Thermal Noise in Thin Silicon Structures

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August 20, 2015

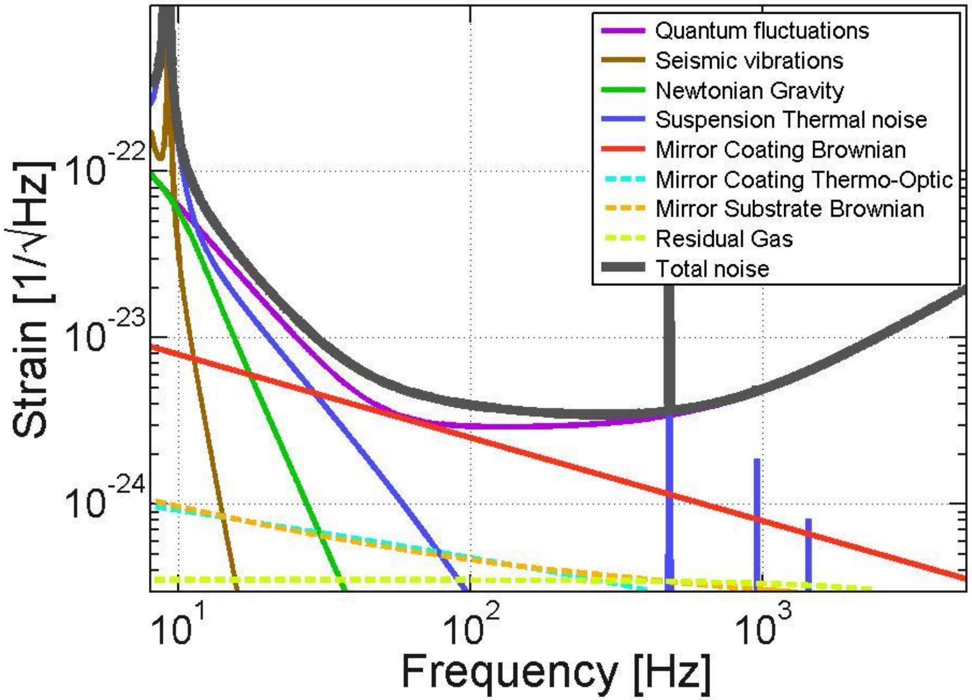
LIGO-G09xxxxx-v1

Overview

- Motivation
- Thermal noise
- Damped oscillator review
- Fluctuation-Dissipation Theorem
- Experimental design
- Measurements
- Conclusions

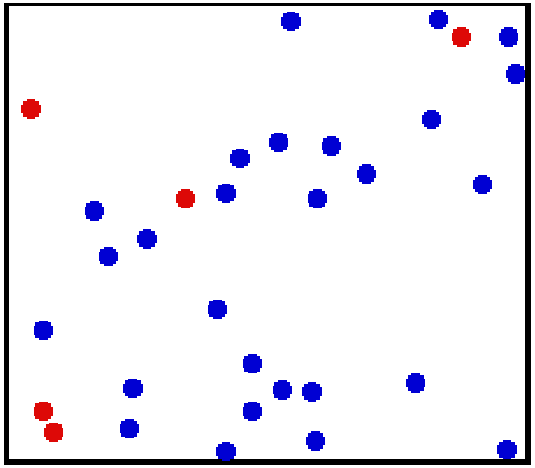
Motivation – Increase Future Detector Sensitivity

- Reduce thermal noise in suspensions and test masses
- Current detectors use fused silica
- Crystalline silicon is being considered for future detectors
  - » Favorable material properties (more on this later)



Thermal Noise in LIGO

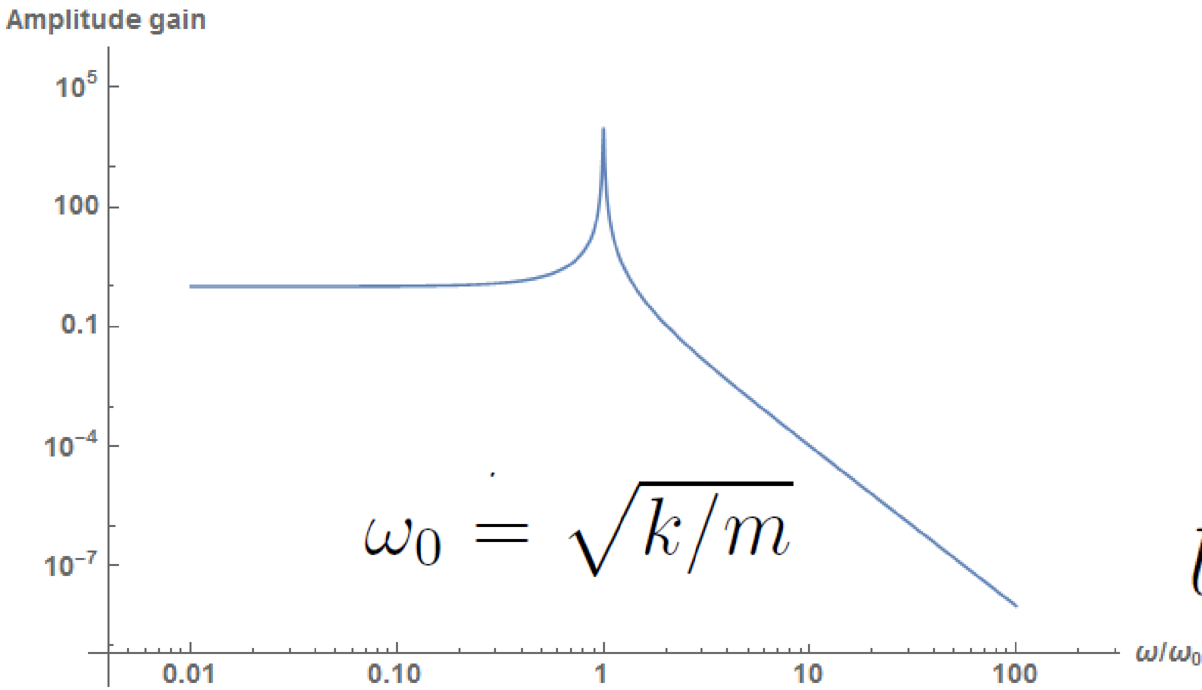
- Temperature is a measure of average kinetic energy
- Random thermal fluctuations couple to displacement noise in detector
- How do we reduce the shaking in our test masses? (without cooling everything to 0K)



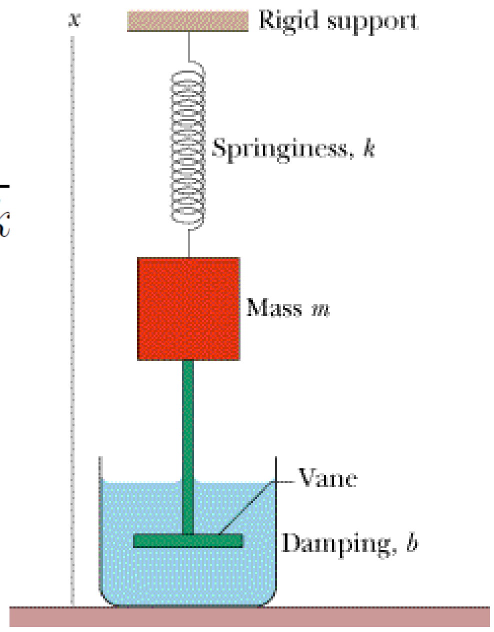
Underdamped Oscillator Review

Equation of motion: $m\ddot{x} + b\dot{x} + kx = f_{ext}$

Frequency response: $H(s) = \frac{X(s)}{F_{ext}(s)} = \frac{1}{ms^2 + bs + k}$



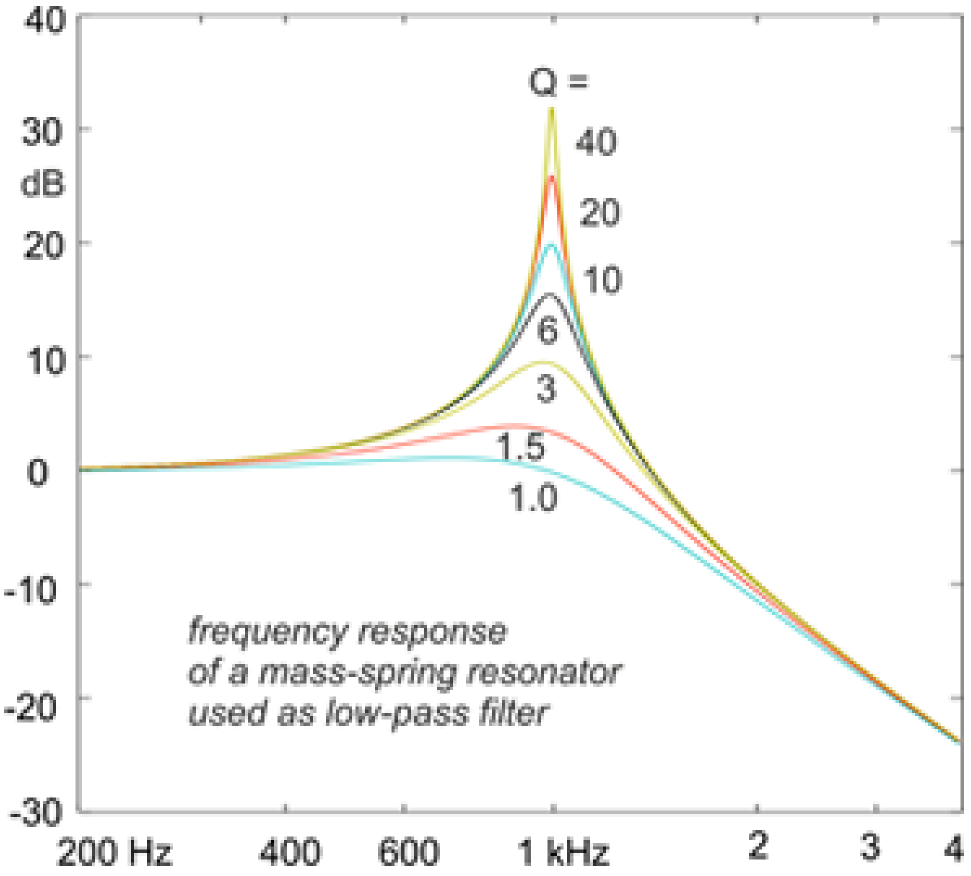
$\omega_0 = \sqrt{k/m}$



Underdamped:

$b^2/4km \ll 1$

Quality Factor Q



$$Q = \frac{\omega_0}{\Delta\omega}$$

$$Q = \frac{\omega_0 m}{b}$$

Internal Damping

$$m\ddot{x} + k(1 + i\phi)x = f_{ext} \longleftrightarrow m\ddot{x} + b\dot{x} + kx = f_{ext}$$

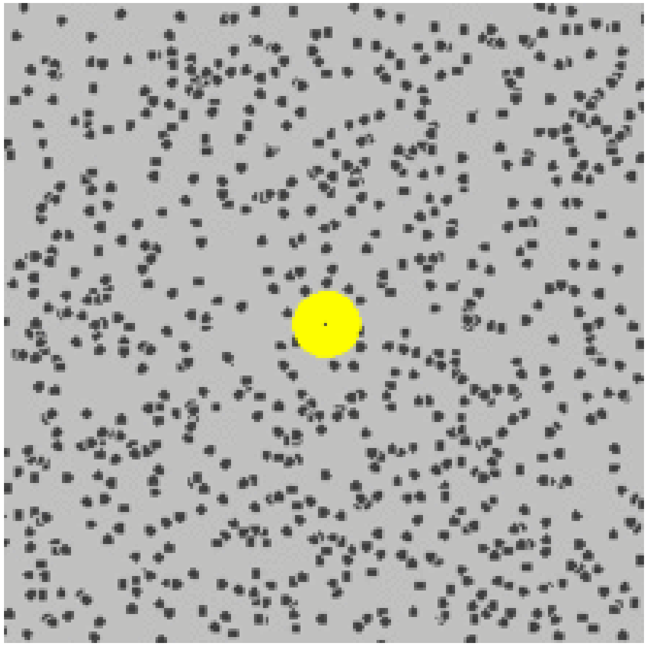
Restorative force leads displacement by loss angle ϕ

$$\boxed{Q = \frac{1}{\phi}}$$

How are losses in oscillators related to thermal noise?

Fluctuation-Dissipation Theorem

- FDT relates thermal fluctuations (noise) to dissipation (damping/resistance/loss)



$$\langle x^2 \rangle = 2Dt$$

Brownian motion

$$\langle V^2 \rangle = 4k_B T R \delta v$$

Johnson noise

Thermal Noise in Damped Oscillators

Thermal noise power spectrum:

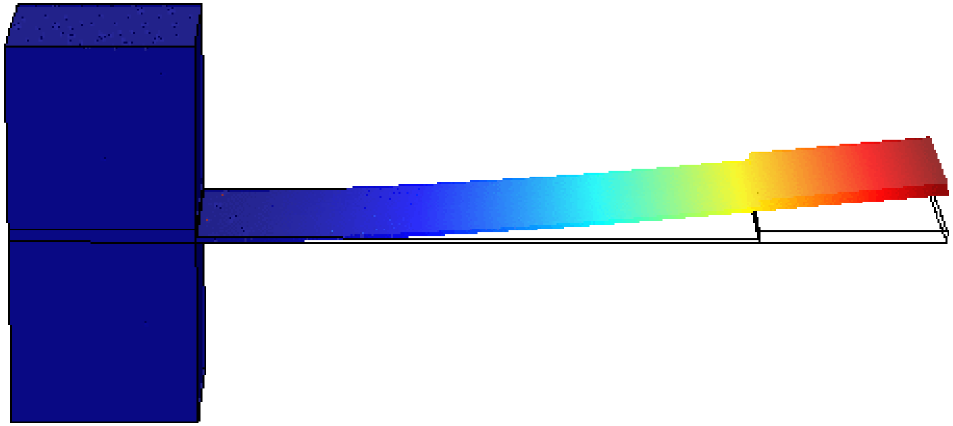
$$x^2(\omega) = \frac{4k_B T k \phi}{\omega[(k - m\omega^2)^2 + k^2\phi^2]}$$

Away from resonance:

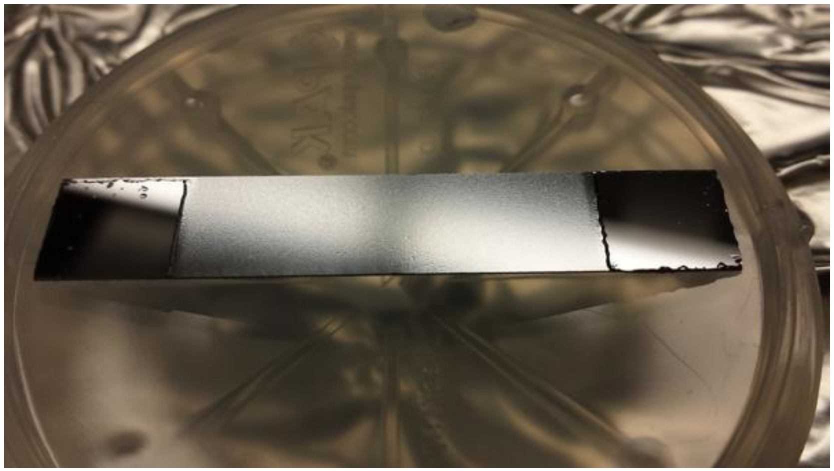
$$\langle x_{th}^2 \rangle \propto \frac{T}{Q}$$

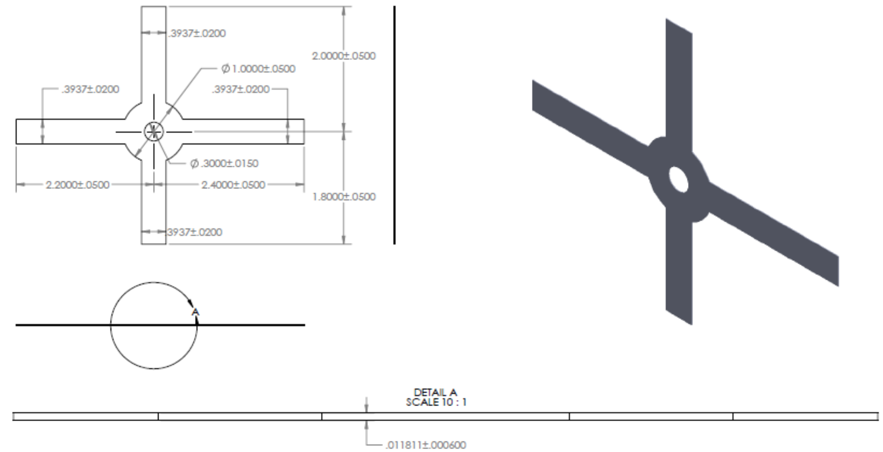
We want to design high Q, low loss silicon resonators

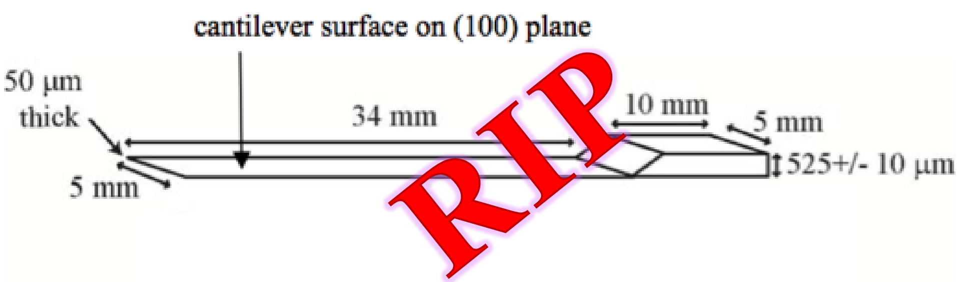
Our Resonators – Silicon Cantilevers

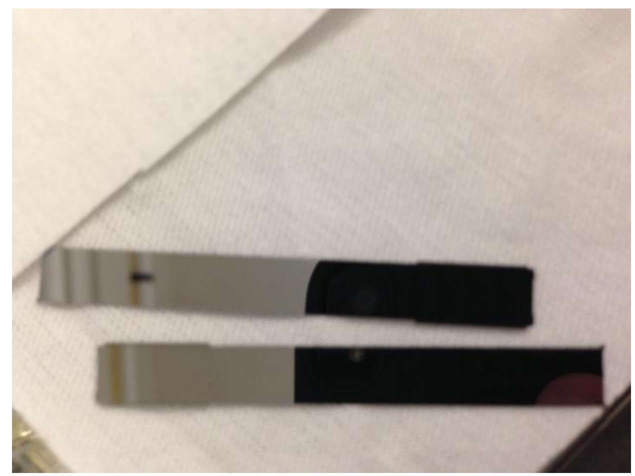


Silicon Cantilever Design

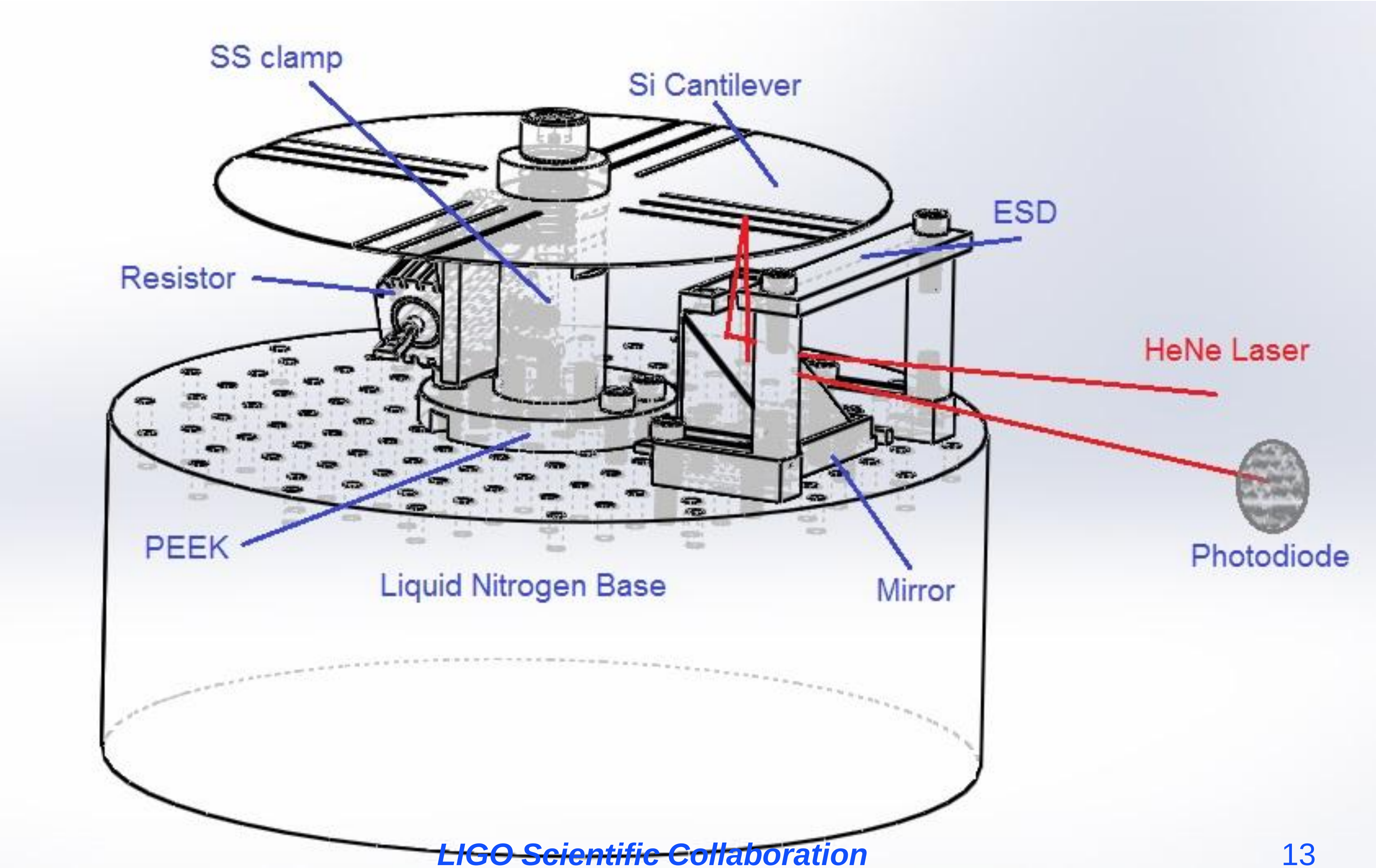




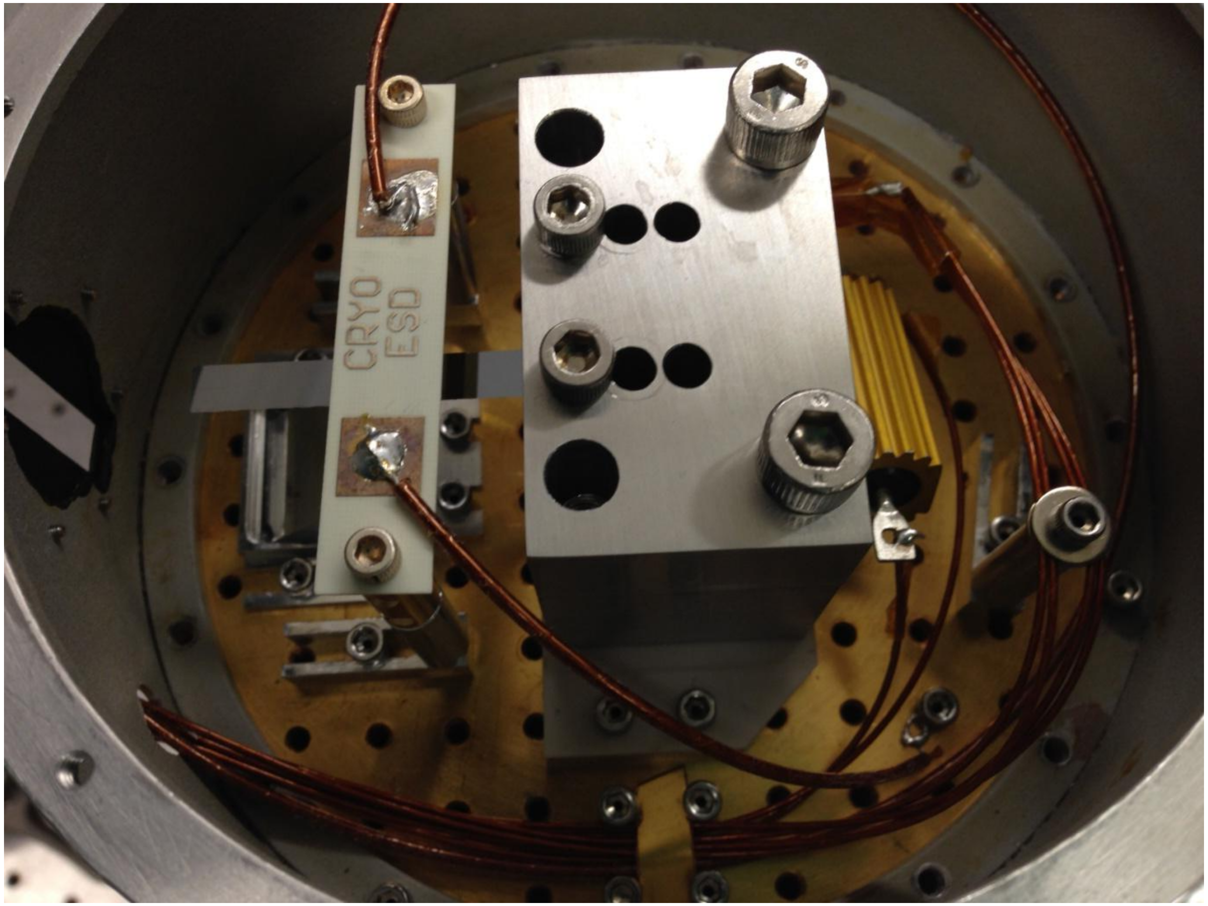




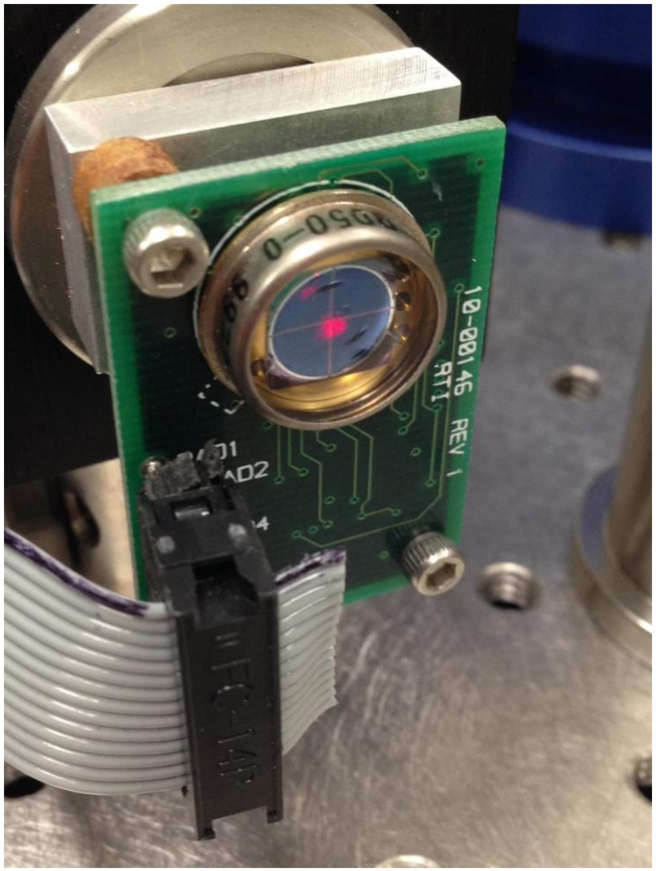
Experimental Design

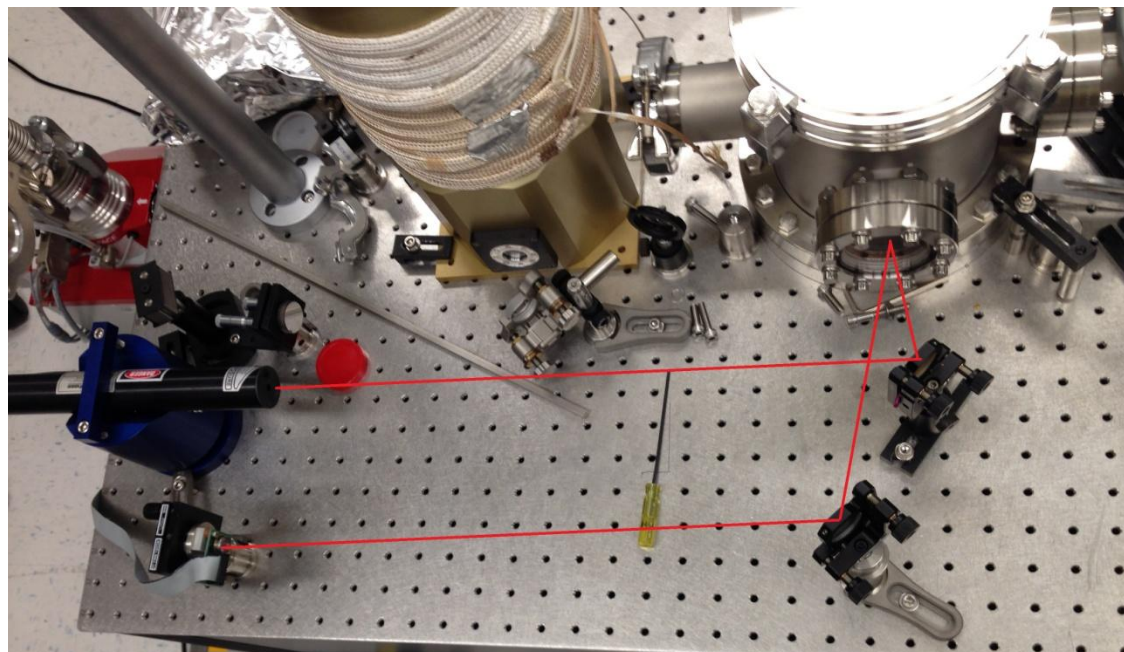


Experimental Design

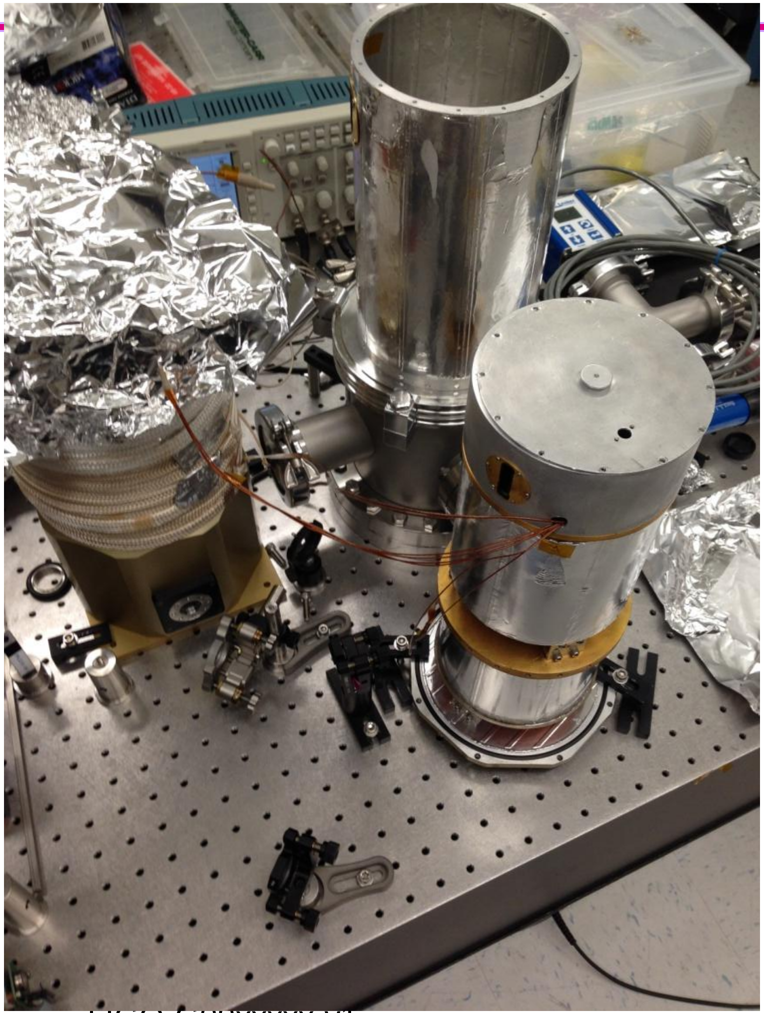


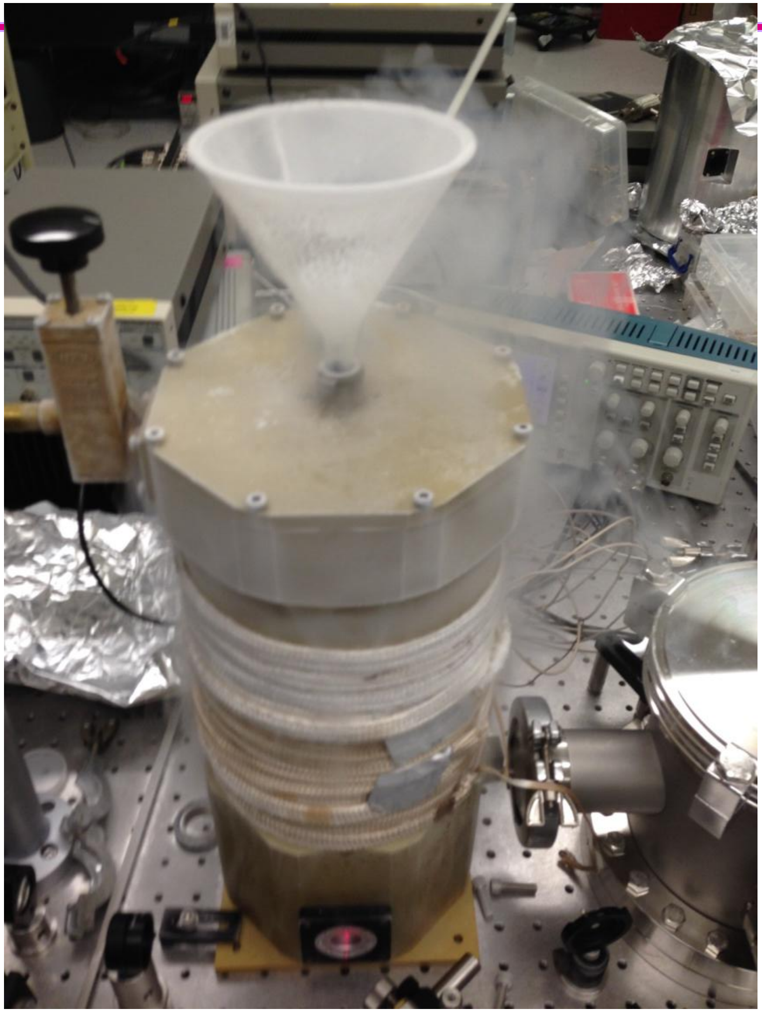
Experimental Design - Optics





Experimental Design – Insulation





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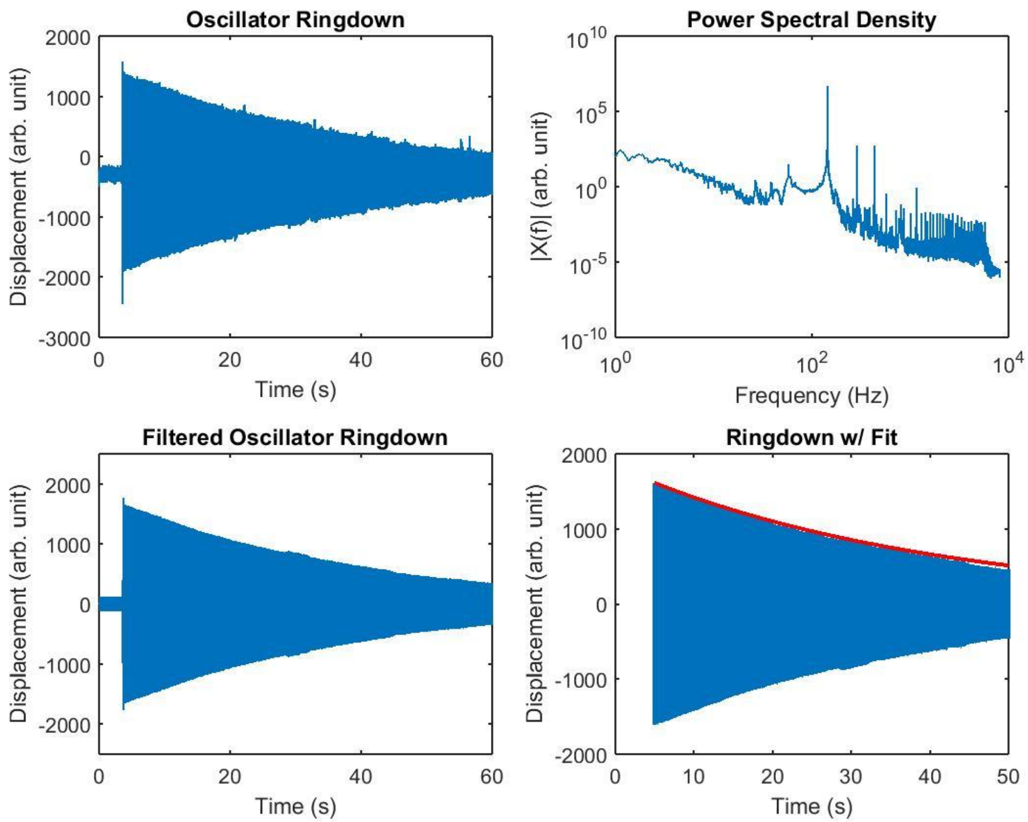
Ringdown Measurement

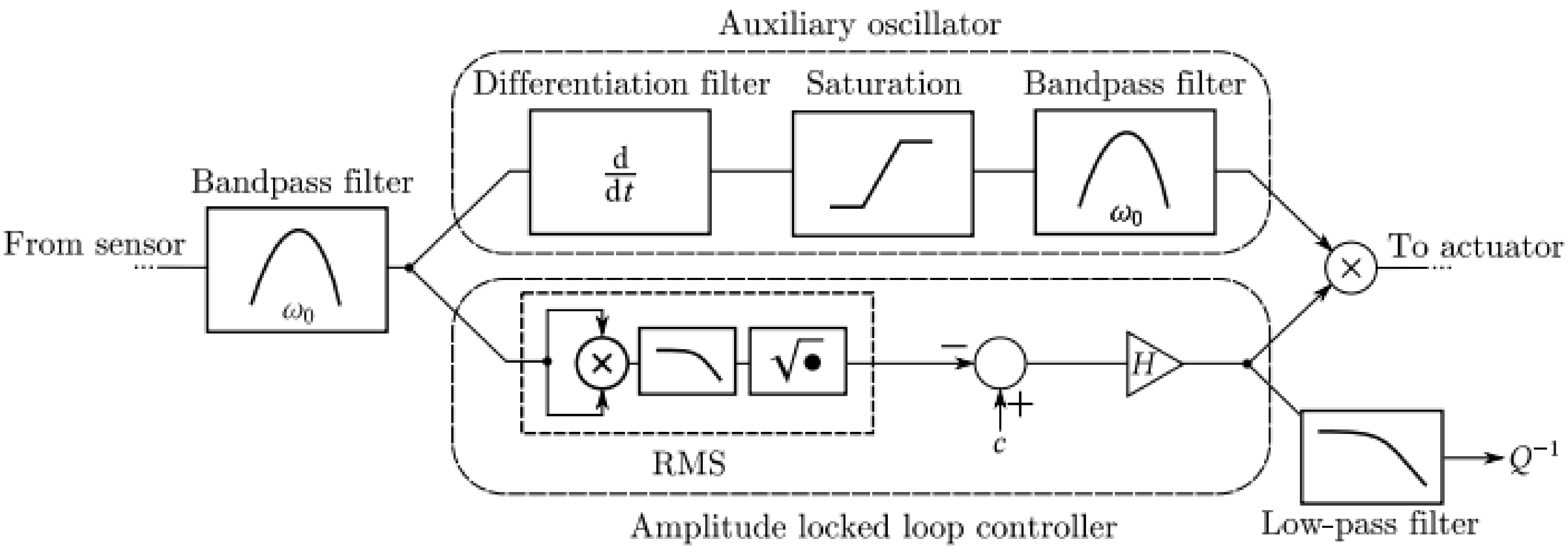
Excite cantilever with impulse and watch free decay



$$x(t) = e^{-t/\tau} \sin(\omega_0 t)$$

$$Q = \omega_0/\gamma = \frac{\omega_0 \tau}{2}$$

Continuous Measurement



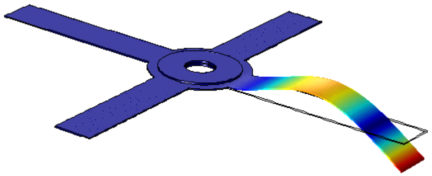
- Cantilever is continuously driven at constant amplitude
- Less sensitive to background excitations
- Measures Q over temperature sweeps, etc.

Making Conclusions about Losses

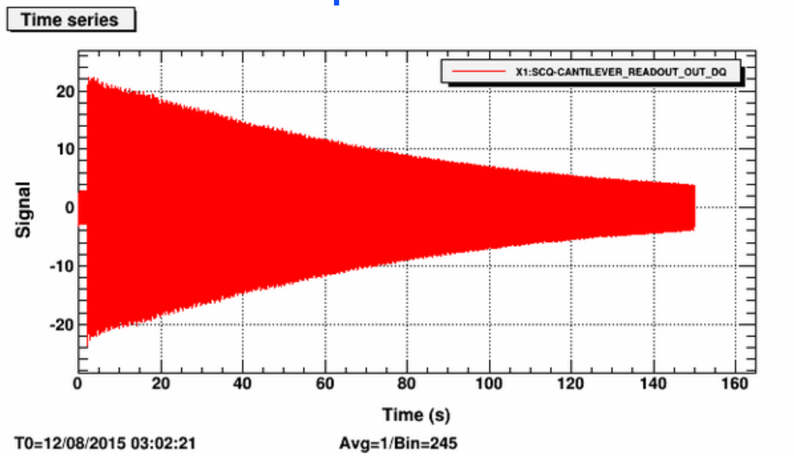
Theory

$$\phi_{TE} = \frac{\alpha^2 Y T}{\rho C_p} \frac{\omega\tau}{1+\omega^2\tau^2}$$

Simulation



Experiment



Sources of Loss

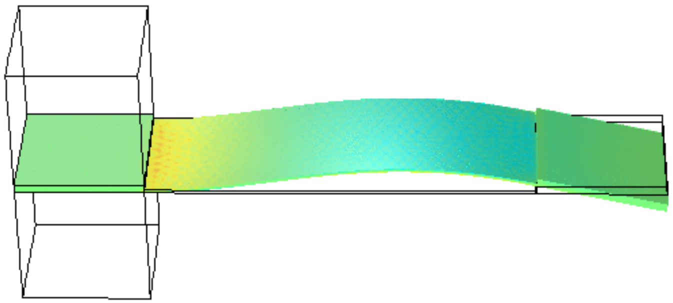
- **Thermoelastic loss**
- **Clamp loss**
- **Surface loss**
- Phonon-phonon loss
- Gas damping
- Bulk loss
- Excess losses

Thermoelastic Loss

- Coefficient of thermal expansion couples strain to temp.
- Heat fluxes driven by temp. gradient dissipate energy

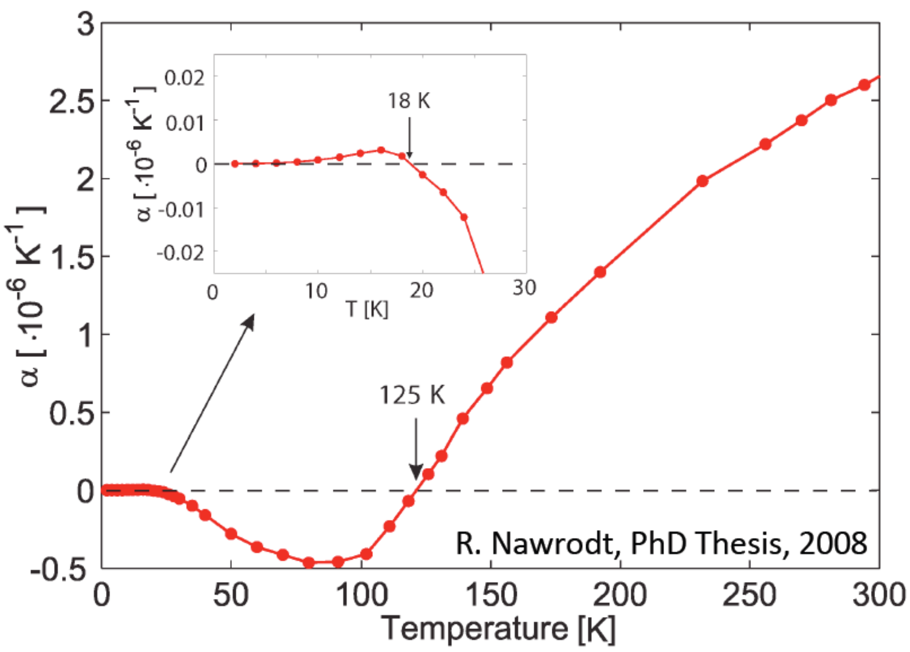
$$\phi_{TE} = \frac{\alpha^2 Y T}{\rho C_p} \frac{\omega\tau}{1+\omega^2\tau^2}$$

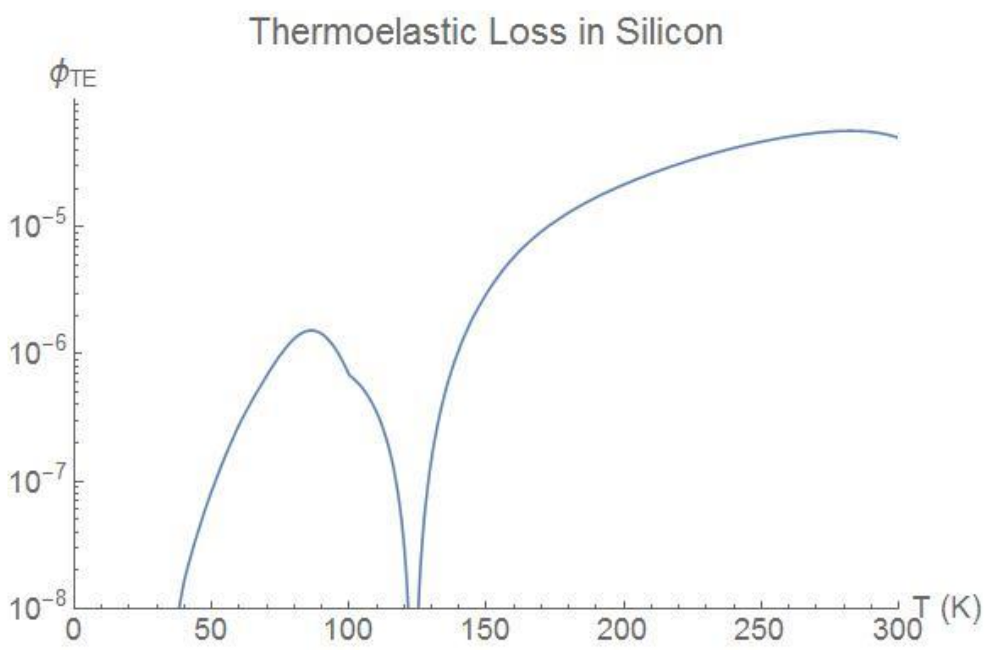
$$\tau = \frac{\rho C_p t^2}{\pi\kappa}$$



Thermoelastic Loss cont.

- Silicon has a vanishing α at T=125K





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Form F0900041-v1

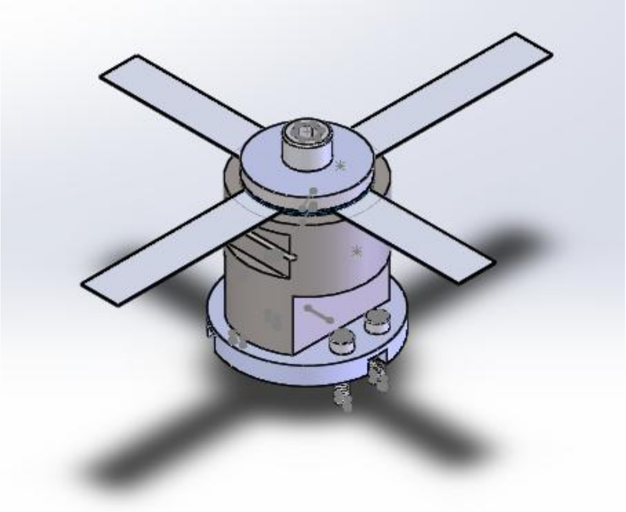
TE Loss Conclusions

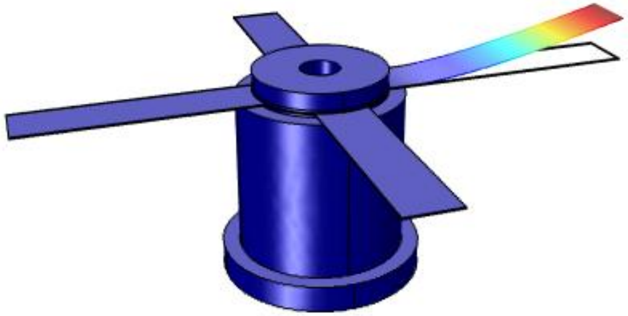
- Don’t see dramatic increase in Q at 125K
- Q is ~2 orders of magnitude lower than TE loss limited prediction
- Not TE loss limited

Clamp Loss

- Energy is transferred from cantilever and stored in clamp, base, etc. as strain energy

$$\phi_{measured} \approx \phi_{Si} + \frac{E_{clamp}}{E_{total}}\phi_{clamp} + \frac{E_{PEEK}}{E_{total}}\phi_{PEEK} + \dots$$





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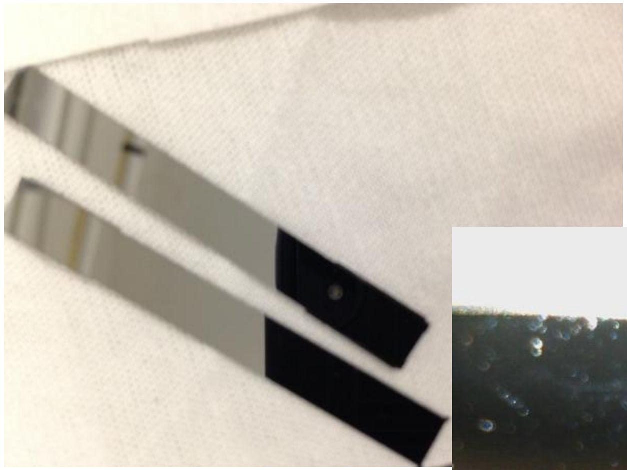
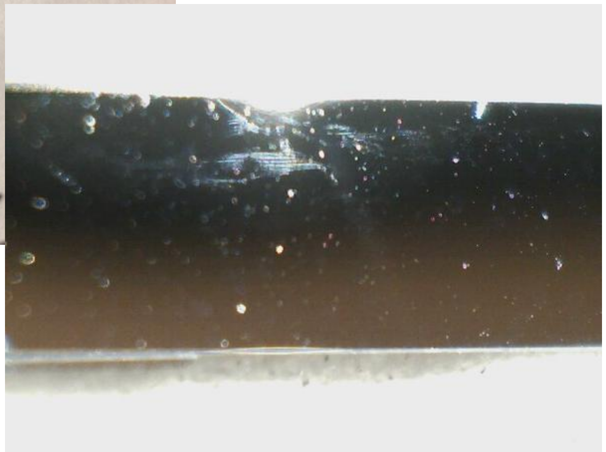
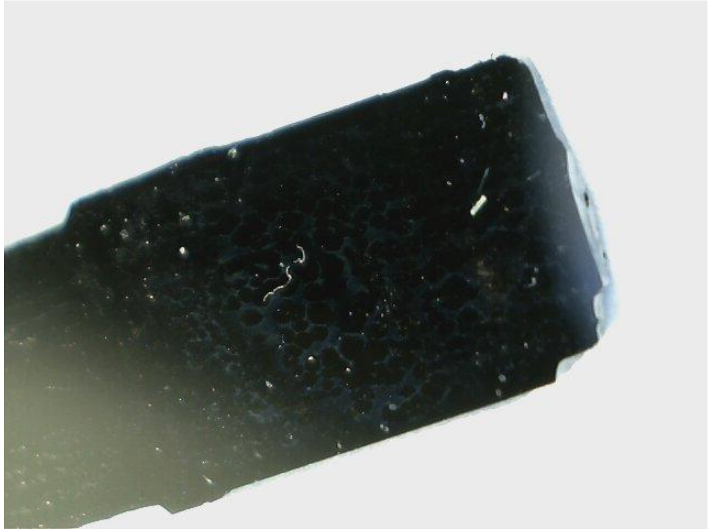
Clamp Loss Conclusions

- Simulations predict that very little strain energy is stored in clamp, washers, base, etc.
- New clamp designs don't significantly improve Q
- Reclamping doesn't change Q
- Not clamp loss limited

Eigenfrequency (Hz)	$E_{pinwheel}$ (arb. unit)	E_{clamp} (arb. unit)	Ratio
161	3016	3.4	1.1e-3
1009	120264	139	1.2e-3
1449	212434	160	0.8e-3

Surface Loss

- Surface roughness, lattice imperfections, adsorbed surface materials, etc. contribute to a lossy surface

Surface Loss Conclusions

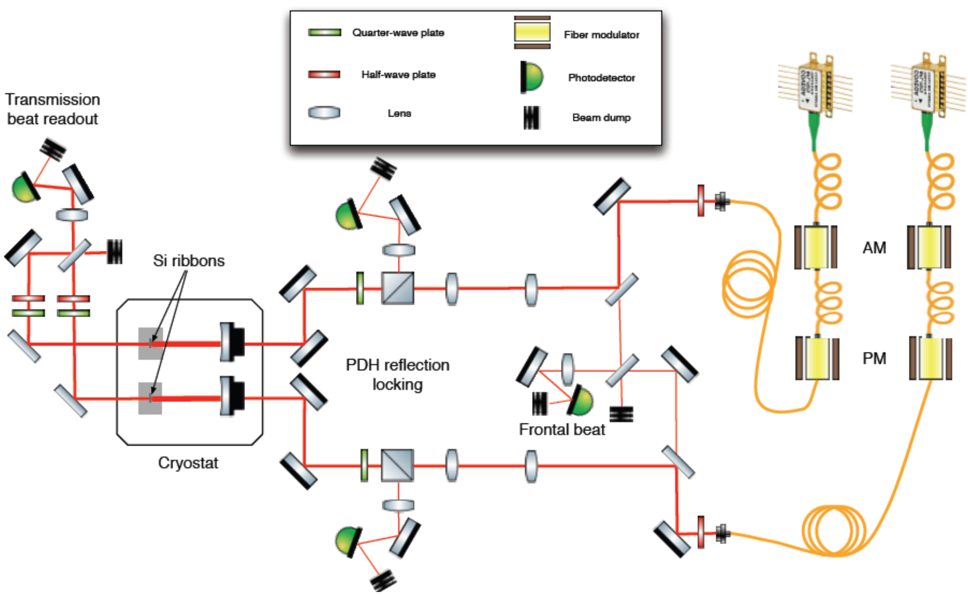
- Cantilever surfaces clearly show surface defects
- Cleaning doesn't improve Q (?)
- Lossy surface layers in simulation accurately predict experimental results
- Much more work to be done investigating different etching techniques
- Candidate for dominant loss source

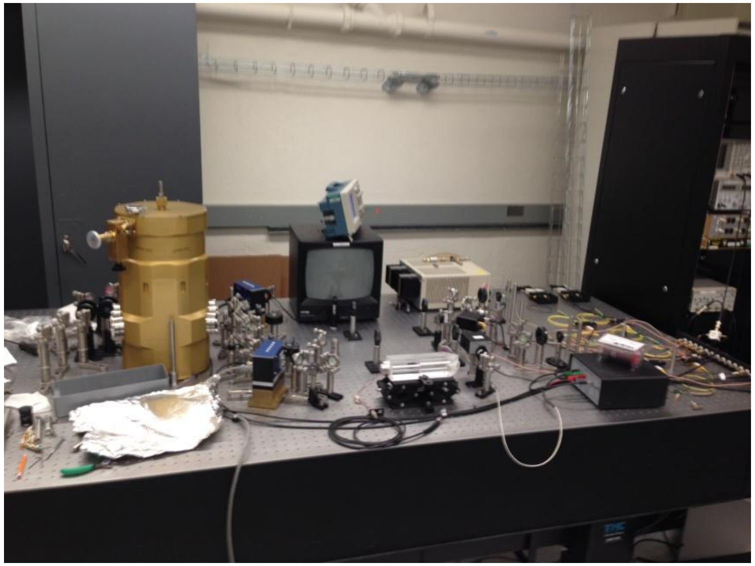
Summary

- No quality factor peak at 125K – not TE limited
- Simulation shows low strain energy ratios – not clamp loss limited
- Most likely surface loss limited – lossy surfaces in simulation match experiment
- Developed technique for continuous measurements of several resonant modes simultaneously – speed up future Q measurements

Future Work

- Experiment with different etching techniques
- Investigate loss in thin films and optical coatings
- Direct thermal noise measurements





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Acknowledgements

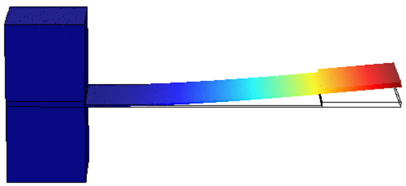
- My mentors: Zach, Nic, and Rana
- Fellow SURF students
- LIGO Scientific Collaboration faculty
- NSF and Caltech

Thanks for a great summer!

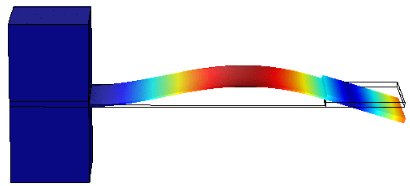
Questions?

COMSOL Mode Simulations

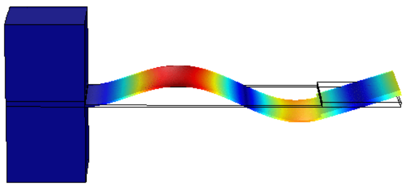
Normal Modes



Fundamental mode

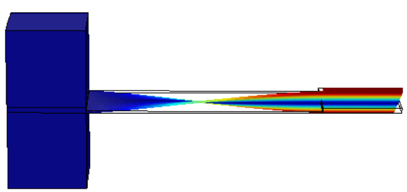


2nd mode

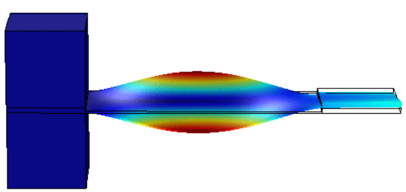


3rd mode

Torsional Modes



Fundamental mode



2nd mode