Sharper TF Representations of GW150914

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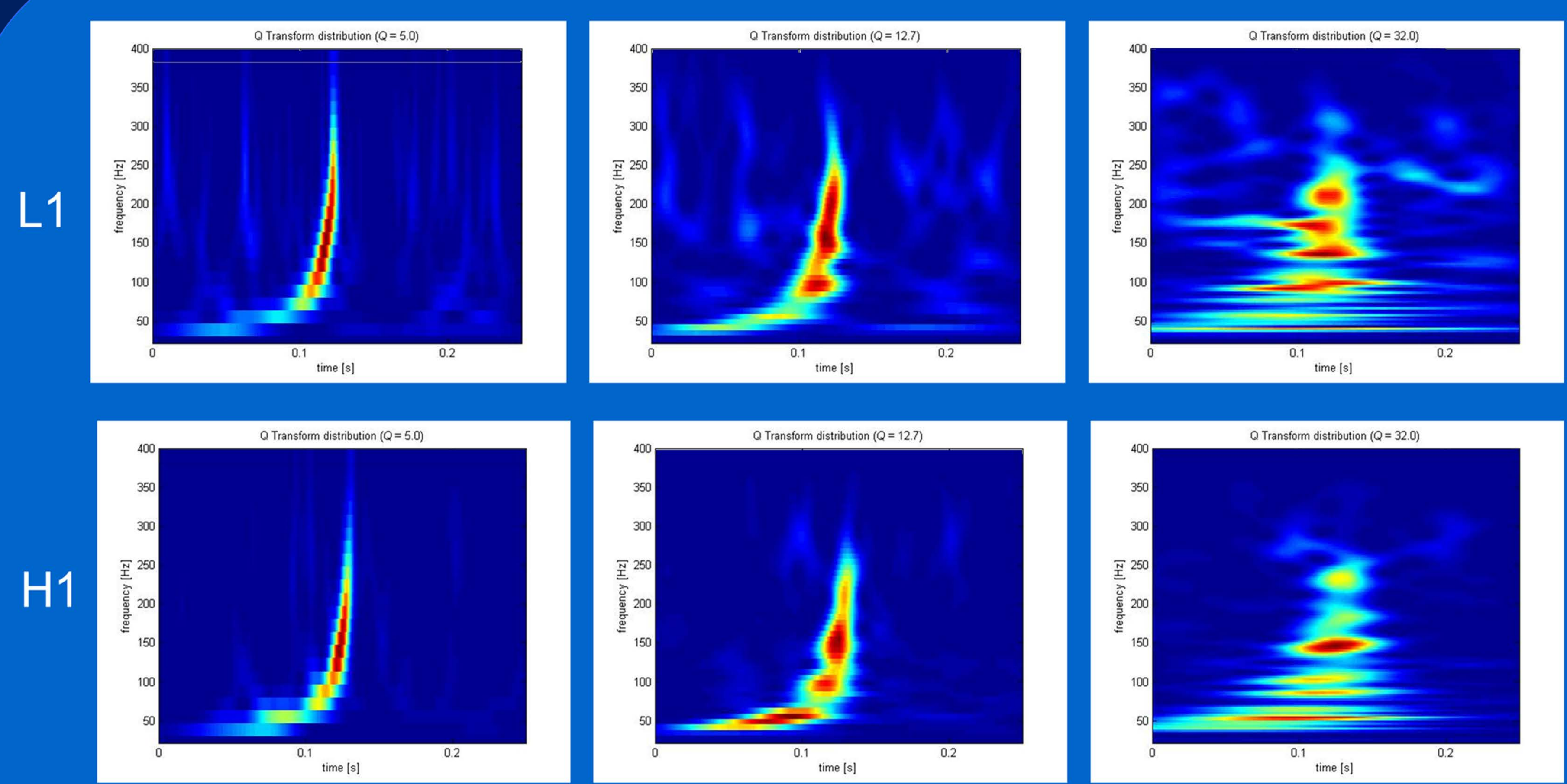
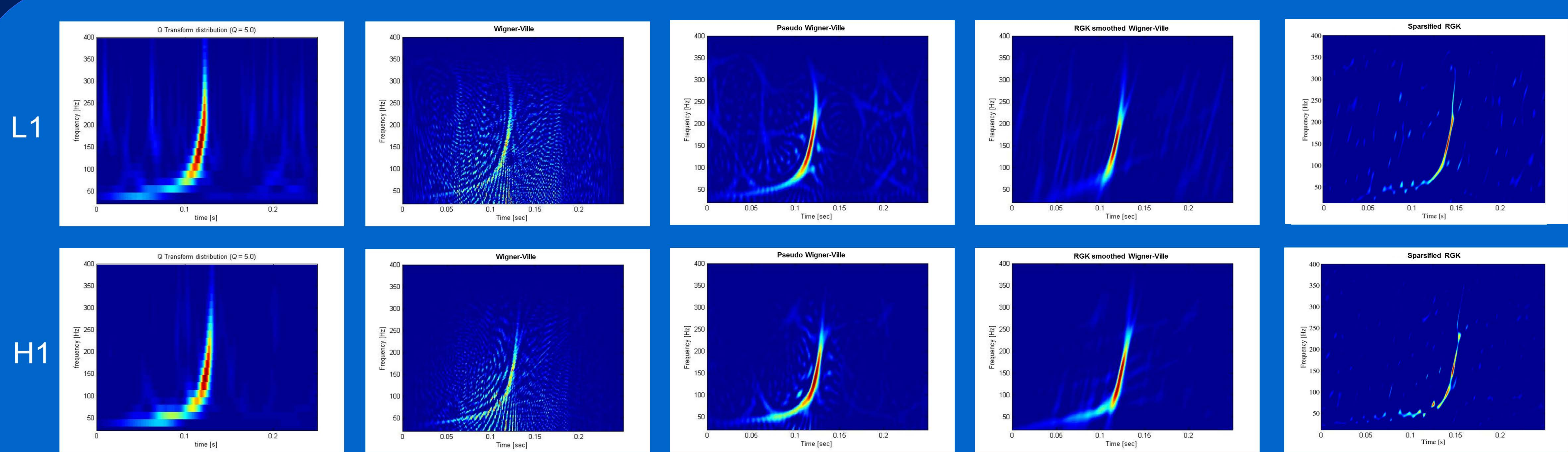


Constant-Q transforms of GW150914 data from L1 and H1 for various Q values.

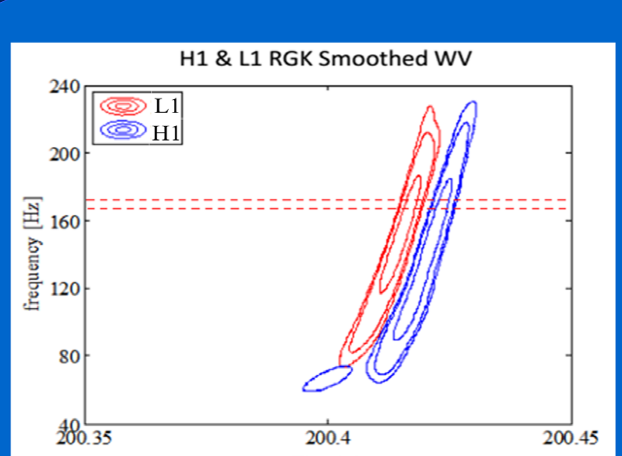
Most time-frequency representations in use today in GW data analysis (including the Q, Omega and Omicron pipelines) are based on the Q-transform [1].
Many alternative TF representations exist. The Wigner-Ville transform features the uniformly largest TF resolution, and nice marginal properties, but is plagued by intermodulation artifacts, due to its nonlinearity [2]. The radial-gaussian kernel (RGK) smoothed Wigner-Ville [3] offers a clever tradeoff between artifact suppression and loss in resolution. Resolution can be restored, in part, using reassignment [4], or enforcing a sparsity constraint [5].

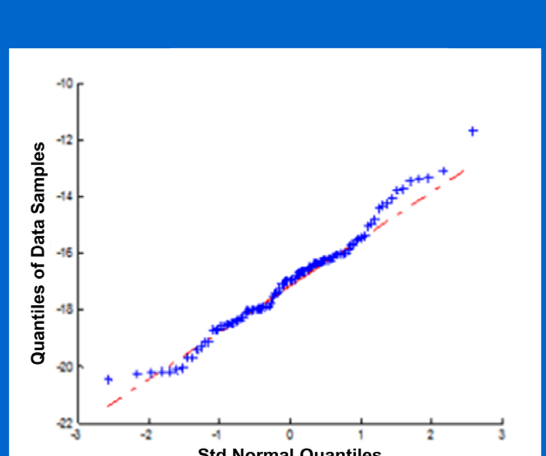
The (sharpest) Q-transform, the Wigner-Ville (WV), and some related TF distribution are visually compared below.



Alternative time –frequency representations of GW150914 (data resampled at 4096 Hz).
Left to right: Q-transform (Q=5); Wigner-Ville; pseudo Wigner-Ville (121 bin Hanning window); radial-gaussian kernel smoothed Wigner-Ville (a la Baraniuk – Jones); sparsified radial-gaussian kernel smoothed Wigner-Ville.

Improved visual sharpness allows accurate estimation of H1-L1 delay using sub-pixel accurate image co-registration algorithms. It also makes easy to estimate the TF distribution *ridge (peak locus)*, representing the *bona-fide* frequency evolution of the signal. The ridge fits well the instantaneous frequency line computed, e.g., from an IMR-PhenomB model using the official estimated chirp-mass of GW150914.



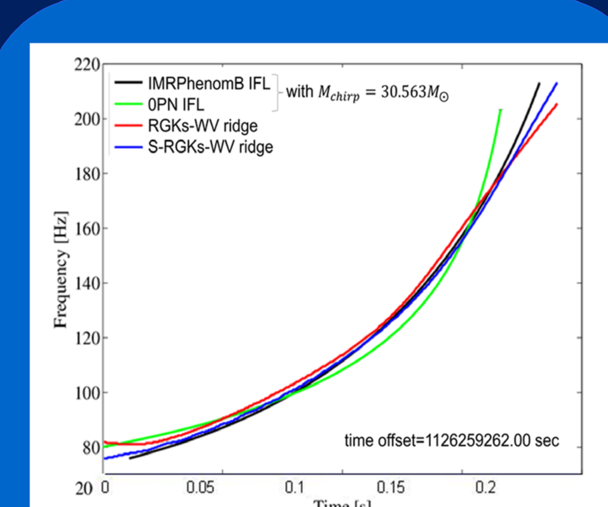


Estimated Delay (& 95% c.i.)

QT (Q= 5.0)	6.57 ms (6.29 ms ÷ 6.97 ms)
WV	7.01 ms (6.86 ms ÷ 7.20 ms)
RGKs WV	6.83 ms (6.78 ms ÷ 6.89 ms)
S-RGKs WV	7.01 ms (6.85 ms ÷ 7.19 ms)

QT= Q Transform
WV = Wigner-Ville
RGKs WV = RGK smoothed WV
S-RGKs WV = sparsified RGKs-WV

Arrival time delay estimation based on TF contour co-registration of GW150914 data from L1 and H1.
Left: the RGK-smoothed WV contour plots of the GW150914 data from L1 and H1 are segmented into 78 horizontal stripes between 70Hz and 250Hz, and for each stripe the time-shift is estimated using the PCR algorithm [5].
Center: quantile-quantilethe plot of the time-shift population residuals, showing almost Gaussian behaviour.
Confidence intervals are estimated using Efron bootstrap technique. Right: the results for different TF representations.



IMR-PhenomB and 0PN frequency evolutions (both with $\mathcal{M}_{chirp} = 30.563 M_{\odot}$) and ridges of the TF RGK-smoothed and sparsified RGK-smoothed Wigner Ville distributions (GW150914 H1 data)

References

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[4] P. Flandrin and P. Borgnat, "Time-Frequency Energy Distributions Meet Compressed Sensing," IEEE Trans. **SP-58** (2010) 2974.
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