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LIGO SURF Proposal: Studying the effects of higher-order modes on the parameter estimation of precessing low-mass binary black holes

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I. INTRODUCTION

LIGO, the Laser Interferometer Gravitational-Wave Observatory, has recently made a huge breakthrough in the field of gravitational wave (GW) astronomy with the first ever detection of gravitational waves [1]. The observed waves were generated by a merging binary black hole system with a total mass of about $70M_{\odot}$ [2]. As theoretically predicted, the detected GW signal evolved in three stages: the inspiral, the merger, and the ringdown. The inspiral waveform matched the waveform calculated using post-Newtonian approximations, while the merger-ringdown waveforms matched the waveforms calculated using numerical relativity simulations.

The inspiral stage of a black hole binary's evolution is characterized by a large separation of the binary and orbital speeds much smaller than the speed of light. Because the motion of the binary is non-relativistic during the inspiral phase, the binary dynamics and the inspiral waveform can be calculated by expanding the Einstein field equations in terms of v/c using post-Newtonian formalism. However, as gravitational waves carry angular momentum and orbital energy away from the system and the orbit of the binary decays, the separation of the binary becomes small and the speeds of the black holes approach the speed of light. At these speeds, which are characteristic of the late inspiral and merger phases of the binary's evolution, the non-linearity of General Relativity becomes significant. At this point, the waveform must be calculated using the numerical solution to the relativistic two-body problem, or numerical relativity. Finally, after the binary black holes merge into a single black hole, the system enters its ringdown stage, which corresponds with the quasi normal modes emitted by the perturbed black hole until it "settles down" to the stationary Kerr solution for spinning black holes. The waveform emitted during the ringdown stage of the binary's evolution must also be calculated using numerical relativity or via perturbation theory.

Because gravitational waves are so weak, compact binary coalescences, such as the merger of two black holes, are among the few systems that can currently be observed by LIGO. Nonetheless, the gravitational waveforms of these objects encode a huge amount of information about each system that would otherwise be inaccessible, as systems like binary black holes cannot be observed through electromagnetic astronomy. Thus, the gravitational waveforms of these systems can help us answer astrophysically interesting questions, like how compact binaries are formed, what are the masses and spins of objects in compact binaries, how they evolve in time, and how these types of systems are distributed in space. Ongoing observations from LIGO will help answer these sorts of questions.

However, while LIGO has now proven that it is capable of detecting gravitational waves from astronomical sources, our ability to use observed gravitational waveforms to analyze the properties of the systems they originated from is still relatively limited. This is due to a number of factors, including the fact that many of the currently used waveform models use the quadrupole approximation [11]. This approximation may lead to parameter degeneracies that prevent us from conclusively determining the physical parameters of an observed system. Such degeneracies include the spin-mass ratio degeneracy, the orientation-distance degeneracy, and many more. For example, Figures 3 and 4 in [3] illustrate the spin-mass degeneracy for binary black holes.

Thus, in order for us to begin to use gravitational-wave observations to answer astronomically interesting questions, we must first improve our ability to probe the properties of the systems we observe. One potential way to break parameter degeneracies and thus improve our parameter estimation abilities is the inclusion of higher modes in our template waveforms [4, 5]. While the quadrupole approximation may be sufficient for highly symmetric binaries, binary systems that contain asymmetries, such as unequal mass ratios and unequal spins, emit more efficiently gravitational-wave energy in higher modes, as, for example, shown in Figures 1 and 2 in [6]. Thus, higher modes carry a significant amount of information about the source system, and the inclusion of higher modes in template waveforms has the potential to allow us to recover large amounts of information from an observed waveform.

II. RESEARCH GOALS

We propose that including higher-order modes in our template waveforms has the potential to break parameter degeneracies, thus allowing us to more accurately recover the properties of an observed system through its detected gravitational waveform. Similar work has been done by O'Shaughnessy et al. [4, 7], who analyzed the inspiral waveforms of select black hole–neutron star binary systems to investigate the possibility of breaking some of these parameter degeneracies. O'Shaughnessy et al. found that for the fiducial system they analyzed, the inclusion of higher order modes broke degeneracies regarding the direction of the system's angular momentum, but otherwise had little impact on the estimation of the systems' parameters.

Our goal in this project is to extend this analysis to a range of precessing binary black hole systems to systematically assess what impact higher order modes have on parameter estimation for the systems we will consider. In particular, we will be analyzing precessing, low-mass binary black holes with various spin configurations and a total mass of less than $12M_{\odot}$. We will not analyze higher total masses as we will only be considering inspiral waveforms, and for black hole binaries with a total mass $\geq 12M_{\odot}$, the merger phase appears in the LIGO band. Using pure inspiral waveforms for such system may result in biased results during parameter estimation [8], hence the restricted analysis. While it is expected that the effects of higher modes will be more significant during the merger phase than during the inspiral phase, we do not currently have models for merger waveforms that would enable us to study these effects, which is why our analysis will be limited to inspiral waveforms.

III. METHODS AND PROJECT OUTLINE

A. Approach

Once we have modified pre-existing waveform-generating codes to allow the inclusion of higher modes, we will construct a set of model precessing low-mass black hole binaries to analyze. In addition to the total mass constraint of $12M_{\odot}$ mentioned previously, we will construct our binary cases based on the parameter degeneracies we want to address in this study: the distance–orientation degeneracy and the spin–mass-ratio degeneracy. To study the effects of higher modes on those degeneracies, we will choose our binary cases accordingly, varying the mass ratio, the spins and the binary orientation for a fixed signal-to-noise ratio. We will then generate the gravitational waveforms produced by these systems, including higher modes. For our higher modes analysis, we will be using the SpinTaylorT4 post-Newtonian waveform model [9].

We will then analyze the resulting simulated waveforms, which contain higher modes, to recover the parameters of our model systems. Our parameter recovery will be performed once using the quadrupole approximation and once again including higher modes. This will enable us to compare the results from each parameter estimation run and study the effects of including higher modes.To measure the source parameters, we will be performing a *Bayesian analysis* using the tools offered in the LALInference software library for Bayesian parameter estimation [10].

In general terms, the process of Bayesian analysis is as follows. First, a probability distribution, known as the *prior distribution* and represented as $p(\vec{\theta})$, is constructed for each set of parameters $\vec{\theta}$ in the binary parameter space. Each set of parameters in the binary parameter space includes values like component masses, spins, position on the sky, distance to the source, orientation of the binary, and many more. The prior distributions we will use in our project consist of uniform distributions over a given range for parameters like component masses and spin magnitudes, and isotropic distributions over the unit sphere for the spin orientation.

After a prior distribution has been constructed, the likelihood $L(d\,|\,\vec{\theta})$ of the observed data d given the parameters $\vec{\theta}$ is calculated. Then, we multiply the likelihood function by the prior distribution $p(\vec{\theta})$ and normalize by $p(d)$, the probability of the data independent of the distribution of the parameters also known as the evidence. This gives us the posterior probability distribution, $p(\vec{\theta}\,|\,d)$, as described by Bayes' Theorem given in Eq. (1):

$$p(\theta\,|\,d) = \frac{L(d\,|\,\theta)\; p(\vec{\theta})}{p(d)} \tag{1}$$

The parameter estimate is then given by the median value of the posterior distribution, and the error is given by the values that correspond to the 90% credible interval, or the parameter values that enclose 90% of the probability of the distribution. We will then compare the results of the two parameter estimation runs for each binary case, and analyze how the inclusion of higher modes affects our ability to accurately constrain the parameters of the model system. This will enable us to determine whether or not the inclusion of higher modes breaks various parameter degeneracies in the inspiral case.

B. Timeline

An estimated timeline of our project is as follows:

- **Pre-start**: Prior to the start date of the program, we will select our binary cases, construct waveform templates for the binary cases using the quadrupole approximation, and begin parameter estimation runs.
- **Weeks 1-2**: First, we will get started with `lalsuite` on a computing cluster. Afterwards, the time will be dedicated to modifying current code infrastructures to enable the inclusion of higher modes in our waveform templates, using the SpinTaylorT4 waveform model
- **Weeks 3-6**: We will simultaneously run parameter estimation algorithms using higher modes, and begin the analysis of the results from the parameter estimation runs that used the quadrupole approximation.
- **Weeks 7-8**: We will work on comparing the results from the quadrupole approximation parameter estimation with the results including the higher modes and write up our findings.
- **Weeks 9-10**: Prepare the final presentation and report.

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[11] The gravitational wave radiation field can be decomposed into many various modes, and, depending on the binary configuration, some modes may carry more energy than others. As a result, current template waveforms include only a few, dominant modes of the gravitational wave radiation field, in what is called the quadrupole approximation.