Multi-carrier optimization of future laser gravitational-wave detectors

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Abstract

Sensitivity of second generation of GW detectors will be quantum noise limited. In this project we assume two main methods to improve the sensitivity: the first approach comprise the use of double optical spring in order to increase the dynamical response of test objects to gravitational wave. The effect is called "negative optical inertia" and it allows to reduce total quantum noise in low frequency band. The second method is based on use of double antisymmetric carriers in order to reach high frequency improvement. We propose to combine two method mentioned above in the xylophone configuration to achieve better broadband sensitivity.

1 Introduction

Currently, the second-generation large-scale laser interferometric gravitational-wave (GW) detectors — Advanced LIGO [1, 2], Advanced VIRGO [3, 4], and KAGRA [5, 6]— are under construction. In particular, aLIGO interferometer in Livingston is beginning to collect scientific data at the end of 2015 [7]. The sensitivities of these detectors are expected to be limited by the quantum noise. Namely, at higher frequencies the shot noise will dominate, originating from the quantum fluctuation of the phase of the optical field inside the interferometer. At lower frequencies, the radiation pressure noise created by the amplitude fluctuations will constitute the significant part of the noise budget [8]. The point where the contribution of these noises becomes equal is known as Standard Quantum Limit [9].

2 Negative optical inertia

Several methods of improving the sensitivity of GW detectors were proposed: one of the approaches is to enhance the test object response to gravitational wave by changing its dynamics, due to the fact, that SQL depends on test mass susceptibility [10]:

$$\chi(\Omega) = \frac{x(\Omega)}{F(\Omega)}, \tag{1}$$

and

$$S_F^{\mathrm{SQL}}(\Omega)_{f.m.} = 2\hbar \left|\chi^{-1}(\Omega)\right|. \tag{2}$$

For example, for harmonic oscillator, the susceptibility reads:

$$\chi(\Omega) = \left[m\left(\Omega_m^2 - \Omega^2\right)\right]^{-1}, \tag{3}$$

thereby

$$S_F^{\mathrm{SQL}}(\Omega)_{h.o.} = 2\hbar \left|\chi^{-1}(\Omega)\right|, \tag{4}$$

where Ω_m is an eigenfrequency of the oscillator, at which it has much stronger response to the GW force.

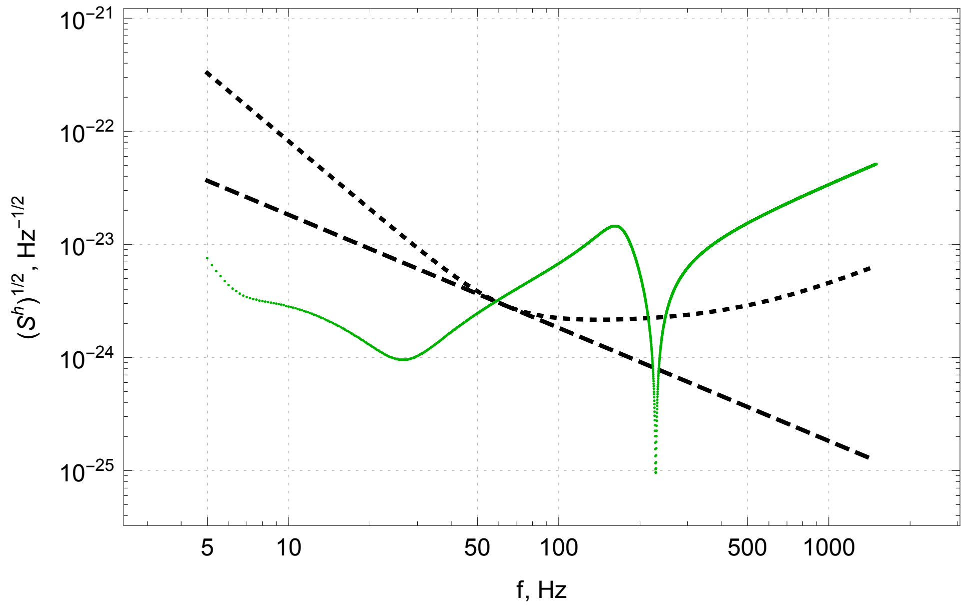

Figure 1: Optimization result for negative optical inertia effect. Dashed line: SQL, dotted line: baseline configuration, green: pair of carriers with parameters, described by (6). All parameters are presented in Table 1.

Implementation of mechanical rigidity in GW detector is impossible, due to the high level of thermal noise, thus optical rigidity, which arises in detuned Fabry-Perot cavities could be used instead. Moreover, using frequency dependance of such a rigidity and combining several pumps, it is possible to create effective negative inertia, which cancels the positive inertia of test mass in some frequency domain. The main disadvantage of such a scheme is the fact, that upper bound of this frequency band scales as $I_c^{1/3}$ of optical power, thus for reasonable values of laser power, it cannot exceed 50 Hz [10].

Assuming $\Omega \ll \delta$ and $\gamma \ll \delta$, optical rigidity could be expanded in Taylor series [11]:

$$\mathcal{K} \approx \bar{\mathcal{K}} - i\Xi_{opt}\Omega - m_{opt}\Omega^2 + O(\Omega^3), \tag{5}$$

where $\bar{\mathcal{K}} = \frac{mJ\delta}{\Gamma^2}$ — static rigidity, $\Xi_{opt} = -\frac{2mJ\gamma}{\Gamma^4}$ — optical damping and $m_{opt} = -\frac{mJ\delta}{\Delta^4}$ — optical inertia. Thus, combining parameters of two carriers in a special way, static rigidity will cancel each other $\bar{\mathcal{K}}_1 + \bar{\mathcal{K}}_2 = 0$, and total optical inertia will cancel positive inertia of the test mass $m + m_{opt1} + m_{opt2} = 0$. The parameters for such a condition are [11]:

$$J_1 = \frac{\Gamma_1^4\Gamma_2^2}{\delta_1(\Gamma_2^2 - \Gamma_1^2)} \qquad \text{and} \qquad J_2 = \frac{\Gamma_1^2\Gamma_2^4}{\delta_2(\Gamma_1^2 - \Gamma_2^2)}. \tag{6}$$

It could be seen that: $|\delta_1| < |\delta_2|$, $\delta_1 > 0$, $\delta_2 < 0$. Eventually, we will obtain test object, which has higher response to force in low-frequency domain.

3 Antisymmetric pairs

3.1 Main idea

The second group of methods of improving sensitivity is based on cross-correlation between shot and back action noises. It was proposed to use double optical spring configuration in annihilation regime [8]:

$$J_1 = J_2 \tag{7}$$

$$r_1 = r_2 \tag{8}$$

$$\Gamma_1 = \Gamma_2 \tag{9}$$

$$\beta_1 = -\beta_2 \tag{10}$$

$$\zeta_1 = -\zeta_2 \tag{11}$$

$$\theta_1 = -\theta_2 \tag{12}$$

Configuration mentioned above cancels the effective cross-correlation between two pumps and optical rigidity. Effective shot noise spectral density decreases as $\frac{1}{\Omega^2}$ and increases as Ω^2, with the minimum at some Ω_0. Effective radiation pressure noise mirrors this dependence. Thus, it is possible to obtain tunable shape of curve for desirable Ω_0 [8].

3.2 Xylophone configuration

It is possible to combine several pairs of antisymmetric carriers in xylophone configuration with quantum noise spectral densities equal to [8]:

$$S_{xx}^{eff}(\Omega) = \left[\sum_{n=1}^{N} \frac{1}{S_{xx}^{(n)}}\right]^{-1}, \tag{13a}$$

$$S_{xF}^{eff}(\Omega) = S_{xx}^{eff}(\Omega)\left[\sum_{n=1}^{N} \frac{S_{xF}^{(n)}(\Omega)}{S_{xx}^{(n)}(\Omega)}\right], \tag{13b}$$

$$S_{FF}^{eff}(\Omega) = \sum_{n=1}^{N}\left[S_{FF}^{(n)}(\Omega) - \frac{\left|S_{xF}^{(n)}(\Omega)\right|^2}{S_{xx}^{(n)}(\Omega)}\right] + \frac{\left|S_{xF}^{eff}(\Omega)\right|^2}{S_{xx}^{eff}(\Omega)}, \tag{13c}$$

Since our goal is to improve high frequency sensitivity, let's consider effective shot noise, due to the fact, that it prevails at higher frequencies:

$$S_{xx}^{eff}(\Omega) = \left[\frac{1}{S_{xx}^{(1)}} + \frac{1}{S_{xx}^{(2)}}\right]^{-1}, \tag{14}$$

where $S_{xx}^{(1)}$ is the shot noise for main pair of carriers and $S_{xx}^{(2)}$ is shot noise for pair of high frequency carriers. Adding another pair, which shot noise is described by the second term in (x), we eventually would decrease the total shot noise, thus improving high frequency sensitivity. Results of this procedure are presented in next section.

4 Numerical Optimization

In presence of several pairs of pumps analytical optimization of configuration is impossible, thus we show numerically, that adding one high frequency pair of antisymmetric carriers will drastically improve the broadband sensitivity. Starting off with optimized pair of carriers with negative optical inertia effect, we include one more pair of antisymmetric carriers. The results are presented on Figure 2.

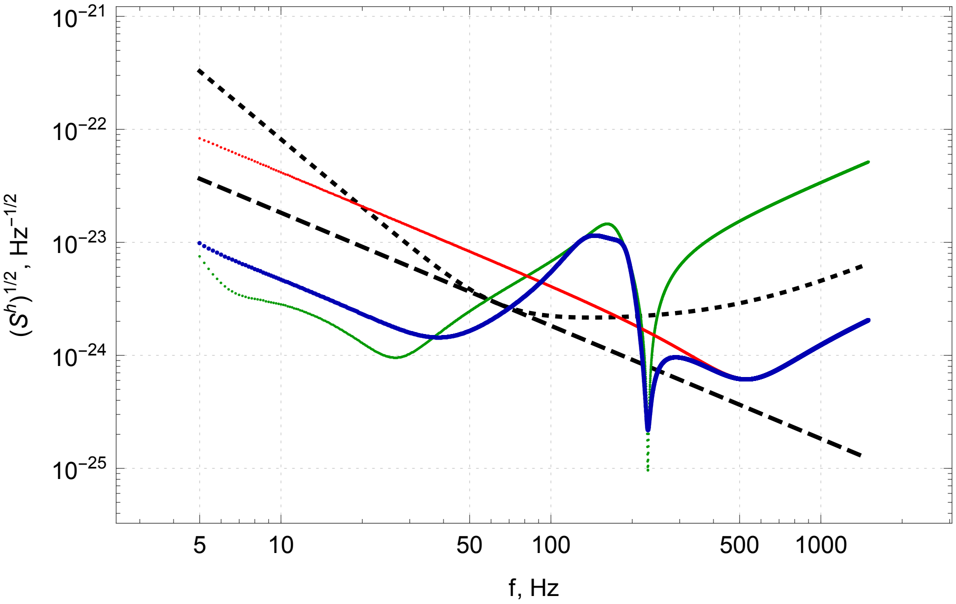

Figure 2: Total noise curve for two pairs of carriers (thick blue). Dashed line: SQL, dotted line: baseline configuration, green: individual noise for negative optical inertia pair, red: individual noise for antisymmetric pair. All parameters are presented in Table 1.

	Baseline	Negative Optical Inertia		Antisymmetric carriers	
I_c, kW	1680	6263.33	9382.98	11986.26	11986.26
γ	$2\pi \times 500$	9.91	13.14	1166.96	1166.96
δ		800.26	-1198.90	-1513.84	1513.84
ζ	$\pi/2$	-1.5702	1.5708	-0.9132	0.9132
r, dB		6	6	6	6
θ		-0.6084	-1.4350	0.8898	-0.8898

Table 1: Results of numerical optimization $\eta = 1$ [12, 13].

5 To do

- Add another pair for middle frequency sensitivity,
- Add plots and result with losses $\eta = 0.95$,
- Show possibility / inability of improvement with reasonable values of optical power (maximum 1680 kW for one pump),
- Include factor α for stabilizing negative optical inertia for high values of power / reasonable values of power,
- Analytically give estimates for parameters of additional antisymmetric pairs [8],
- Increase squeezing factor up to 12 db.

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