Test of the Second Law of Black Hole Thermodynamics with the LIGO event GW150914

Unnikrishnan. C. S.
Tata Institute of Fundamental Research,
Homi Bhabha Road, Mumbai 400005, India

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Abstract

The recent LIGO discovery of the binary black holes merging and forming a single Kerr black hole provides the first and unique opportunity to test the black hole area-entropy theorem or the second law of black hole thermodynamics. We discuss the test of the entropy law using the mass and spin estimates from the LIGO event GW150914. Because both the initial and final states consist only of black holes with high entropy and coherent gravitational waves with very low entropy, the test is essentially geometrical and ideal. However, the precision and the test itself are limited by interdependencies and errors in parameter estimation. Future studies on similar BBH events are critical precision tests of black hole area-entropy theorem.

I. BLACK HOLE AREA-ENTROPY THEOREM

The relation between the classical area of the black hole horizon and to the entropy in black hole thermodynamics is well known [1]. The second law of black hole thermodynamics states that the area-entropy $S_{bh} = A/4l_p^2$, where A is the area of the black hole horizon and l_p the Planck length , cannot decrease in any classical physical process. The generalized second law then is that the sum of the entropies of the thermodynamical system outside the black hole and that of the black hole cannot decrease in any physical process that includes interactions of the two. For a real astrophysical Kerr black hole with spin (and relatively negligible charge as we expect), the entropy law includes the angular momentum, with the area given by

$$A = 4\pi(r_h^2 + r_j^2) \tag{1}$$

where $r_h = r_M + \left(r_M^2 - r_j^2\right)^{1/2}$ with $r_M = GM/c^2$ and $r_j = J/Mc$ is the specific intrinsic angular momentum. The dimensionless spin parameter $a = r_j/r_M$.

For the special case of two black holes in a binary system spiralling in emitting gravitational waves, the initial entropy is purely geometrical, that of the two black holes, with no other form of entropy present. Since the final state also is a black hole, the final entropy is also purely geometrical, but for the entropy in the emitted gravitational waves. However, in this specific case of binary black holes or neutron stars, the gravitational waves are coherent and quasi monochromatic, with a chirp waveform that increases in frequency and amplitude all the way to merger. Since the gravitational wave coherent state is quantum mechanically a pure state [3], its entropy is zero. Even if the actual wave is mildly multi-modal, the entropy is negligible, at most the logarithm of number of modes. This then is the most ideal situation to test the area-entropy law for black holes and we expect that the sum of the geometrical entropies of the binary black holes is nearly equal to that of the final black hole. This puts fairly severe constraints on the spin of the final black hole, especially if the initial spins are insignificant compared to the total orbital angular momentum.

Remarkably, the recent LIGO gravitational wave event GW150914 [4] is exactly such a geometrically pure astrophysical event in which to stellar mass black holes with comparable masses coalesced into a single Kerr black hole emitting about 5% of the total gravitational energy in gravitational waves. We discuss how well this event tests the second law of black hole thermodynamics and show that such events will provide precision tests of the area-entropy law as the sensitivity of multiple detectors improves.

II. LIGO EVENT GW150914

The LIGO event GW150914 is the centre of attention at present and has been discussed extensively. For our analysis we only need the set of parameters estimated from the analysis of the chirp waveform reconstructed from the signals detected in the aLIGO detectors at Hanford and Livingston [4, 5]. The masses are determined to about 10% accuracy. These are, for the BH in the binary, $M_1/M_\odot = 36.3^{+5.3}_{-4.5}$, $M_2/M_\odot = 28.3^{+4.4}_{-4.2}$ and $M/M_\odot = 62.0^{+4.4}_{-4.0}$. The initial spins are not well-determined, except as an upper limit $a_1 < 0.65$ for the heavier BH in the binary. The actual parameter estimates are consistent with nearly zero or spin smaller than $a \simeq 0.1$. The spin of the final BH is much better determined because it is dominated by the orbital angular momentum of binary, which is well determined as $a = 0.67^{+0.06}_{-0.08}$ from the chirp waveform and mass parameters.

III. TEST OF BH AREA-ENTROPY THEOREM

The validity of the black hole area-entropy theorem and the second law of BHT demands that $S_{bh} + S_{GW} \geq S_1 + S_2$. We have already reasoned that S_{GW} is negligible for the coherent quasi-monochromatic waves. Therefore we test for $S_{bh} \geq S_1 + S_2$ noting that S_{GW}, if any, makes the inequality more valid. This is equivalent to testing $A_f \geq A_1 + A_2$. The area of the horizon of a Kerr black hole is given by

$$A = 4\pi(r_h^2 + r_j^2) \tag{2}$$

with

$$r_h = \frac{GM}{c^2} + \left[\left(\frac{GM}{c^2}\right)^2 - \left(\frac{J}{Mc}\right)^2\right]^{1/2} = r_M + \left(r_M^2 - r_j^2\right)^{1/2} \tag{3}$$

We get

$$A = 4\pi(r_h^2 + r_j^2) = 8\pi r_M^2\left(1 + \frac{\left(r_M^2 - r_j^2\right)^{1/2}}{r_M}\right) = 8\pi r_M^2\left(1 + (1 - a^2)^{1/2}\right) \tag{4}$$

For the Schwarzschild BH without spin, $a = 0$, and horizon radius $r_S = 2GM/c^2$, the area $A = 16\pi r_M^2 = 4\pi r_S^2$.

For GW150914, we have $A_1 \leq 16\pi \left(GM_1/c^2\right)^2$ and $A_2 \leq 16\pi \left(GM_2/c^2\right)^2$, where $M_1 = (29 \pm 4)\, M_\odot$ and $M_1 = (36 \pm 5)\, M_\odot$. For the final black hole, the estimated parameters for mass $M_1 = (62 \pm 4)\, M_\odot$ and spin $a \leq 0.7$. In principle, and ideally, the mass of the final black could be estimated from the ring-down part of the detected waveform, which makes it independent of the chirp part of the waveform. The frequency of the lowest mode (n=0,l=2), is inversely proportional to the mass, and one expects about 300-350 Hz for the gravitational waves for the spinning 60 solar mass black hole [6] . However, for this particular first detection, detailed numerically relativity fits with general relativistic continuity of the whole waveform, with parameters of the binary, had to be used to obtain accurate estimate of the parameters of the final black hole [5]. This already implies an agreement with the area-entropy law [7]. However, the stand taken in this paper is that the area-entropy law as a law of black hole thermodynamics is a general deep feature of gravity, independent of general relativity and that it could be as fundamental as the conservation laws. In this sense, the law is a fundamental constraint on all theories of gravity (there could be small quantum corrections, irrelevant for astrophysical situations). For example, one can think of estimating the allowed mass-spin parameter space of the final black hole using only the thermodynamical relation and not the explicit general relativistic calculations and fits, when the final part of the waveform is not available. We assume that future improvements in sensitivity will allow fairly good estimates of the mass of the final black hole from ring down and does not use the masses of the initial black holes in an essential way, allowing genuine precision tests. To indicate how the test goes, we nevertheless use the mass and spin estimates for the final black hole as estimated using numerical relativity fits [5] and get

$$A_f = 8\pi r_0^2 \left[1 + (1 - a^2)^{1/2}\right] \geq (3.61 \pm 0.47) \times 10^{11} m^2 \tag{5}$$

Similarly we can estimate the sum of the areas of the (non-spinning) black holes in the binary using the their mass estimates and the knowledge that spin values are consistent with $a < 0.1$ as,

$$A_1 + A_2 < (2.36 \pm 0.48) \times 10^{11} m^2 \tag{6}$$

This is very interesting because the fairly close (blurred by the statistical uncertainties) boundaries show that the the final black hole could not be spinning much faster if the smaller two had no significant spin to start with. For example, with both initial spins near zero, the final black hole spinning at $a > 0.95$ may indicate a violation of the second law of black hole thermodynamics. In other words, the second law of black hole thermodynamics is useful to constrain spin value of the final black hole, with only partial knowledge of its other relevant parameters. We can already see that if the spin parameter is known, the mass parameter indeed can be estimated using the second law alone because with only black holes and waves in the initial and final states, we expect the area-entropy theorem satisfied as a near equality.

There is another interesting issue that could be raised. In such a clean system with only geometric entropies, one does not expect the final black hole to have significantly larger entropy than the sum of the two initial entropies. This suggests that the central value of the final black hole could be somewhat smaller or that the spin could be slightly larger than the estimated value. A proper treatment of this should estimate the relveant probabilities for the agreement with the area law and this has been addressed in the context of the test of the area theorm for GW150914 by W. del Pozzo and S. Vitale [8]. However, a good test requires higher precision in parameter estimates and this is expected in some future such events, when the detector is operating at its full sensitivity. Yet, it is clear that the first detection by LIGO has shown the opportunity to test the second law of black hole thermodynamics for the first time in a real astrophysical event.

IV. CONCLUDING COMMENTS

The LIGO event GW150914 is remarkable in that apart from the double discovery of action of gravitational waves on a terrestrial detector, it also discovered the first stellar mass binary black hole system that spiralled in and merged to form a single Kerr black hole, promptly ushering in a new wave in astronomy. While straightforward, it is truly interesting that the first observation already allows physical tests only imagined so far in highly theoretical and hypothetical scenarios. This by itself does not prove that ideas of black hole thermodynamics and its hypothetical, yet essential, quantum basis are correct. However, this seems to be a first step with its own deep significance. No doubt, future BBH events in aLIGO and other future detectors in the network will allow precision tests of the second law of black hole thermodynamics, and perhaps more, and a violation would of course be highly significant and paradigm changing.

[1] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).

[2] S. W. Hawking, Phys. Rev. Lett. 26, 1344 (1971).

[3] C. S. Unnikrishnan and G. T. Gillies, Class. Quantum Grav. **32,** 145012 (2015).

[4] B. P. Abbott, *et al.,* LIGO Scientific Collaboration and Virgo Collaboration, Phys. Rev. Lett. **116**, 061102 (2016).

[5] B. P. Abbott, *et al.*, LIGO Scientific Collaboration and Virgo Collaboration, *Properties of the binary black hole merger GW150914*, LIGO document P1500218 (2016).

[6] K. D. Kokkotas and B. G. Schmidt, *Quasi Normal Modes of Stars and Black Holes*, Living Reviews in Relativity, lrr-1999-2.

[7] I thank Badri Krishnan for emphasizing and pointing this out to me.

[8] W. del Pozzo *et al.*, GW150914: experimental verification of Hawking's area theorem, LIGO Document P1600008-v2 (22/01/2016).