LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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Adaptive Feedforward Seismic Noise Cancellation at the 40m Interfermeter

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1 Background

1.1 Noise in LIGO Interferometers

LIGO Interferometers send a laser beam through a beam splitter to be locked in Fabry-Perot cavities and recombined so that the effects in the cavity can be observed [1] However, many other factors besides just gravitational waves affect the resulting signal from the lasers. These outside factors are noise that must be filtered out either through the construction of a mathematical filter or physically building a filter to dampen the noise. Many kinds of noise affect LIGO Interferometers, but Seismic and Newtonian noise are most problematic because they affect frequencies LIGO is trying to detect and are more difficult to mechanically control precisely [1]. Mechanical filters, including stabilizing the concave mirrors in each cavity through magnetic fields and using oscillators to dampen seismic noise, are already in place but are always being improved.

1.2 Feedforward vs Feedback Filtering

Mathematical filters are just as important as physical filters. Feedforward techniques predict what noise will be in the signal and adjust the detector so that this particular noise is not part of the output. Feedback adjusts the detector after noise has propagated through the system [2]. The differences between feedforward and feedback are depicted below (figure 1). This is because, with feedback, the disturbances and noise sources must pass through the

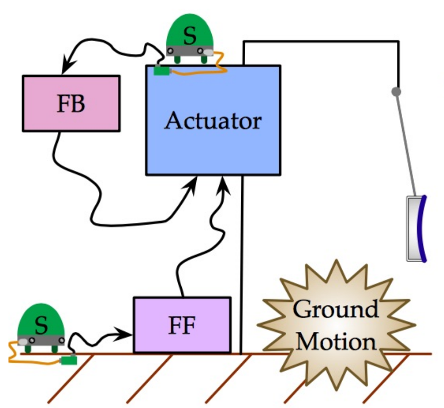


Figure 1: The differences between a feedback loop and a feedforward loop [4]

system in order to be detected, whereas with feedforward these noise sources are predicted and preemptively filtered out. Thus, there is no time lag that is associated with feedback techniques [2]. Wiener filters in particular are desirable to use for feedforward techniques because they determine the value of an unknown signal given known signals. Using feedback, one can determine the noise of the system and then construct a Wiener filter based off of that [4]. A feedforward Wiener filter can then be created. These filters will have an extensive impact, minimizing seismic noise through multiple degrees of freedom [5]. Therefore, we will only need to construct one Wiener filter to filter out noise from several sources.

1.3 IIR Wiener Filters

In this case, IIR Wiener filters are ideal because they require fewer parameters than FIR filters, which is important because Wiener filters have to create a very precise model of the system, so the fewer parameters required means that there is less room for error. This also drastically reduces computational time of the filters by using IIR instead of FIR coefficients [5]. This is especially important for a feedforward system, which must predict how to filter the signal prior to receiving it. Also, IIR filters are more likely to achieve the lowest mean-square error because they are computed by integral, while a FIR filter is a summation [3]. Therefore, FIR filters can come close to achieving the results of IIR filters, but will never be quite as good. However, IIR filters are difficult to calculate, and they also can introduce noise into controls of the system because of how each control interacts with others [5]. Therefore, it will be difficult to implement an IIR filter that also optimizes subtraction and introduces minimal noise into other parts of the system.

2 Goals During the First Weeks

During the first several weeks of the summer, I compared various algorithms and techniques of filtering noise from our sensors to determine self-noise. Self-noise is important to know because it cannot be mechanically be filtered out and gives us a minimum goal. We use both seismometers and piezoelectric accelerometers to detect seismic noise, so it is important to know the self-noise of both. Piezoelectric accelerometers are solid state devices, consisting of a crystal and a seismic mass. When a force is applied to the crystal by seismic motion, a voltage is created across it which we then measure.[6]. A seismometer consists of a weight on a spring, with the attached frame connected to the earth's surface. Seismometers act as a transducer between the input acceleration and the position of the mass [7]. They also behave similarly to accelerometers at small frequencies. In either case, we can measure the input and output of individual sensors, but have no way to physically isolate the noise from a single sensor. As a result, we must use multiple sensors, which all measure the same signal, and then use the differences in these results to determine the noise in an individual sensor. I applied the Three-Cornered Hat Technique and Wiener filtering to accelerometers which we had performed a huddle test on. Wiener filtering was applied to seismometers.

3 Three-Cornered Hat Technique

The Three-Cornered Hat Method is an algorithm to determine the error of one measurement of a given quantity after multiple measurements were taken. This is useful when, experimentally, we have no way to measure the quantity in a control environment, but must always deal with noise. As accelerometers will always detect vibrations, this method is useful to determine the self-noise of a single accelerometer. The propagation of uncertainty, or how the uncertainty of a variable x affects the uncertainty of f(x), is integral to understanding the Three-Cornered Hat Technique. Allan and Gray explain this method by having a single variable "x" of unknown quantity and then taking three measurements of it [8].

$$u1 = x \pm e1 \qquad u2 = x \pm e2 \qquad u3 = x \pm e3 \tag{1}$$

The only information we can directly measure are the three quantities u1, u2, and u3 in equation (1). From there however, we can calculate the variations between these measurements: σ_{12}^2, σ_{13}^2, and σ_{23}^2. We also know that the sigmas add linearly such that

$$\sigma_{12}^2 = \sigma_1^2 + \sigma_2^2 \tag{2}$$

$$\sigma_{13}^2 = \sigma_1^2 + \sigma_3^2 \tag{3}$$

$$\sigma_{23}^2 = \sigma_2^2 + \sigma_3^2 \tag{4}$$

We can then obtain the expression for σ_1^2 by combining equation (2), (3), and (4):

$$\sigma_1^2 = \frac{1}{2}(\sigma_{12}^2 + \sigma_{13}^2 - \sigma_{23}^2) \tag{5}$$

[8]. We have now obtained a sigma for just one signal, successfully extracting the noise for one measurement by analyzing the differences between three observed quantities.

4 Application of Three-Cornered Hat to Huddle Test

I used the Three-Cornered Hat method to determine the noise in accelerometers subject to the huddle test. To perform a huddle test, the six accelerometers were grouped together so that each measures the same quantity. They were also clamped to the table to reduce differences in seismic noise between them. Given the results of the huddle test, I was able to calculate the Power Spectral Density (PSD) line for each accelerometer and determine its self-noise using the Three-Cornered Hat technique. Following the steps outlined in Section 3, I made the plots in two sets of three, determining the noise of accelerometers 1-3 separately from accelerometers 4-6. Figure 2 shows the noise levels for the first accelerometer using the Three-Cornered Hat method. It can be seen here that at lower frequencies, especially

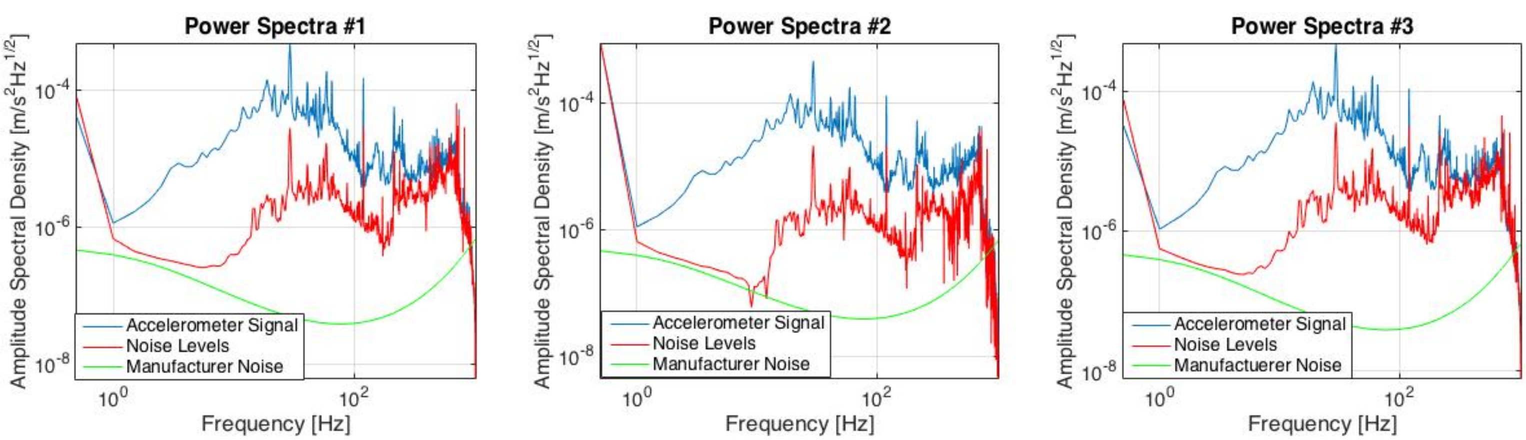


Figure 2: PSD plots for the noise in the first three accelerometers in the huddle test, made by the Three-Cornered Hat method

between 1 and 10 Hz, the Amplitude Spectral Density (ASD) has been significantly lowered, while at higher frequencies the Three-Cornered Hat method did not filter as much out. Comparing this to the expected noise, which is the green line, it can be seen that what has been filtered out by the Three-Cornered Hat method is still significantly higher than is desired between 10 and 1000 Hz.

5 Constructing a Wiener Filter

Wiener filters are used to calculate noise by comparing several input signals to an output signal. This is useful in determining the self-noise of accelerometers and seismometers, and has the potential to filter noise better than the Three-Cornered Hat method can. The thought behind constructing a Wiener filter is to solve this RMS minimization equation:

$$\xi = < d^2 > -2\vec{\omega}^T \vec{p} + \vec{\omega}^T R \vec{\omega} \tag{6}$$

where ξ is what we will minimize to calculate the Wiener coefficients, d is the signal we wish to filter, $\vec{\omega}$ is the Wiener filter we are solving for, $\vec{p}$ is the cross-correlation vector between witness and target signals, and R is the matrix [10]. To do this and construct a Wiener filter, the input data needs to be a covariance matrix. This matrix is constructed as a Toeplitz-block matrix but is converted to a block-Toeplitz matrix. Knowing that a block matrix has the form

$$M = \begin{pmatrix} a & a & b & b \\ a & a & b & b \\ c & c & d & d \\ c & c & d & d \end{pmatrix}$$

which can be written as

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

and a Toeplitz matrix has items repeating down the diagonals,

$$N = \begin{pmatrix} a & b & c \\ b & a & b \\ c & b & a \end{pmatrix}$$

it can be seen that a block-Toeplitz matrix will have blocks repeating down the diagonals [9]. The dimensions of this matrix are determined by the number of inputs and the filter order. Filter order is important because I am focusing on FIR (Finite Impulse Response) Wiener filters now. To calculate the coefficients for the Wiener filter, a cross-correlation vector also needs to be calculated. The same parameters are used to calculate this vector, but with respect to the input matrix. From here, two methods can be used to calculate the final coefficients. The first, more complex but more exact method is the Block-Levinson method, which uses the calculated matrix and vector to invert the block-toeplitz matrix. This method solves the equation

$$M_n \vec{x}_n = \vec{y}_n \tag{7}$$

where M_n is the first column of input matrix and $\vec{y}_n$ is the cross-correlation vector, for $\vec{x}_n$ [9]. From here, we will begin by solving for the matrix M by dividing it into two components, one multiplied by $\begin{matrix} a_{n-1} \\ 0 \end{matrix}$ and the other multiplied by $\begin{matrix} b_{n-1} \\ 0 \end{matrix}$, where a and b are series. The results of this are:

$$M_n \begin{pmatrix} a_{n-1} \\ 0 \end{pmatrix} = \begin{pmatrix} \epsilon_{n-1} \\ 0_{n-1} \\ \xi_n * \epsilon_{n-1} \end{pmatrix} \tag{8}$$

$$M_n \begin{pmatrix} 0 \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} -\nu_n * \epsilon_{n-1} \\ 0_{n-1} \\ \epsilon_{n-1} \end{pmatrix} \quad (9)$$

ϵ, λ, ν, ξ are all equations that depend on a_{n-1} or b_{n-1}. From here, the vector $\vec{x}_n$ can be solved for by observing that the solutions to a and b are recursive, resulting in

$$\vec{x}_n = \begin{pmatrix} \vec{x}_{n-1} \\ 0 \end{pmatrix} + \frac{\lambda_n}{\epsilon_n}\vec{b}_n \quad (10)$$

This vector contains the coefficients for the FIR Wiener filter [9]. This technique was used to calculate a SISO (Single Input Single Output) filter, while the simpler method was used when calculating MISO (Multiple Input Single Output) Wiener filters.

6 MISO Filtering of Accelerometers

I first created a MISO Wiener filter and applied it to the accelerometers subject to the huddle test. This way, I can later compare the Three-Cornered Hat method of determining self-noise to the ability of the Wiener filter to do the same. To create a Wiener filter for these accelerometers, I used a function that implemented the matrix algebra described in the section above. Each accelerometer was a separate input. There was no specific output. Instead, I used 2 accelerometers as input and a third as output to determine the self-noise of that particular accelerometer. The results of this (figure 3) were successful. At low frequencies, the Wiener filtered noise was still above the expected ASD, shown by the green line below, but it filtered out much more than the Three-Cornered Hat method did at higher frequencies. Comparing the Wiener filter of the accelerometers to the Three-Cornered Hat

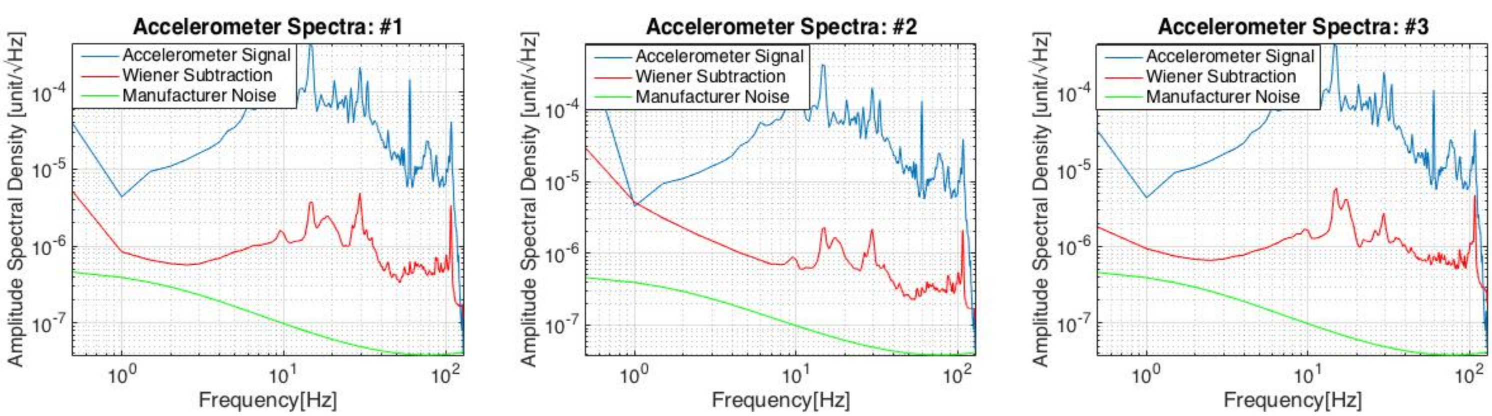
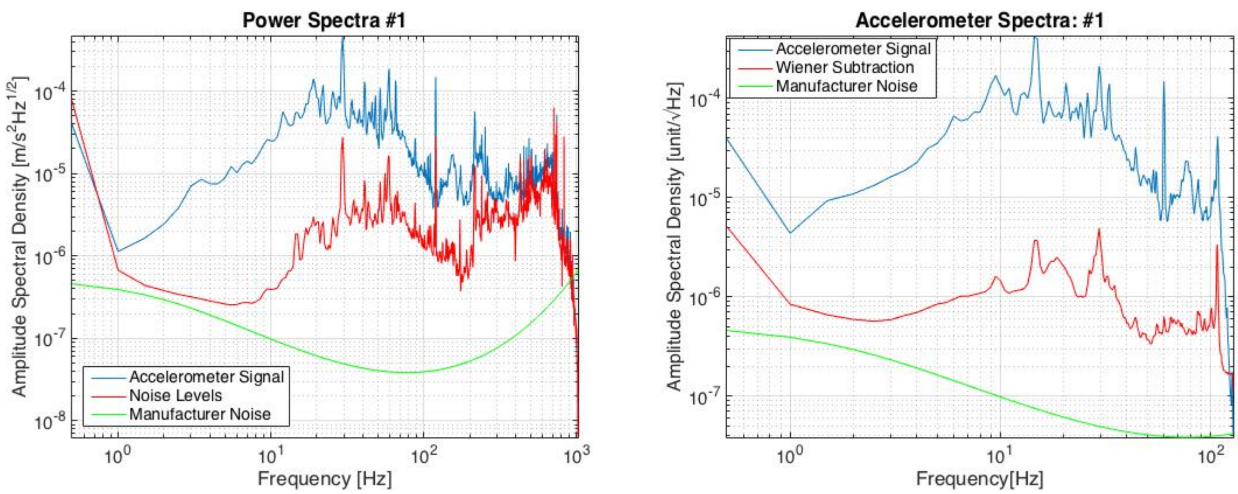


Figure 3: MISO Wiener filtering of the first three accelerometers in the huddle test

method of determining self-noise of accelerometers, as is pictured below (figure 4), I found that the Wiener filter worked much better, especially taking high frequencies into account. In low frequency ranges both methods produce similar results, but the significantly lower noise curve at high frequencies produced by the Wiener filter is promising. It motivates further exploration into the Wiener filter, including adaptive as opposed to static filters. A combination of Wiener filtering and the Three-Cornered Hat method also holds potential to filter noise down to the manufacturer's predicted level.



(a) Three Cornered Hat method applied to the first accelerometer in the huddle test

(b) MISO Wiener filtering of the first accelerometer in the huddle test

Figure 4: Comparison between the Three-Cornered hat method (a) and MISO filtering (b)

7 MISO Filtering of Seismometers

I then used matlab to apply MISO filtering to a seismometer having components in the x, y, and z directions. The seismometer was measuring noise around the mode cleaner, so all three components had an output corresponding to the mode cleaner. I then used a Wiener filter to determine how much noise came from each component. I then subtracted each noise component from the total signal and found the power spectral density of that. The noise filtered from the seismometer is pictured below (figure 5). However, there are also two

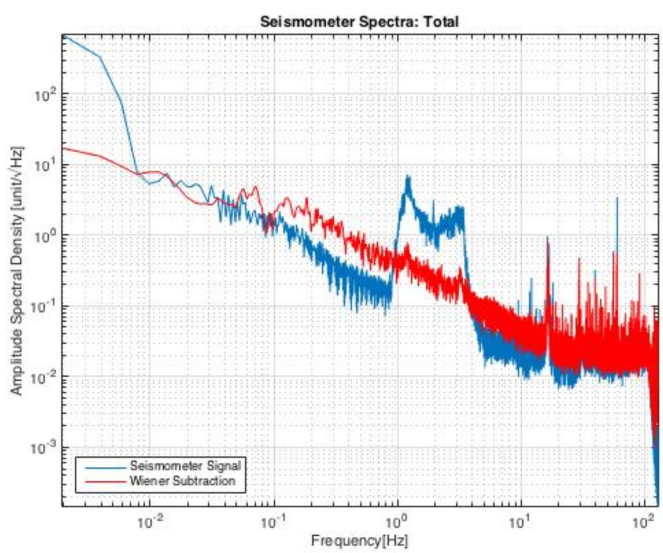


Figure 5: total MISO filtering of seismometers in 3-Dimensions

accelerometers, each with three components, that are measuring noise at different ends of the mode cleaner. I then filtered noise from each component in both accelerometers and then added this to the seismometer noise. Subtracting all of this from the output and then finding the PSD resulted in the plot below, which was much better than just the seismometer (figure 6). At frequencies bel5ow 1Hz, the Wiener filter of the seismometer alone actually increased the noise. Adding in the accelerometers reduced this, providing a better overall filter.

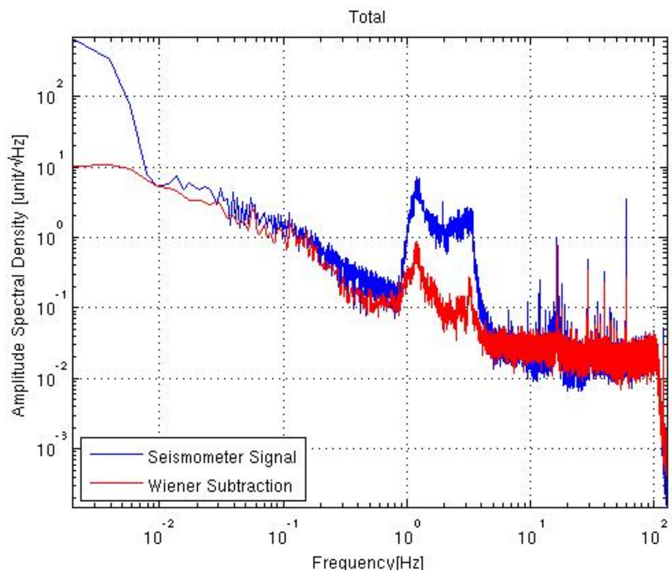


Figure 6: Filtering noise in seismometer and 2 accelerometers out of mode cleaner

8 Long Term Goals

Generally, by the end of summer I want to construct a static offline IIR filter, then create a static online IIR filter and an adaptive online IIR filter based off of that. I will also redo the huddle test in more isolated conditions at some point for both accelerometers and seismometers.

Week 4: I will look up the manufacturer-predicted noise of our accelerometers and determine how close our real noise is to that. If the front panel for the ALS delay line box comes in this week, I will build the box. Otherwise I'll do that next week. I'll start looking into pre-filtering to optimize our Wiener filters more.

Week 5: I will work more on pre-filtering. I'll also start looking into static offline IIR Wiener filters. I will test out both isolated and conductive connectors this week for the ALS delay line box and decide which work better or if it matters.

Week 6: Continue working on pre-filtering and construction of static offline IIR filter. We also have a trip to LLO this week.

Week 7: Create static offline IIR filter and learn the math for creating an adaptive IIR filter and an online IIR filter. Work on Progress Report 2 this week.

Week 8: Construct a static online IIR filter, finish up and turn in Progress Report 2

Week 9: Using the static IIR filter, I will begin working on the adaptive online IIR Wiener filter.

Week 10: Finish up the IIR adaptive online filter and work on my presentation and final paper.

9 Appendix

9.1 ALS Delay Line Time Delay

During my first three weeks working on filtering, I also worked on another project trying to physically isolate the ALS Delay line cables. My goal was to keep the cables seismically and electrically isolated. To do this, I wanted to put them inside the same conductive box so that they feel the same seismic vibrations and are inside a Faraday cage to keep them electrically isolated. When the cables arrived, I had to determine the time delay in the cables. To do this, I created Bode plots for each cable. I then used this data to write a code in matlab that found the time delay in each cable. Both had a time delay of 127 ns.

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