Selecting the Best Coatings to Precisely Measure Thermo-Optic Coefficients

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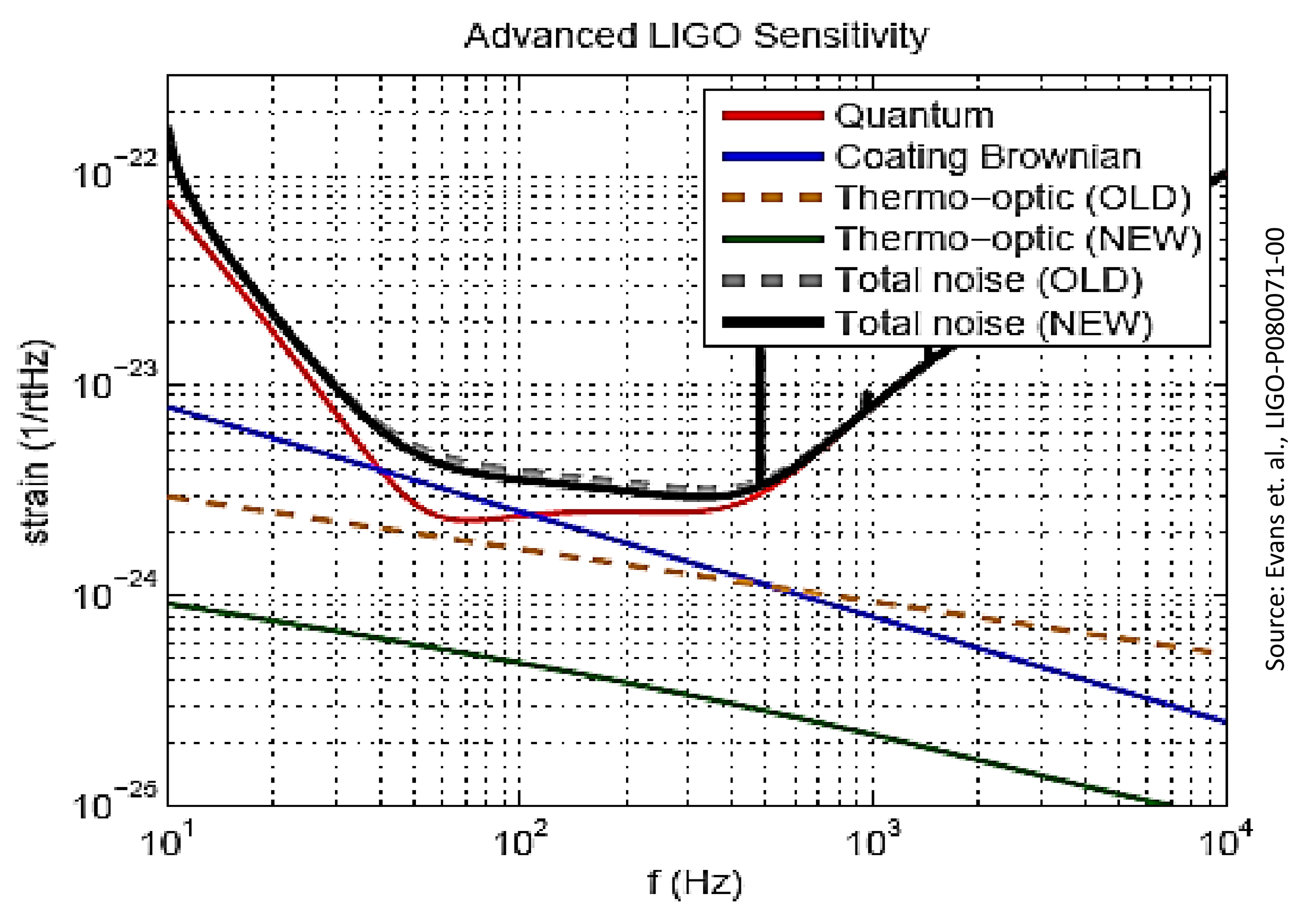
Background:



In the LIGO noise curve seen above, there are two predictions for the thermo-optic noise. The old thermo-optic noise curve assumed that the thermo-elastic (TE) noise (pictured below, left), and thermo-refractive (TR) noise (pictured below, right) would add incoherently. However, the new noise curve is based on the understanding that the thermo-elastic and thermo-refractive noises should be added coherently since they are driven by the same underlying temperature change and thus can partially cancel. The equation for this coherent noise is:

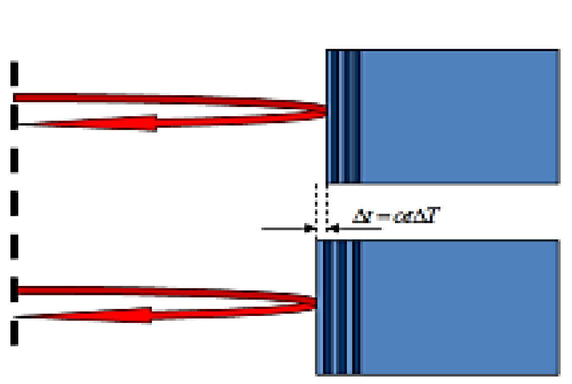
$$S_{\Delta x}(\omega) \approx \left(\alpha_{eff} d - \lambda \beta_{eff}\right)^2 S_{\Delta T}(\omega)$$

LIGO predicts that the α_{eff}, (the combination of α, the thermo-elastic response, in each material) and β_{eff} (the combination of β, the thermo-refractive response ,in each material) terms will at least partially cancel. However, to realistically predict this cancellation we need to know the coefficients to a high degree of certainty. Additionally, although it is below the current AdLIGO noise floor, eventually LIGO may decrease the Brownian noise enough that thermo-optic noise becomes a concern.

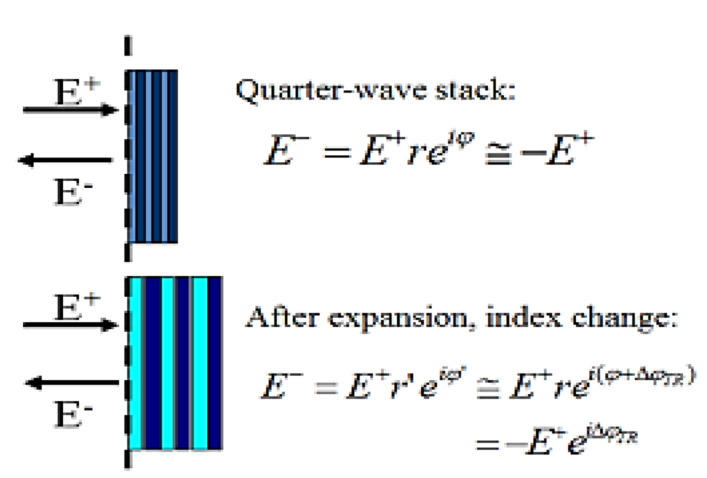
To determine the coefficients, we first have to understand the thermo-elastic and thermo-refractive responses in mirror coatings. Thermo-Elastic noise is a physical change in position of the surface of the mirror due to the thermal expansion of the coating. Thermo-Refractive noise is a change in the complex reflection coefficient of the coating when temperature changes cause the optical thickness of the layers to change.

$$d\varphi/dT\Big|_{TE} = \frac{-4\pi}{\lambda}\sum_{index}\alpha_i l_i \qquad d\varphi/dT\Big|_{TR,QWL} = \frac{\pi(\beta_H + \alpha_H n_H + \beta_L + \alpha_L n_L)}{{n_H}^2 - {n_L}^2}$$

Thermo-Elastic:



Thermo-Refractive:



Source: Ogin et.al, LIGO-**G1200269-v1**

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Abstract:

One possible source of noise in interferometric experiments such as LIGO is thermo-optic noise which is caused by variations in the local temperature of the mirror coatings. To fully characterize this noise source we need to know α (the coefficient of expansion), and β (the change in index of refraction with temperature, dn/dT) for all materials used in the coatings. We are setting up an experiment at Whitman College to measure these coefficients, but we would like to determine which coatings will give us the lowest fractional error on each parameter. We wrote code in Mathematica to analyze sets of coatings, and determine the set that will give us the smallest errors on α and β. After running seventy initial coatings we found a number of good options.

The Problem:

The main difficulty in getting α and β for a particular coating is that we can only actually measure dφ/dT which depends on all four parameters. This means we need four linearly independent equations, or four different coatings to create a "measurement matrix" (see below) which we can invert and use to solve for the coefficients:

$$\begin{bmatrix} d\varphi_1/dT \\ d\varphi_2/dT \\ d\varphi_3/dT \\ d\varphi_4/dT \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \cdot \begin{bmatrix} \alpha_L \\ \alpha_H \\ \beta_L \\ \beta_H \end{bmatrix}$$

After we invert the matrix, we need to add and subtract combinations of our dφ/dT values, and if the matrix is poorly constructed, small numbers can subtract while errors on measurements (around five percent) add in quadrature, resulting in huge fractional errors on α and β. For example:

$$(1.01 \pm 0.1) - (1.0 \pm 0.1) = \left(0.01 \pm 0.1(\sqrt{2})\right) \neq (0.01 \pm 0.0) \text{ (as we may naively expect)}$$

We would like to find a set of coatings that give us the smallest errors for α and β.

Part 1: Calculate Parameters for the Measurement Matrix:

The code reads in a coating configuration.

The configuration is in the form of a list for each layer, with the first element closest to the vacuum. For example, a Quarter Wavelength stack is a series of alternating high index (tantala, n=2.065) and low index (silica, n=1.45) layers, each a quarter of a wavelength thick. The input list form would be {layer length (optical wavelengths), layer index of refraction}:

{{¼, 1.45},{¼,2.065},{¼, 1.45},{¼,2.065},{¼, 1.45},{¼,2.065},{¼, 1.45},{¼,2.065}.....}

It then calculates the expected values for thermo-elastic and thermo-refractive response.

If our equation is:

$$d\varphi/dT = a_1\alpha_L + a_2\alpha_H + a_3\beta_L + a_4\beta_H$$

To find, for example, a_1, we calculate:

$$a_1 = (1/\alpha_L)(d\varphi/dT) \text{ at } \alpha_H = \beta_L = \beta_H = 0$$

Where α_L and dT are nominal values for the coefficient of expansion and a change in temperature.

We then find the change in phase, $d\varphi$, by:

1. Calculating the initial reflectance ,r_0.
2. Calculating the new reflectance, r', with $l \to l(1 + \alpha * dT)$ for all low-index layers.
3. Calculating $d\varphi = r' - r_0$, adding the additional phase from the thermo-elastic response.

Then find a_2, a_3, a_4in a similar way.

Finally, it outputs a potential row of the measurement matrix: {a1,a2,a3,a4}.

For the Quarter Wavelength stack given above, with 11 doublets, the output is:

Row = {-21.7282,-13.7364, 4.43988,1.45144}

Part 2: Choose the Best Coating Combinations:

This code imports all the generated coatings and creates all unique combinations of four coatings. However, this is a huge number of combinations so we only ask it to choose the N best combinations with the lowest error on the αs and βs.

Import a list of possible coefficient sets generated from the code above.

For each unique combination of four coefficient sets calculate a figure of merit (see right). If it is smaller than one on the list of the N best vectors then use it to replace the worst value on the list.

After going through all possible combinations the program will output an N-length list of the best figures of merits and their associated coating sets.

Figure of Merit Calculation:

Generate a matrix of the four vectors currently under consideration.

If the determinant of the matrix is non-zero then invert the matrix and continue. If it is zero then the calculation is finished and we set the figure of merit to infinity.

Calculate the expected dφ/dT values using nominal thermo-optic parameters.

Set the error in dφ/dT as some fraction of dφ/dT (0.05 in this case).

Use the inverse matrix to propagate these errors to errors on the coefficients.

Calculate the figure of merit.

Figure of Merit: What do we mean by "Best Coating"?

It's obvious that the best coating choice is one that has the smallest errors on the parameters. However, we need to define what constitutes smallest error and there are multiple ways to do so. For example, imagine that we had two measurements:

$$\begin{cases} 20.0 \pm 1.0 \\ 1.2 \pm 0.8 \end{cases} vs. \begin{cases} 20.0 \pm 3.0 \\ 1.2 \pm 0.2 \end{cases}$$

Here, taking the sum of the two uncertainties (1.8 vs.3.2) is a completely different result than taking the sum of fractional errors (72% vs. 32%). Depending on how we calculate the lowest error either of these two sets could be considered the best.

In our code, we decided to take fractional errors because we believe that having low fractional errors is important. We make the four fractional errors into a vector and calculate the length to get the figure of merit:

$$F.O.M. = \sqrt{\left(\frac{\sigma_{\alpha L}}{\alpha_L}\right)^2 + \left(\frac{\sigma_{\alpha H}}{\alpha_H}\right)^2 + \left(\frac{\sigma_{\beta L}}{\beta_L}\right)^2 + \left(\frac{\sigma_{\beta H}}{\beta_H}\right)^2}$$

Defined this way, a small figure of merit will ensure that all four fractional errors are small.

Initial Results:

We tried this procedure for the following coatings:

-Quarter-quarter stack with 6, 11, and 14 doublets

-Bragg Stack (⅛ high index, ⅜ low index) with 6, 11, and 14 doublets

-Reverse-Bragg(⅜ High index, ⅛ low index) with 6, 11, and 14 doublets

-All of the above with a half-wavelength cap of low index material

-All of the above with an additional half-wavelength layer of high or low index inserted at every point in the coating list.

In our calculations the best coating sets were the quarter-quarter and Bragg stacks, while the reverse-Bragg stacks were never in the top choices. Furthermore, the coatings with 11 or 14 layers seemed to be preferred, while coatings with 6 layers were rarely chosen.

The set that gave us our lowest figure of merit was the following:

- A quarter-quarter stack with a half-wave cap and a half-wave high index at the last layer
- A quarter-quarter stack with a half-wave cap and a half-wave low index at the twenty-first layer
- A quarter-quarter stack with a half-wave cap and a half-wave low index at the ninth layer
- A Bragg stack with a half-wave cap and a half-wave low index at the nineteenth layer

For this set of coatings we found the following fractional errors:

$$\left(\frac{\sigma_{\alpha L}}{\alpha_L}\right) = 0.0385, \left(\frac{\sigma_{\alpha H}}{\alpha_H}\right) = 0.0336, \left(\frac{\sigma_{\beta L}}{\beta_L}\right) = 0.0414, \left(\frac{\sigma_{\beta H}}{\beta_H}\right) = 0.0360$$

And the figure of merit is 0.0749.

Our preliminary results are fairly unsurprising and they seem to support the experiments that have already been done (with quarter-quarter and Bragg stacks). It seems logical that the code would select coatings with an extra layer at the top, bottom and middle of the stack, because those have the most diverse placement and thus form the best measurement matrix. However, there are a few things in our preliminary results that are particularly interesting to note. Firstly, as mentioned, the reverse-Bragg coating never showed up in the coating sets with the best figures of merit. Furthermore, for our best set of coatings, the fractional errors are all less than 5 percent even though we added a nominal error of 5 percent early on in the code. We would have expected the fractional error to increase and we are currently looking into why these errors are smaller than expected.

Next Steps:

Although we have eliminated the reverse-Bragg coating as a likely candidate and we have found that roughly 11 layers gives us the best results there are still many more coatings to try. Next we will evaluate coatings with two or more half-wave layers inserted in various locations in the stack as well as some less conventional doublets that still add to a half wavelength (such as 1/10 – 4/10 and 1/6 – 1/3 coatings).