Parametrization of the effect of weak interactions on the production of heavy elements in binary neutron star mergers.

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1 Abstract.

Recent research[1] has shown that material ejected during binary neutron star mergers (BNSMs) is a likely source of about half of our galaxy's heavy nuclei created. These nuclei are synthesized via rapid neutron capture, or the r-process. We investigate the effect of positron, electron, electron neutrino and antineutrino capture reactions on the r-process in BNSM ejecta, which is uncertain and has only received significant attention recently. To do this, we modify the nuclear reaction network SkyNet to include the aforementioned interactions. Then, we parametrize the neutrino luminosities and entropy of the outflow to study their effect on the evolution of electron abundance, isotopic abundances, and reaction rates in the BNSM ejecta. We then compare our results to the observed solar abundances of r-process elements to gauge the accuracy of our simulations. Results show that, for a reasonable set of parameters, neutrino luminosities at or above 10^{53} ergs/s may significantly limit the production of heavy elements of mass number >120. We also find that increasing entropy decreases the sensitivity of the electron abundance to neutrino luminosities. Similarly, at high neutrino luminosities (around 10^{53} ergs/s), the final electron abundance becomes less sensitive to entropy. The increase in electron abundance caused by increasing luminosity and entropy is also related to how rapidly the ejecta expands away from the collision, parametrized as τ. Some further research may investigate the ratio of neutrino to antineutrino luminosity as a parameter, the inability to form heavy elements >200 in our neutrino-including r-process simulations, and more accurate modeling of neutrino outflow.

2 Introduction.

The formation of heavy elements with mass number greater than Fe-56 is perhaps the last remaining mystery about the origin of the elements in the universe. While elements up to $^{56}_{26}$Fe are created via stellar fusion, the fusion of heavier elements is blocked by a decreasing binding energy per nucleon and increasing repulsive force between free protons and the nuclei[2]. Thus, these heavy elements are thought to form through what are called the slow and rapid neutron capture processes, or the s and r-processes. In the r-process, neutrons are captured on heavy nuclei on a timescale that is much shorter than the beta-decay half-lives of the heavy nuclei, forming heavy unstable nuclei on a short timescale, which then decay into stable heavy elements after neutrons are exhausted. Recent research has identified binary neutron star mergers as strong candidates for the primary source of heavy elements in the universe, in part because their high neutron flux makes them a suitable environment for the r-process[1]. Simulations have shown that binary neutron star mergers can produce 10^{-2} to 10^{-3} $M_\odot$ of r-process material[1].

Not everything about the potential r-process in binary neutron star mergers is well-known. In particular, the impact of neutrino interactions on the neutron density and the resultant composition of the ejecta has not been closely investigated. Neutrino capture reactions and their inverses are given by the following equations[2],

$$n + e^{+} \rightleftharpoons \bar{\nu}_e + p$$

$$p + e^{-} \rightleftharpoons \nu_e + n.$$

Neutrinos and antineutrinos can interact with nucleons to form electrons ($\nu_e + n \rightarrow p + e^{-}$) or destroy protons ($\bar{\nu}_e + p \rightarrow n + e^{+}$). Their inverse reactions, the positron and electron captures, can create protons or destroy electrons. In this way the weak interactions can change the initial neutron abundance, i.e. the total number of neutrons present prior to the r-process, in the binary neutron star merger ejecta. The effect neutrino interactions have on the r-process have only just begun to be studied within the past year, in part because the outflow and behavior of neutrinos are difficult to constrain in highly dense matter like a binary neutron star merger.

In this study, we present simulations of r-process nucleosynthesis using the nuclear reaction network Skynet, developed by Jonas Lippuner and Luke Roberts. Adjustments were made to Skynet's code such that its r-process simulations would incorporate the effect of weak interactions. The rate of weak interactions depends on the temperature, electron density, and the neutrino density of the ejected material. Using Skynet, we parametrized how neutrino interactions affect the r-process. To do this, we chose neutrino luminosity, entropy, and dynamical time scale as parameters to vary over a range of physically reasonable values. We then ran simulations with these values and study how these parameters affect the r-process by examining the data output by Skynet.

Through our parametrization of the effect of neutrino interactions, we observe that the ejecta electron abundance increases with increasing values of neutrino luminosity and fluid entropy. We find that between entropy and luminosity, the increase of one parameter decreases the sensitivity of electron abundance to the other parameter. We find that this is due to the competition between the forward and reverse weak reaction rates. At high luminosity, neutrino/antineutrino captures dominate the creation and destruction of neutrons . At high entropy, positron/electron captures dominate. We find that, for a chosen set of parameters, the critical luminosity at which the r-process fails is 10^{53} ergs/s. However, it is important to note that the critical value of luminosity or entropy at which heavy elements fail to be made, depends on the rate at which the ejecta expands, τ.

3 Modeling the neutrino interactions.

As previously mentioned, neutrino interactions can affect the formation of heavy elements by raising or lowering the neutron density.

The mechanism by which neutrino interactions do this involves raising or lowering the electron abundance of the ejecta. Abundance describes the number of a given particle within the matter being studied, and changes only when the particle is destroyed or created. Abundance is unaffected by changes in the density of the matter. In order to understand the relationship between neutron density and electron abundance, we begin with the statement that the protons and neutrons essentially make up the total mass of the ejecta:

$$X_p + X_n = 1$$

Where X_y is the mass fraction of species y. The mass fraction of protons can be written as the sum total of protons in the matter over the total mass in the matter:

$$X_p = \frac{\sum_i Z_i Y_i}{\sum_i X_i}$$

Where, being the total mass of the matter, $\sum_i X_i = 1$. Matter tends to be electrically neutral because charged particles experience electromagnetic force that quickly corrects charge imbalances. Thus, there are as many electrons as there are protons in the matter:

$$Y_e = \sum_i Z_i Y_i$$

$$Y_e = \frac{\sum_i Z_i Y_i}{\sum_i X_i}$$

$$Y_e = X_p = 1 - X_n$$

If we consider that mass fractions are essentially abundances divided by the total mass fraction, 1, then this can be written as:

$$Y_e = Y_p = 1 - Y_n$$

Increasing electron abundance decreases neutron abundance, and thus neutron density.

The rate of change of the electron abundance is related to the capture rates of the neutrino interactions and the amount of heavy nuclei present, as described in Goriely[1]. The importance of heavy nuclei is due to what is known as the alpha effect. As protons become bound in nuclei and less are available to interact, they leave free neutrons as the primary targets of neutrino interactions. These neutrons are destroyed and produce electrons, raising the electron abundance.

For the purpose of showing the relationship between neutrino captures and electron abundance during early times of an NS-NS merger, we will consider an equation describing ejecta with only free neutrons and protons. Neutrino and positron captures both destroy neutrons, which, as we previously discussed, reduces neutron density and increases electron abundance. We will group their capture rates together as $\lambda_+ = \lambda_{\nu_e} + \lambda_{e^+}$. Antineutrino and electron capture rates destroy protons, which reduces the proton mass fraction and thus the electron abundance. Their capture rates will also be grouped together as $\lambda_- = \lambda_{\bar{\nu}_e} + \lambda_{e^-}$. Thus, the following equation for the rate of change of electron abundance can be written as follows:

$$\dot{Y}_e = \lambda_+ Y_n - \lambda_- Y_p$$

Neutrino and positron captures produce electrons drive up Y_e and depend on the amount of neutrons available to be captured on, while antineutrino and electron captures reduce Y_e and depend on the amount of protons available.

In neutrally charged matter, $Y_e = Y_p = 1 - Y_n$.

Thus, this equation can be rewritten entirely in terms of Y_e such that:

$$\dot{Y_e} = \lambda_+ - (\lambda_+ + \lambda_-)Y_e$$

$$\dot{Y_e} = \lambda_{total}(\frac{\lambda_+}{\lambda_{total}} - Y_e)$$

Where $\lambda_{total} = \lambda_+ + \lambda_-$.

The term $\frac{\lambda_+}{\lambda_{total}}$ has no radius-dependent factor and depends solely on the ratio of neutrino and antineutrino luminosity, cross-sections, and mean energies. According to this equation, Y_e is bounded by the value this term takes and will always approach that value. Because of this, $\frac{\lambda_+}{\lambda_{total}}$ is called the Y_e equivalence value or Y_{eq}. Expanded from our notation for collected capture rates,

$$Y_{eq} = \frac{\lambda_{\nu_e} + \lambda_{e^+}}{\lambda_{\nu_e} + \lambda_{e^+} + \lambda_{\bar{\nu_e}} + \lambda_{e^-}}$$

As mentioned in the introduction, the forward and reverse rates of neutrino interactions compete against each other in dominating the evolution of Y_e. In order to determine which rates are dominating, it is useful to look at Y_{eq} only in terms of forward capture rates, or only in terms of reverse capture rates. In that case, we may write these equilibrium values as:

$$Y_{eq,\ nu+nubar} = \frac{\lambda_{\nu_e}}{\lambda_{\nu_e} + \lambda_{\bar{\nu_e}}}$$

$$Y_{eq,\ e+pos} = \frac{\lambda_{e^+}}{\lambda_{e^+} + \lambda_{e^-}}$$

The capture rates of neutrinos, antineutrinos, positrons and electrons can be described in terms of their number density multiplied by their capture cross-section. For neutrinos and antineutrinos, luminosity (total energy), mean energy, and $4\pi r^2$ describes the number density of neutrinos at a radius r in the equations given below:

$$\lambda_{\nu_e} = \frac{L_{\nu_e} < \sigma_{\nu_e} >}{4\pi r^2 < E_{\nu_e} >} \qquad \lambda_e = cn_e < \sigma_e >$$

$$\lambda_{\bar{\nu_e}} = \frac{L_{\bar{\nu_e}} < \sigma_{\bar{\nu_e}} >}{4\pi r^2 < E_{\bar{\nu_e}} >} \qquad \lambda_{e^+} = cn_{e^+} < \sigma_{e^+} >$$

Where for species x, λ is the capture rate, L_x is the luminosity, $< \sigma_x >$ is the capture cross-section, $< E_x >$ is the mean energy, and n_x is the number density.

The given equation gives a simplified radius-dependent fall-off for the neutrino capture rates, which does not hold close to the center of the merger. SkyNet uses a more sophisticated approximation for neutrino outflow. While the capture rates for electrons and positrons are determined by the known temperature, density, and electron fraction of the outflow, the neutrino and antineutrino capture rates are uncertain due to our ignorance of the neutrino fluxes emitted from the remnant of the NSNS merger. This uncertainty is because the neutrino outflow is very dynamic and computationally expensive to calculate, and therefore the neutrino properties are not well known. Because of this difficulty, and in order to control and vary neutrino interaction capture rates, we use neutrino luminosity as a parameter. By choosing values for neutrino luminosity that are physically reasonable for BNSMs, we investigate the impact neutrino interactions have on Y_{eq}, Y_e, and ultimately, the r-process.

4 Methods.

The simulations used for this study were performed using a program called Skynet, developed by Jonas Lippuner and Luke Roberts. Skynet is a nuclear reaction network program written in C++; it holds libraries for strong interactions, weak interactions, and fission events that it uses to evolve a system of nuclei through time. Skynet has a library of nuclei, starting from free protons and neutrons to very heavy elements, that it uses to evolve the production of heavy elements in BNSMs. In order to evolve this system of nuclei, Skynet uses input values for temperature, entropy, density and dynamical time scale τ to solve for nuclear statistical equilibrium and an equation of state for the BNSM ejecta. In this study, we input values for temperature, entropy, and dynamical time scale τ that are physically reasonable for binary neutron star mergers. It should be noted that, given a temperature and an entropy, Skynet will solve for a density. For the simulations discussed in this study, the density Skynet simulated was 2.51×10^{10} g, which is a lower value than typically expected for BNSM density. A continuation of this study may adjust Skynet such that the densities used are appropriately high, around 10^{12} g.

For this study, a neutrino interactions library was developed and implemented into Skynet. Variables for neutrino luminosity and mean energies were introduced. Because of the radius-dependence of neutrino capture rates at distances far away from the center of the merger, we developed a method to evolve the radius of the ejecta during the simulation. This was done using the density of the ejecta output by Skynet at any given time, and the following relation between density and radius:

$$\bar{\rho} = \frac{3M}{4\pi r^3}$$

Where M is the mass of the ejecta. For our study, we assigned the value of the BNSM mass to be five percent of a solar mass.

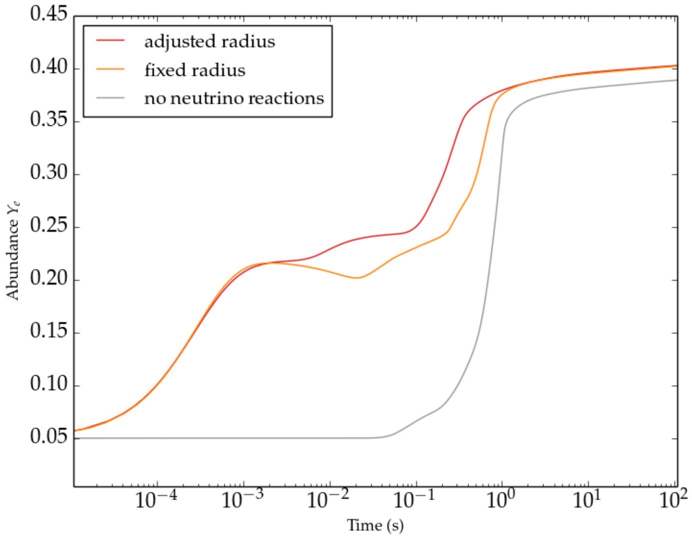


Figure 1: Electron abundance versus time of r-process simulations with (grey) no neutrino interactions, (orange) a constant radius, and (red) an evolving radius. Without neutrino interactions, no initial rise in electron abundance from neutrino/positron capture is present. With no expansion of the ejecta, the second rise in electron abundance is significantly decreased. Developing a model for expanding the radius of the ejecta thus had a significant effect on the results of the simulation. Developing an even more comprehensive model–where we take the derivative of $\bar{\rho}$ and solve for radius for any given point in time–may be of interest for further study.

In addition to these implementations, adjustments were made to the Skynet and r-process code so that it would calculate and output the velocity of the ejecta at certain time steps, as well as output the neutrino interaction reaction rates at each time step in a simulation.

Analysis code for the data output by Skynet was written in Python. Programs were written to create plots of changing electron abundances, nucleic abundances, capture rates, etcetera to help examine how and in what way varying parameters changed the r-process.

5 Results and discussion.

5.1 Neutrino forward and reverse rates.

As previously mentioned, positron and neutrino captures are responsible for creating electrons and driving up the electron abundance value. The relative amount that neutrino luminosity affects the r-process in BNSMs depends on which reaction rate is contributing more to the building electron abundance: the forward (neutrino) rate or the reverse (positron) rate. In order to understand how neutrino interactions impact the r-process, we must investigate under what conditions forward or reverse rates dominate.

Positron and electron capture rates have a strong temperature dependence, $\approx T^5$. At high temperatures (and thus high entropies), electrons become non-degenerateand positron-electron pairs can be created out of the vacuum. Positrons can then capture on a neutron to form protons, which in neutrally-charged matter means more electrons are also formed. Thus, we see that at a high entropy (i.e. high temperature), positron captures dominate neutrino captures in determining and driving up the Y_e and Y_{eq} values. At low entropies, electrons become degenerate and the amount of positrons plummets. Neutrino capture rates are then dominant in driving up Y_e. Neutrino and antineutrino capture rates also dominate at high neutrino/antineutrino luminosities, when the sheer density of available neutrinos allows neutrino captures to dominate the evolution of Y_e.

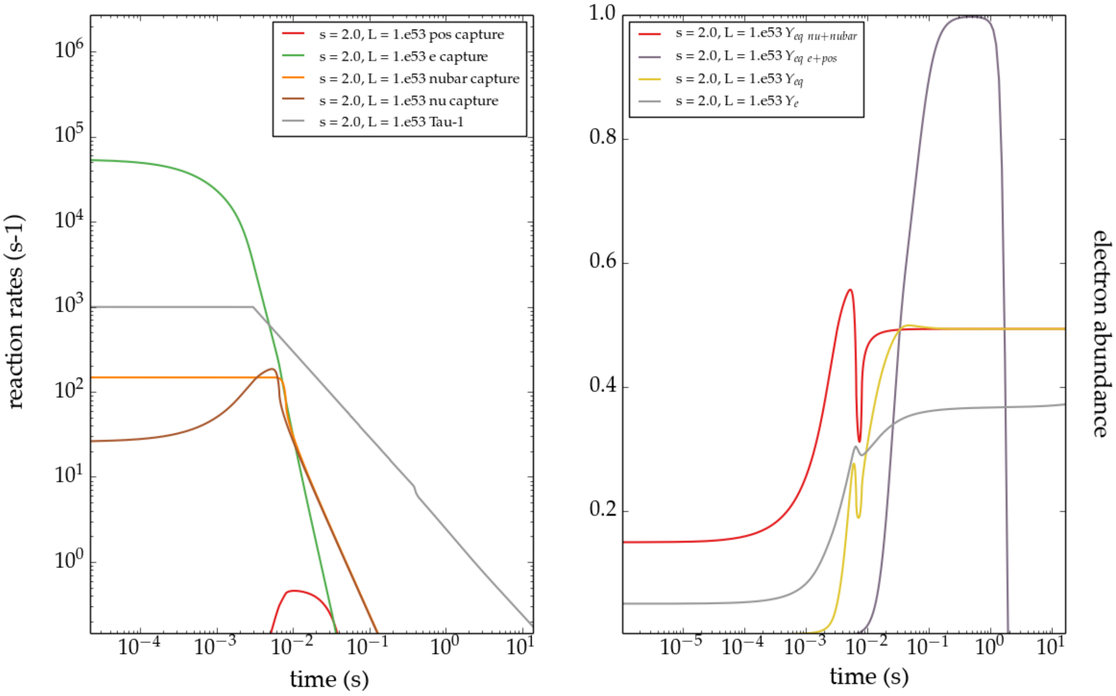


Figure 2: Neutrino interaction reaction rates (left) and their effect on the Y_e and Y_{eq} values (right) for $s = 2.0$ kB/baryon, $L_{\nu_e,\,\bar{\nu}_e} = 1.e53$ ergs/s. At this low entropy, the positron capture rate (red line, left) is nearly nonexistent, and falls to zero far more quickly than neutrino and antineutrino capture rates. Neutrino and antineutrino rates persist past 10^{-2}, when the decrease in τ^{-1} (grey) indicates the ejecta has expanded almost completely. Note how the Y_{eq} (yellow line, right) closely resembles the electron abundance equivalence value using only neutrino and antineutrino capture rates, $Y_{eq\ nu+nubar}$ (red line, right). Thus, in cases of low entropy and high luminosity, neutrino captures almost entirely dominate the process of driving up Y_e.

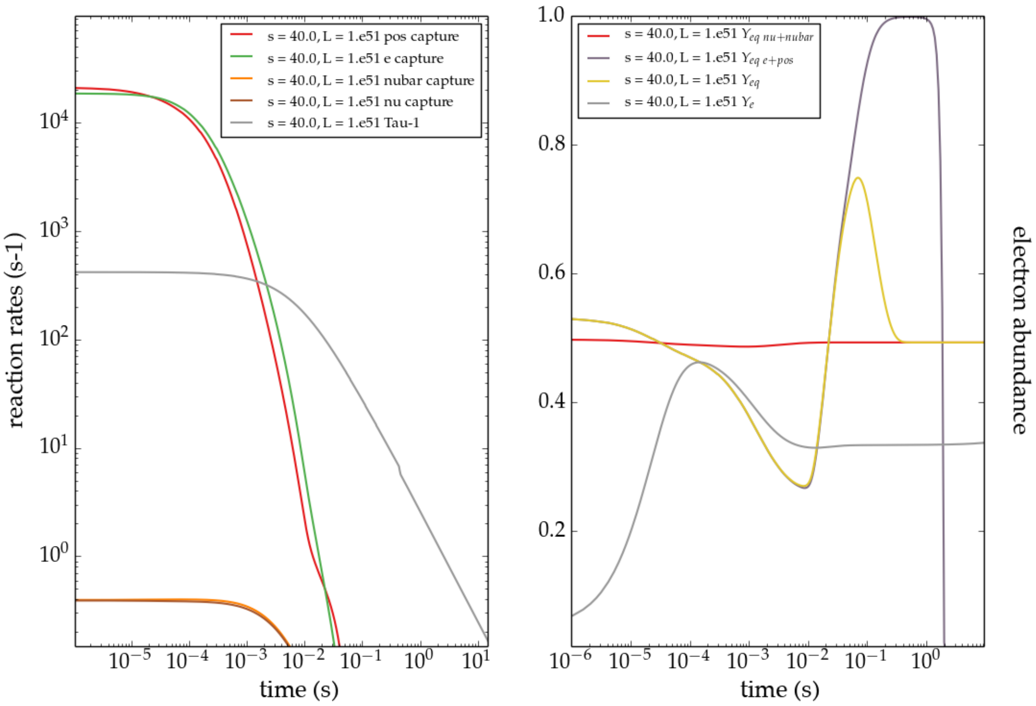


Figure 3: Neutrino interaction reaction rates (left) and their effect on the Y_e and Y_{eq} values (right) for $s = 40.0$ kB/baryon, $L_{\nu_e,\ \bar{\nu}_e} = 1.e51$ ergs/s. At high entropy and thus low degeneracy, positrons are more abundant and capture at high rates. They persist at higher rates past 10^{-2}, where the decrease in τ^{-1} (grey) indicates the ejecta has expanded outwards and become much less dense. Y_{eq} (yellow) follows the $Y_{eq\ e+pos}$ line (in purple), meaning positron/electron captures are the primary drivers of the electron abundance value. This rate plot and the plot before it show how rate dominance changes with different values of entropy and neutrino luminosity.

5.2 Impact of varying different parameters.

In order to get a better idea of how increasing entropy reduces the importance of neutrino luminosity, and vice versa, we ran several simulations with varying values of entropy and neutrino/antineutrino luminosity. As previously discussed, increasing entropy increases the rate of positron/electron captures, while increasing the neutrino luminosity increases the rate of neutrino/antineutrino captures. At a high enough value for entropy, positron captures will push the electron abundance value up so much it becomes insensitive to neutrino luminosity. Similarly, at high enough luminosities, Y_e will be insensitive to changes in entropy. In order to determine what these values were for entropy and neutrino luminosity, we ran 1,600 simulations across a range of entropies and neutrino luminosities. The final electron abundance value for each simulation was then plotted on the following contour plot.

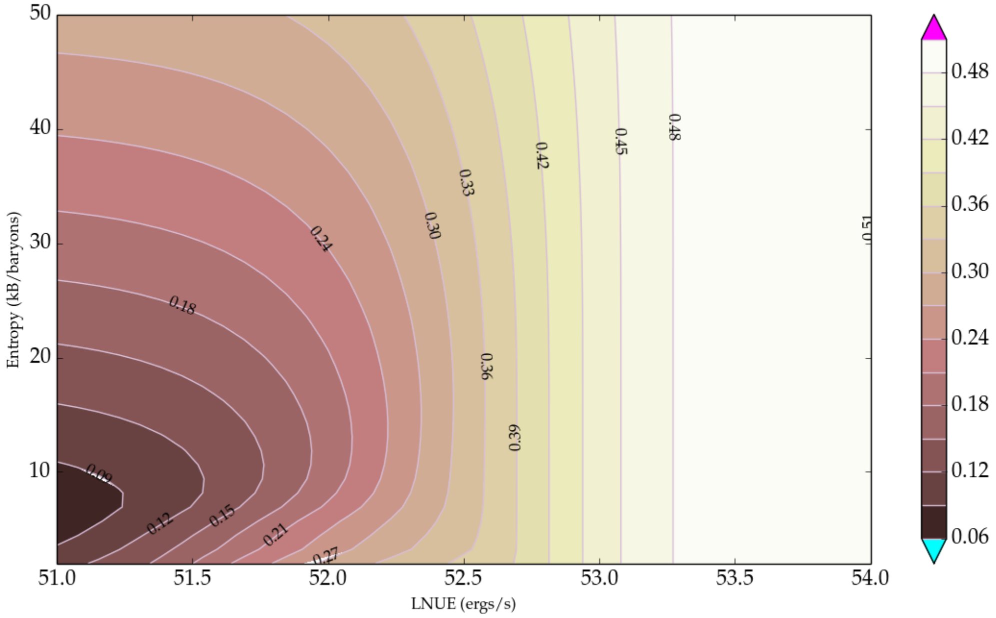
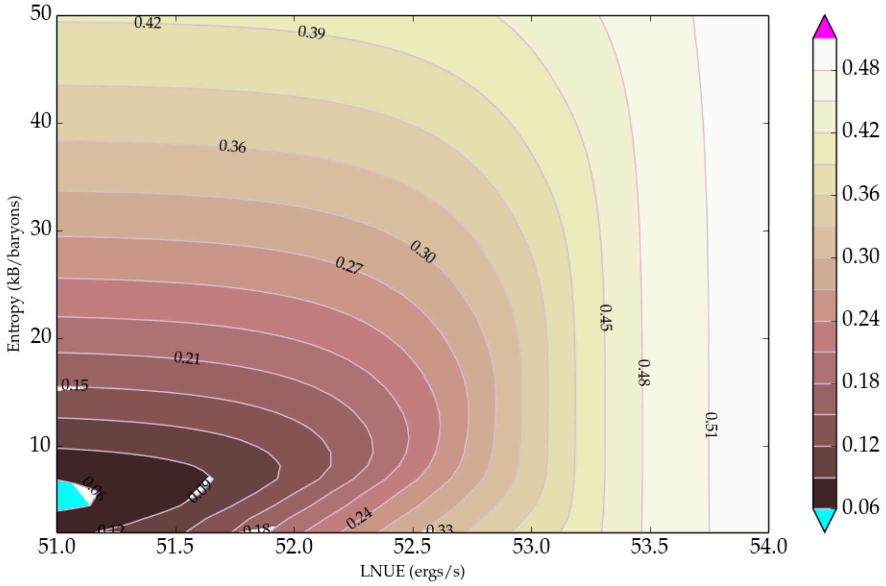
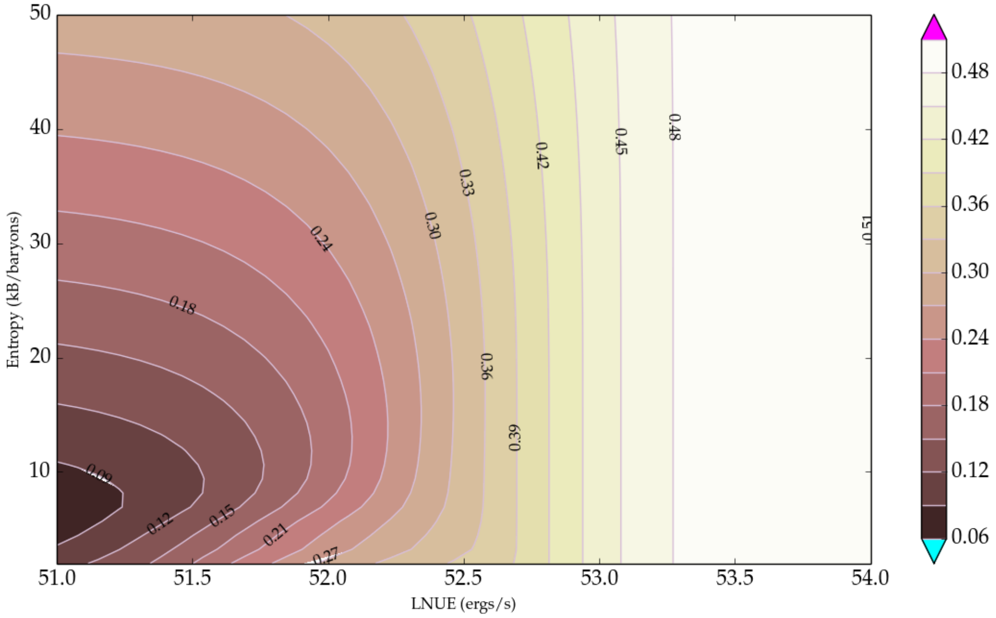


Figure 4: Contour plot with increasing entropy from 2.0 - 50.0 kB/baryon on the y-axis, and increasing luminosity from 1.e51 - 1.e54 ergs/s on the x-axis. τ = 3.0 ms. At low entropy and luminosity, Y_e stays at a low value of 0.06. Future research may investigate why the lowest Y_e values occur at the lowest neutrino luminosity, but not the lowest entropy. At about an entropy of 40.0 kB/baryon, neutrino luminosity must be greater than 1.e52 ergs/s in order to affect the Y_e. While high luminosities still affect Y_e for the chosen range of values for entropy, at a high enough luminosity, Y_e is almost completely insensitive to changes in entropy. This luminosity is around 5.e52 ergs/s. Although this contour plot encompasses a wide range of electron abundance values, previous research has shown that a successful r-process can typically only occur with a final Y_e value less than 0.25. Thus, the creation of heavy elements in BNSMs is always sensitive to both entropy and neutrino luminosity.

It is important to note that the values at which neutrino luminosity or entropy dominates is dependent on the dynamical time scale of the ejecta, τ. τ describes the time it takes for the ejecta to expand away from the center of the collision. As τ decreases, the ejecta becomes rapidly less dense, and all neutrino interactions have less time to interact with nucleons and push up Y_e.



(a) Contour plot of final electron abundances at varying entropies and neutrino luminosities, with a τ of 1.0 ms.



(b) Contour plot of final electron abundances at varying entropies and neutrino luminosities, with τ = 3.0 ms.

Figure 5: τ is the dynamical time scale, or the time it takes for the ejecta to expand a certain distance away from the collision. A larger τ means the ejecta expands slower, allowing neutrinos/antineutrinos and positrons/electrons to capture on nucleons for much longer before the matter becomes too sparse or too cool. τ's effect can be observed by noting how the final Y_e in $\tau = 3.0$ (b) is generally higher than the Y_e observed at the same entropy and neutrino luminosity for $\tau = 1.0$. With a longer timescale to capture on nucleons, neutrino interactions will drive the electron abundance higher than they would have on a short τ.

5.3 Finding the critical luminosity for one set of parameters.

Having explored several parameters' effects on the ejecta electron abundance, we now turn our attention to neutrino luminosity in a specific case. The effect of neutrinos and antineutrinos on the r-process has not been closely examined. We set all other parameters to physically reasonable values for BNSMs. For these simulations, the initial temperature was T0 = 50.0 GK, the entropy was s = 10.0 kB/baryon, there neutrino/antineutrino mean energies were $< E >$ = 12.0 MeV, and the dynamical time scale was τ = 3.0. Neutrino and antineutrino luminosities were equal to each other but were varied from 10^{49} - 10^{54} erg/s. By varying neutrino luminosity for this one set of parameters, we can determine, at least for one case, when neutrino luminosity begins to interfere with the creation of heavy elements in BNSMs. We will call the point at which the r-process is seriously impeded due to neutrino luminosity is called the critical luminosity. After running simulations through this range, a finer grid of simulations with luminosity between 10^{52} - 10^{54} was run in order to close in on this critical luminosity.

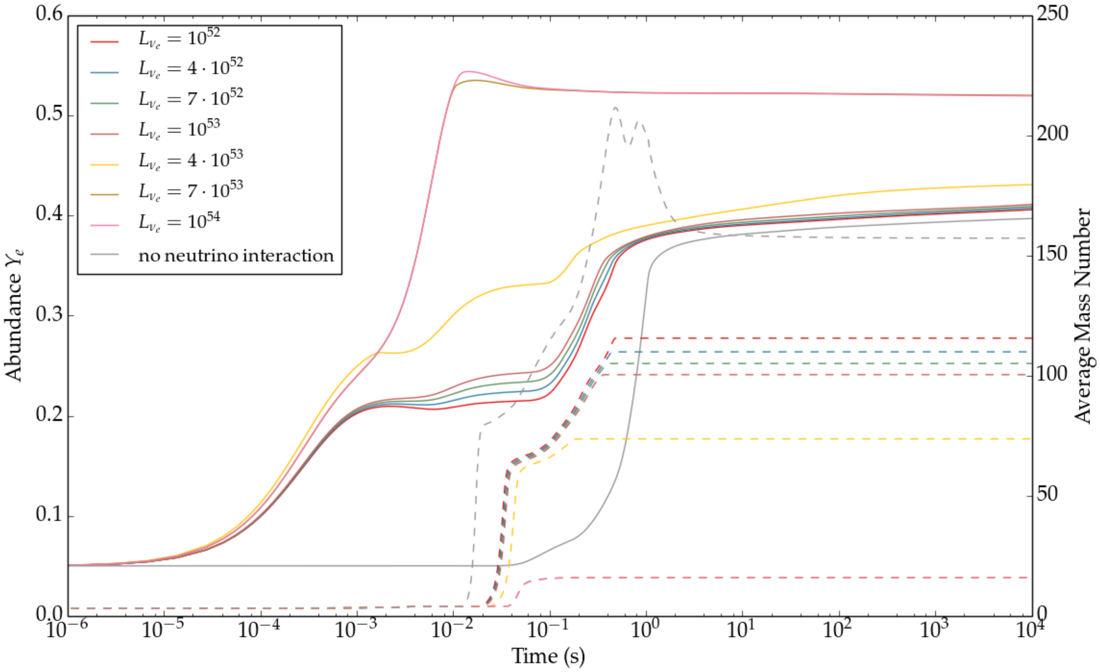


Figure 6: Electron abundance (solid line) and average mass number (dashed line) over time $t = 10^{-10}$ to $t = 10^{9}$, for various neutrino luminosities.

In general, three bumps can be observed in the increasing electron abundance. The first is attributable to positron/electron capture rates, which capture at early-time high temperatures. The second bump is from neutrino interactions, and the third bump (the only rise in Y_e for the no neutrino case) is from the r-process itself when neutrons decay to protons. As neutrino luminosity increases from 1.e52 ergs/s, neutrino captures contribute more to Y_e. At around 4.e53 ergs/s, electron abundance curves transform from a three-bump regime to one very large increase, a result of neutrino captures dominating the rise in Y_e.

Close examination of the average mass number $< A >$ vs time shows a significant drop in final average mass number once neutrino luminosity increases to 4.e53 ergs/s. This suggests that the critical luminosity, i.e. the luminosity when neutrino interactions significantly interfere with the creation of heavy elements, may lie between 1.e53 and 4.e53 ergs/s.

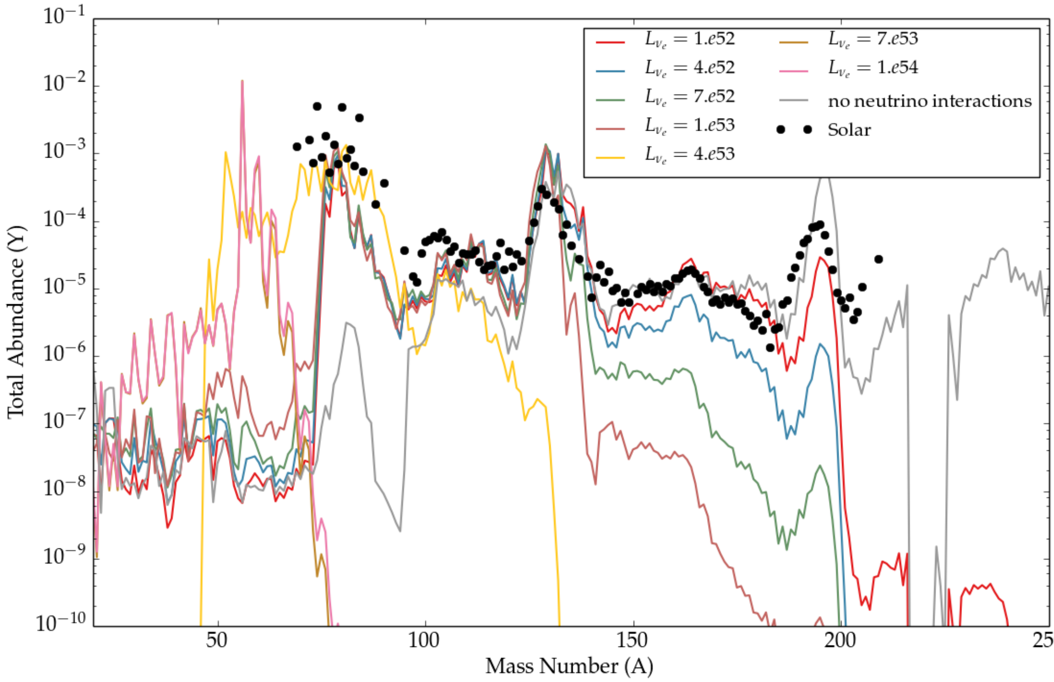


Figure 7: Abundance versus mass number (mass number 25 - 250) for various neutrino luminosities. The solar abundances, which are the observed abundances of r-process heavy elements formed within the Sun, are superimposed above (black dots) for comparison.

As observed in the solar abundances, the r-process forms three distinctive peaks in heavy nuclei abundances. In general, the addition of neutrino interactions reduced the third peak at mass number 190. This is particularly interesting because previous r-process simulations, done without consideration of neutrino interactions, have produced a third peak that is typically larger than the peak in the measured solar mass abundances. It is also important to note that solar abundances show a rise in abundance of nuclei with mass number >200, and that the simulated r-process without neutrino interactions (grey line) also show this extra peak. These heavy elements are a product of fission cycling, which is when exceptionally heavy nuclei spontaneously fission into smaller, more stable nuclei. When neutrino interactions are introduced to the simulation, fission cycling becomes near nonexistent. Exploring how our simulation fails to produce fission cycling, which is observed in solar abundances, merits further study.

As neutrino luminosity increases upwards of 1.e52 ergs/s, the third peak of abundances diminishes significantly. Again, it becomes clear that this third characteristic peak of the r-process fails to be made somewhere between 1.e53 - 4.e53 ergs/s.

6 References.

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2. J. Cowan, F-K. Thielemann. R-Process Nucleosynthesis in Supernovae. Physics Today. Oct 2004; 47-53.

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