Orthogonal Sensor and Actuator, and Automatic Alignment of High Finesse Optical Ring Cavity

by

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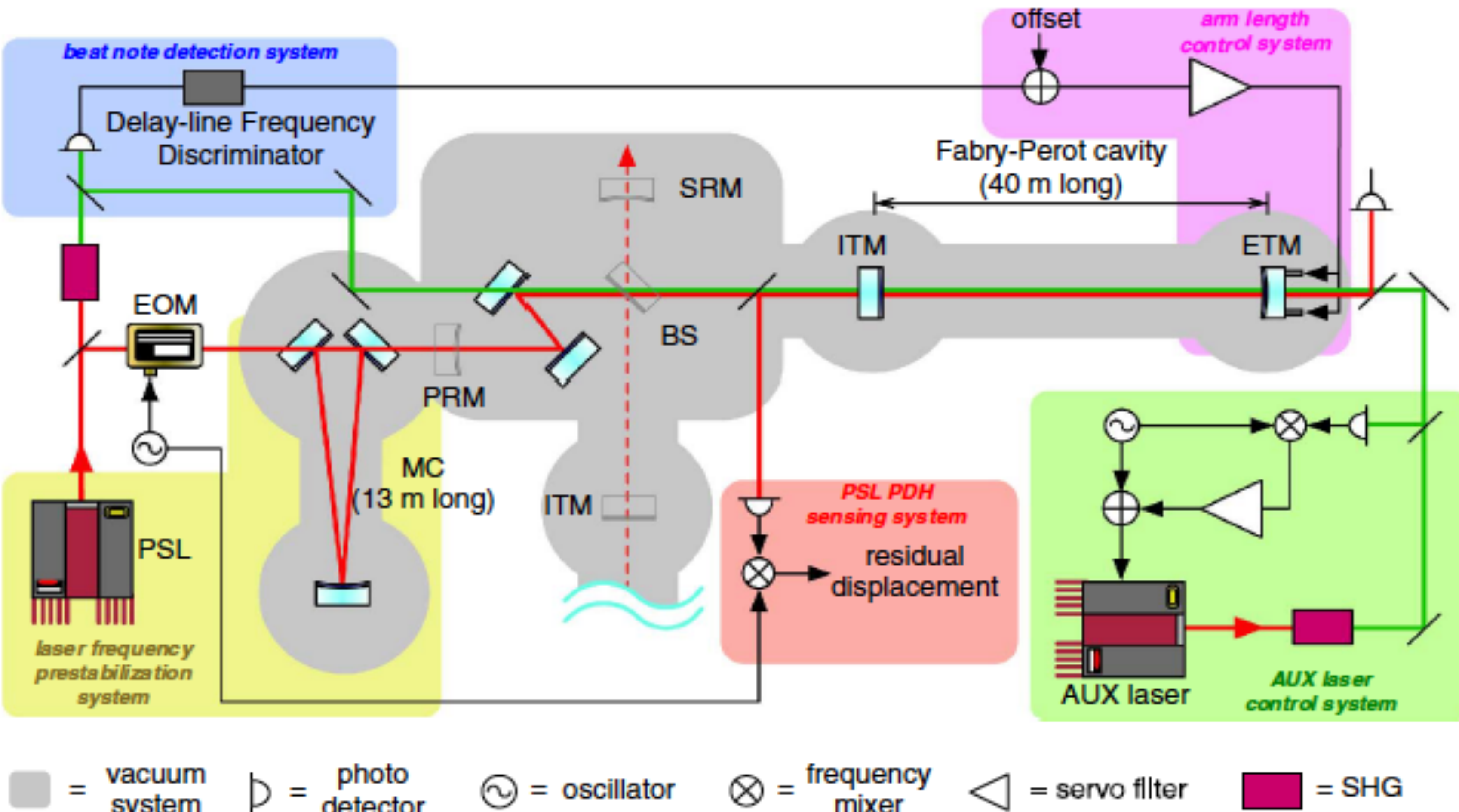
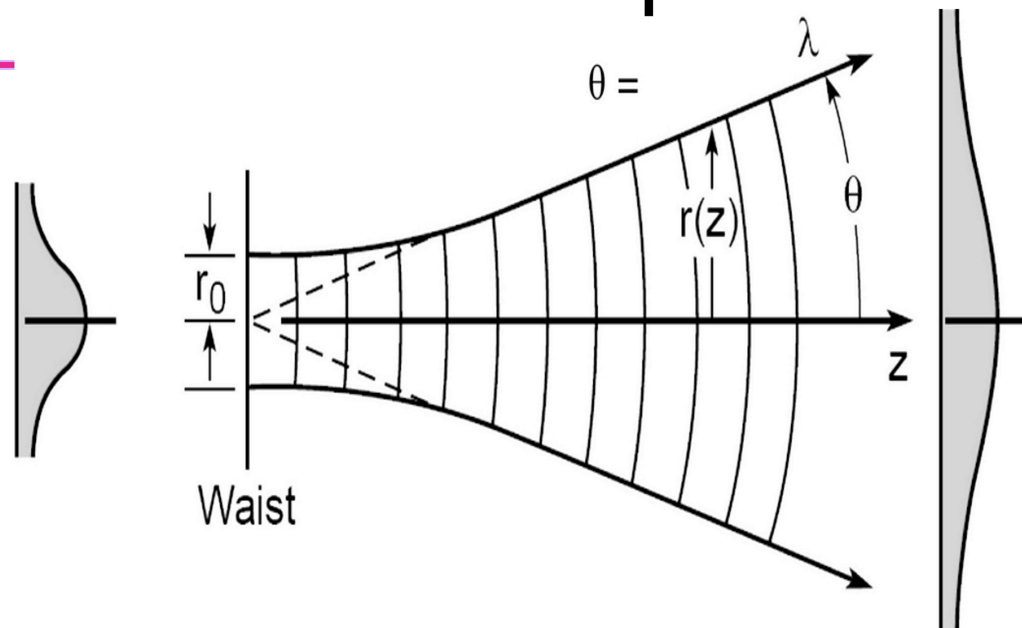
LIGO-G09xxxxx-v1

Gravitational Wave Detector



Gaussian Beam and its Properties



Paraxial Wave Equation

$$[\nabla^2 + k^2]\vec{E}(x,y,z) = 0$$

$$\frac{1}{\tilde{q}(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi w^2(z)}$$

$$w^2(z) = w_0^2\left[1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]$$

$$R(z) = z\left[1 + \left(\frac{\pi w_0^2}{\lambda z}\right)^2\right]$$

ABCD Matrix

$$q = \frac{Aq_0 + B}{Cq_0 + D}$$

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

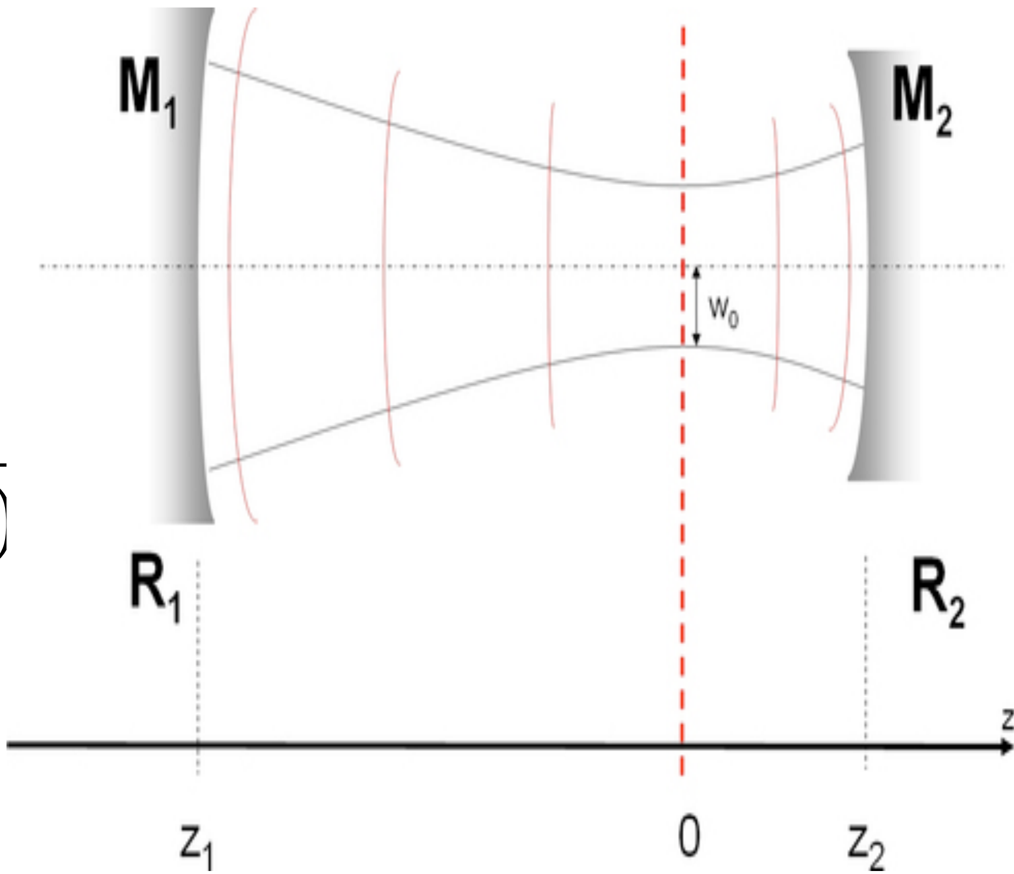
Mode Matching

Amplitude Coupling

$$C = 2\frac{\sqrt{z_{R1}z_{R2}}}{i(z_1 - z_2) + (z_{R1} + z_{R2})}$$

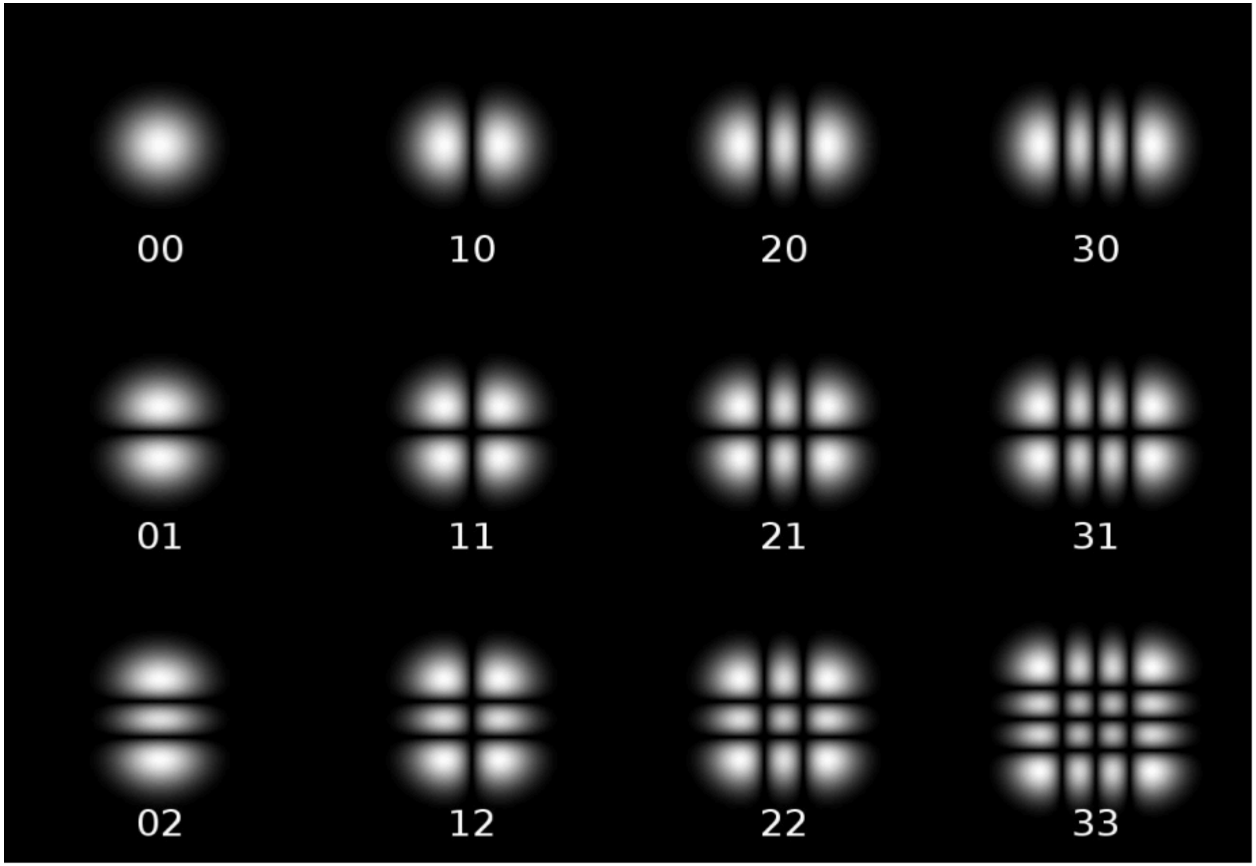
Power Coupling

$$|C|^2 = 4\frac{z_{R1}z_{R2}}{(z_1 - z_2)^2 + (z_{R1} + z_{R2})}$$

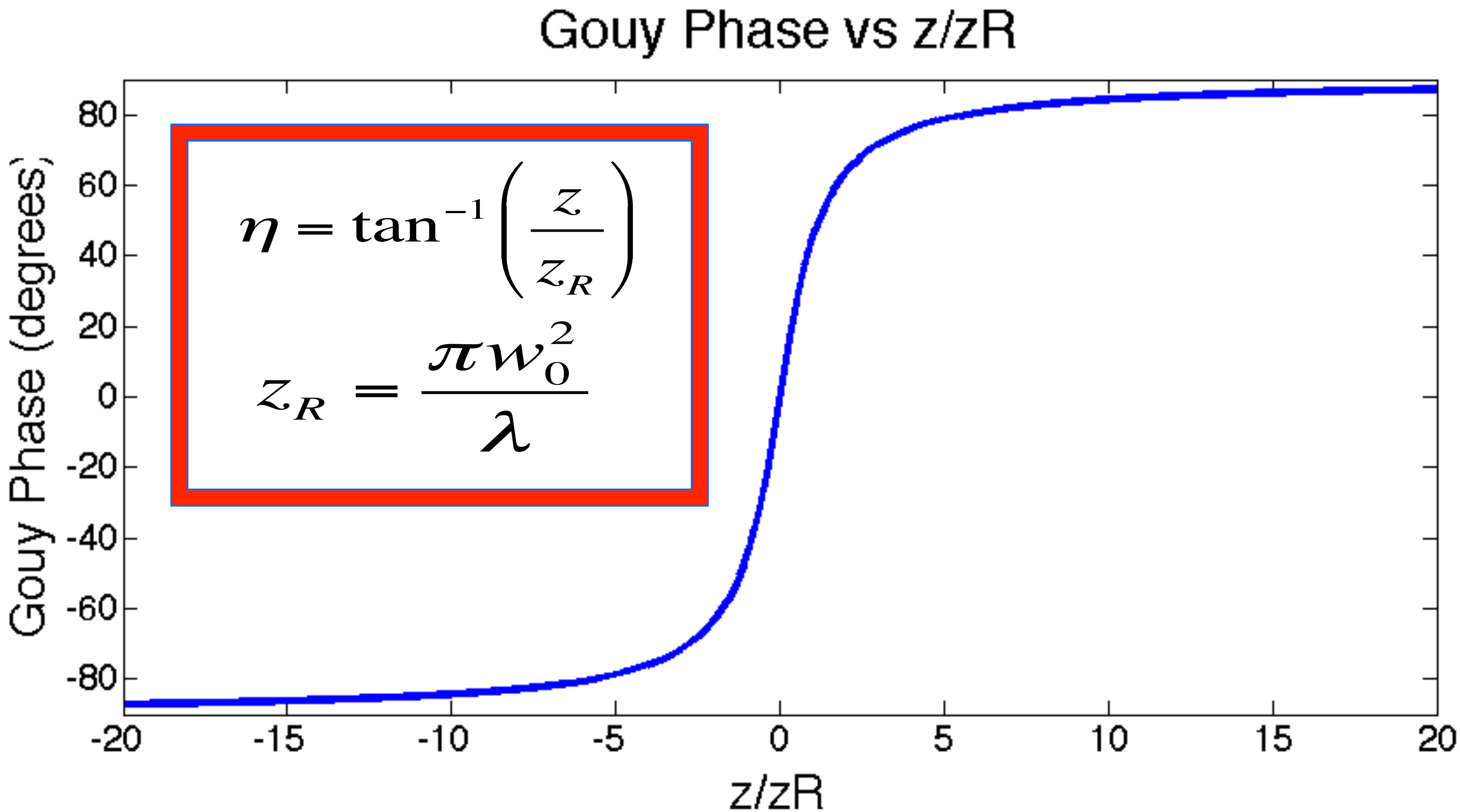


Cavity Modes

Hermite-Gaussian Modes

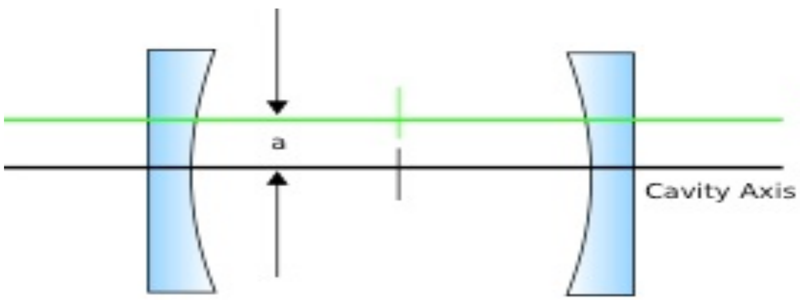


Gouy Phase

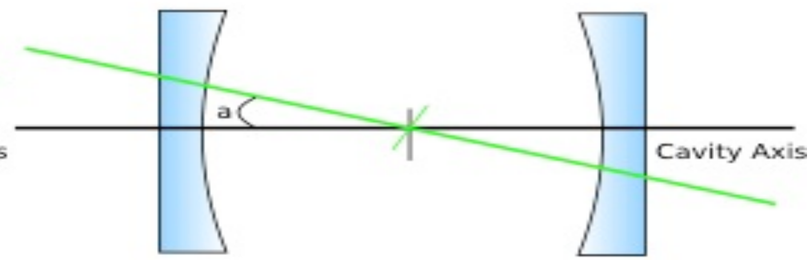


Gouy Phase vs z/zR
$\eta = \tan^{-1}\left(\frac{z}{z_R}\right)$
$z_R = \frac{\pi w_0^2}{\lambda}$
Gouy Phase (degrees)
80
60
40
20
0
-20
-40
-60
-80
-20
-15
-10
-5
0
5
10
15
20
z/zR

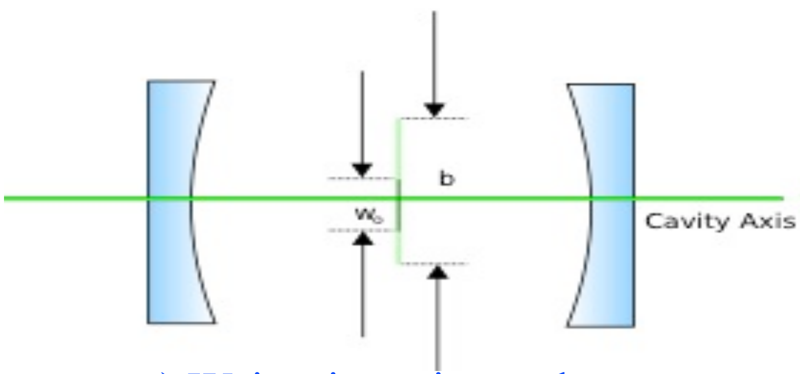
Misalignment



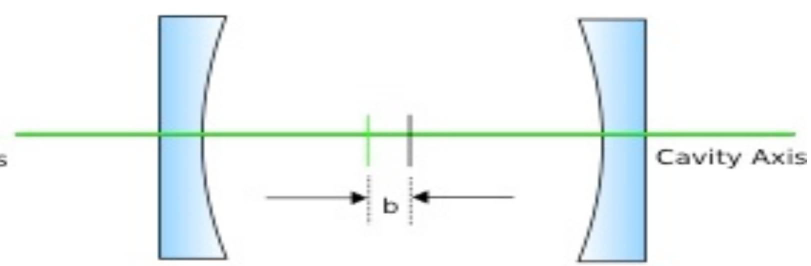
a) Transverse displacement in the x direction



b) Tilt through an angle a



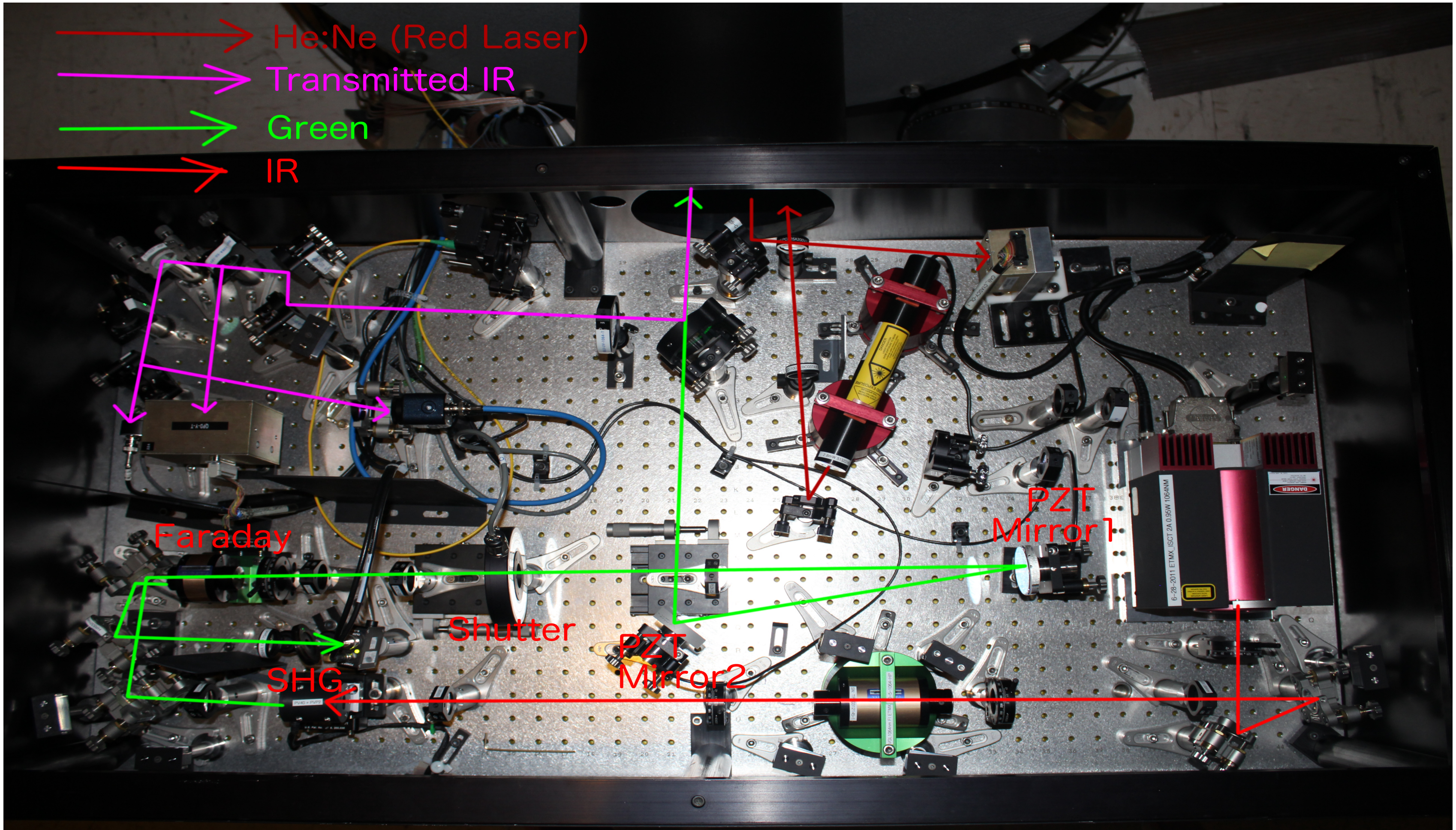
c) Waist size mismatch



d) Axial waist displacement

LIGO

Issues with Current Setup



Understanding Better the Current Issues

Representation of the beam field

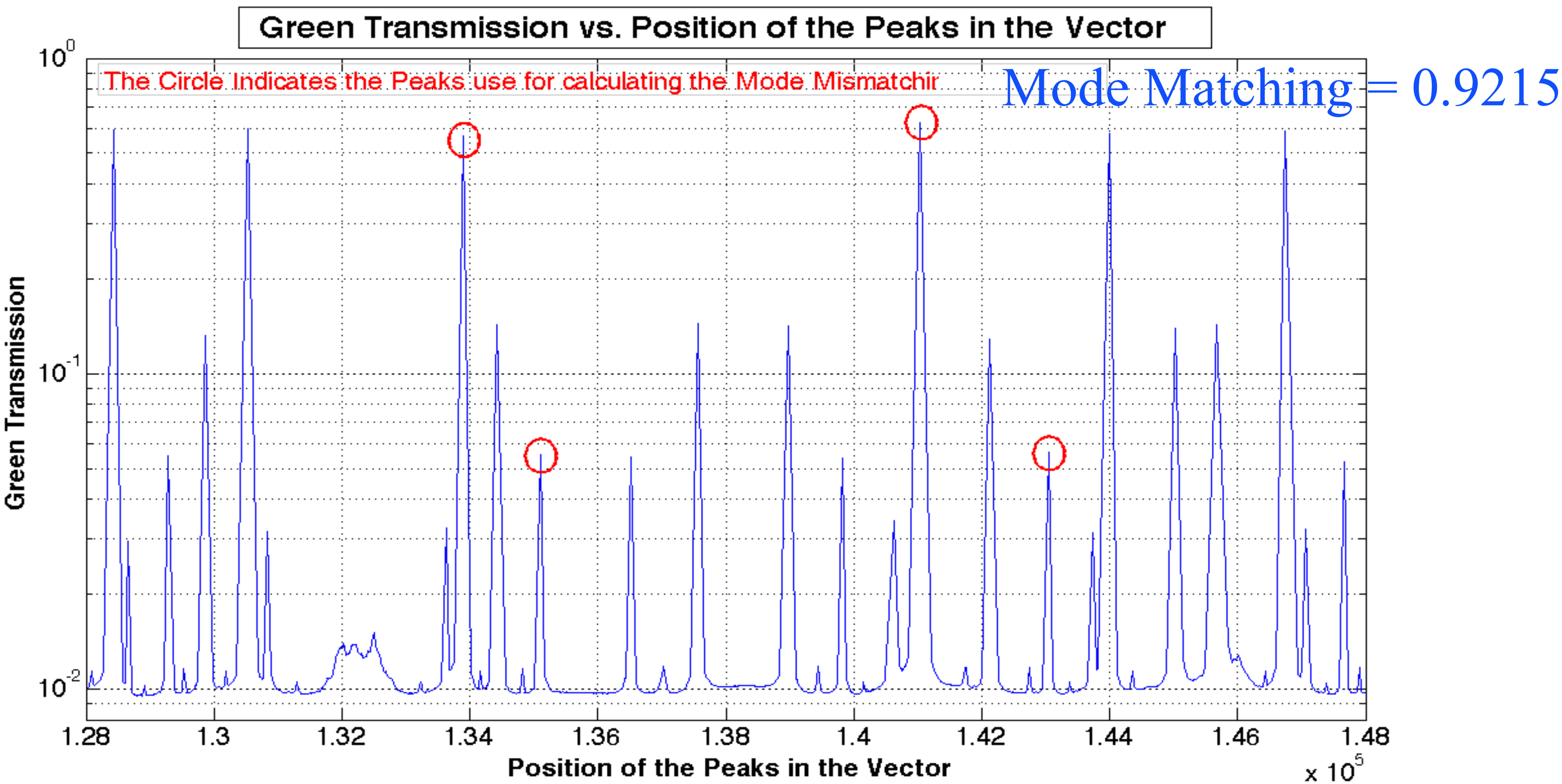
$$|E\rangle = |00\rangle + \left(\frac{\delta x}{w_0} + i\frac{\theta}{\theta_d}\right)|01\rangle$$

Operator of the entire alignment System

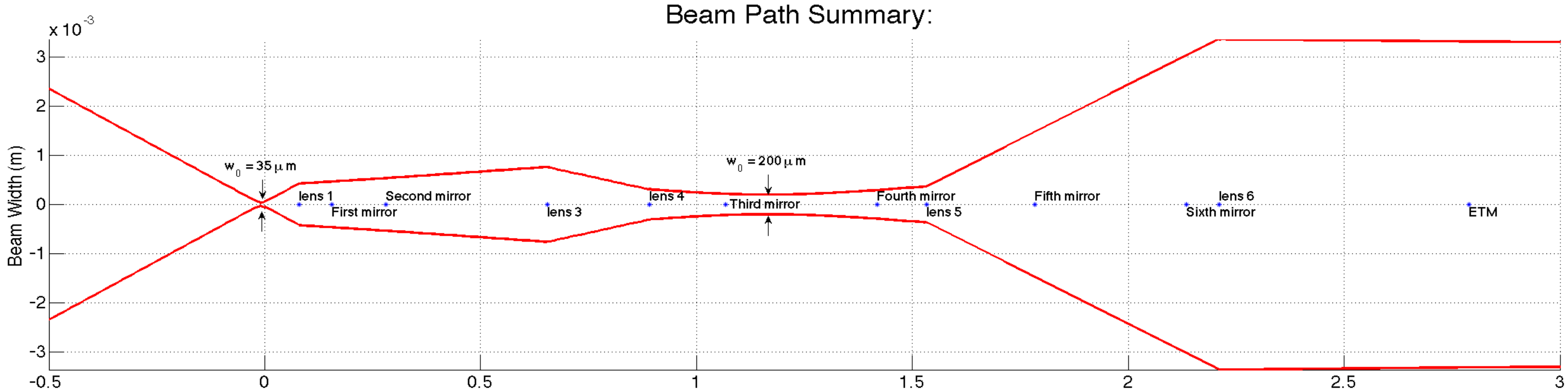
$$M_B P M_A = \begin{bmatrix} 1 & -2i(e^{i\eta}\theta_A + \theta_B) \\ -2i(e^{i\eta}\theta_A + \theta_B) & e^{i\eta} \end{bmatrix}$$

$$|E\rangle_{out} = |00\rangle - 2i\left(e^{i\eta}\theta_A + \theta_B\right)|01\rangle$$

Current Mode Matching

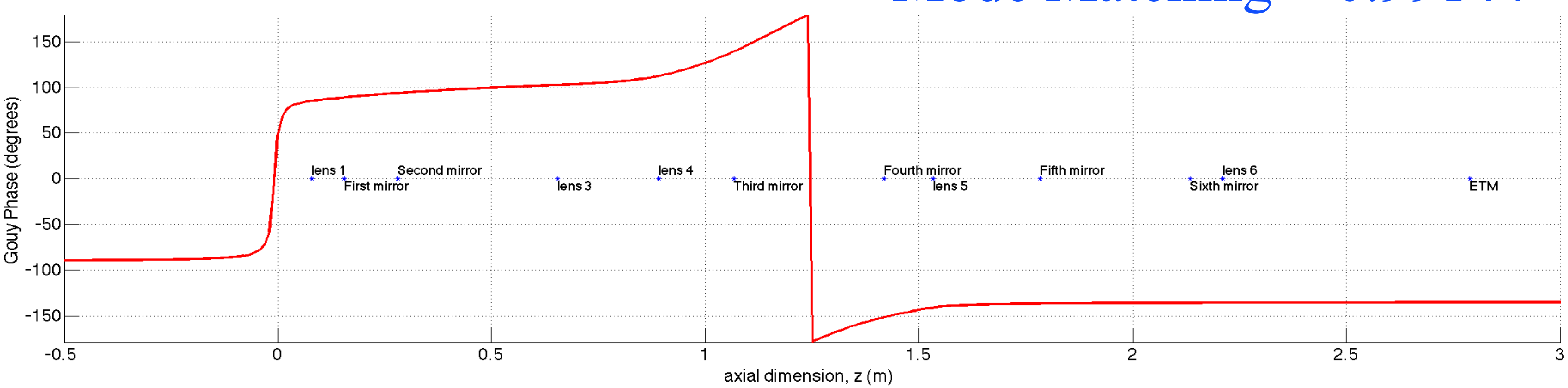


Predicted Mode Matching and Gouy Phase

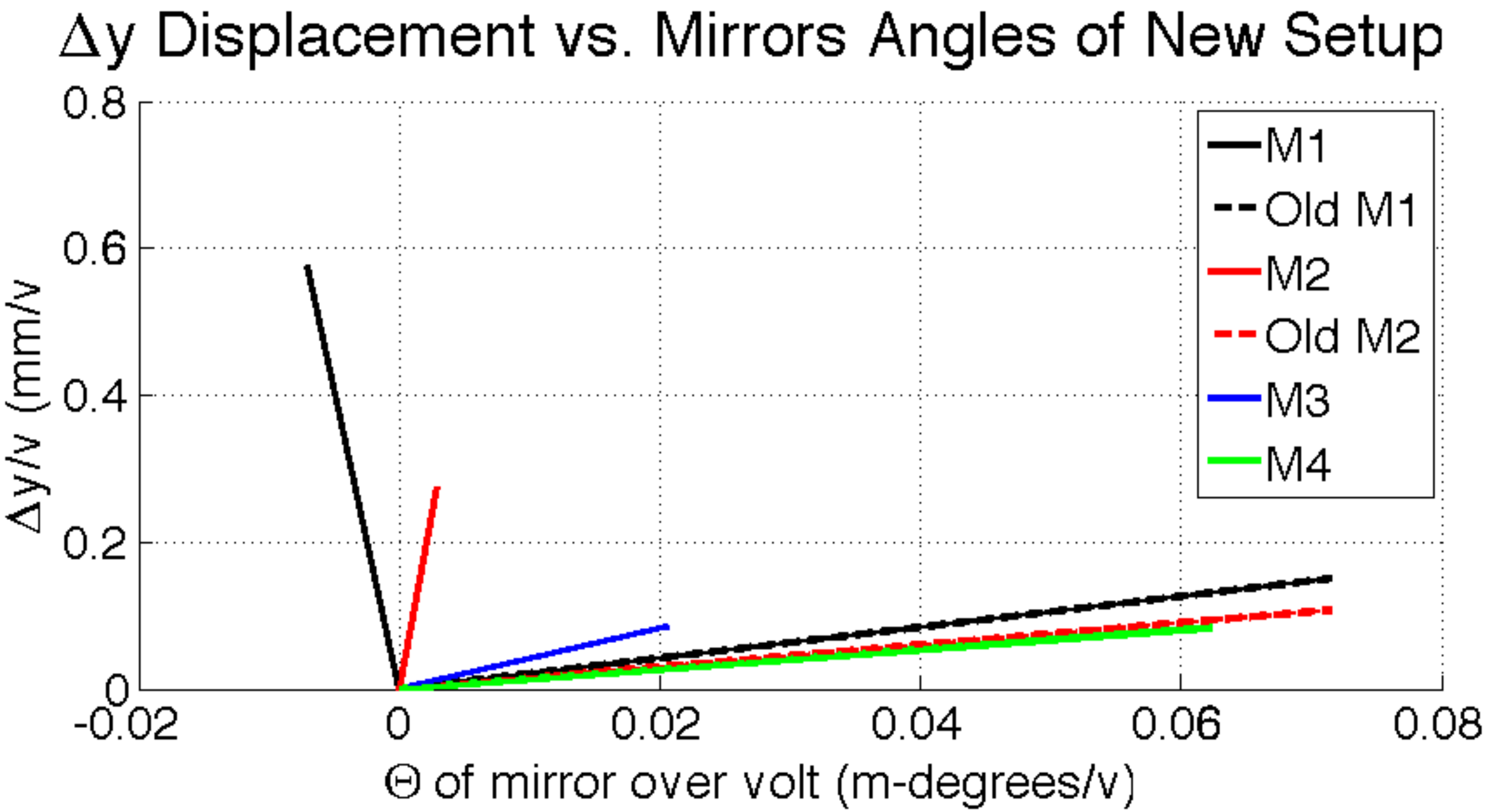


Gouy Phase = 85.23°

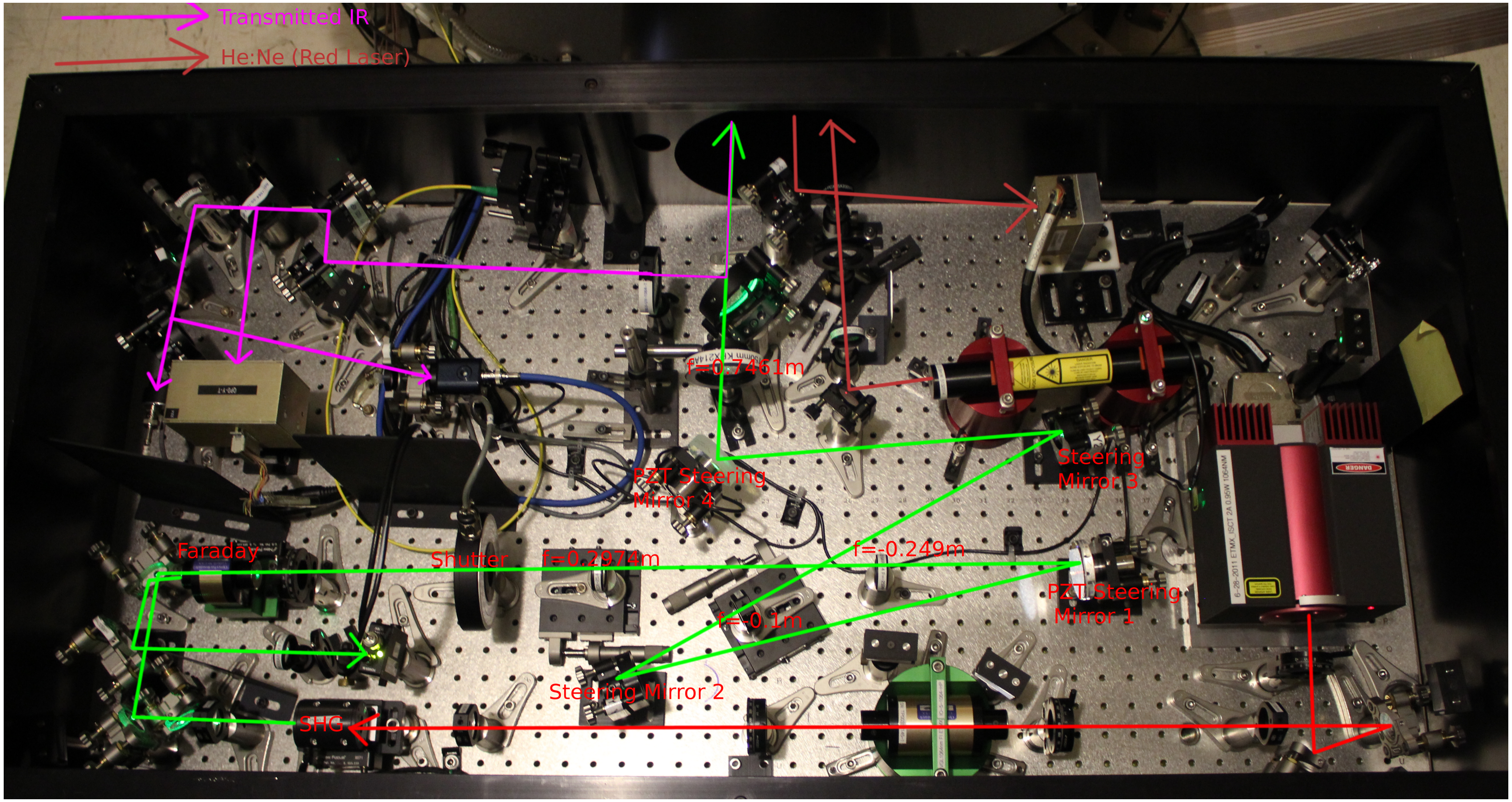
Mode Matching = 0.99144



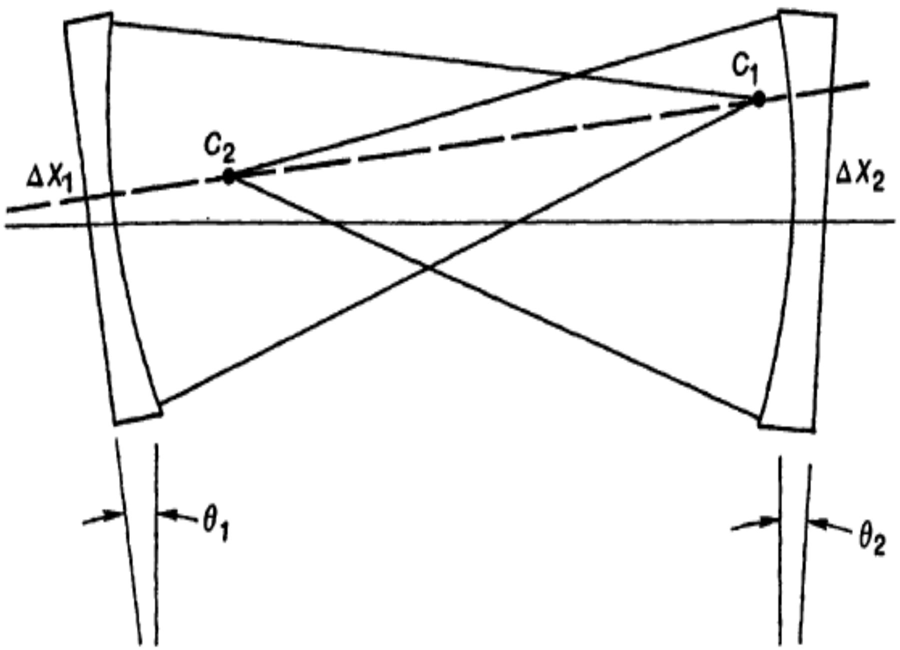
Gouy Phase


Δy Displacement vs. Mirrors Angles of New Setup
0.8
0.6
0.4
0.2
0
Δy/v (mm/v)
-0.02
0
0.02
0.04
0.06
0.08
Θ of mirror over volt (m-degrees/v)
M1
Old M1
M2
Old M2
M3
M4

New Optical Table Setup



Matrix Sensing



$$\Delta x_1 = \frac{g_2}{1-g_1 g_2} d\theta_1 + \frac{1}{1-g_1 g_2} d\theta_2$$

$$\Delta x_2 = \frac{1}{1-g_1 g_2} d\theta_1 + \frac{g_1}{1-g_1 g_2} d\theta_2$$

$$\Delta\theta \equiv \frac{(1-g_2)\theta_1 - (1-g_1)\theta_2}{1-g_1 g_2}$$

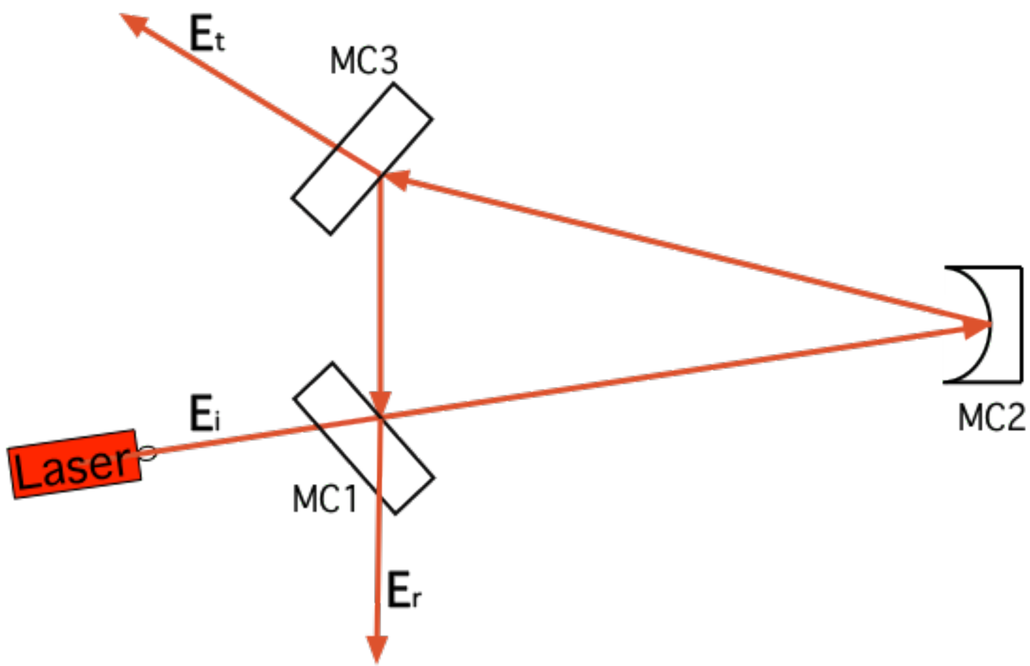
Matrix Sensing

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = M_S \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

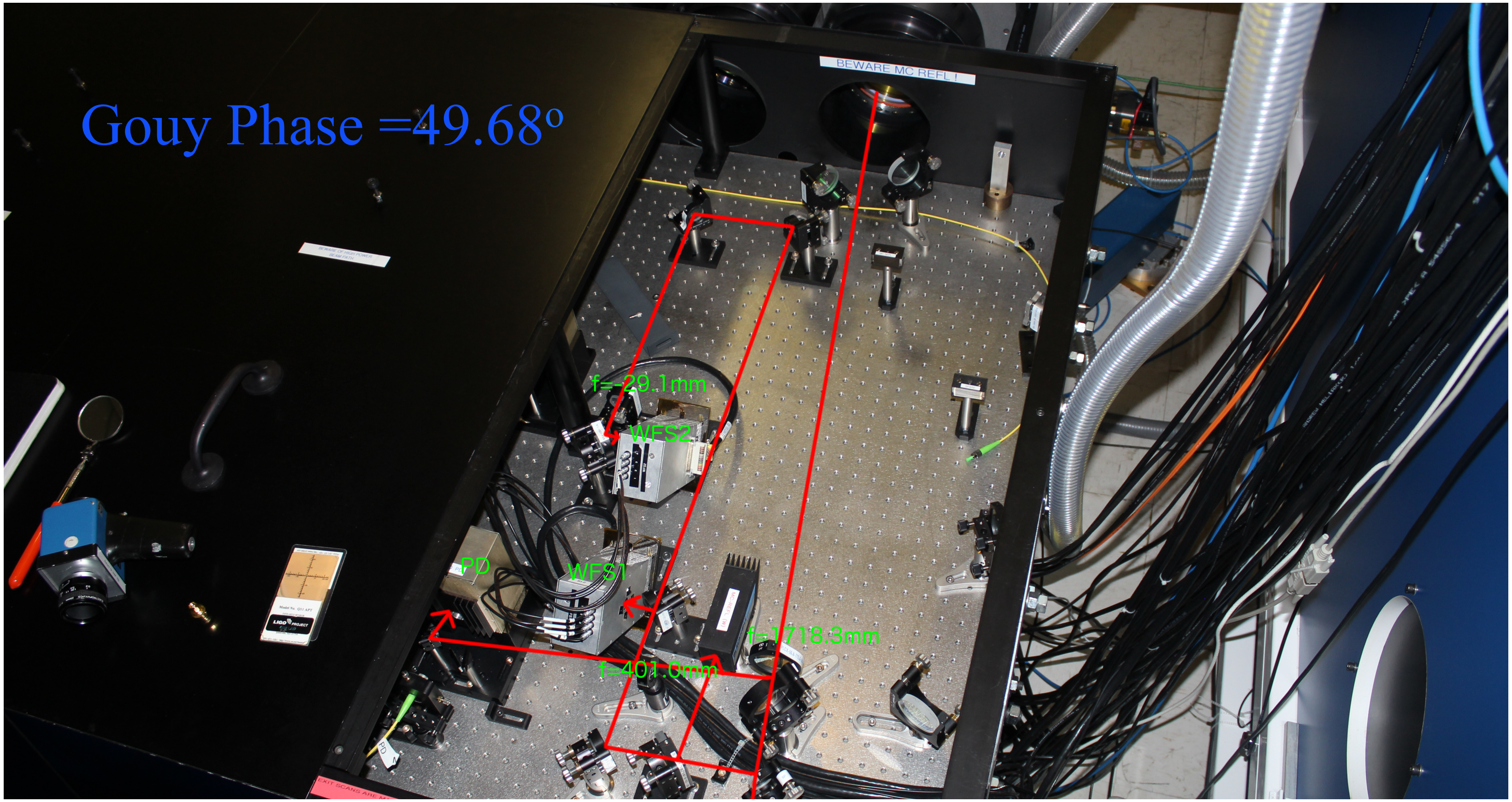
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{g_2}{1-g_1 g_2} d & \frac{1}{1-g_1 g_2} d \\ \frac{1}{1-g_1 g_2} d & \frac{g_1}{1-g_1 g_2} d \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

Input Mode Cleaner

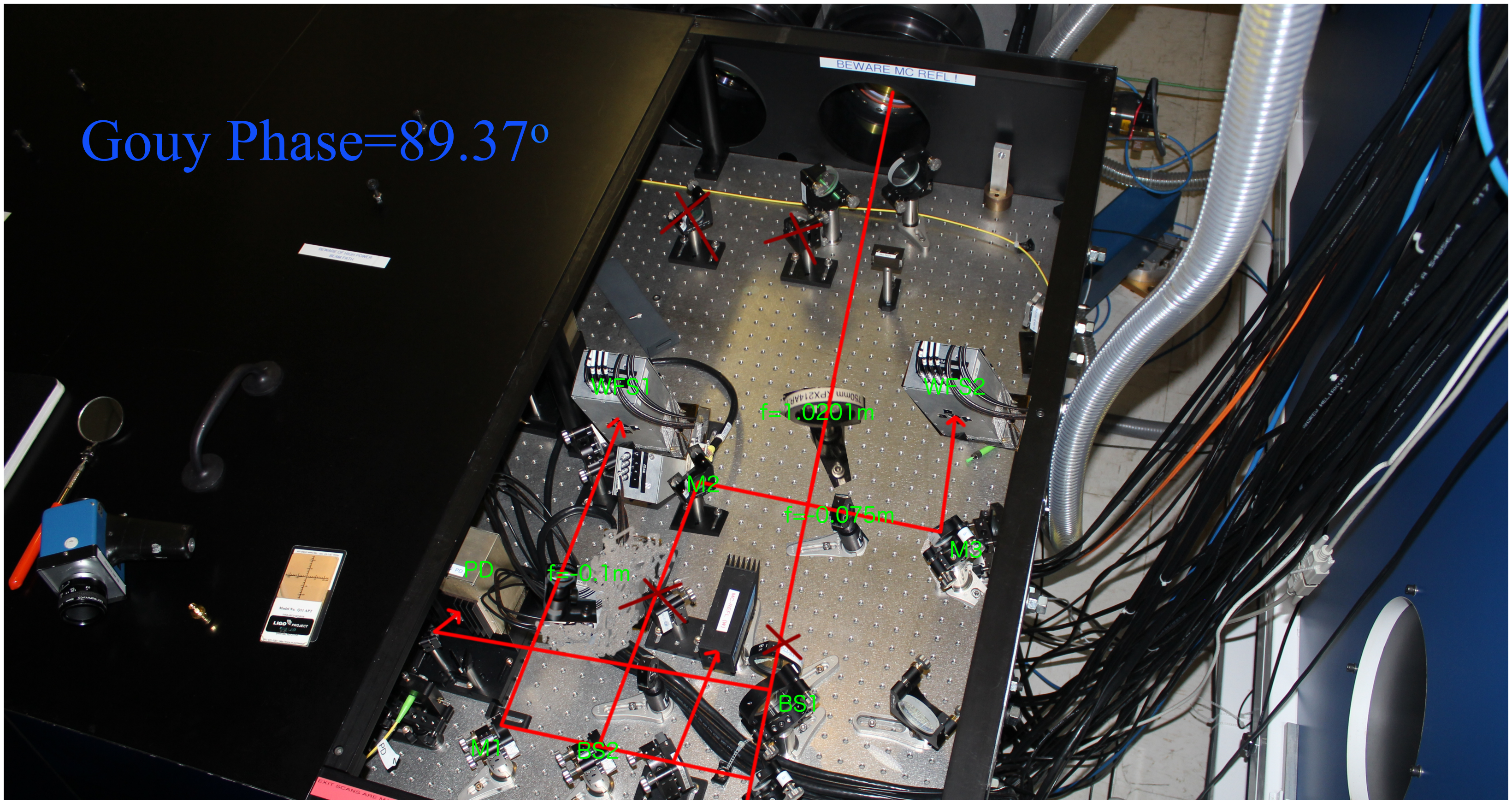
Mode Cleaner Configuration



Current Setup and its Gouy Phase



New Optical Setup and its predicted Gouy Phase



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Future Plan

- Characterize the Xarm Green upgrade, which include measuring how perpendicular the PZTs mirrors are, measure the mode matching
- We need to implement the new optical table layout for the IMC
- The control negative feedback system need to be upgrade so that it keep the mode cleaner lock
- Characterize IMC

Acknowledgements & Thanks

- Nick and Gabriele
- 40m Team
- My Surf teammates
- Caltech LIGO
- NSF

Back Up Slides

Solution of the Wave Equation

$$\tilde{u}(x,y,z) = \frac{1}{\tilde{q}(z)} \exp\left[-jk\frac{x^2+y^2}{2R(z)} - \frac{x^2+y^2}{w^2(z)} \right]$$

Mirror Operator

$$M_A = \begin{bmatrix} 1 & -2i\theta_A \\ -2i\theta_A & 1 \end{bmatrix}$$

Propagation operator

$$P = e^{i\phi} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\eta} \end{bmatrix}$$

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