Thermal Noise in Ultra-Stable Fabry-Pérot Cavities
LIGO SURF 2014: Progress Report 1

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Abstract

The Laser Interferometer Gravitational-Wave Observatory (LIGO) aims to detect gravitational waves, which requires high precision detectors. The test masses of LIGO are mirrors, which have dielectric coatings consisting of SiO_2 and Ta_2O_5. The mirror coatings are a limiting noise source in the most sensitive frequency band (50-300 Hz). Theoretical calculation of thermal noise in this frequency range has yet to be verified. Ultra stable dual Fabry-Pérot cavities can be used to measure the coating noise as well as identify and reduce various noises by using a feedback control system and various laboratory skills. The goal of this project is to determine whether CTN light, which is known to be stable, can be transported to other labs to be used as a stable frequency reference.

1 Introduction

LIGO is a signal recycling Michelson interferometer with FabryPérot arms. The mirrors of the optical cavities consist of SiO_2 and Ta_2O_5 coatings, which are a dominant source of thermal noise. Other sources of noise include seismic vibrations such as earthquakes and microseisms, photon shot noise and Newtonian gravity noise [1]. At frequencies lower than 10 Hz, seismic noise, environmental disturbances and technical noise sources dominate. Radiation pressure noise, thermal noise from the suspensions and Newtonian gravity noise are the limiting sources in the 10-40 Hz frequency range. At frequencies in the 50-200 Hz range, thermal noise from the mirror optical coatings and quantum effects from the light are limiting noise sources. Above 200 Hz, shot noise from the light become a dominant noise source [2]. The dielectric, thin-film coatings on the test masses experience mechanical dissipation, which causes Brownian noise. Additionally, temperature fluctuations in the coatings are the cause of thermo-optic noise [3]. Pound-Drever-Hall Locking is a technique used to obtain stable light from a laser by locking the laser frequency to stable Fabry-Pérot cavities. Various experiments investigating different noise sources need a stable frequency reference to identify and suppress these noises. Transporting stable light via optical fibers will be explored.

1.1 Thermal Noise

In order to detect gravitational waves, high precision is needed since the space-time distortion effects of the strain are so small. However, at low frequencies, thermal noise becomes a limiting noise source in LIGO. Brownian noise occurs as a result of mechanical dissipation in the system, which is represented by the imaginary part of the Young's Modulus for the material: $E = E_0[1 + i\phi(f)]$ where $\phi(f)$ is the loss angle [4]. Random fluctuations in temperature is the source of thermo-optic noise, which is manifested in two forms. Thermo-refractive noise occurs as a result of the temperature dependence of the refractive index. Thermo-elastic noise occurs as a result of the temperature dependence of the linear expansion coefficient, α, which causes random motions on the surface of mirror [3].

1.2 Fluctuation Dissipation Theorem

In order to calculate thermal noise in a system, the Fluctuation-Dissipation Theorem (FDT) can be applied. Callen and Welton's generalized FDT states that when fluctuations in a system occur, a time-dependent dissipation function can be obtained in which the power spectral density is expressed as: $S_x(f) = \frac{k_B T}{\pi^2 f^2}[Re[Y(f)]]$ where $Y(f) = 1/Z(f)$ is the mechanical admittance and Z is the complex impedance, $Z(f) = F(f)/\dot{x}(f)$, where $F(f)$ and $x(f)$ are the Fourier transforms of the driving force $F(t)$ and the response of the observable, $x(t)$. Various approaches of calculating S_x include normal-mode decomposition, a method proposed by Gonzales and Saulson, which involves calculating $[Re[Z(f)]]$ separately for each normal-mode. An alternate approach proposed by Levin suggests that a small applied force to a system is analogous to random fluctuations. This method requires an oscillatory pressure to be applied, which allows for a relationship between the dissipation and PSD to be obtained: $S_x(f) = \frac{2k_B T}{\pi^2 f^2}\frac{W_{diss}}{F_0^2}$ where $S_x(f)$ is the one-sided power spectral density of the displacement x at a certain frequency f, T is the temperature, k_B is Boltzmann's constant, F_0 is the amplitude of the the applied oscillating force, and W_{diss} is the dissipated power when the oscillating force is applied. This method allows for direct calculation of S_x [4].

1.3 Pound-Drever-Hall Laser Frequency Stabilization

It is essential to have frequency stabilized light in order to make precise measurements with gravitational-wave detectors and use of stable light as a reference allows for identification of noise in a system. The Pound-Drever-Hall technique involves locking light emitted by a laser to a stable Fabry-Pérot cavity to stabilize the frequency of this light. Fabry-Pérot cavities can be used to measure the frequency of a laser beam by examining how much light is transmitted or reflected. Light that is an integer number times the free spectral range of the cavity, which is dependent on the length of the cavity is transmitted. If the cavity is perfectly resonant, then the laser frequency is indeed an integer number times the free spectral range. If this is not the case, some light will be reflected off the cavity. However, there will always be some reflected light unless the following two conditions are met: (1) the cavity mirrors have identical reflectivity, and (2) the spatial mode of the laser beam is perfectly matched to the mode of the cavity, which means it is entirely TEM_{00}, has the correct waist at the correct position, and the axis of the laser beam is the same as the axis of the cavity. In the case that the laser frequency is perfectly resonant with the cavity, the two beams destructively interfere since the reflected beam and light from the standing wave inside the cavity that is leaked through the mirror are 180 degrees out of phase with each other. If the reflected beam and leakage beam do not completely cancel each other out, which means the light is not perfectly resonant with the cavity.

The phase of the reflected beam must be measured in order to determine whether or not the laser is perfectly resonant with the cavity. A phase modulator can be used in which the sidebands produced are interfered with with the reflected beam, thus producing a beat pattern. From this beat pattern, the phase of the reflected beam can be determined with a photodetector. The signal from the photodetector passes through a mixer in which the RF signal is mixed with the same waveform that is used to drive the phase modulator. If the local oscillator (LO) port of a mixer is driven with $\cos(\omega_1 t)$ and the radio frequency (RF) port of the mixer is driven with $\cos(\omega_2 t)$, then the intermediate frequency (IF) port will be proportional to:

$$\cos(\omega_1 t)\cos(\omega_2 t) = \frac{\cos[(\omega_1 + \omega_2)t] + \cos[(\omega_1 - \omega_2)t]}{2}.$$

In the CTN experiment, the LO port, which generates signal from the sidebands and RF port, which generates signal from the photodiode voltage, are sinusoids with frequency Ω_m where $\Omega_m/2\pi = 14.75$ MHz. The mixer gives the components at the sum and difference, $\Omega_m + \Omega_m = 2\Omega_m$ and Ω_m - $\Omega_m = 0$. After the signal passes through the low-pass filter, the sum component is removed since the error signal is contained in the difference term. Examining the error signal will determine whether or not the laser is resonant with the cavity [5].

2 Progress and Goals

The cavity light from the CTN experiment is known to be stable, having a beat note of (0.5 Hz)/$f^{1/2}$ from 10 Hz to 1 kHz. This experiment contains two separate cavities, each with their own stabilized laser. The beams from the two lasers are interfered on an RF photodiode, which produces a beat note. The power spectral density of the frequency noise of the beat note is measured with a spectrum analyzer. Since the laser frequency noise is stable in this experiment, it can be used as a frequency reference for a number of of labs conducting various experiments including investigation of noises in gravitational-wave detectors. That being said, coupling the cavity light into optical fibers that lead to other labs is a goal of this project. Optical fibers are extremely sensitive to environmental perturbations, which generates phase noise. It has been demonstrated that phase modulators, specifically acousto-optic modulators, can be used as a method of phase-noise cancellation. A phase-locked loop (PLL) is used to read out the frequency fluctuation of light returning from the fiber.[6].

2.1 Work Completed

Thus far, we have created a table top experiment to prove that that transporting stable light through optical fibers will work. We have installed various optics on the table for the fiber phase noise measurement. Light passes through the Pre-mode Cleaner (PMC), a half wave plate, Faraday isolator, lens 1, another half wave plate, an electro-optic Modulator (EOM), Electro-optic Amplitude Modulator (EOAM), another half wave plate and then through a polarizing beamsplitter (PBS). Light is reflected off a the beamsplitter and directed toward lenses and a steering mirror to mode-match the light into the fiber. The round trip length of the PMC is 42 cm and the radius of curvature of the concave mirror is 1 m. Using the formula $w_0^2 = \frac{\lambda}{2\pi}\sqrt{d(2R-d)}$, we found that a waist of 370 microns coming out of the PMC. Since we wanted a waist of 50 microns going into the fiber input, we calculated the proper lenses needed as well as the distances in which they should be placed from each other. Following the beamsplitter, we placed down lens 2, with a focal length of 124 mm and lens 3 with a focal length 250 mm. Between the two lenses is a mirror, which is tilted at a 45 degree angle to guide the light from lens 2 to lens 3. Lens 2 and Lens 3 are roughly 2 inches away from each other.

A fiber coupler was placed 4 inches away from the second lens so that a 50-micron waist occurs at the input of the fiber. Following the fiber output, which has a waist of 50 microns, we calculated the proper lens to use as well as the proper distance to place the objects so that we would have a waist of approximately 150 microns going into the Acousto-optic Modulator (AOM). Roughly 3.5 inches from the fiber output, we placed a lens with focal length of 50.2 mm and a half wave plate followed by an AOM. After the light passes through the AOM, we placed another lens that yields a waist at the mirror placed at the end of this setup. The light is then reflected back, so it double passes the AOM, and then passes back through the fiber. The returning light goes through a 50/50 beam splitter and is guided to the photodetector, so that a beat frequency measurement can be done.

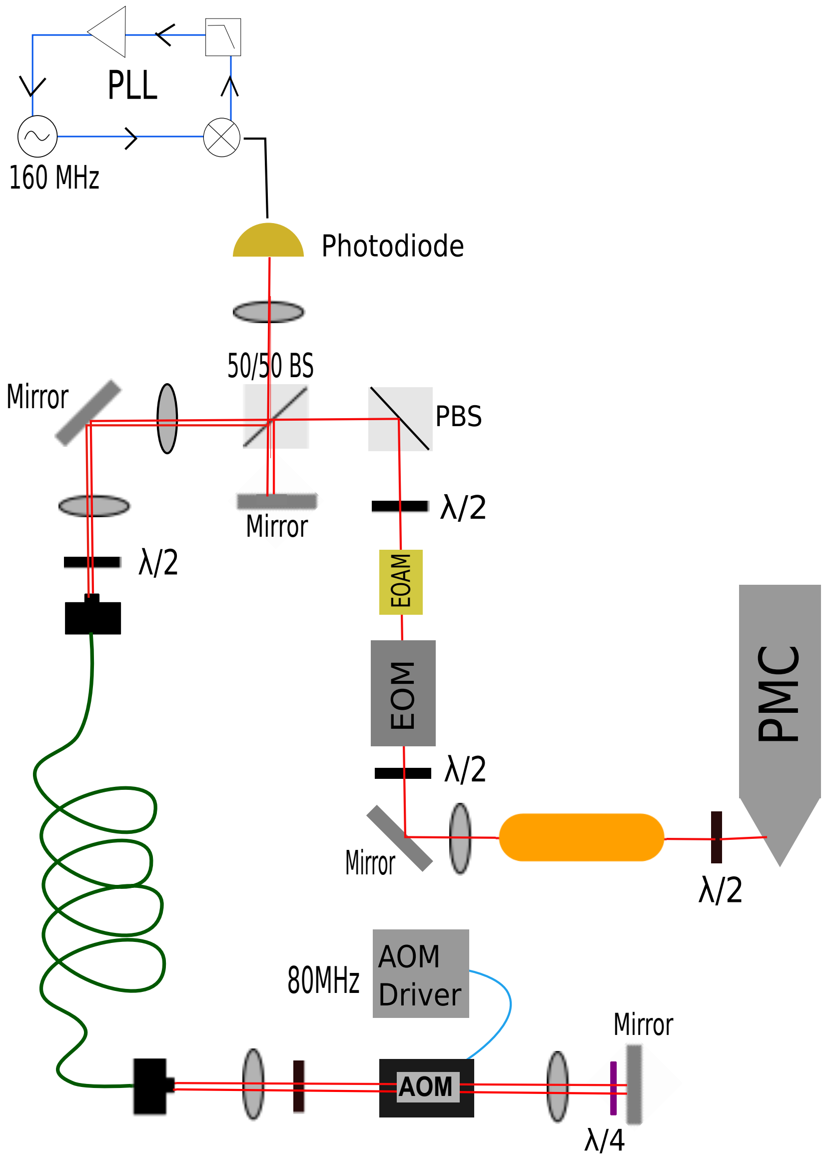

Fig 1. Table top set up for measuring fiber phase noise. The photodetector is connected to a phase-locked loop (PLL) to read the beat frequency, which gives fiber phase-noise information.

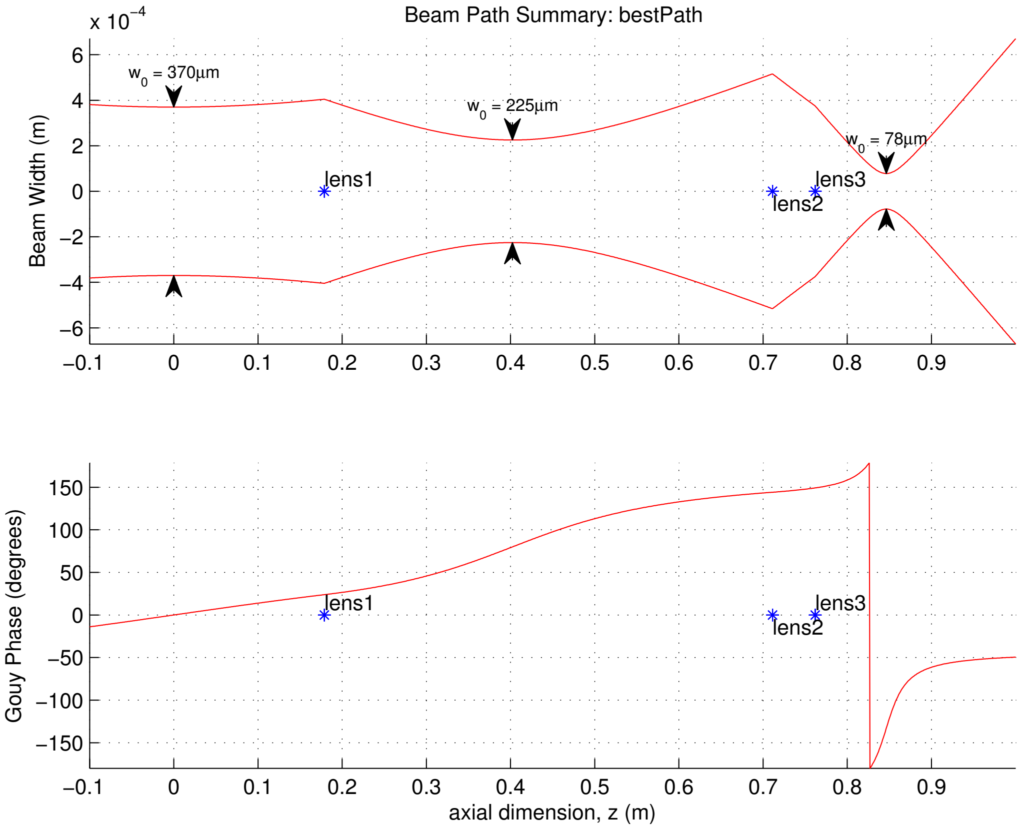

Fig 2. With a waist of approximately 370 microns coming out of the PMC, two lenses were placed roughly 2 inches away from each other along with a steering mirror to send light into the fiber at roughly 50 microns.

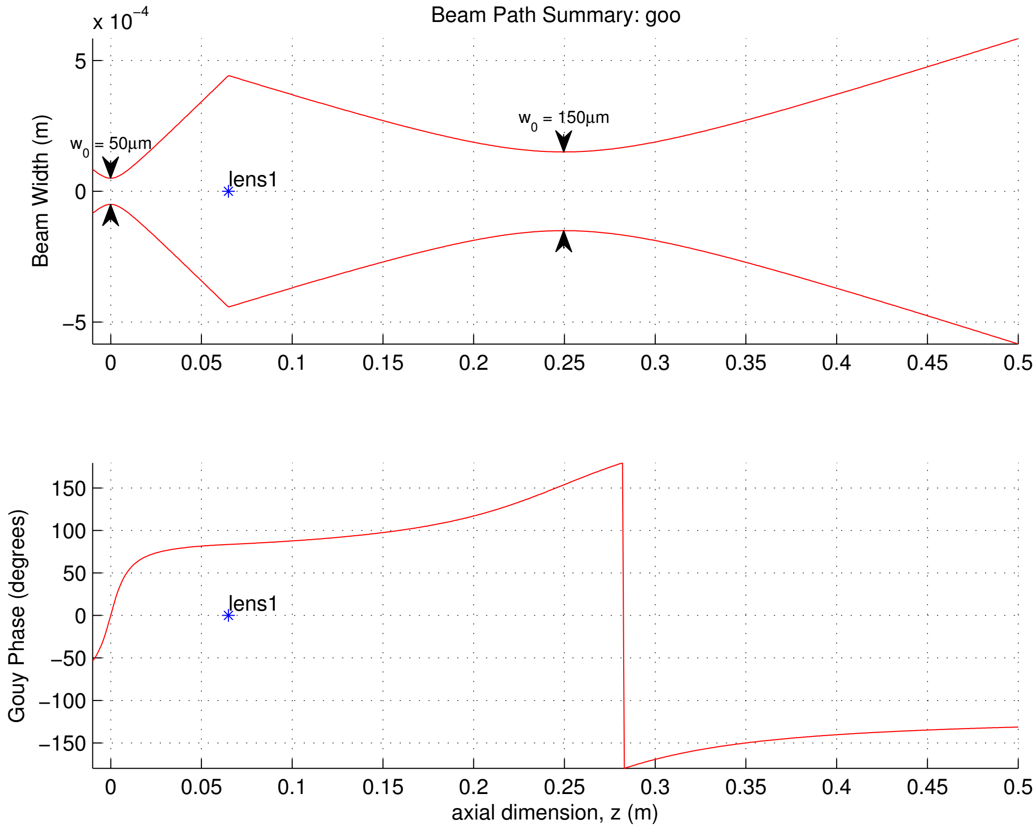

Fig 3. At the fiber output, a waist of approximately 50 microns is coming out and a waist of 150 microns going into the AOM is desired, so the proper lens of focal length 50.2mm and 3.5 inches away from the output was placed down onto the table.

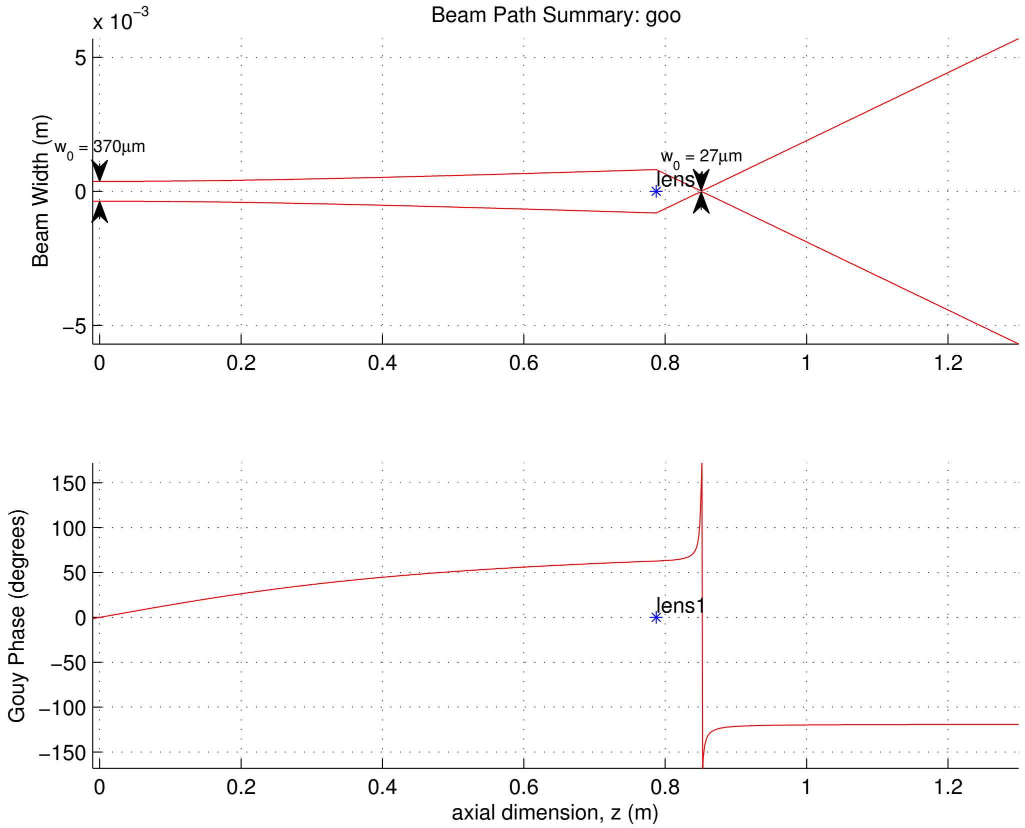

Fig 4. After the beam from the PMC goes through the PBS, it goes through a 50/50 beamsplitter where some light is reflected onto a mirror and then passes through the RF photodiode. It is required that the beam spot size is 1/3 the diameter of the photodetector. A lens with a focal length of 33 mm was placed on the table at the appropriate distance, which gives a waist of 27 microns. The RF photodiode was placed past the 27 microns waist point, so there would be a beam with a radius of approximately 50 microns going into the detector.

2.2 Future Work

The next step is to develop a noise budget and determine various noise sources such as fiber noise, electronics noise and phase noise. After demonstrating that transporting stable frequency light through optical fibers is plausible and that phase noise can be cancelled out, light will be transported from the CTN lab to crackle lab, cryo lab and gyro lab.

3 References

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