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# Multi-carrier optimization of future laser gravitational-wave detectors

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## Abstract

Sensitivity of second generation of GW detectors will be quantum noise limited. In this project we consider two methods of suppression of this noise in future GW detectors. The first approach comprise the use of double optical spring in order to increase the dynamical response of test objects to gravitational wave. The effect is called "negative optical inertia" and it allows to reduce total quantum noise in low frequency band. The second method is based on use of double antisymmetric carriers in order to reach high frequency improvement. We propose to combine two method mentioned above in the xylophone configuration to achieve better broadband sensitivity and perform the numerical optimization of this scheme.

# Introduction

Currently, the second-generation large-scale laser interferometric gravitational-wave (GW) detectors — Advanced LIGO [1, 2], Advanced VIRGO [3, 4], and KAGRA [5, 6] —are either starting to collect scientific data [7] or under construction. The sensitivities of these detectors are expected to be limited by the quantum noise. Namely, at higher frequencies the shot noise will dominate, originating from the quantum fluctuation of the phase of the optical field inside the interferometer. At lower frequencies, the radiation pressure noise created by the amplitude fluctuations will constitute the significant part of the noise budget [8]. The point where the contribution of these noises becomes equal is known as Standard Quantum Limit [9].

# 1 Negative optical inertia

Several methods of reducing the quantum noise in GW detectors were proposed: one of the approaches is to enhance the test object response to gravitational wave by changing its dynamics, due to the fact, that SQL depends on test mass susceptibility [10]:

$$\chi(\Omega) = \frac{x(\Omega)}{F(\Omega)}, \tag{1}$$

and

$$S_F^{\mathrm{SQL}}(\Omega)_{f.m.} = 2\hbar\left|\chi^{-1}(\Omega)\right|. \tag{2}$$

For example, for harmonic oscillator, the susceptibility reads:

$$\chi(\Omega) = \left[m\left(\Omega_m^2 - \Omega^2\right)\right]^{-1}, \tag{3}$$

thereby

$$S_F^{\mathrm{SQL}}(\Omega)_{h.o.} = 2\hbar\left|\chi^{-1}(\Omega)\right|, \tag{4}$$

where $\Omega_m$ is an eigenfrequency of the oscillator, at which it has much stronger response to the GW force. Implementation of mechanical rigidity in GW detector is impossible, due to the high level of thermal noise, thus optical rigidity, which arises in detuned Fabry-Perot cavities could be used instead. Moreover, using frequency dependance of such a rigidity and combining several pumps, it is possible to create effective negative inertia, which cancels the positive inertia of test mass in some frequency domain.

Assuming $\Omega \ll \delta$ and $\gamma \ll \delta$ (see the notations in Table 1), optical rigidity could be expanded in Taylor series [11]:

$$\mathcal{K} \approx \bar{\mathcal{K}} - i\Xi_{opt}\Omega - m_{opt}\Omega^2 + O(\Omega^3), \tag{5}$$

where $\bar{\mathcal{K}} = \frac{mJ\delta}{\Gamma^2}$ — static rigidity, $\Xi_{opt} = -\frac{2mJ\gamma}{\Gamma^4}$ — optical damping and $m_{opt} = -\frac{mJ\delta}{\Delta^4}$ — optical inertia. Thus, combining parameters of two carriers in a special way, static rigidity will cancel each other $\bar{\mathcal{K}}_1 + \bar{\mathcal{K}}_2 = 0$, and total optical inertia will cancel positive inertia of the test mass $m + m_{opt1} + m_{opt2} = 0$. The parameters for such a condition are [11]:

$$J_1 = \frac{\Gamma_1^4\Gamma_2^2}{\delta_1(\Gamma_2^2 - \Gamma_1^2)} \qquad \text{and} \qquad J_2 = \frac{\Gamma_1^2\Gamma_2^4}{\delta_2(\Gamma_1^2 - \Gamma_2^2)}. \tag{6}$$

It could be seen that: $|\delta_1| < |\delta_2|$, $\delta_1 > 0$, $\delta_2 < 0$. Eventually, we will obtain test object, which has higher response to force in low-frequency domain.

The main disadvantage of the negative inertia is the fact, that upper bound of this frequency band scales as $I_c^{1/3}$ of optical power, thus for reasonable values of laser power, it cannot exceed $\sim 100\,\mathrm{Hz}$ [10].

| Quantity | Description |
|---|---|
| $m = 40$ kg | Test mass (aLIGO) |
| $L = 4000$ m | Arm length (aLIGO) |
| $\omega_p$ | Pump laser frequency |
| $\Omega$ | GW frequency |
| $\gamma$ | Bandwidth |
| $\delta = \omega_p - \omega_0$ | Detuning |
| $\beta = \arctan\frac{\delta}{\gamma}$ | Effective detuning |
| $\Gamma = \sqrt{\gamma^2 + \delta^2}$ | Effective bandwidth |
| $I_c$ | Optical power, kW |
| $J = \frac{4\omega_p I_c}{MLc}$ | Normalized optical power |
| $\eta$ | Quantum efficiency |
| $r = \ln 2\,(6\,\mathrm{db})$ | Squeezing factor |
| $\theta$ | Squeezing angle |
| $\zeta$ | Homodyne angle |

Table 1: Main notations

# 2 Antisymmetric pairs

## 2.1 General idea

The second group of methods of improving sensitivity is based on cross-correlation between shot and back action noises. It was proposed to use double optical spring configuration in annihilation regime [8]:

$$J_1 = J_2 \tag{7a}$$
$$r_1 = r_2 \tag{7b}$$
$$\Gamma_1 = \Gamma_2 \tag{7c}$$
$$\beta_1 = -\beta_2 \tag{7d}$$
$$\zeta_1 = -\zeta_2 \tag{7e}$$
$$\theta_1 = -\theta_2 \tag{7f}$$

Configuration mentioned above cancels the effective cross-correlation between two pumps and optical rigidity. Effective shot noise spectral density decreases as $\frac{1}{\Omega^2}$ and increases as $\Omega^2$, with the minimum at some $\Omega_0$. Effective radiation pressure noise mirrors this dependence. Thus, it is possible to obtain tunable shape of curve for desirable $\Omega_0$ [8].

### 2.2 Xylophone configuration

It is possible to combine several pairs of antisymmetric carriers in xylophone configuration with quantum noise spectral densities equal to [8]:

$$S_{xx}^{eff}(\Omega) = \left[\sum_{n=1}^{N} \frac{1}{S_{xx}^{(n)}}\right]^{-1}, \tag{8a}$$

$$S_{xF}^{eff}(\Omega) = S_{xx}^{eff}(\Omega)\left[\sum_{n=1}^{N} \frac{S_{xF}^{(n)}(\Omega)}{S_{xx}^{(n)}(\Omega)}\right], \tag{8b}$$

$$S_{FF}^{eff}(\Omega) = \sum_{n=1}^{N}\left[S_{FF}^{(n)}(\Omega) - \frac{\left|S_{xF}^{(n)}(\Omega)\right|^2}{S_{xx}^{(n)}(\Omega)}\right] + \frac{\left|S_{xF}^{eff}(\Omega)\right|^2}{S_{xx}^{eff}(\Omega)}, \tag{8c}$$

Since our goal is to improve high frequency sensitivity, let's consider effective shot noise, due to the fact, that it prevails at higher frequencies:

$$S_{xx}^{eff}(\Omega) = \left[\frac{1}{S_{xx}^{(1)}} + \frac{1}{S_{xx}^{(2)}}\right]^{-1}, \tag{9}$$

where $S_{xx}^{(1)}$ is the shot noise for main pair of carriers and $S_{xx}^{(2)}$ is shot noise for pair of high frequency carriers. Adding another pair, which shot noise is described by the second term in (9), we eventually would decrease the total shot noise, thus improving high frequency sensitivity. Results of this procedure are presented in next section.

# 3 Numerical Optimization

We carried out numerical optimization of the Advanced LIGO interferometer with two pairs of carriers: the first one satisfying the negative inertia condition (6) and the second one — the antisymmetric carriers condition (7). As the figure of merit, we used the following cost function:

$$\mathcal{C}(\mathbf{x}) = \int_{f_{min}}^{f_{max}} \log_{10}\left[S^h(\mathbf{x}, 2\pi f)\right] d(\log_{10} f) = \int_{f_{min}}^{f_{max}} \frac{\log_{10}\left[S^h(\mathbf{x}, 2\pi f)\right]}{f \ln 10} df, \tag{10}$$

where $f_{min} = 5$ Hz, $f_{max} = 1500$ Hz, and

$$\mathbf{x} = \{I_c, \gamma, \delta, \zeta, r, \theta\} \tag{11}$$

is the vector of the optimized parameters (for each of the four carriers).

In order to decrease execution time we used C language and GNU scientific library [12] with interface [13]. The algorithm of optimization is Nelder-Mead simplex method. It has one well-known disadvantage, namely, depending on the starting point in the parameter space, it can return some local minimum instead of the global one. Therefore, we used grid of initial values for the procedure, and then chose the best output, based on cost function value.

We tried two strategies of the optimization: the one-step optimization of the all four carriers together (the joint optimization) and the two-step process, consisting of: (i) finding the optimal parameters for the negative optical inertia pair, with $f_{max}$ limited by 100 Hz, due to the reason that it is a low-frequency effect, and (ii) optimization

| | Baseline | Negative Optical Inertia | | Antisymmetric carriers | |
|---|---|---|---|---|---|
| $I_c$, kW | 1680 | 6263.33 | 9382.98 | 11986.26 | 11986.26 |
| $\gamma$ | $2\pi \times 500$ | 9.91 | 13.14 | 1166.96 | 1166.96 |
| $\delta$ | 0 | 800.26 | -1198.90 | -1513.84 | 1513.84 |
| $\zeta$ | $\pi/2$ | -1.5702 | 1.5708 | -0.9132 | 0.9132 |
| r, dB | 0 | 6 | 6 | 6 | 6 |
| $\theta$ | 0 | -0.6084 | -1.4350 | 0.8898 | -0.8898 |

Table 2: Results of numerical optimization $\eta = 1$.

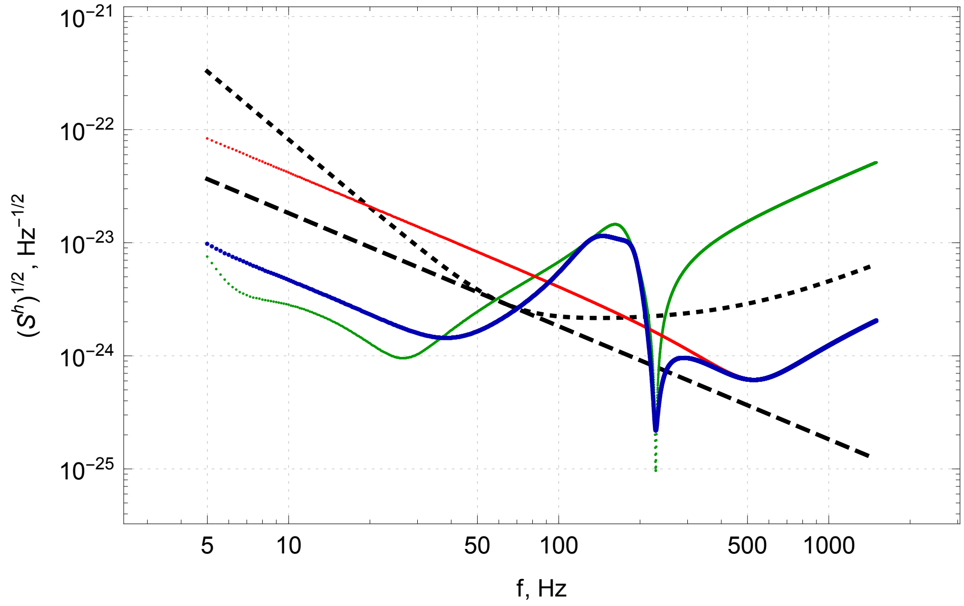

Figure 1: The optimized quantum noise spectral densities. Thick blue line: total quantum noise of 2 pairs of carriers; thin green and thin red lines: contributions of the respective negative inertia and antisymmetric carriers components; Black dashed line: SQL, black dotted line: baseline configuration. The corresponding parameters of the interferometer are presented in Table 2.

of antisymmetric carriers pair with the given first (the separate optimization) in the broad range from 5 to 1500 Hz. The separate optimization showed better results than the joint one in terms of value of cost function ($-116.126$ opposite to $-115.614$).

Thus, we performed two separate launches of program. First for negative optical inertia pair from 5 to 100 Hz. We applied boundaries to obtain realistic parameters: positive values of bandwidth, $-\frac{\pi}{2} < \zeta < \frac{\pi}{2}$, fixed squeezing factor $r$ corresponding to 6 dB squeezing and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Given parameters for the first pair, we added the second pair of antisymmetric carriers and carried out the second optimization. The results are shown in Table 2 and in Figure 1.

We compare all plots with the configuration of Advanced LIGO interferometers, which we assume to have 1680 kW of input optical power, $\pi/2$ homodyne angle, no squeezing and no detuning — the baseline configuration.

# 4 Conclusion

The main results of this work are the following. We developed the software written in C which can be used for numerical optimization of multi-carrier configurations of GW detectors. Using this software, we performed numerical optimization of the Advanced LIGO interferometer with two pairs of carriers: the first one satisfying the negative inertia condition (6) and the second one — the antisymmetric carriers condition (7).

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# References

[1] `http://www.advancedligo.mit.edu/`.

[2] G.M.Harry (for the LIGO Scientific Collaboration), Classical and Quantum Gravity **27**, 084006 (2010).

[3] `http://wwwcascina.virgo.infn.it/advirgo/`.

[4] F. Acernese et al., J. Phys.: Conf. Ser. **32**, s223 (2006).

[5] `http://gwcenter.icrr.u-tokyo.ac.jp/en/`.

[6] Y. Aso et al., Phys. Rev. D **88**, 043007 (2013).

[7] `https://ligo.caltech.edu/LA`.

[8] M. Korobko, N. Voronchev, H. Miao, and F. Y. Khalili, Phys. Rev. D **91**, 042004 (2015).

[9] V.B.Braginsky, F.Ya.Khalili, *Quantum Measurement*, Cambridge University Press (1992).

[10] F. Khalili et al., Phys. Rev. D **83**, 062003 (2011).

[11] N. Voronchev, S. Danilishin, F. Y. Khalili, Optics and Spectroscopy **112**, 377 (2012).

[12] GNU scientific library for C, `http://www.gnu.org/s/gsl/`.

[13] GSL interface, `http://cafim.sssup.it/~giulio/software/multimin/multimin.html`.