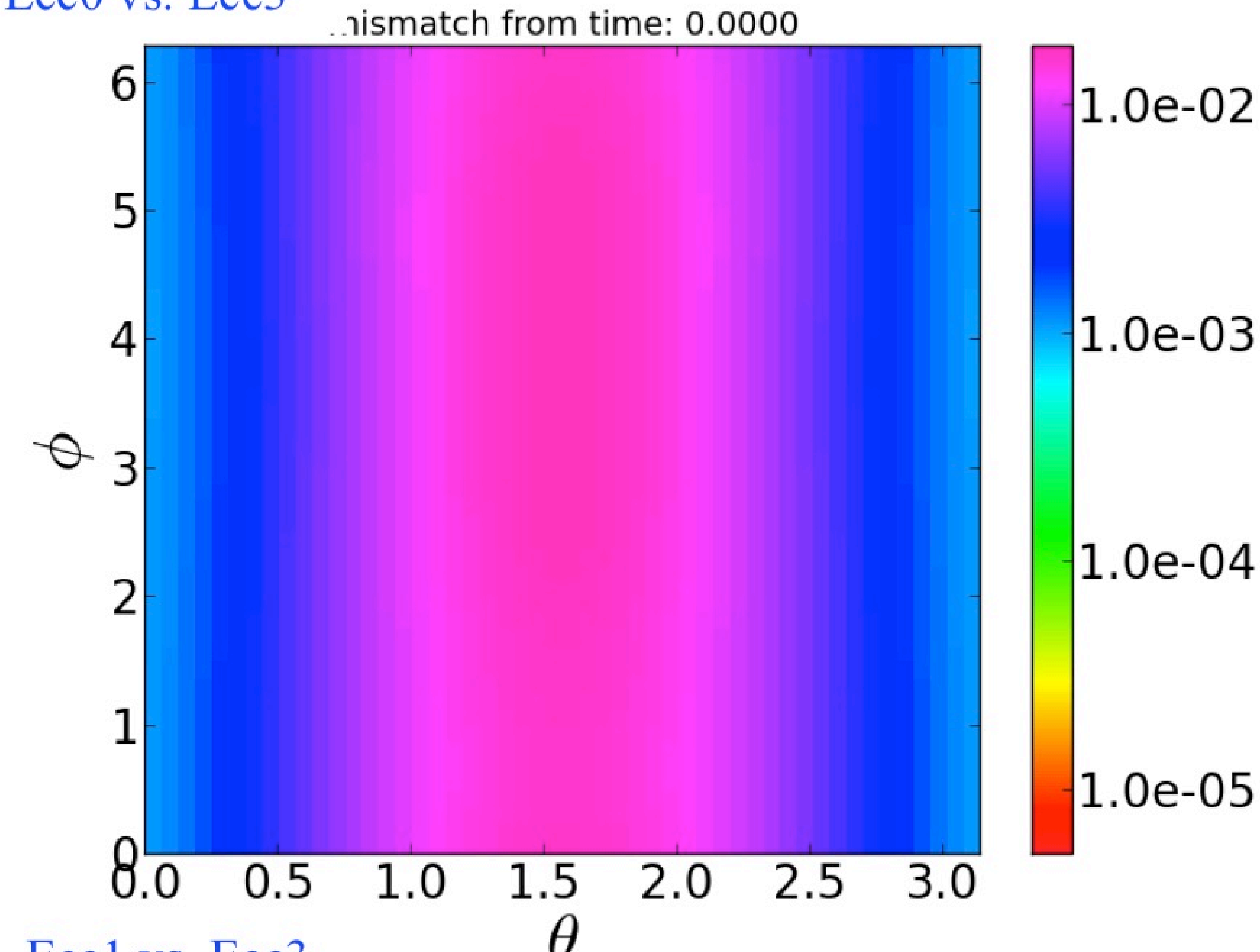
- Need to establish a reference level for GW mismatch
  - Plotted for most circular simulation, Ecc3

Highest vs. 2nd highest resolution mismatch, Flat noise

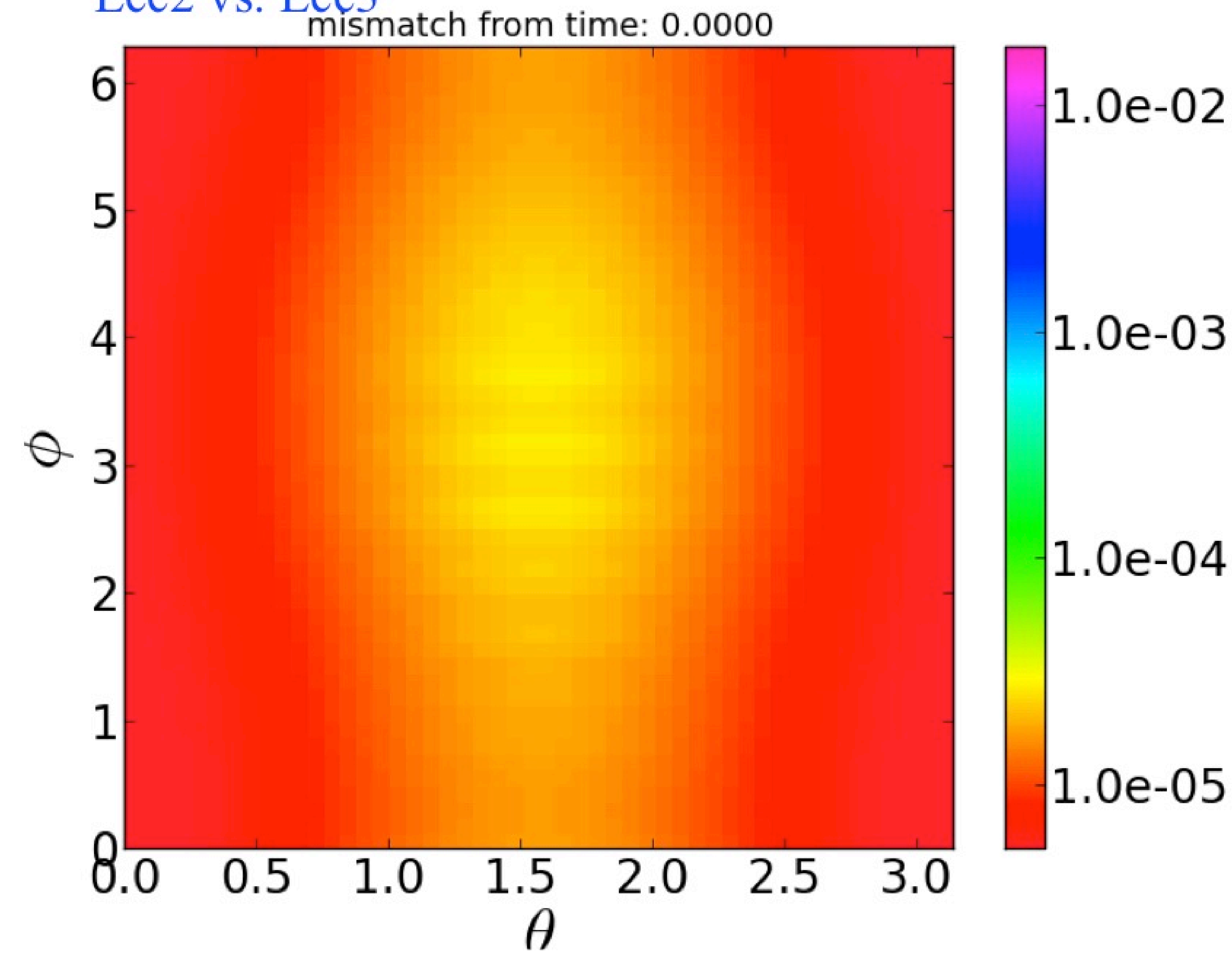


< Max Mismatch = 5.6 x 10^{-4}

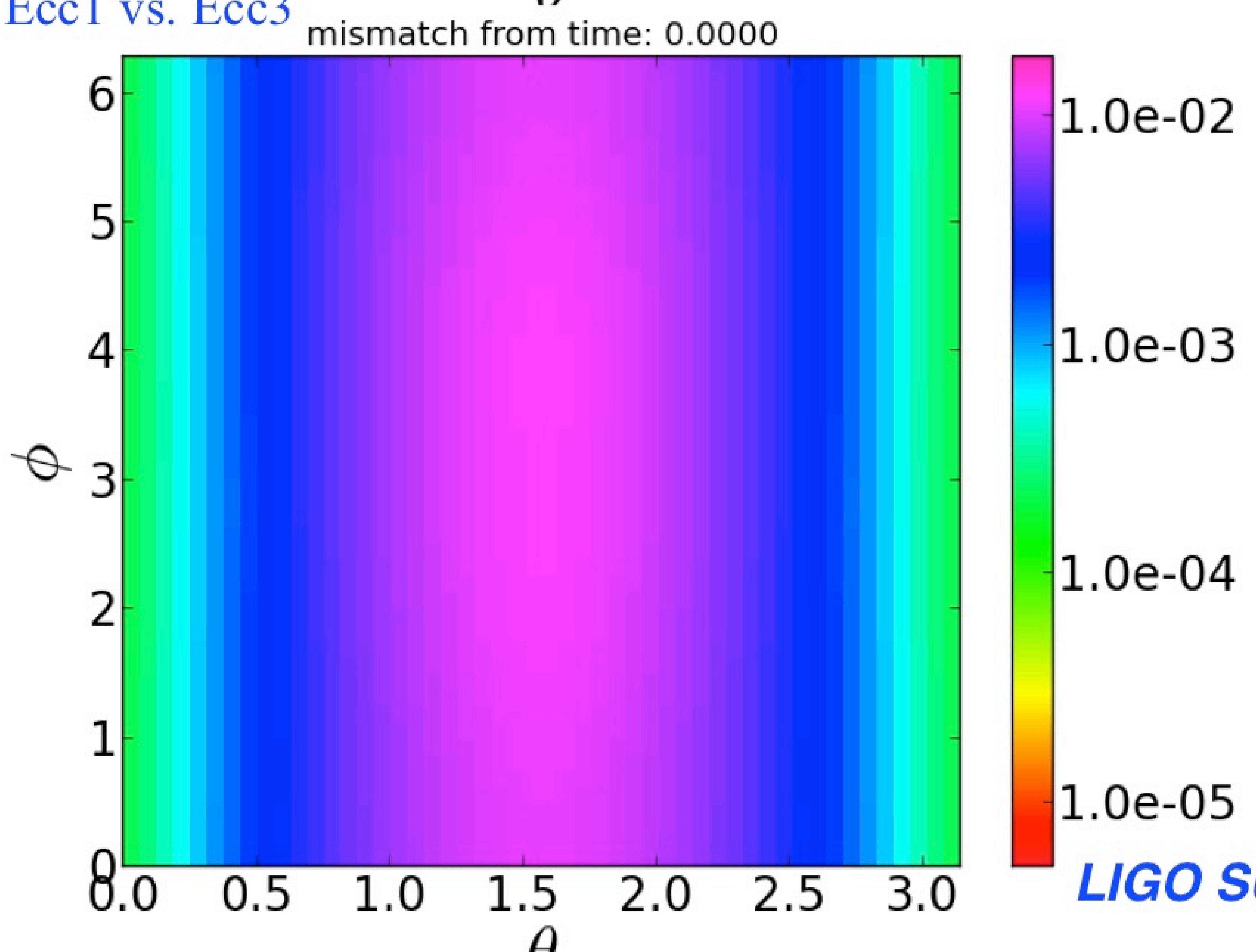
Ecc0 vs. Ecc3



Ecc2 vs. Ecc3



Ecc1 vs. Ecc3



GW Mismatch Skyplots, Flat noise

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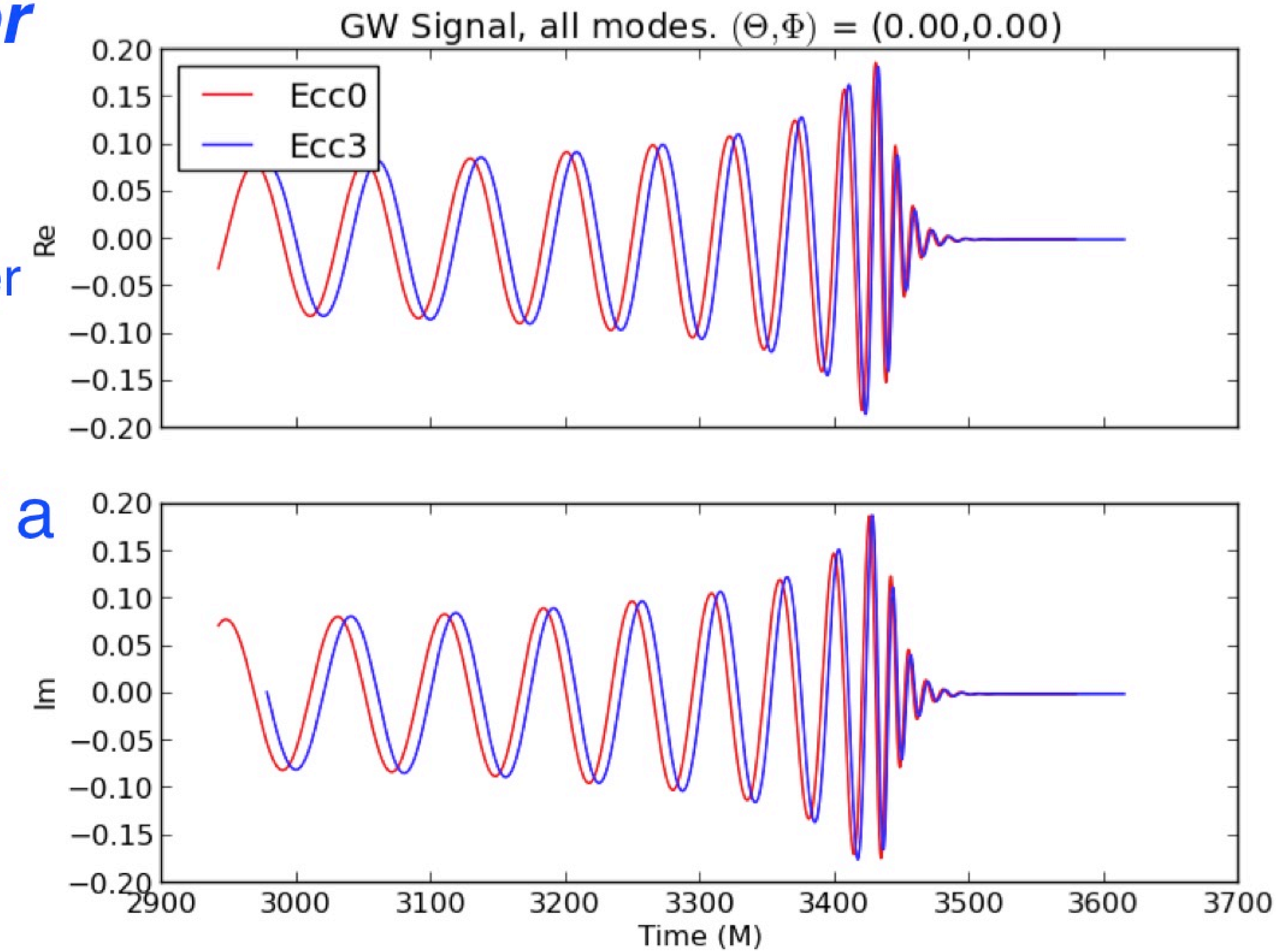
Mismatch dependence on waveform length

- Does comparing ***longer*** waveforms lead to a ***greater*** mismatch?
  - More cycles means a larger cumulative phase error…
- Can calculate match of a ***truncated*** waveform
  - "Low frequency cutoff"
  - Match aligns waveforms where power is highest, at merger



Max and Min GW Mismatch over all (θ, ϕ)

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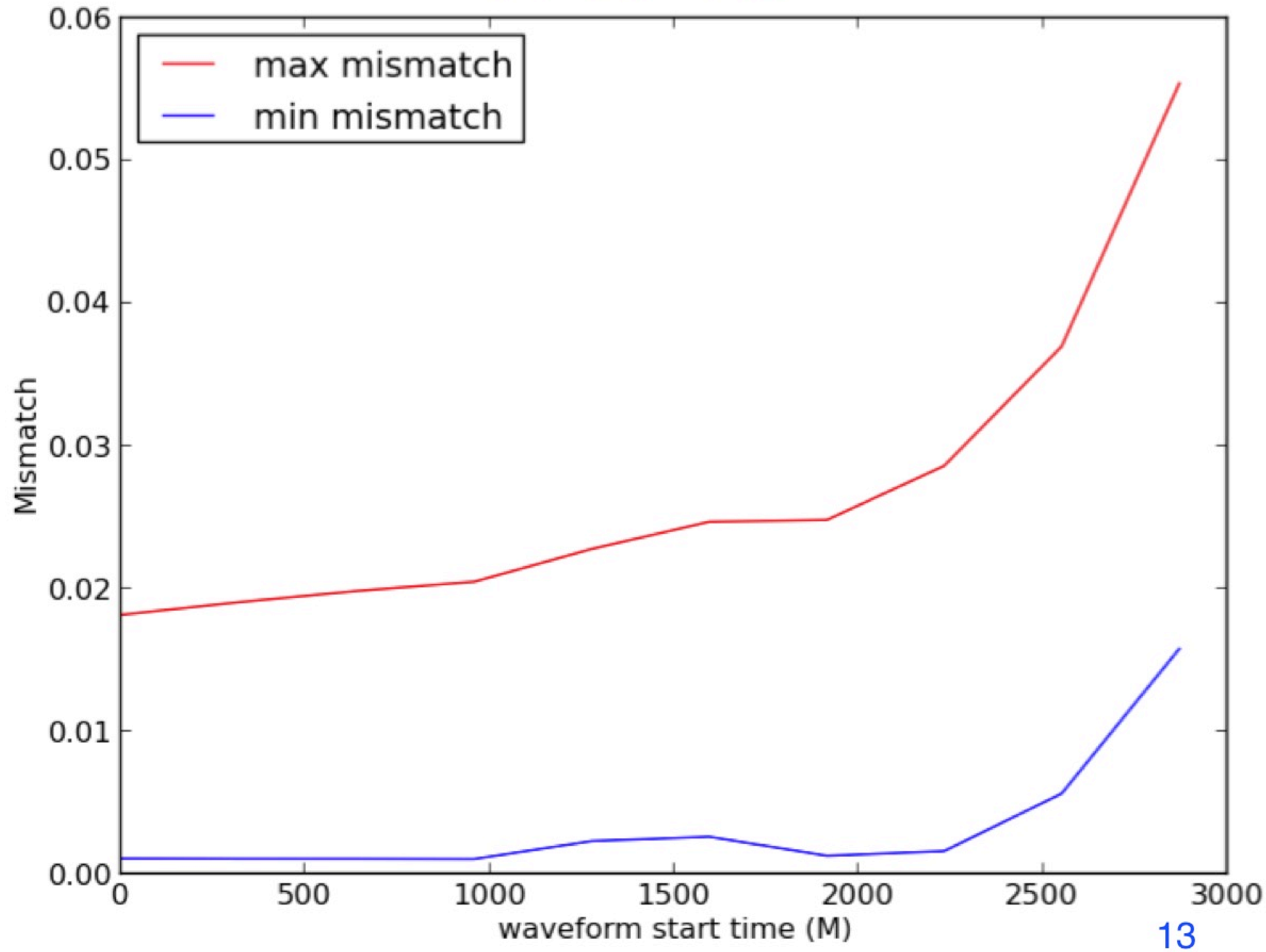
Ecc3	0.00628

Ecc0 vs. Ecc3



GW Mismatch
Adv Ligo Strain Noise Power Spectral Density

Ecc0 vs. Ecc3

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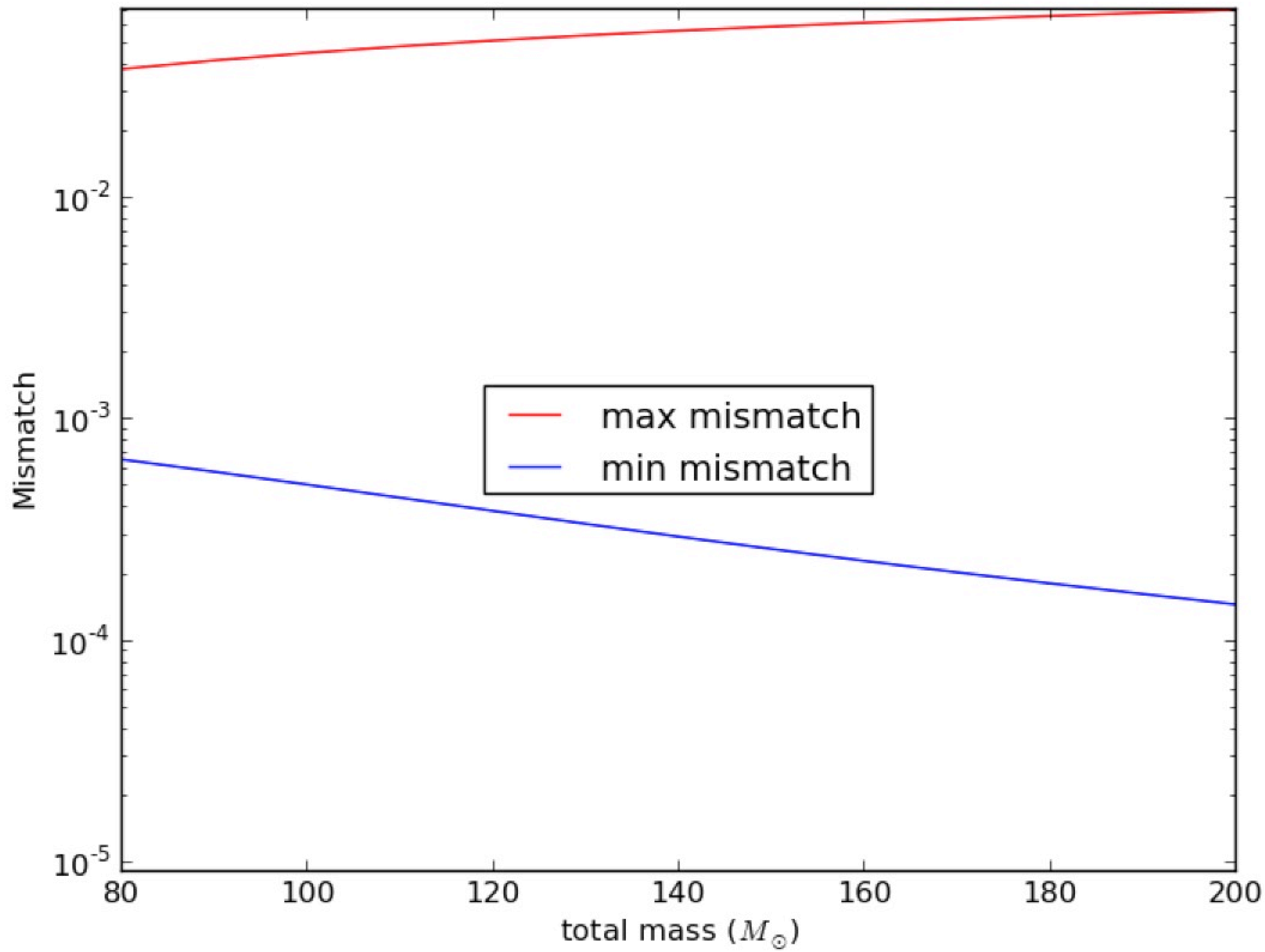
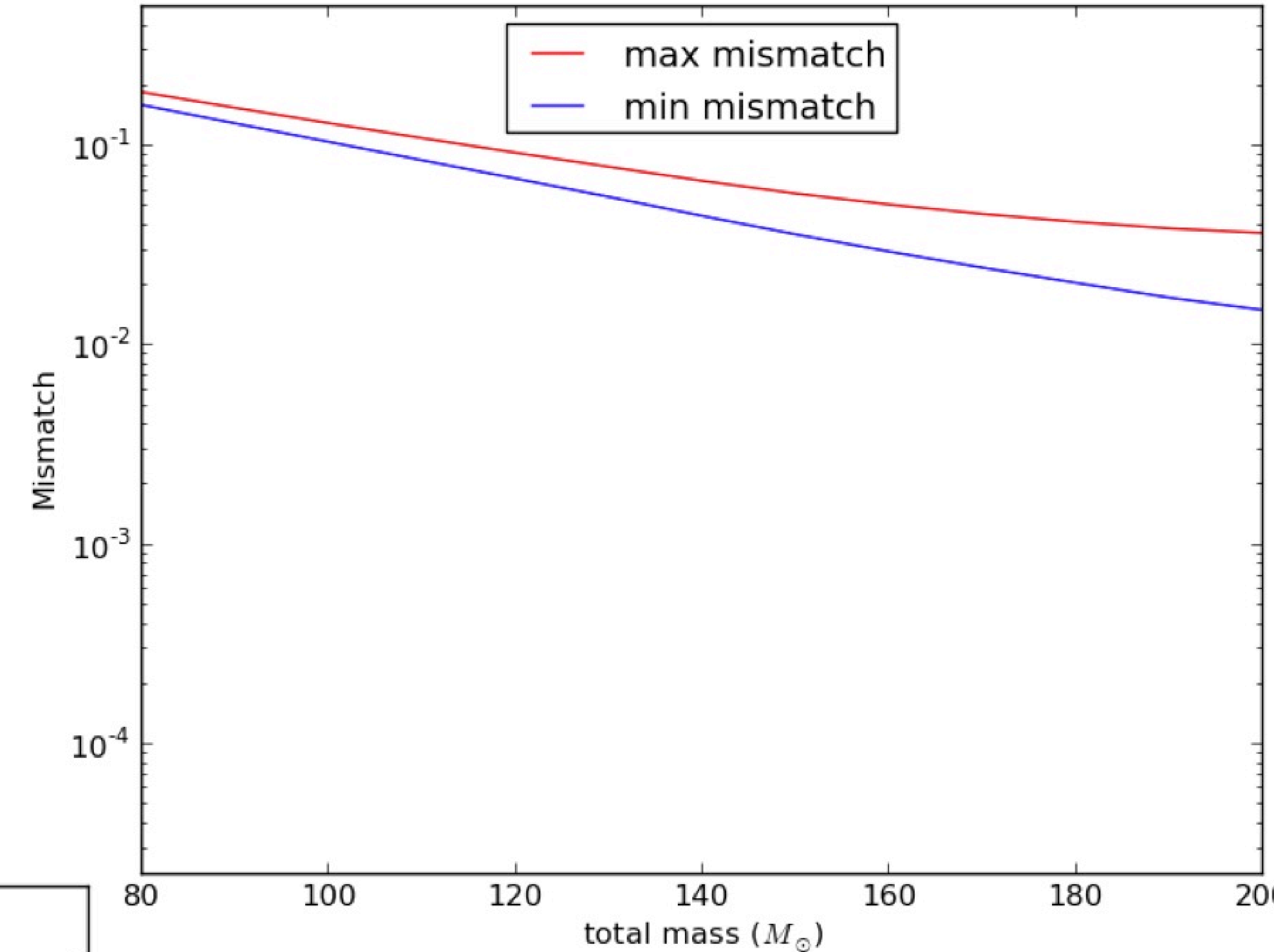
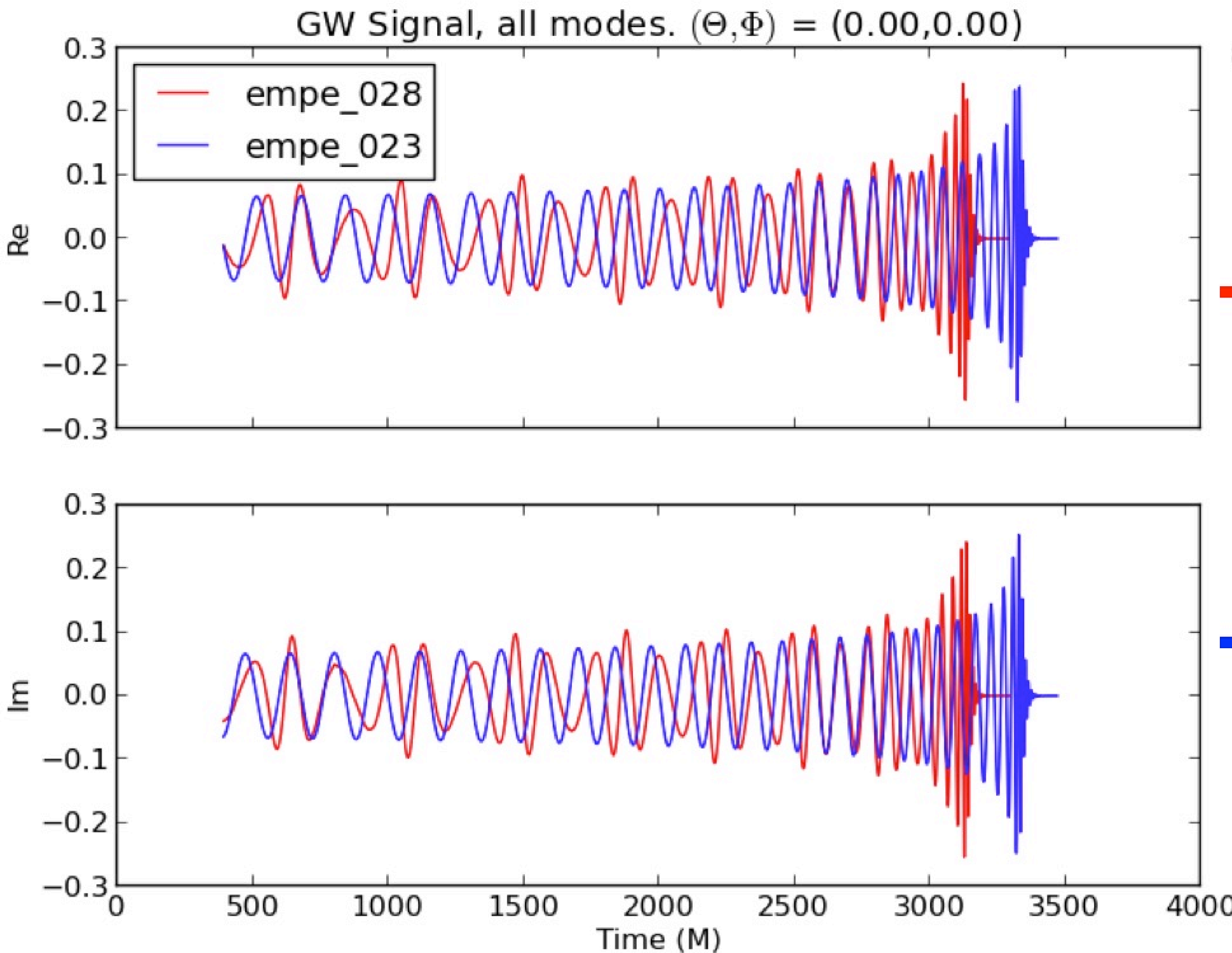


Existing data in "AEIRuns"

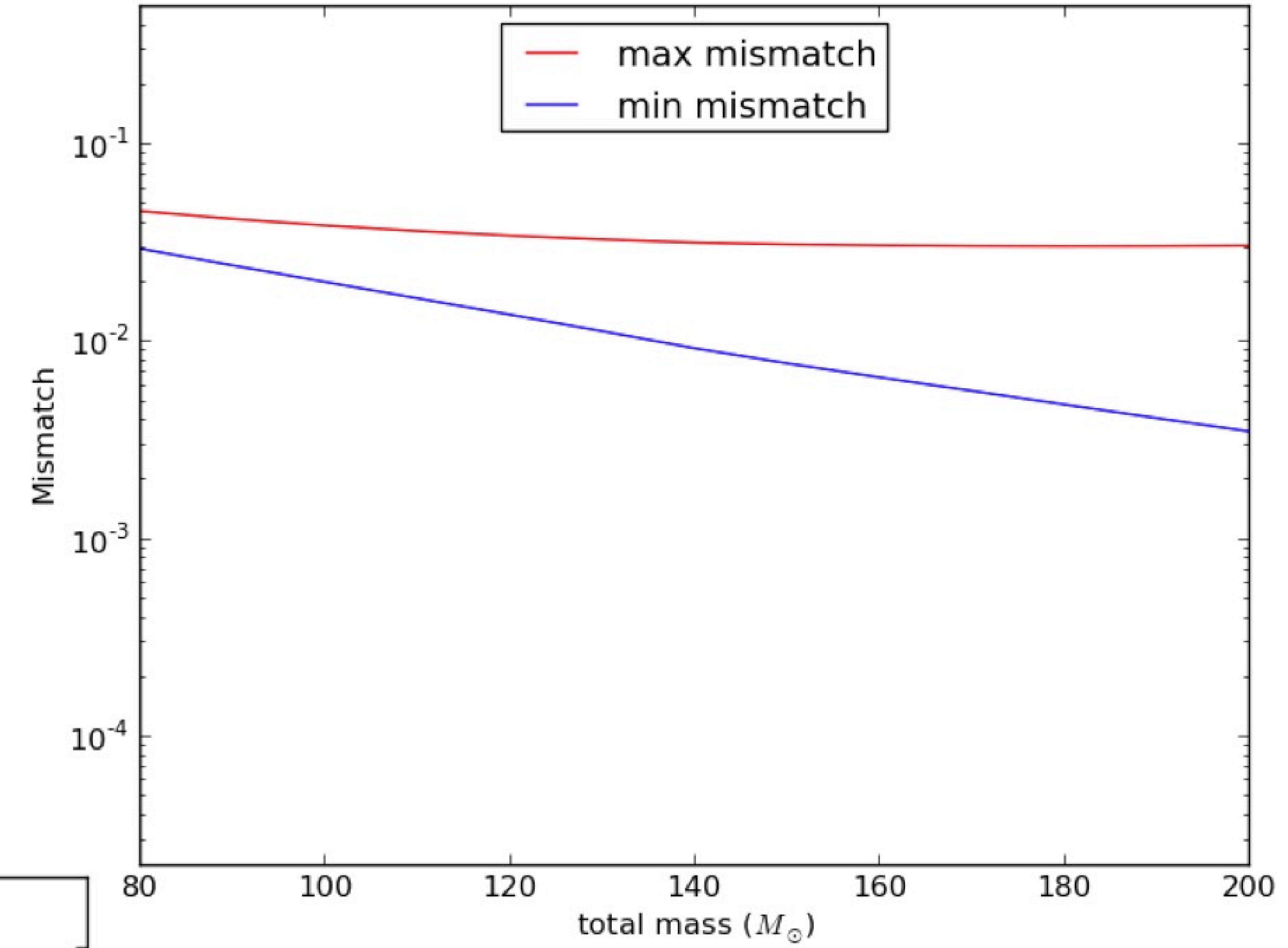


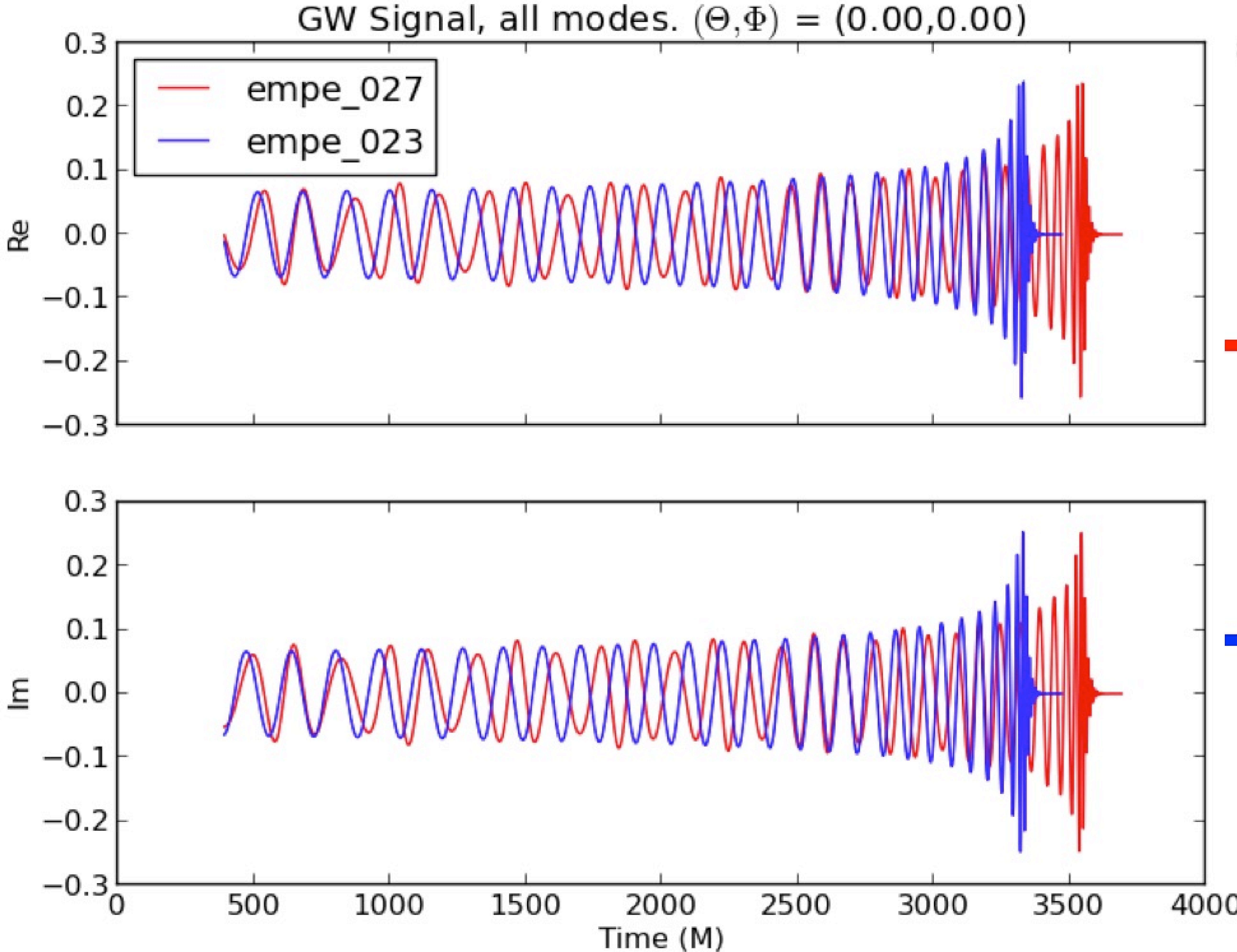


Name	e, Lev3	e, Lev4
empe_028	0.38	0.38
empe_027	0.1891710	no Lev 4
empe_026	0.1012370	no Lev4
empe_034	0.0629434	no Lev4
empe_035	0.0190964	no Lev4
empe_036	0.0082493	no Lev4
empe_023	0.0000983	0.0001083

$q = 1.22,\ \chi^A = (0, 0, 0.0996)$ and $\chi^B = (0, 0, -0.0893)$

Existing data in “AEIRuns”





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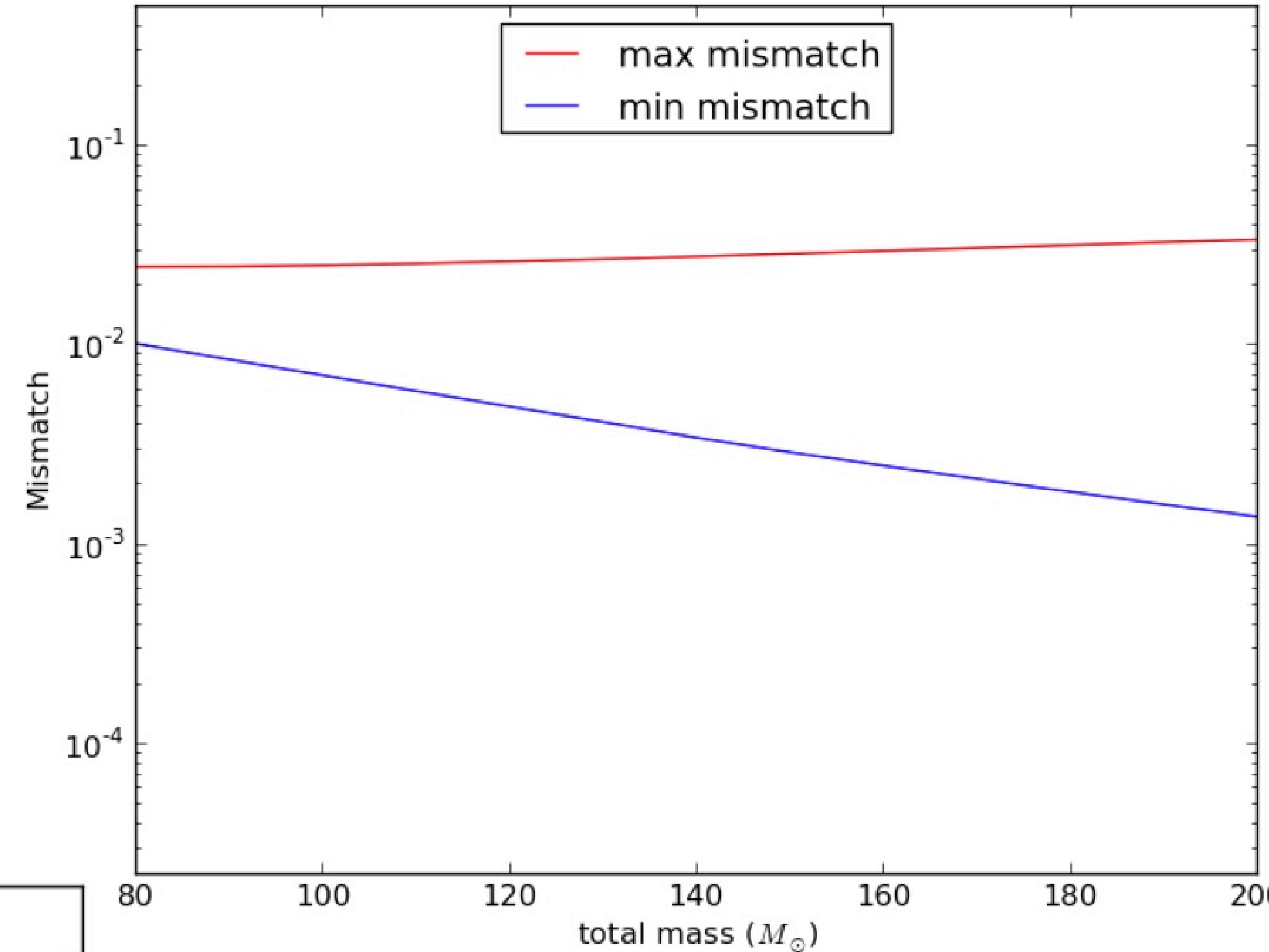

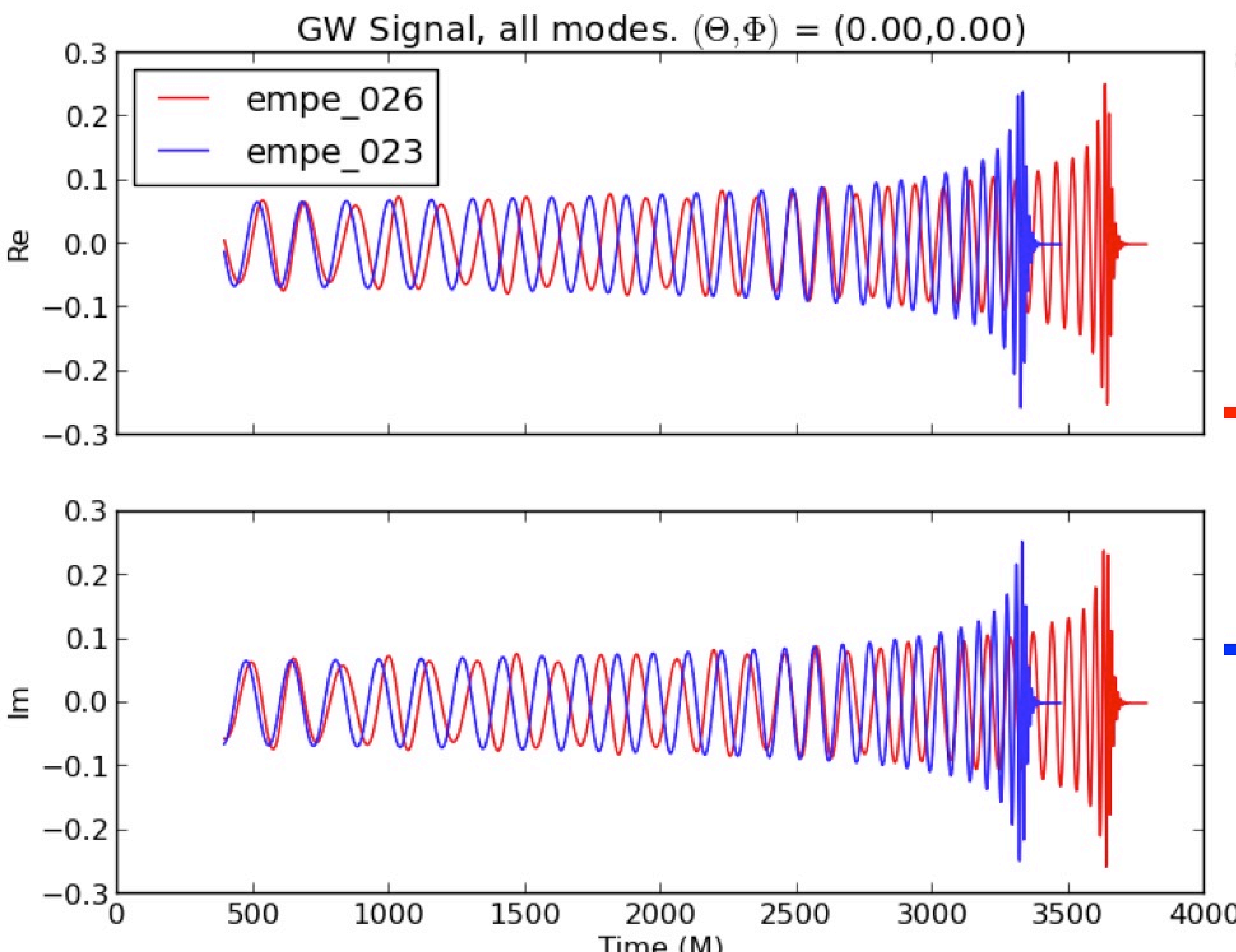

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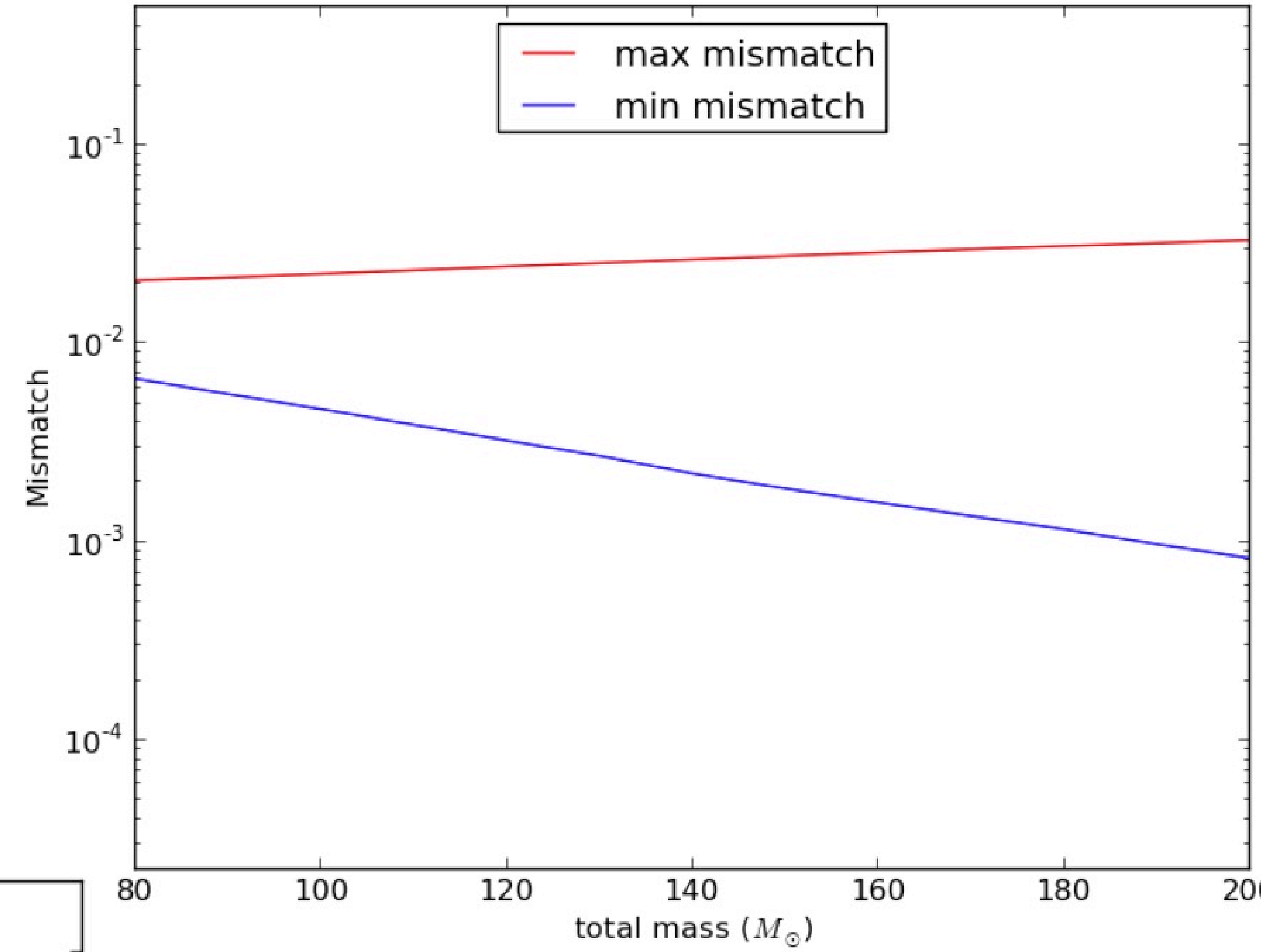

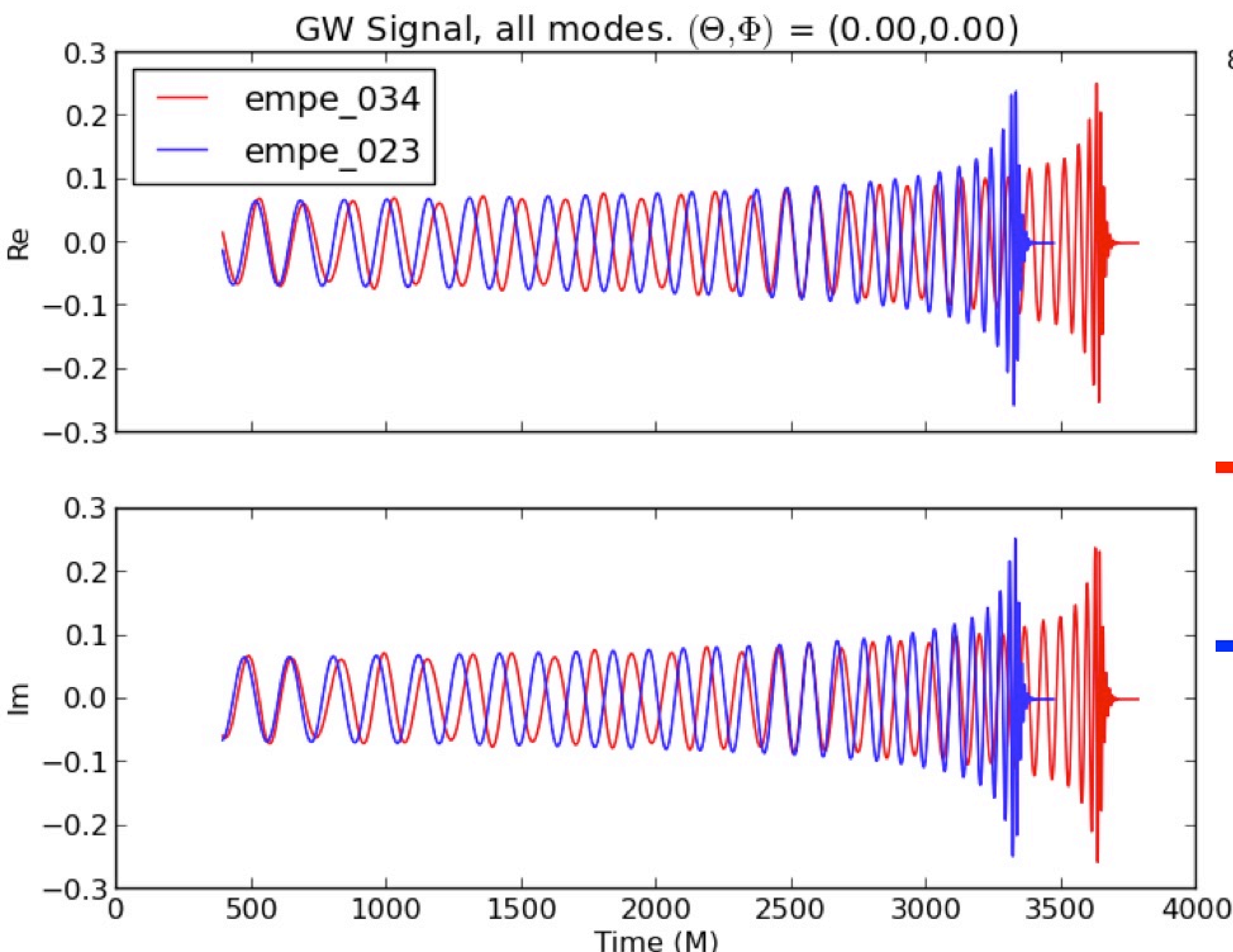

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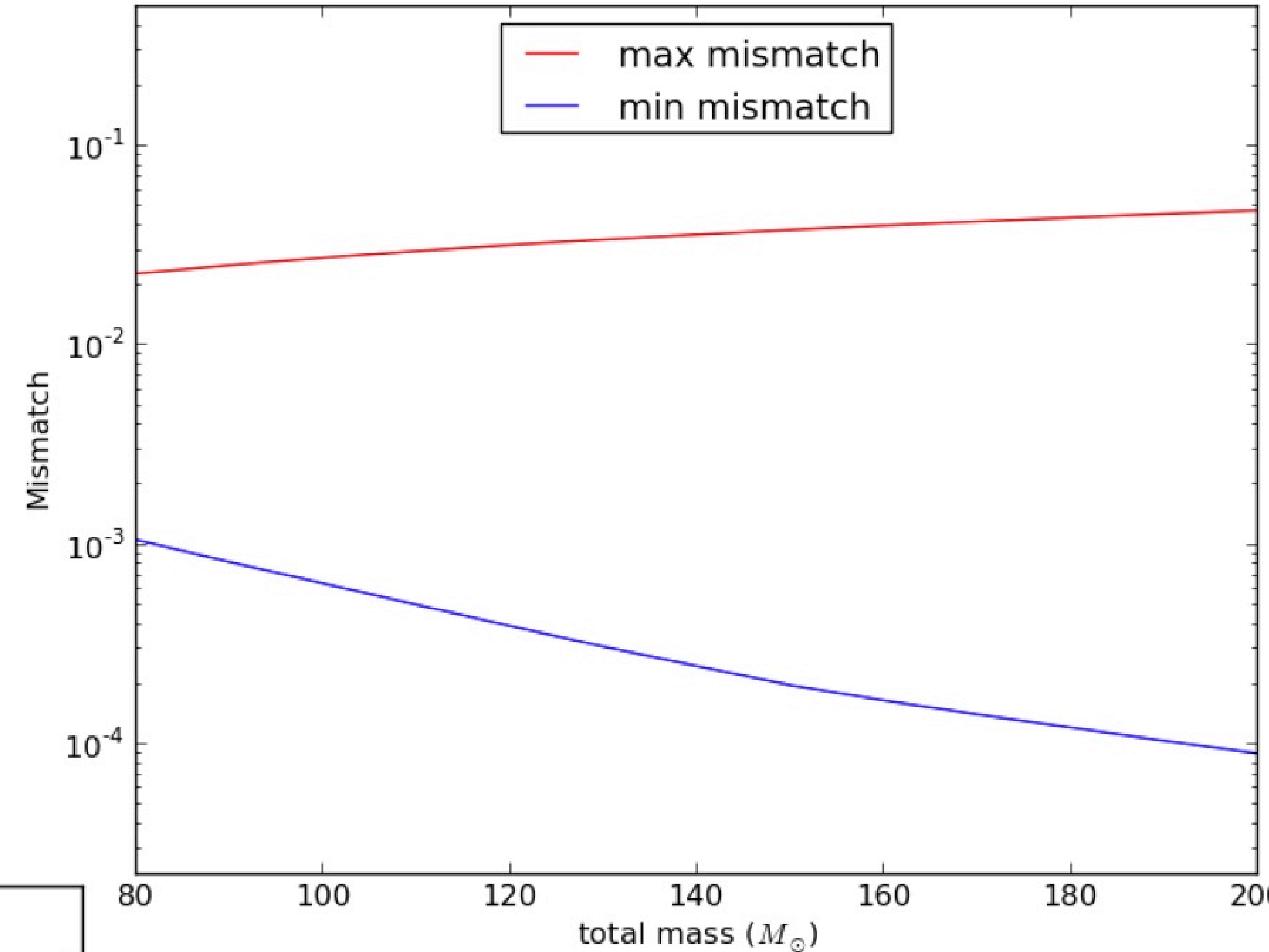

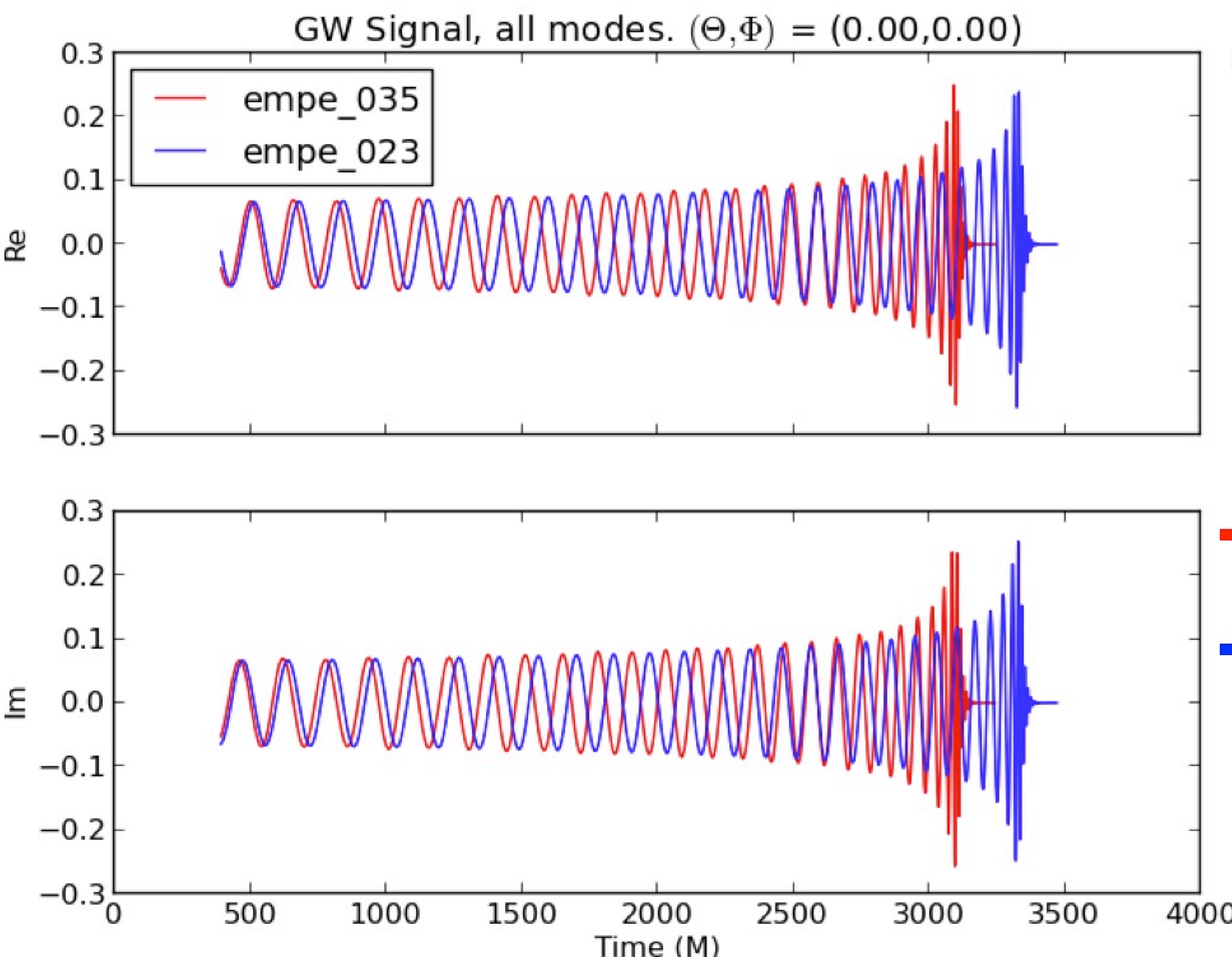

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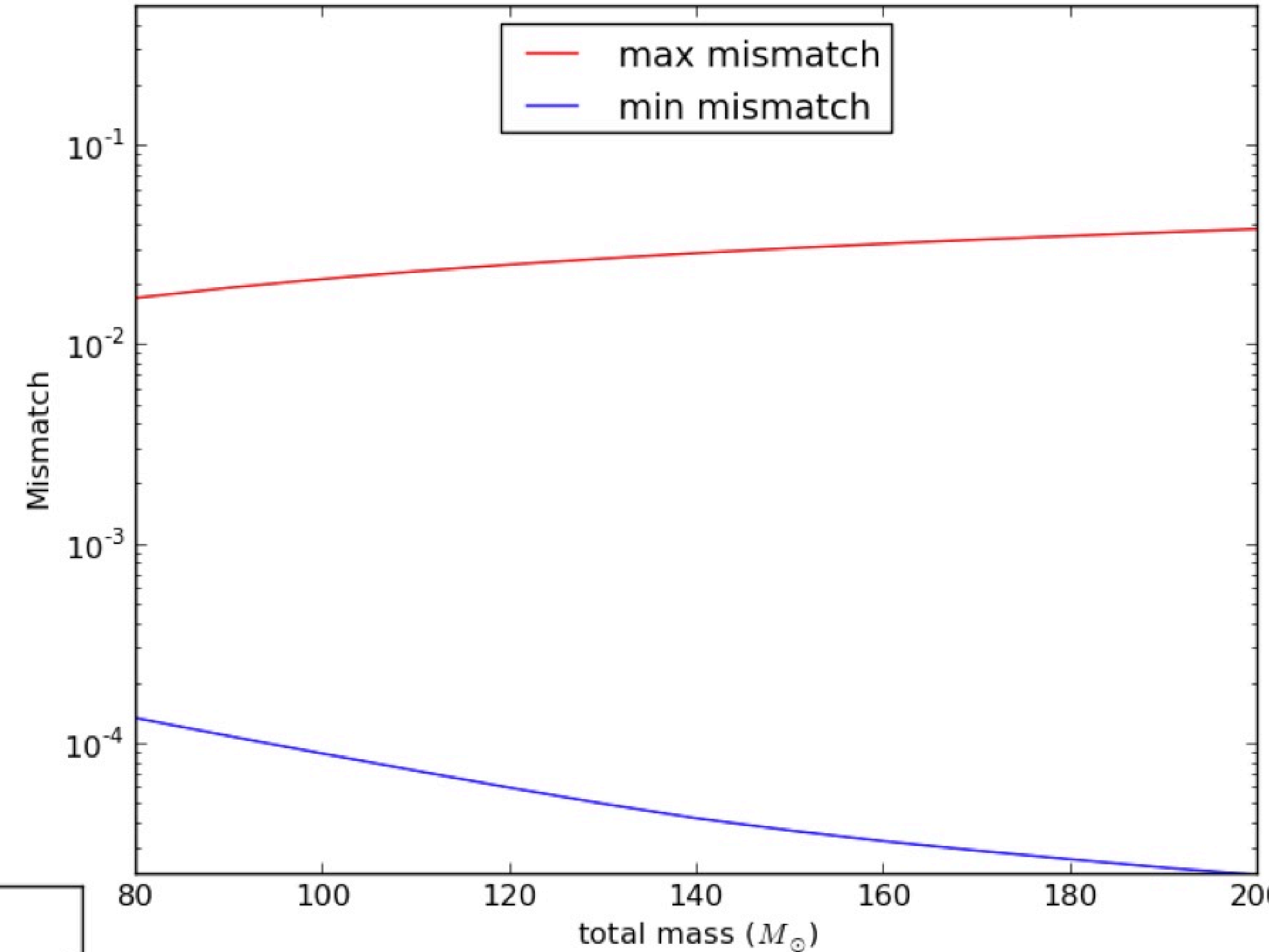

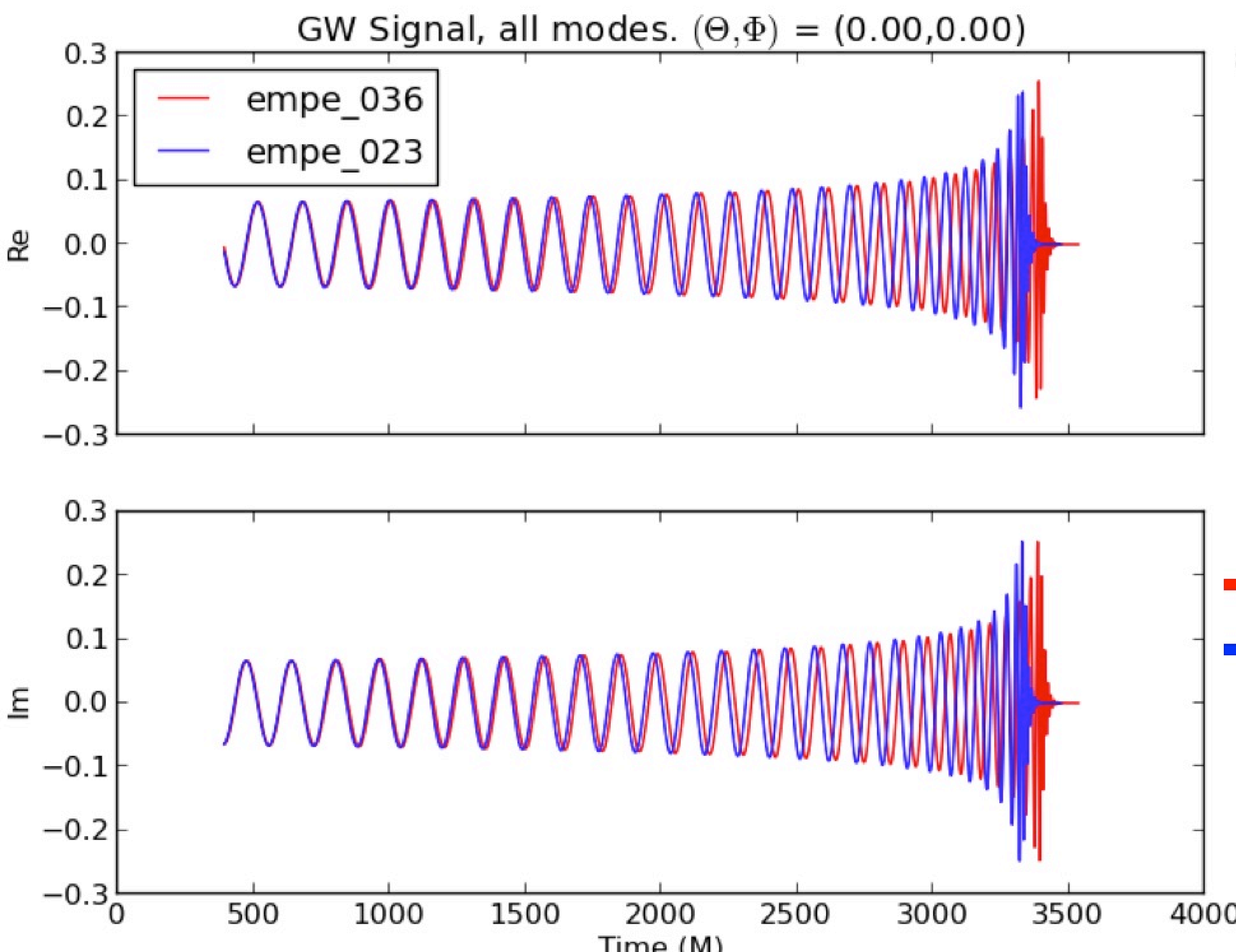

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$q = 1.22$, $\chi^A = (0, 0, 0.0996)$ and $\chi^B = (0, 0, -0.0893)$

Conclusions

- Investigated dependence of GW mismatch on eccentricity, total mass, and BBH orientation
- Significant mismatch – larger than numerical errors – for $\Delta\varepsilon > 0.001$
- GW mismatch mass profile shape is complicated for high eccentricities

  ($\varepsilon > 0.01$)

Future Work

- Continue analysis with the unfinished runs:

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7.2	3.0	0.7	3.14159	0	0.6	3.14159	0
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7.4	3.0	0.7	1.0	0.5	0.6	2.0	4.0
7.5	2.0	0.0	0.0	0.0	0.0	0.0	0.0
7.6	1.0	0.0	0.0	0.0	0.0	0.0	0.0

- How do Bayesian estimates of intrinsic parameters vary between for orbits of different eccentricity?

Acknowledgements

- Caltech Numerical Relativity Group
- Mellon Mays Undergraduate Fellowship Program
- Caltech Student-Faculty Programs
- Dan Hemberger

Introduction to Numerical Relativity

	Description	Caveat
Post-Newtonian (PN)	Perturbative analytic solution to Einstein's equations, in powers of $M\frac{v}{c}$	Fast, so useful for pipeline detection and data analysis (DA) Describes inspiral, ringdown well, but not merger
Numerical Relativity (NR)	Exact numerical solution to Einstein's equations	Computationally intensive and plenty of numerical challenges
NR + PN	Analytic waveform models, like EOB and PhenomP. Need to calibrate against NR waveforms	Still requires NR, but fast enough for DA

Analyzing Waveforms with GWFrames

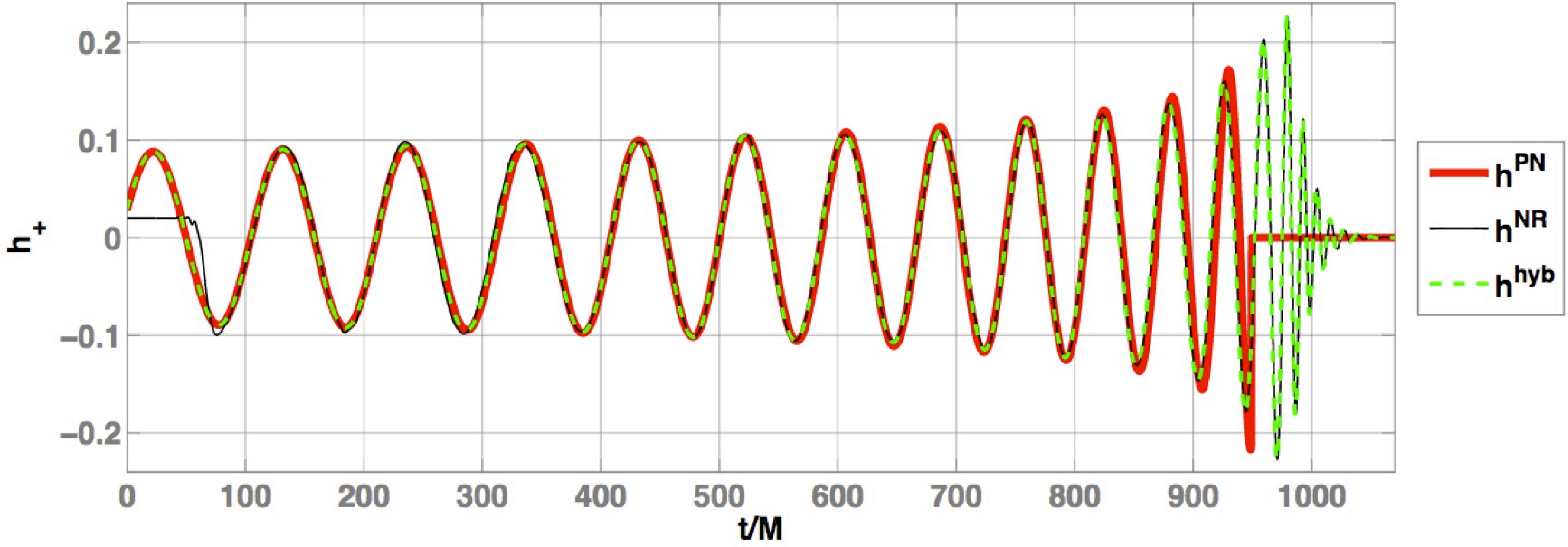
- Module containing operations on time series waveforms
  - Mike Boyle Phys. Rev. D **87**, 104006 (2013)
  - Complex number operations, derivatives, noise-weighted inner products, FTs
  - Match function:

$$\mathrm{Match}(u_1, u_2) = \max_{t_c, \phi_c} \langle u_1 | u_2 e^{i(2\pi f t_c - \phi_c)} \rangle, \tag{3.1}$$

$$\langle x | y \rangle = 4\mathrm{Re} \int_0^\infty \frac{\tilde{x}(f) \tilde{y}^*(f)}{S_n(f)} df. \tag{3.2}$$

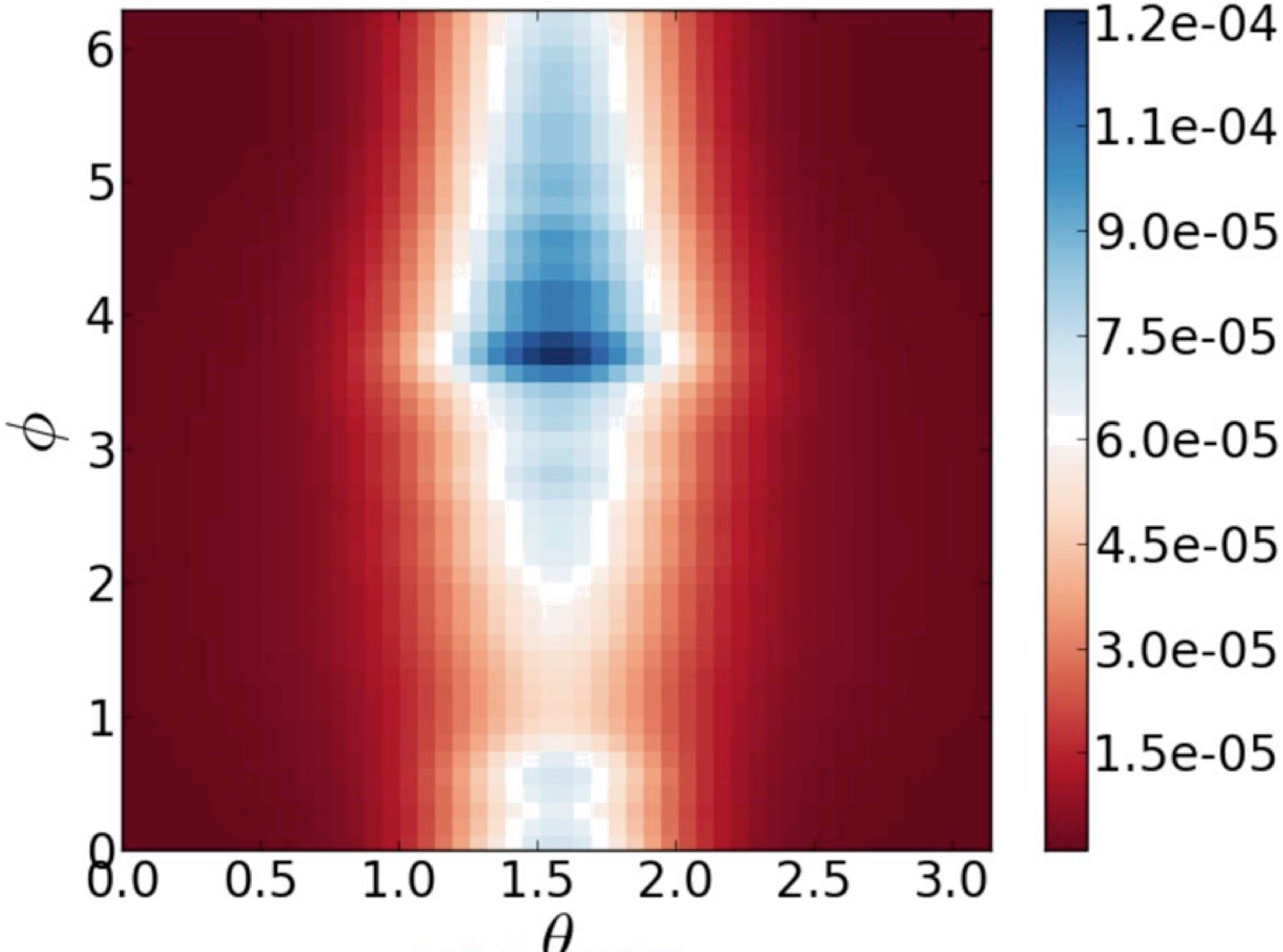
- GW emission decomposed into spherical harmonic modes, (l,m). (2,2) is most important for CBCs
  - Can look at one particular mode, over all directions
  - Can look at sum of modes, in a particular direction

Constructing a hybrid waveform with PN + NR

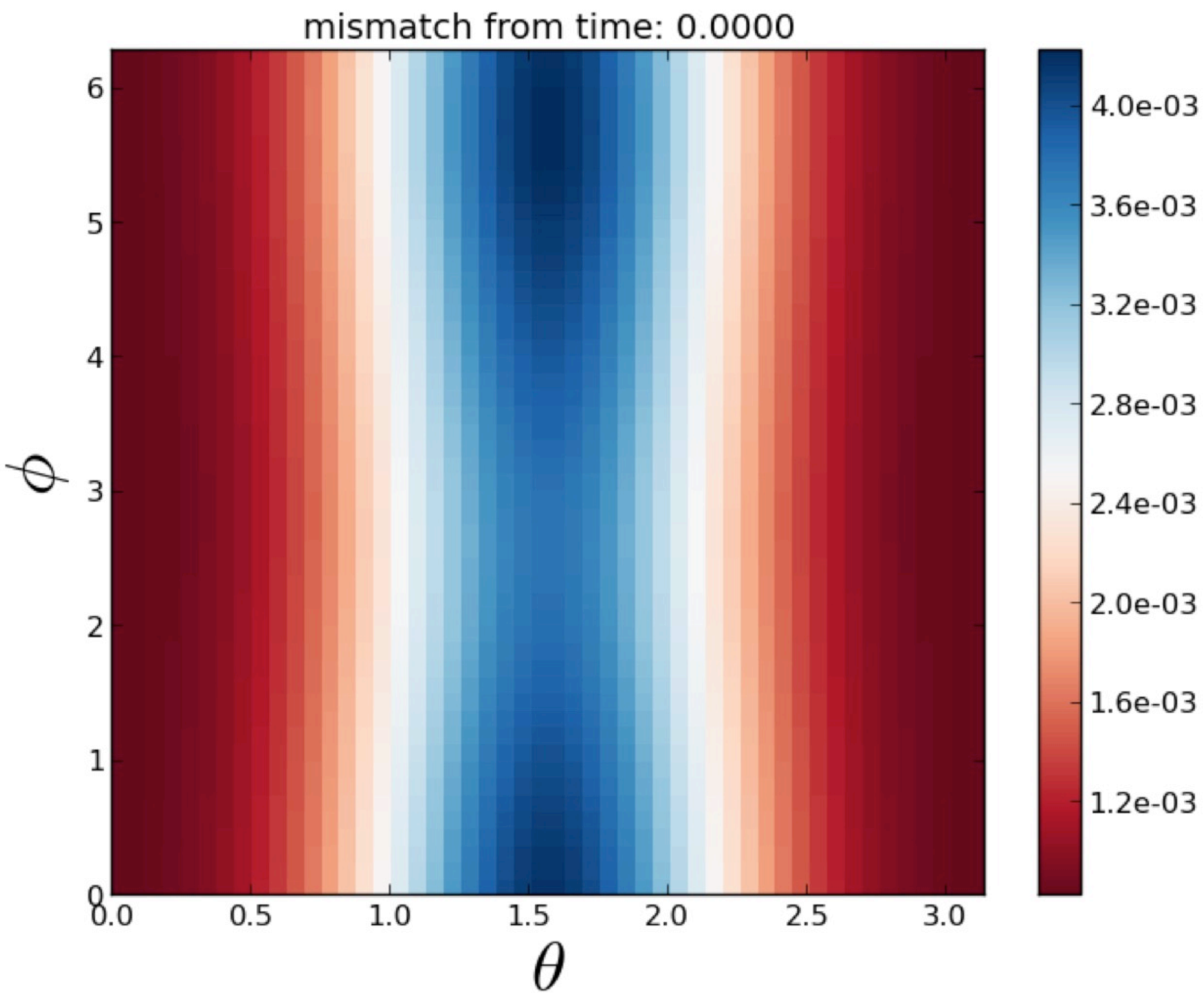


Phenomenological hybrid waveform for an equal mass BBH (green), along with a 3.5PN inspiral waveform (red) and numerical waveform (black). Plot taken from P Ajith et al. 2007 Classical and Quantum Gravity **24** (2007) S680-S699

Ψ_4 (Weyl scalar) vs. $\ddot{h}_+ - i\ddot{h}_\times$



gw_comparison50_mass10_aeiruns_mass_0_ecc_empe_036_empe_023.png



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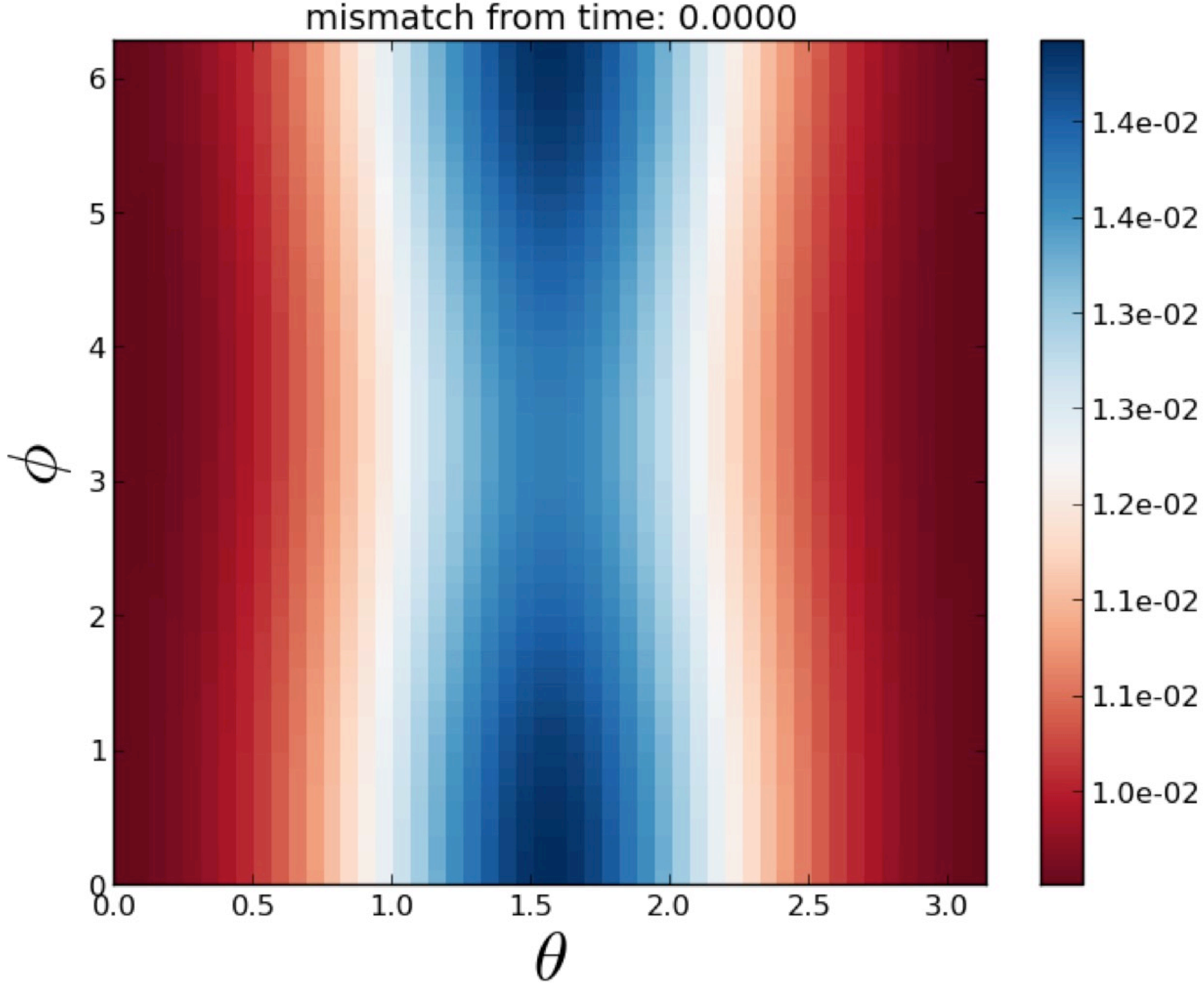

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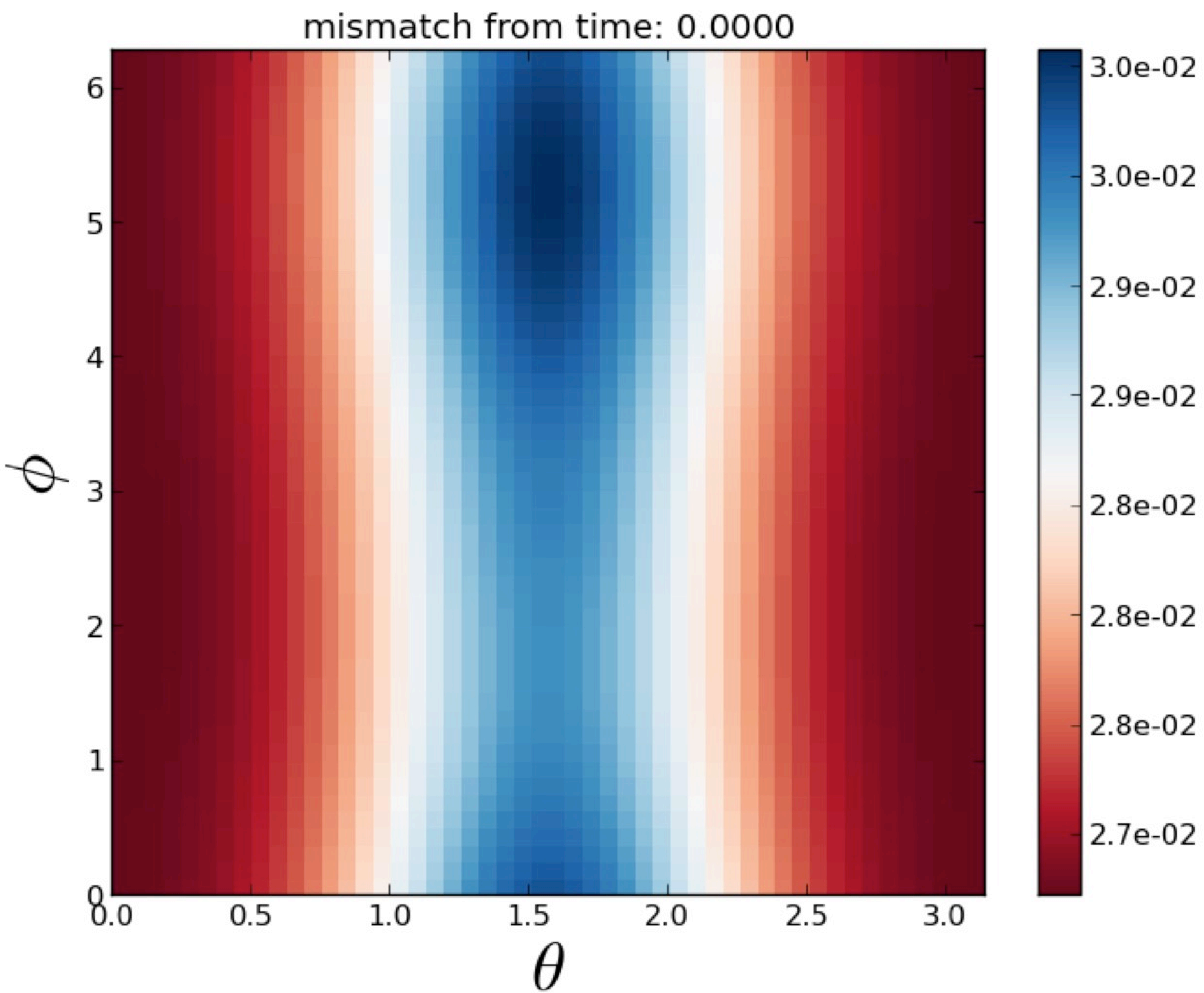

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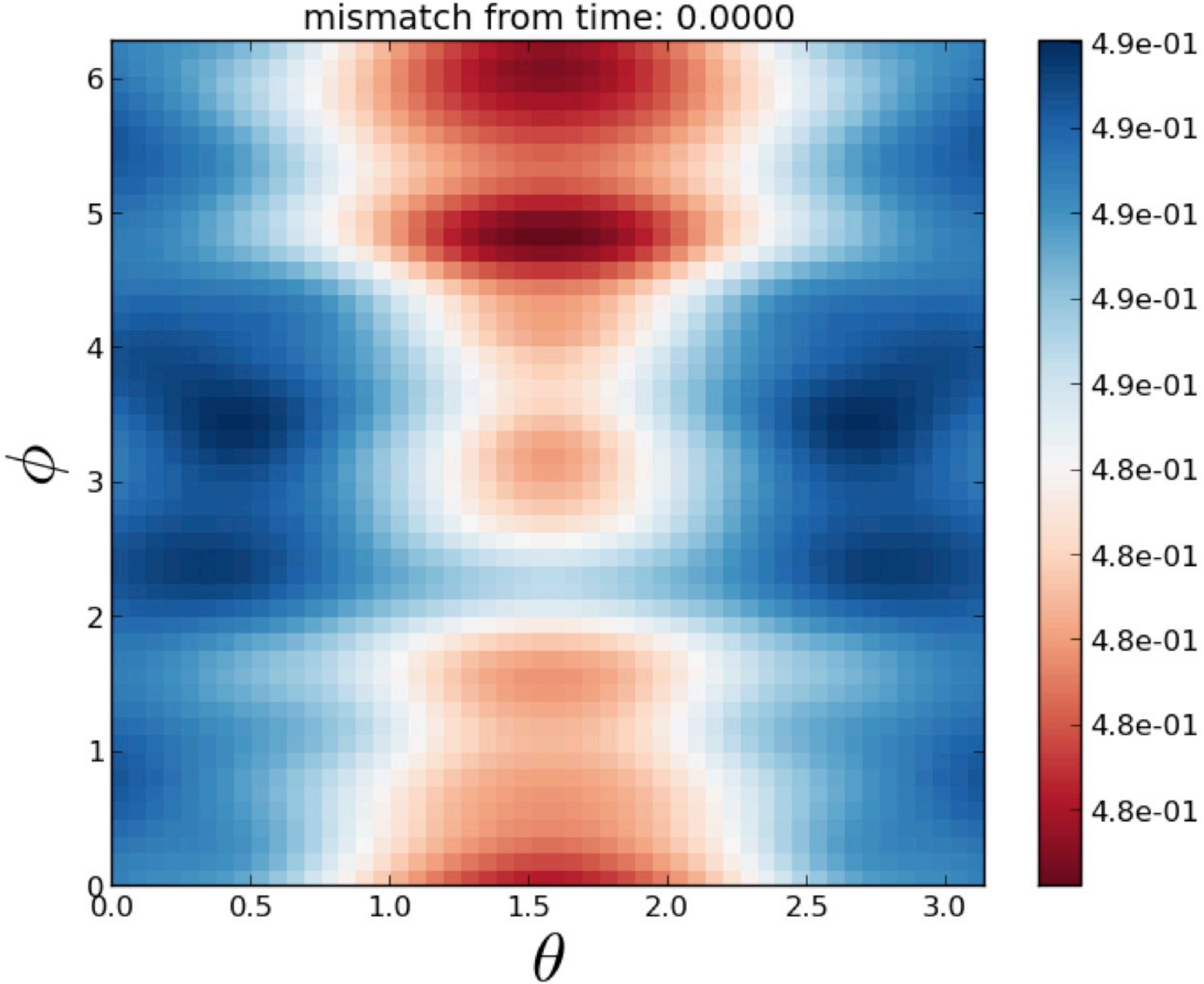

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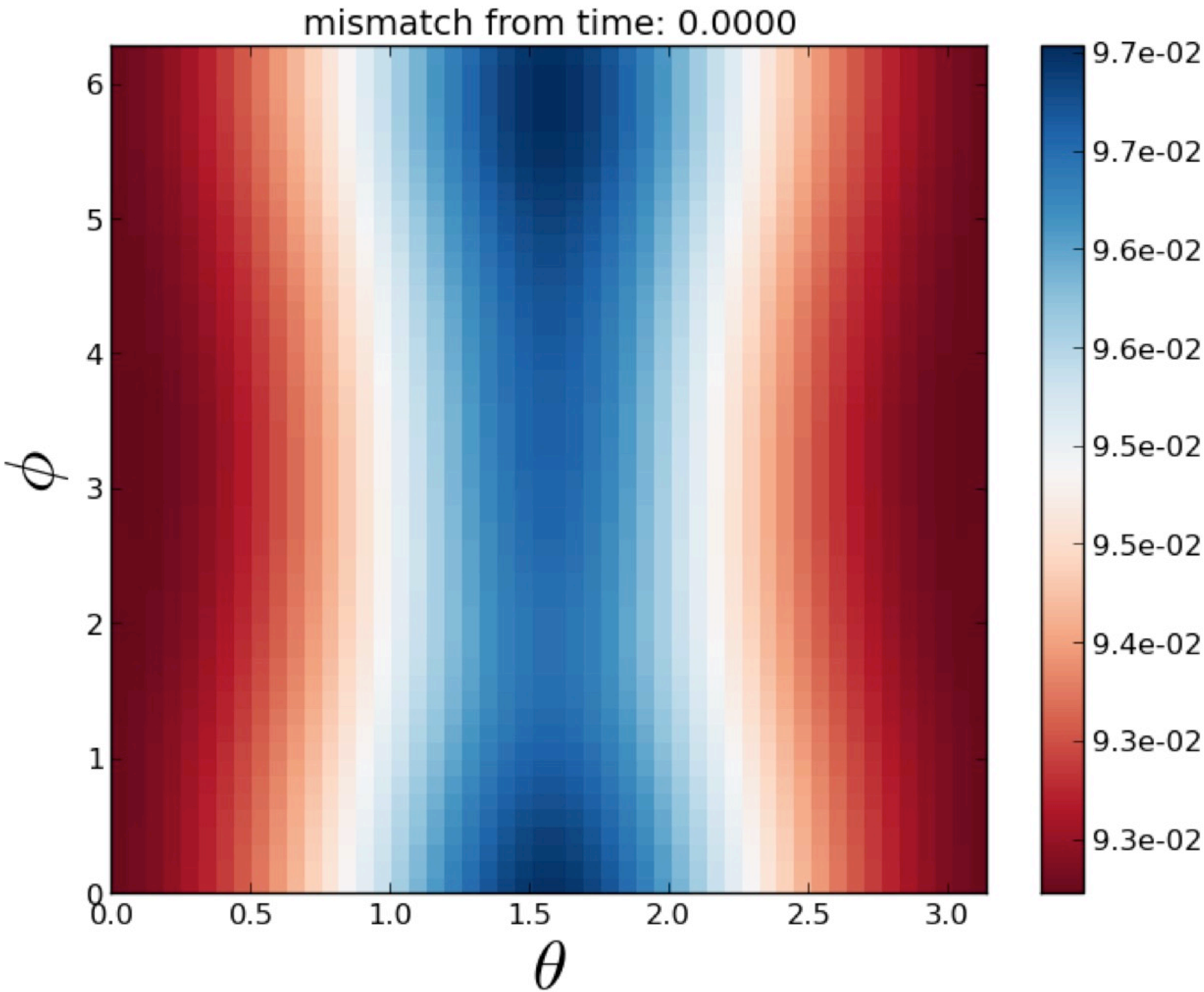

LIGO

gw_comparison50_mass10_aeiruns_mass_0_ecc_empe_026_empe_023.png