Construction of Full Hybrid CBC Waveforms: effect of Higher Modes in Detection and Parameter Estimation

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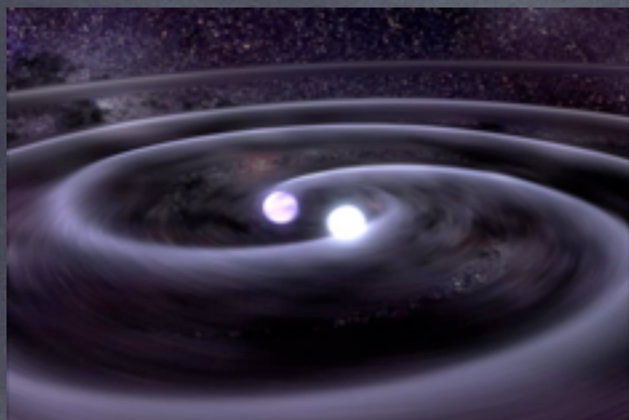
Outlook

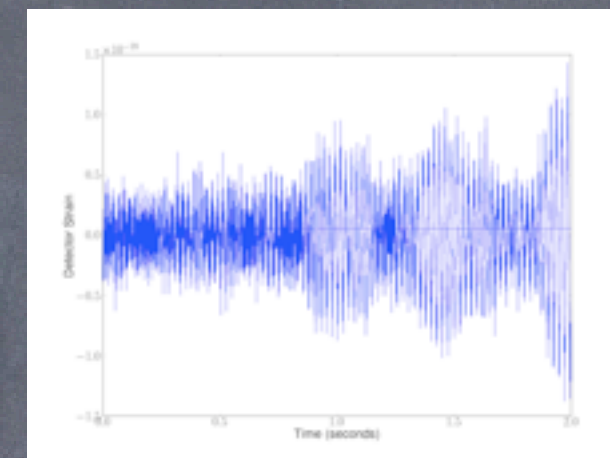
- Introduction: CBC Searches and Higher Modes.
- Hybrid Waveforms: mono-mode and multi-mode.
- Data analysis: Overlap and Fitting Factor.
- Detection Rates - Visible Volumes.
- Parameter Estimation.

A CBC search:sketch

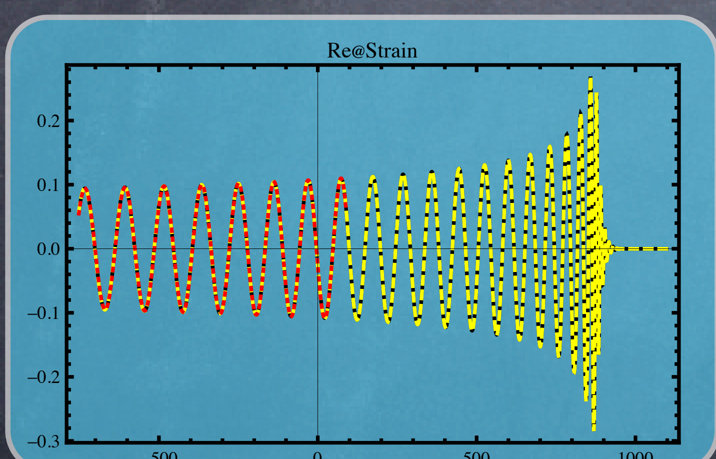


A source emits a GW

reaches a detector



and changes the data stream



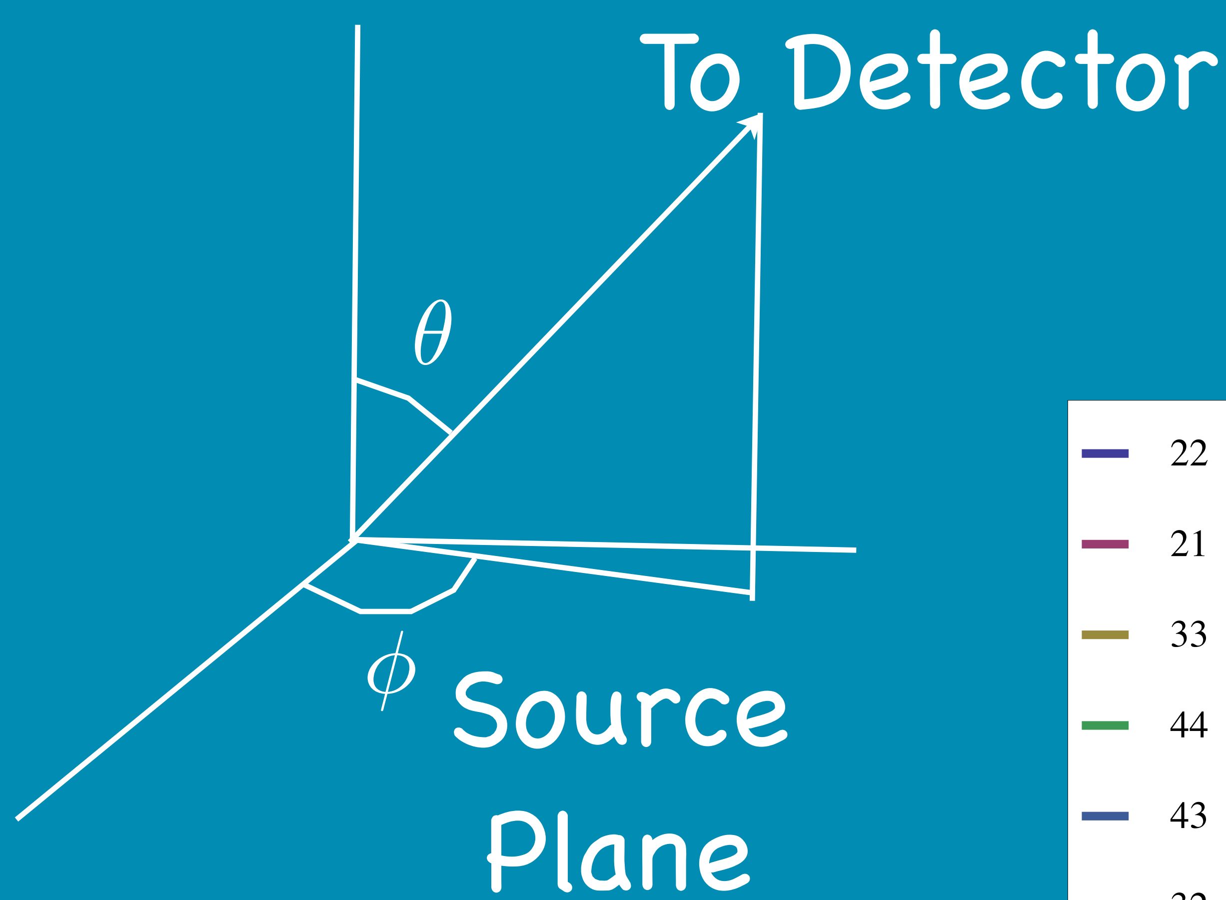
which is matched filtered with a template

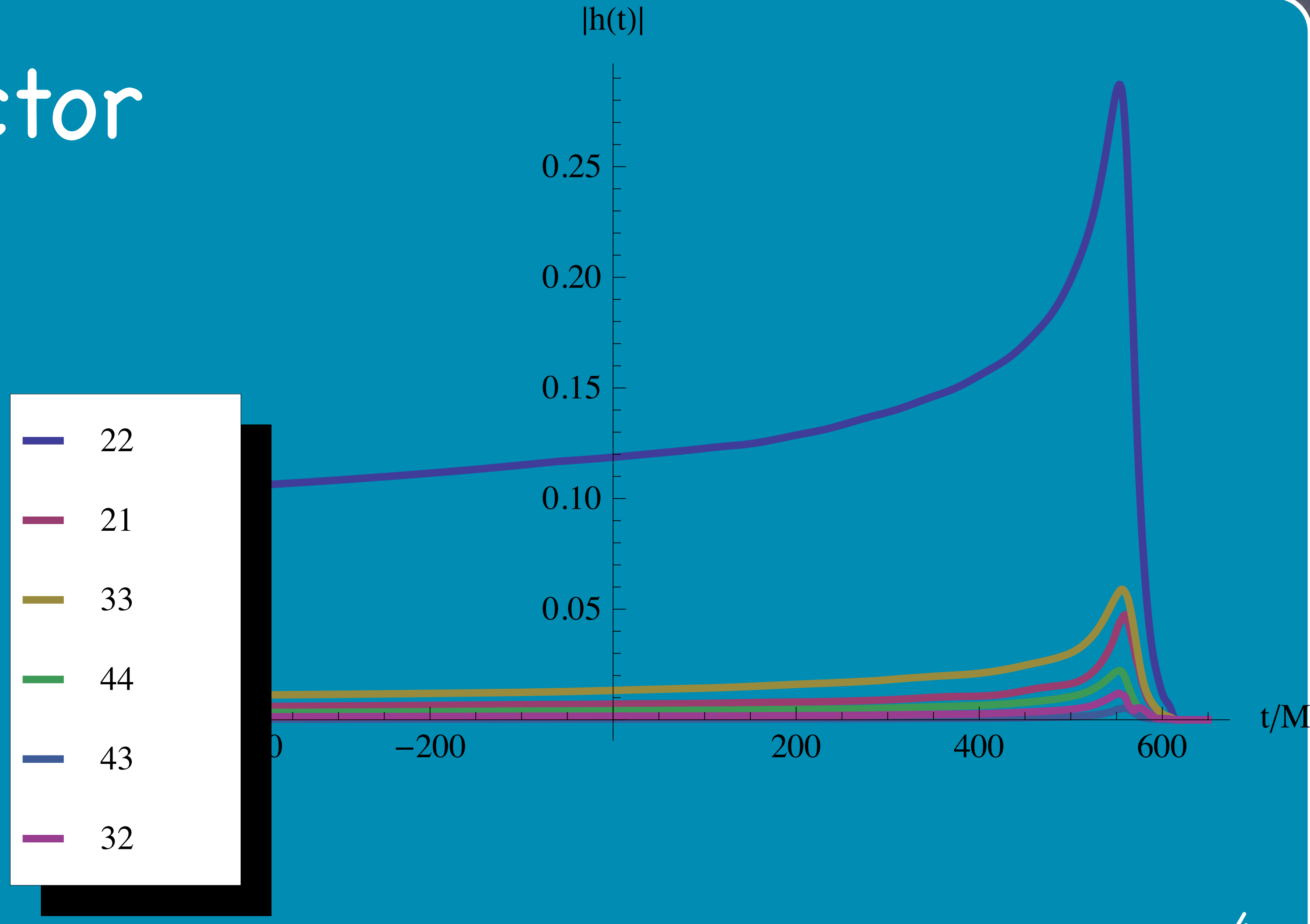
Signal-to-noise-Ratio (SNR)

$$\rho = 2\Re \frac{\int_{f_m}^{f_M} \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)} df}{\sqrt{\int_{f_m}^{f_M} \frac{\tilde{h}(f)\tilde{h}^*(f)}{S_n(f)} df}}$$

Higher Order Modes

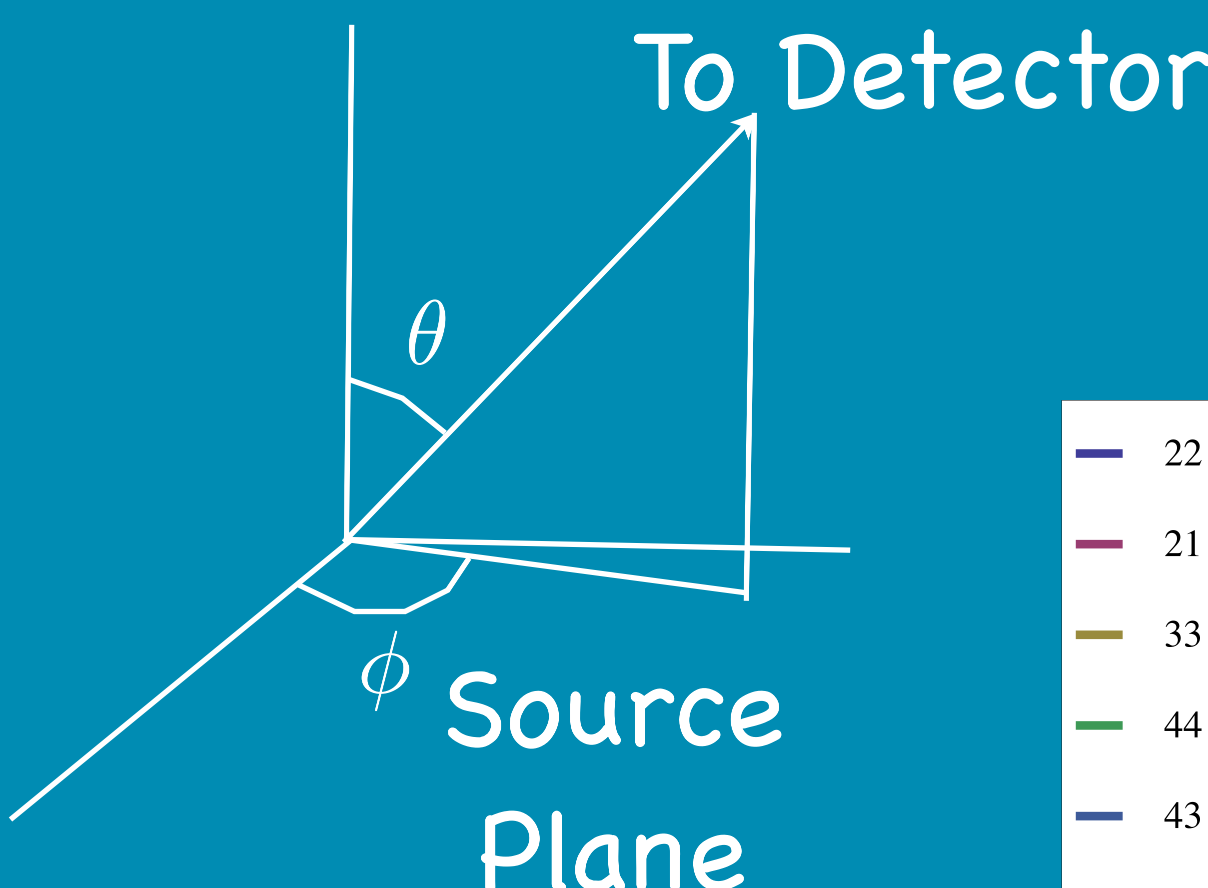
$$h(\theta, \phi, t) = \sum_{l,m} Y^{-2}_{l,m}(\theta, \phi) h_{l,m}(t)$$

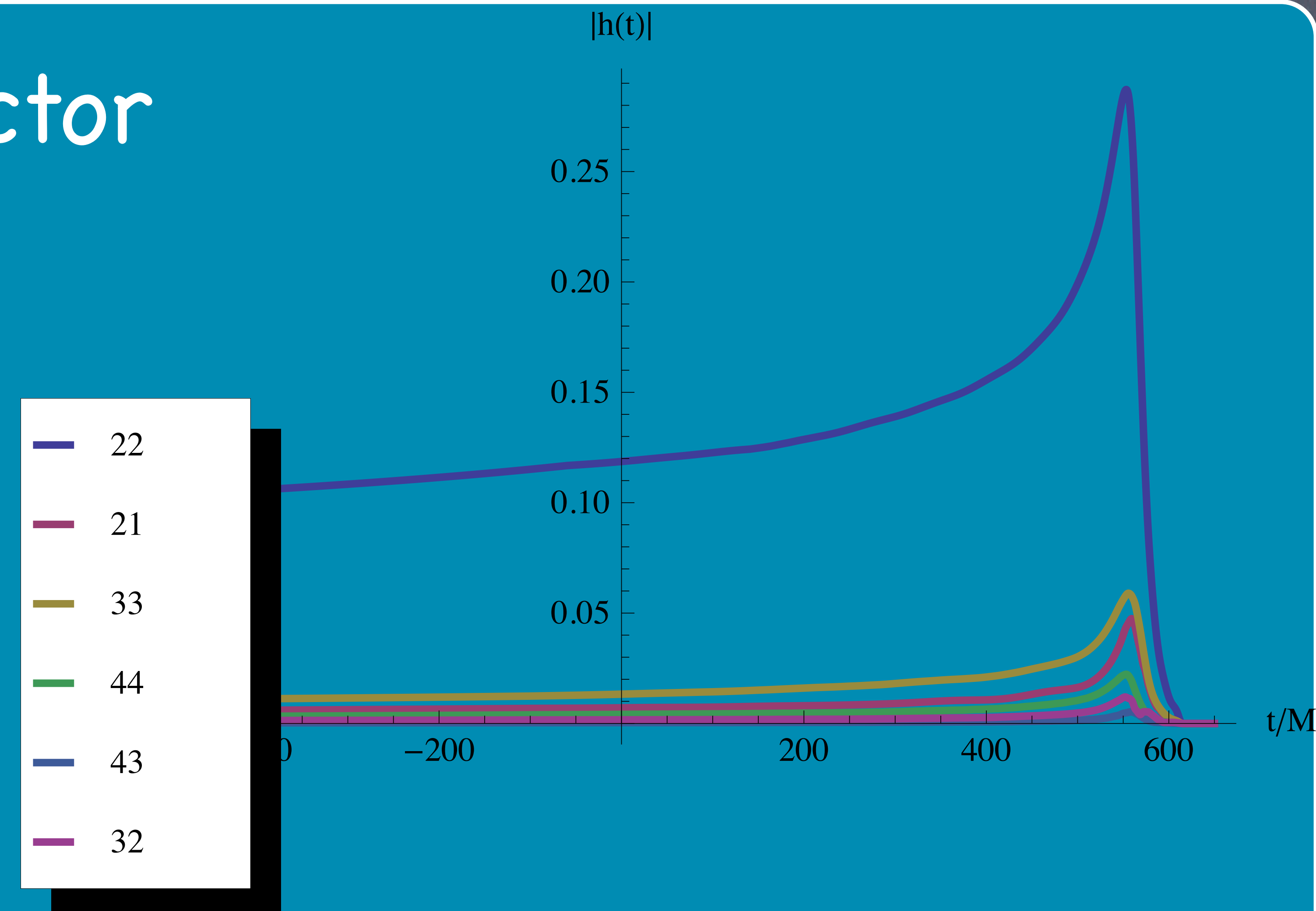




CURRENT SEARCHES ONLY INCLUDE DOMINANT 22 and 2-2 MODES

$$h(\theta, \phi, t) = Y^{-2}_{2,2}(\theta, \phi) h_{2,2}(t) + Y^{-2}_{2,-2}(\theta, \phi) h_{2,-2}(t)$$





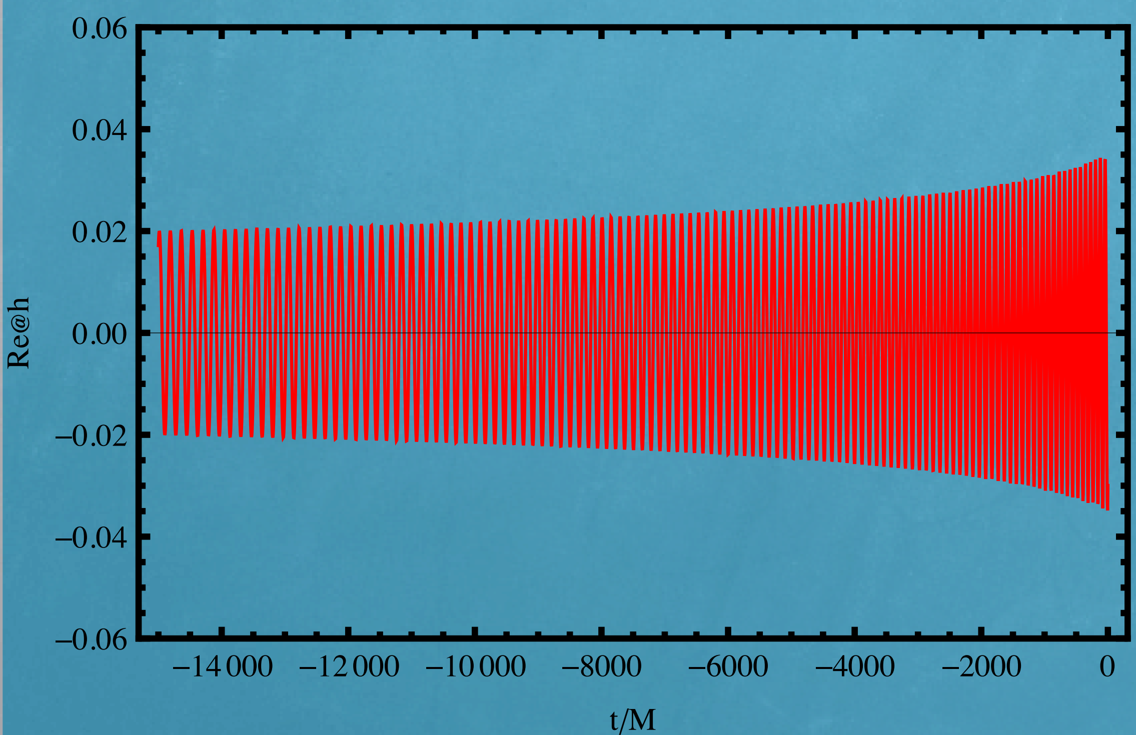
When are Higher Modes relevant?

- 22 and 2,-2 Spherical Harmonics are weak on the orbital plane, while higher harmonics are strong.
- Very Massive radiate at low frequencies which makes their 22 mode invisible ......higher modes will have higher frequencies and make them visible.
- Parameter Estimation.

Hybrid Waveforms

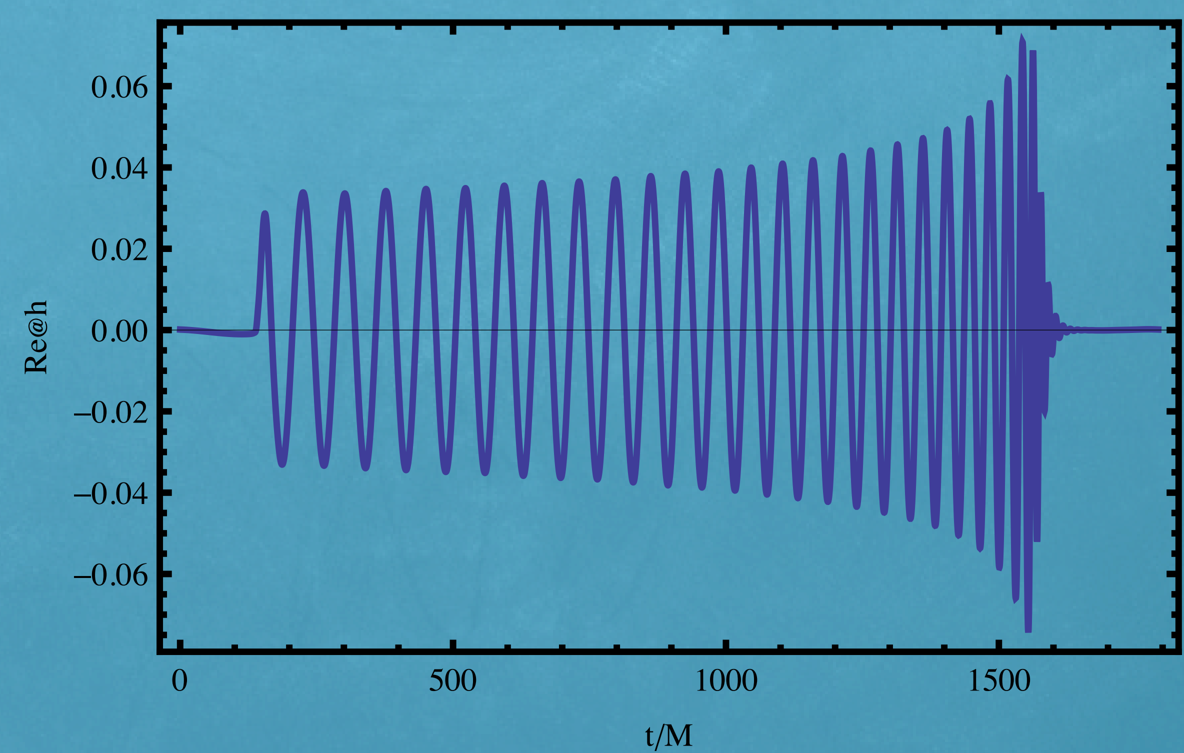
- Inspiral stage: anlytic Post-Newtonian approximats T1,T4,TaylorF2....
- Merger + Ringdown: Full Numerical Relativity

Post-Newtonian



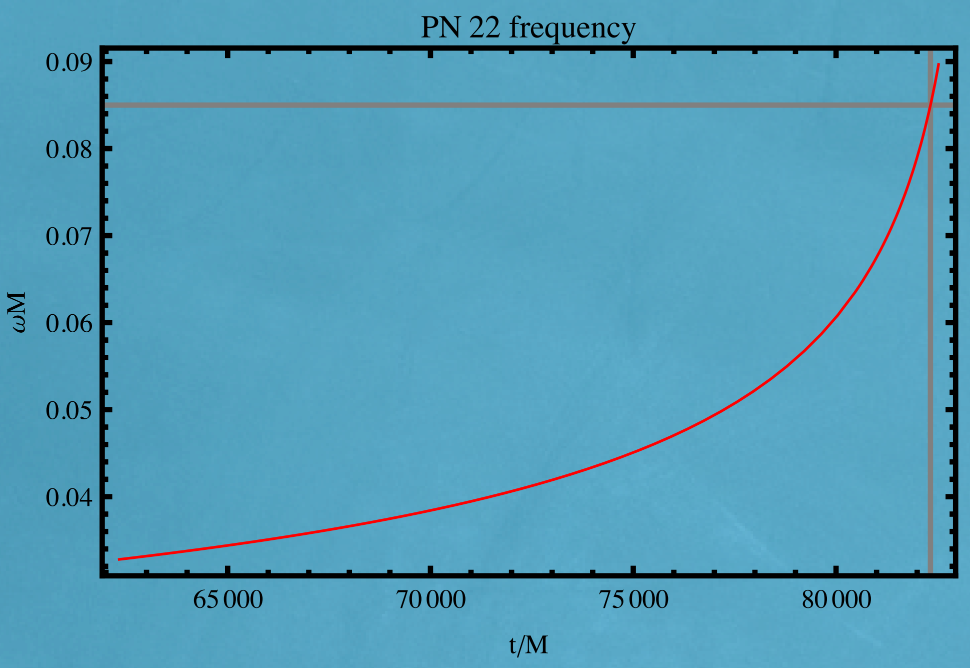
+

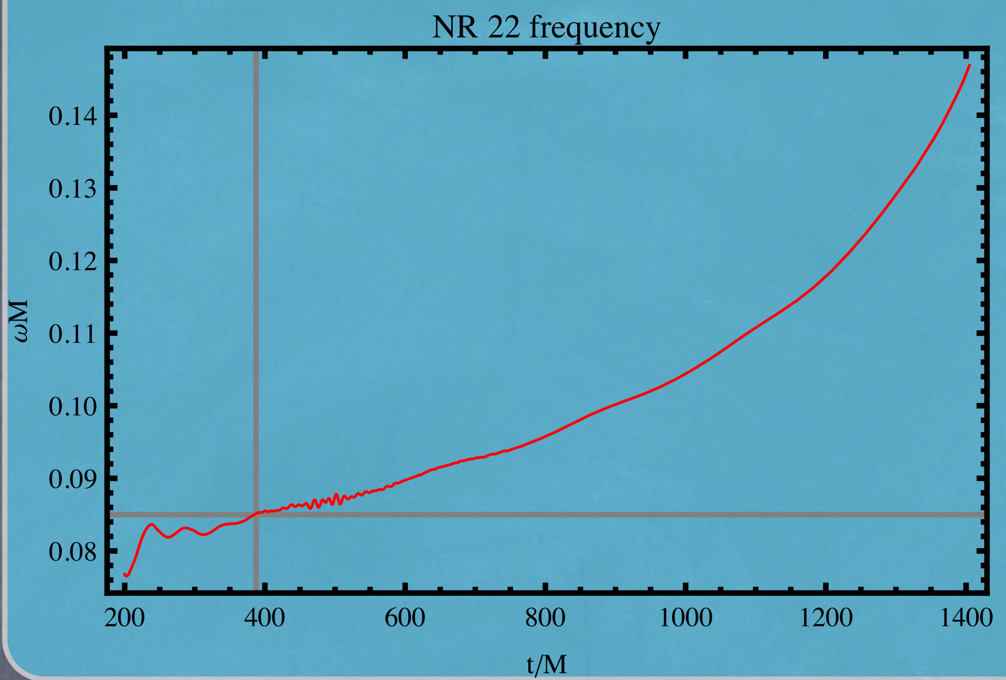
Numerical Relativity



Hybrid Waveform Recipie

- 1. Choose a frequency such that both waves do reasonably agree -> NR data should not be noisy, and PN data should not be too late.


PN 22 frequency
0.09
0.08
0.07
0.06
0.05
0.04
ωM
65 000
70 000
75 000
80 000
t/M


NR 22 frequency
0.14
0.13
0.12
0.11
0.10
0.09
0.08
ωM
200
400
600
800
1000
1200
1400
t/M

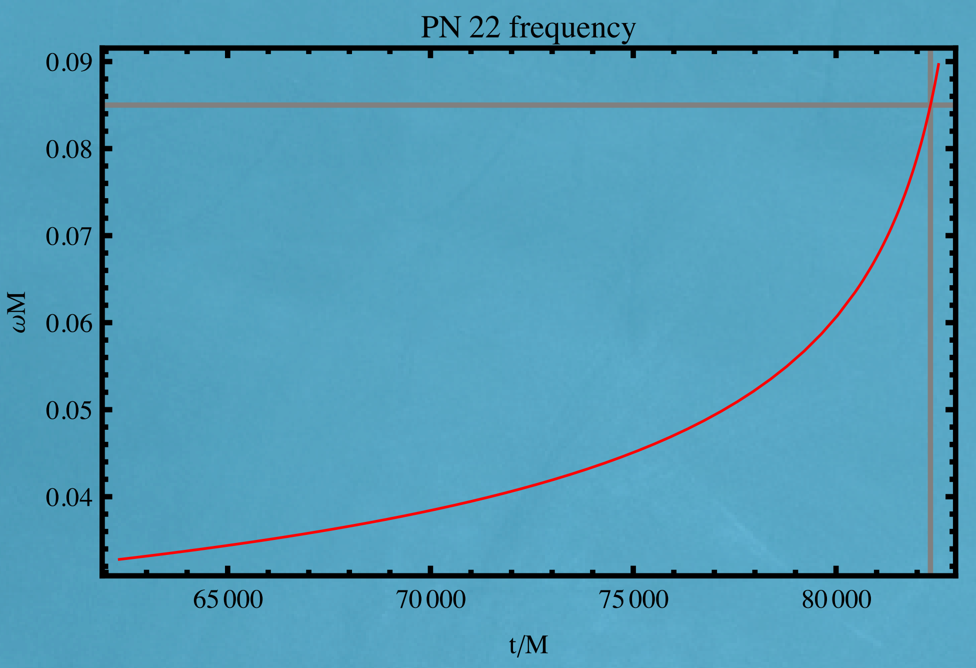
Hybrid Waveform Recipie

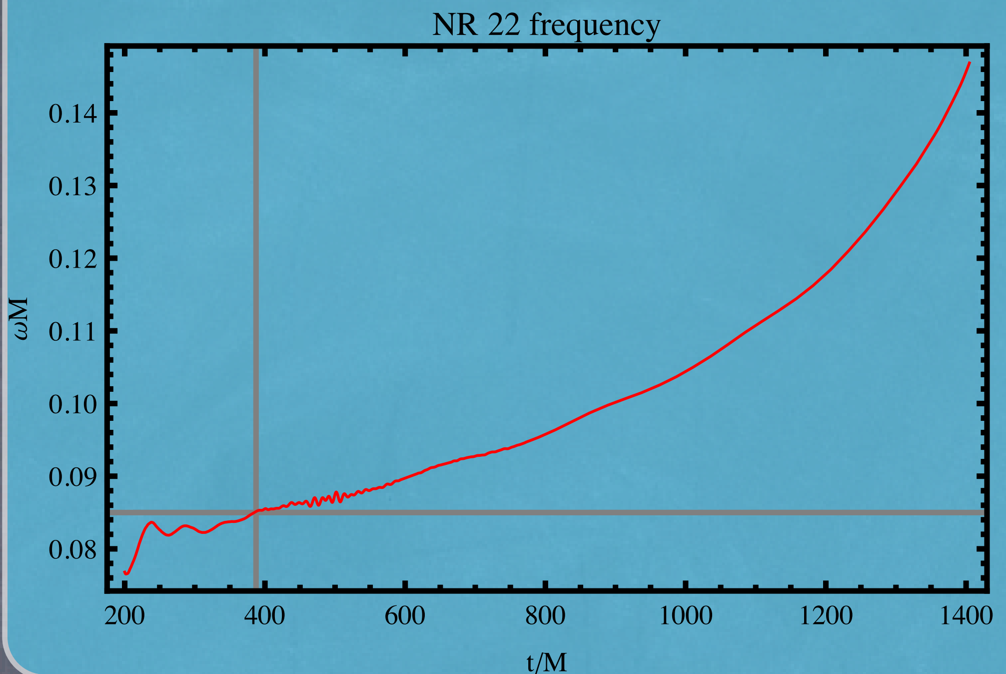
- 1. Choose a frequency ω_m such that both waves are reasonably good.
- 2. Find the corresponding $t_{NR}(\omega_m)$ and $t_{PN}(\omega_m)$ times (this can involve lots of technical details)

Hybrid Waveform Recipie

- 1. Choose frequency such that both waves are reasonably good.
- 2. Find corresponding NR and PN times
- 3. Apply a time shift such that

$$t_{PN}(\omega_m) = t_{NR}(\omega_m)$$



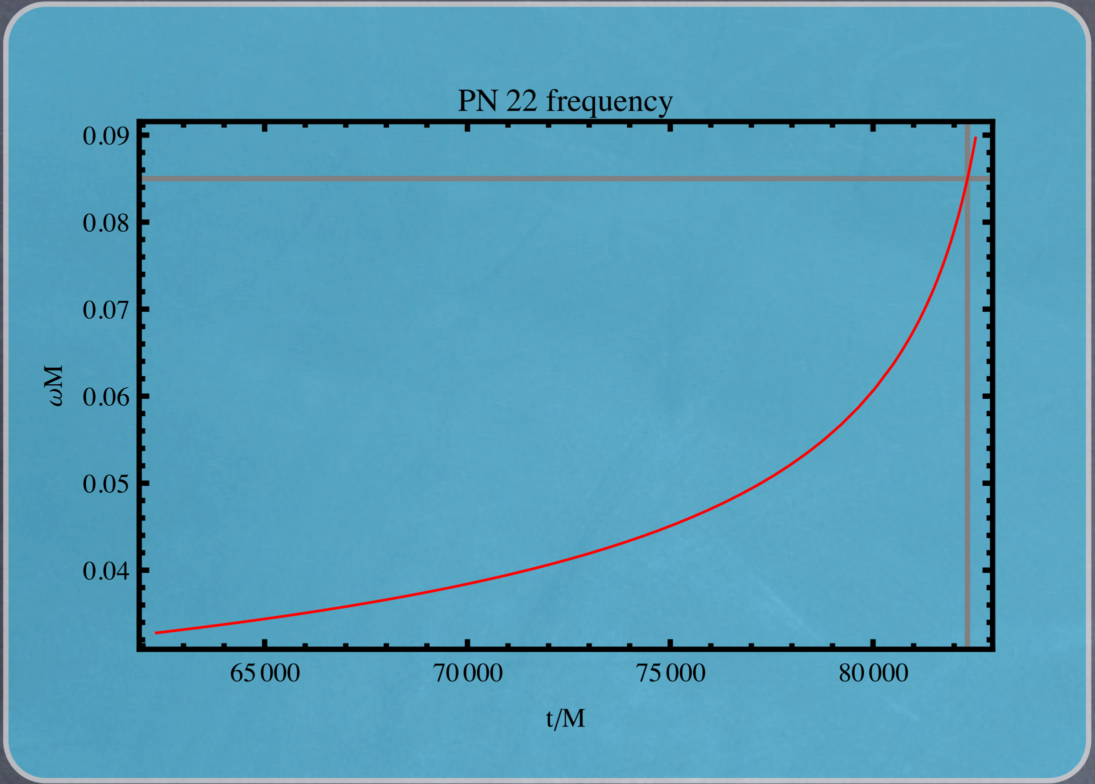


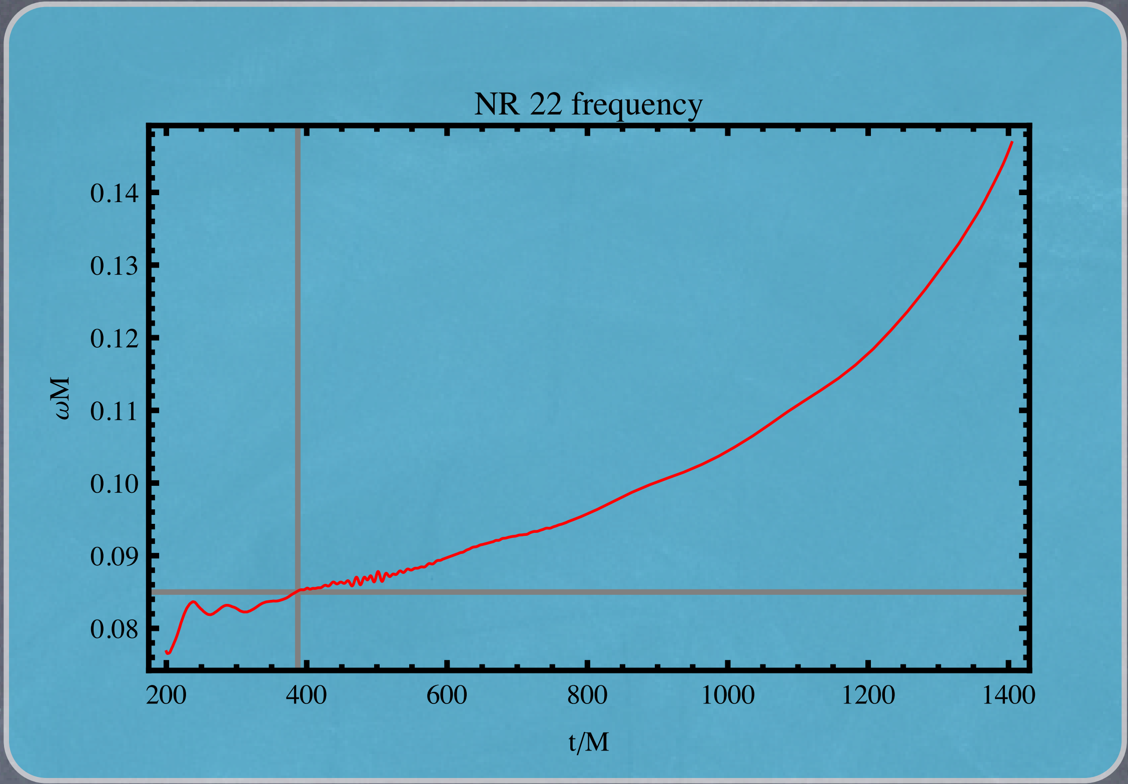
$$t_{pn} = 82327.3t/M$$

$$t_{nr} = 387.296t/M$$

Hybrid Waveform Recipie

- 1. Choose frequency such that both waves are reasonably good.
- 2. Find corresponding NR and PN times
- 3. Apply time shift such that matching time is equal in both waves.
- 4. Apply a phase shift such that phase is continuous at ω_m.





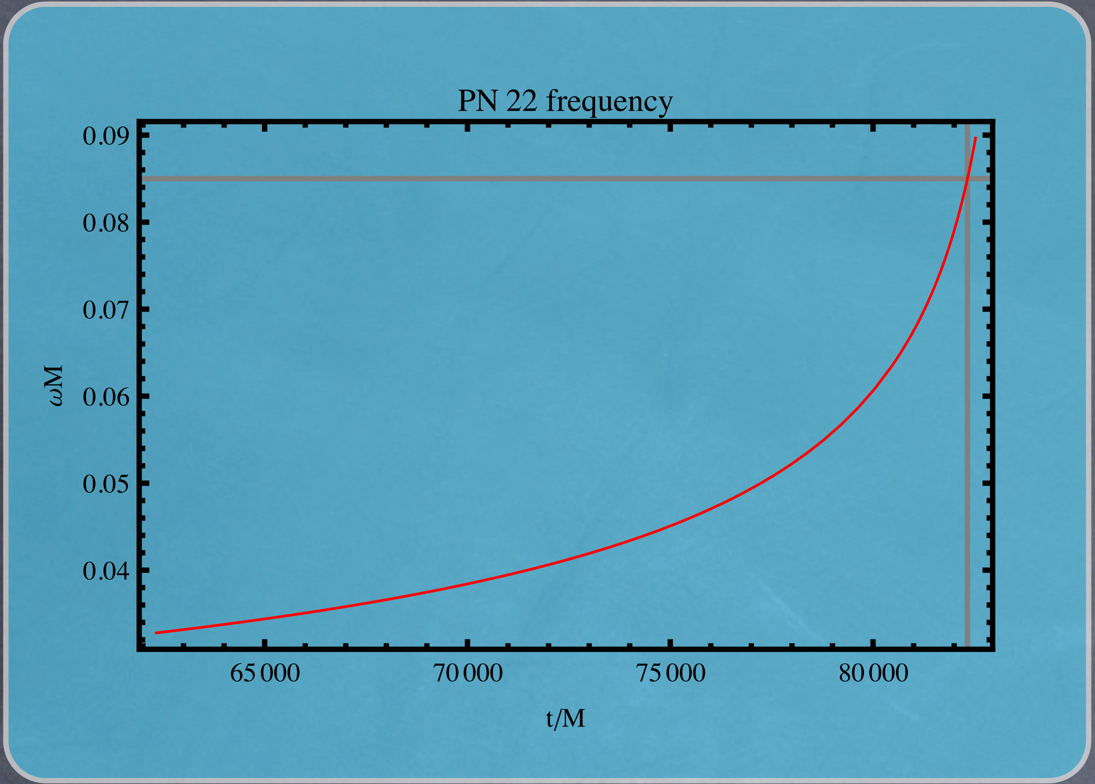
$$t_{pn} = 82327.3t/M$$

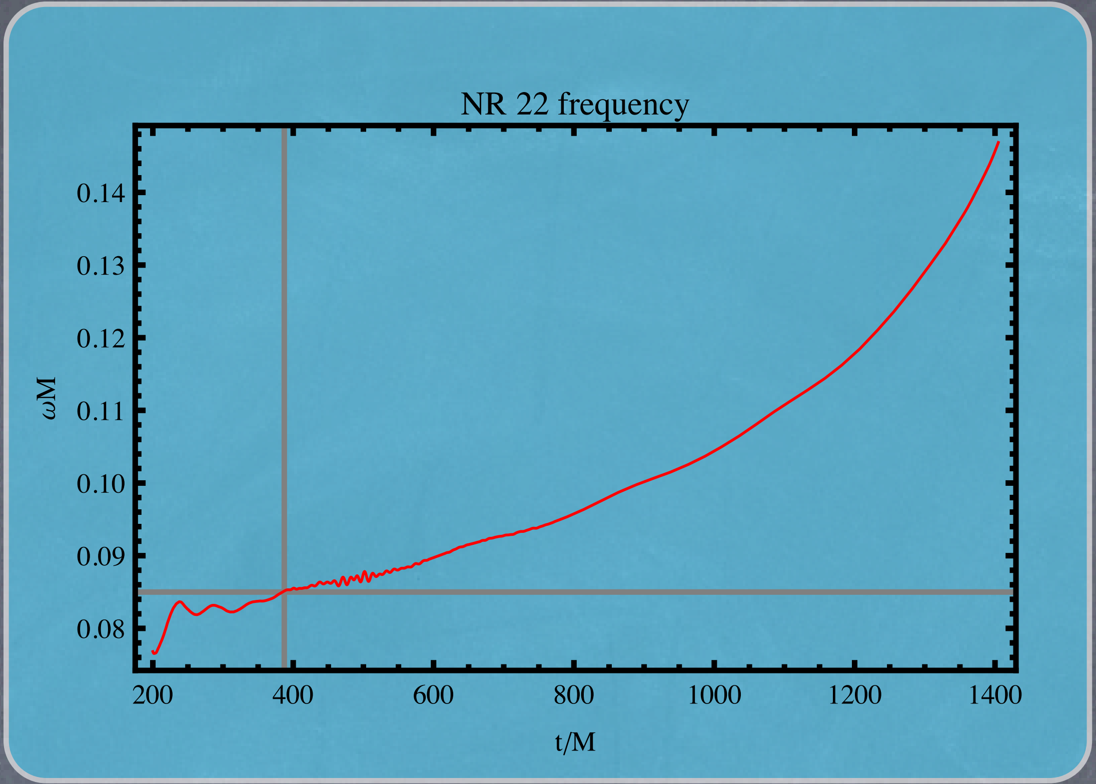
$$t_{nr} = 387.296t/M$$

$$\Delta\phi_{22} = -2042.99$$

Hybrid Waveform Recipie

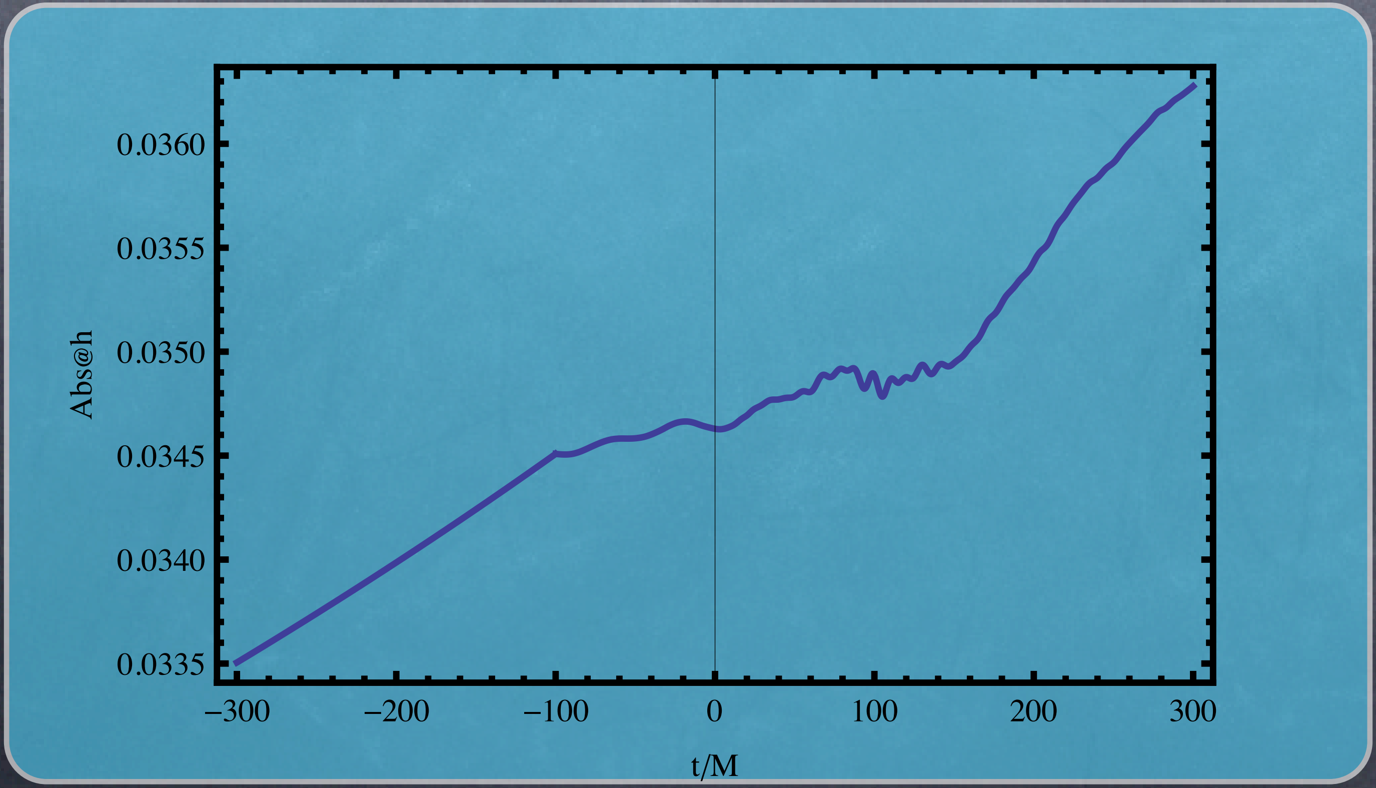
- 1. Choose frequency such that both waves are reasonably good.
- 2. Find corresponding NR and PN times
- 3. Apply time shift such that matching time is equal in both waves.
- 4. Apply a phase shift such that phase is continuous at f0.
- 5. Smooth the amplitude difference over a time window.



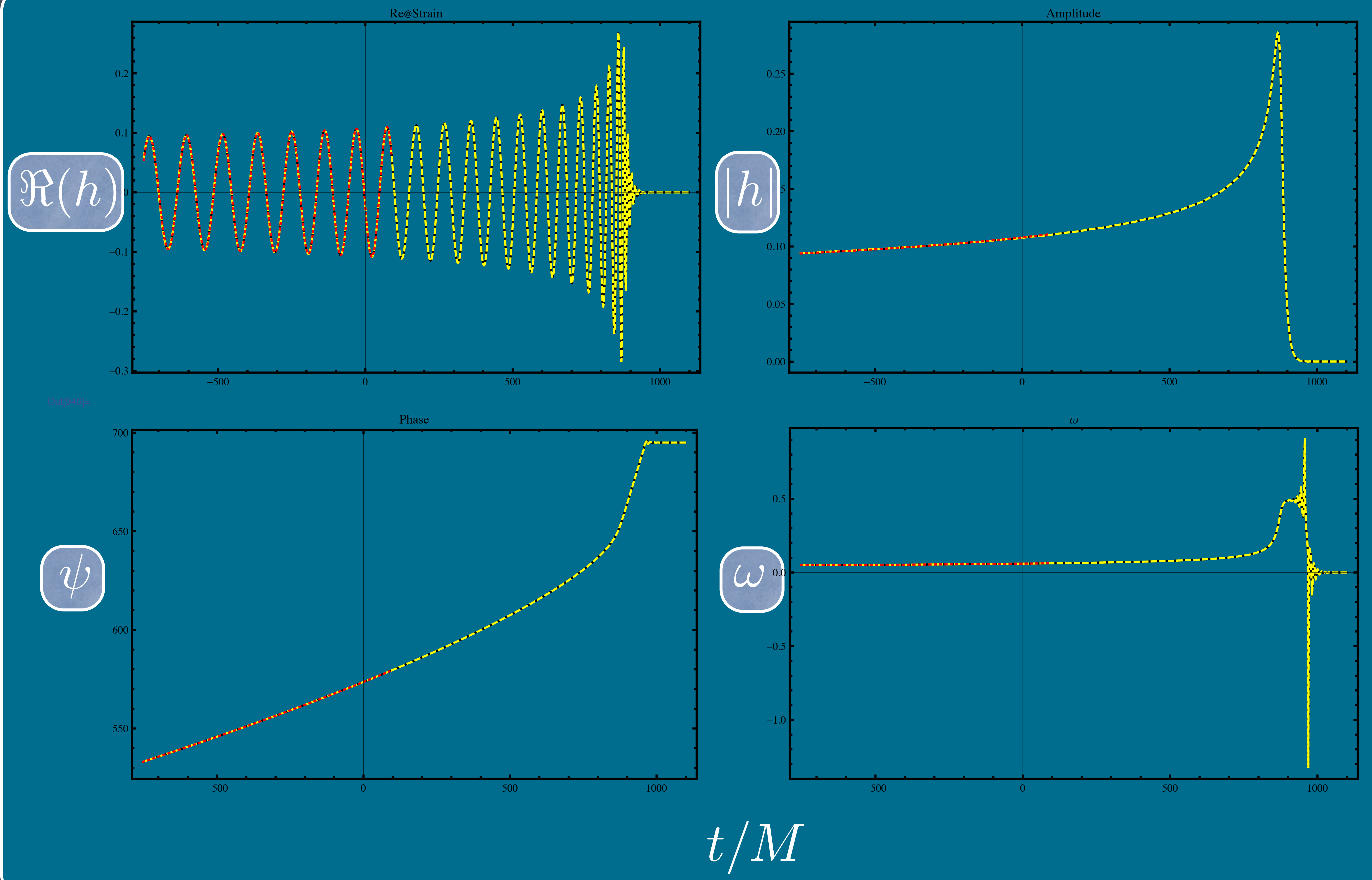


$$t_{pn} = 82327.3t/M \qquad t_{nr} = 387.296t/M$$

$$\Delta\phi_{22} = -2042.99$$



HYBRID WAVEFORM SXS q=3, s=0



Hybrid Higher Modes

- Once 22 hybrid is built, matching times are fixed.
- Phase shift of the lm mode is a little more involved: we have two degrees of freedom.

Hybrid Higher Modes

- Selection of coordinates: shift of $m\phi$ for the lm mode.
- Selection of the tetrad in which the GW is computed: shift of ψ_0 for all modes

$$h(\theta,\phi,t) = \sum_{l,m} |Y^{-2}_{l,m}|(\theta) e^{-im(\phi-\phi_0)} |h_{l,m}(t)| e^{-i(\psi(t)-\psi_0)}$$

Hybrid Higher Modes

Given 2 modes, typically 22 and 33:

$$\Delta\phi_{22} = \psi_0 + 2\phi$$

$$\Delta\phi_{22} = \psi_0 + 3\phi$$

And

$$\boxed{\Delta^t\phi_{lm} = \psi_0 + m\phi}$$

Hybrid Higher Modes

Given 2 modes, typically 22 and 33:

$$\Delta\phi_{22} = \psi_0 + 2\phi$$

$$\Delta\phi_{22} = \psi_0 + 3\phi$$

And

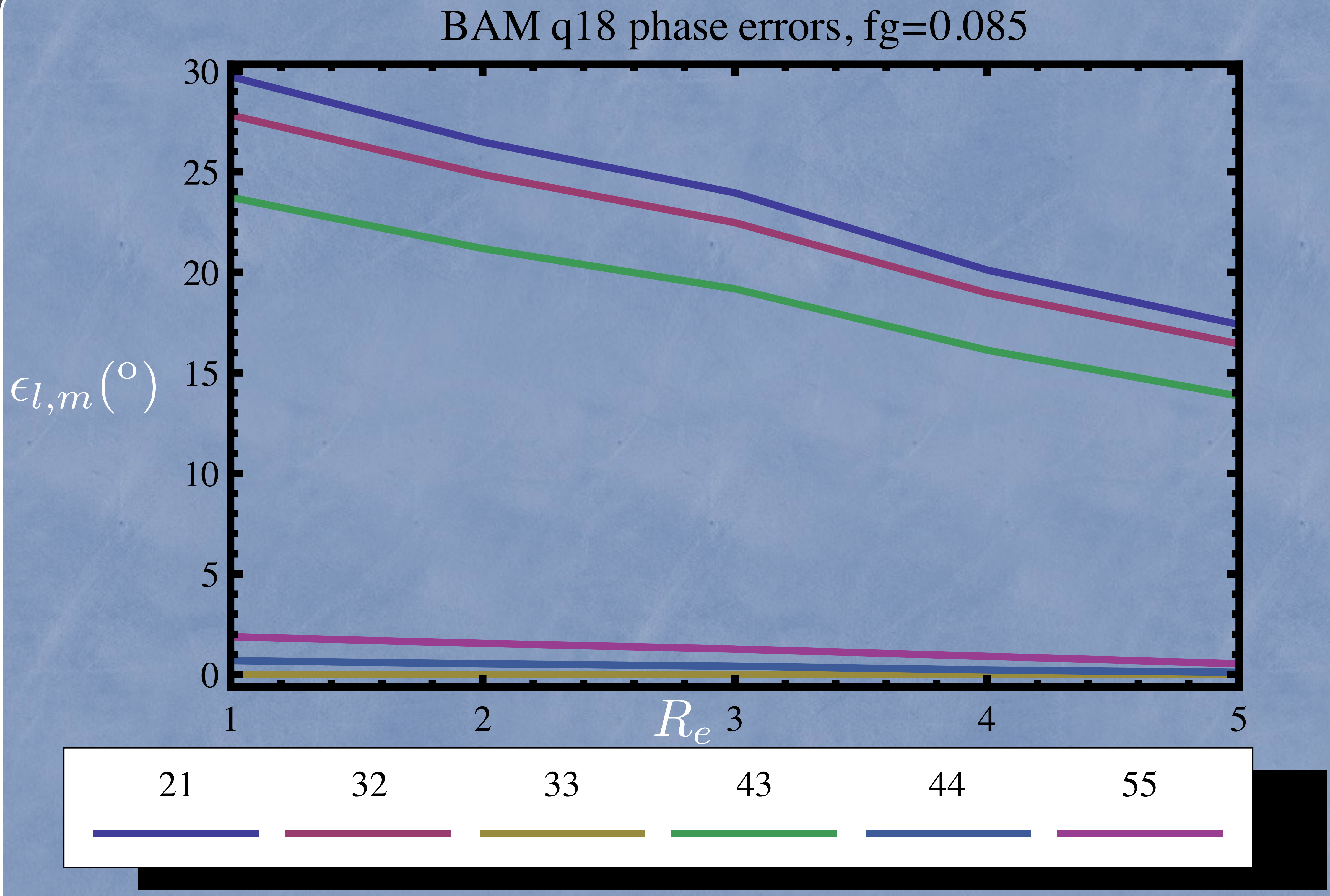
Phasing Error

$$\boxed{\Delta^t\phi_{lm} = \psi_0 + m\phi}$$

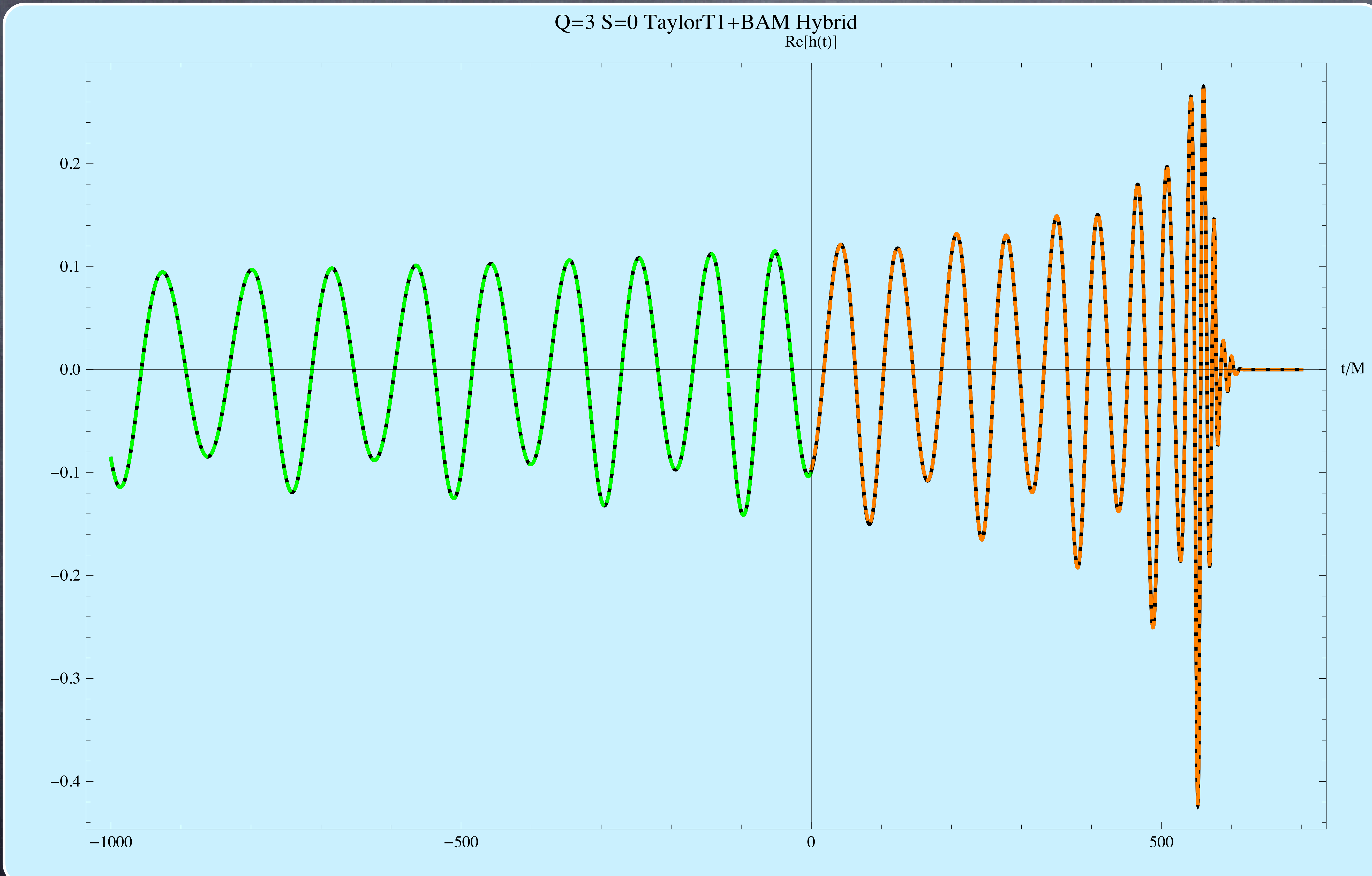
But

$$\boxed{\epsilon_{lm} = \Delta^t\phi_{lm} - \Delta^t\phi_{lm}}$$

Phase Errors, q=18



Full Hybrid Waveform



Data Analysis

$$(h(\theta,\phi,\nu)|g(\theta^{'},\phi^{'},\nu^{'})) = \frac{\max_{t_0,\theta',\phi',\nu'} \Re \int_{fm}^{fM} \frac{\tilde{h}(\theta,\phi,\nu)(f)\tilde{g}^*(\theta^{'},\phi^{'},\nu^{'})(f)}{S_n(f)} df}{\sqrt{< h|h >< g|g >}}$$

$$\rho(h)(\theta,\phi,\nu) = \frac{\max_{t_0} \Re \int_{fm}^{fM} \frac{\tilde{h}(\theta,\phi,\nu)(f)\tilde{h}^*(\theta,\phi,\nu)(f)}{S_n(f)} df}{\sqrt{< h|h >}}$$

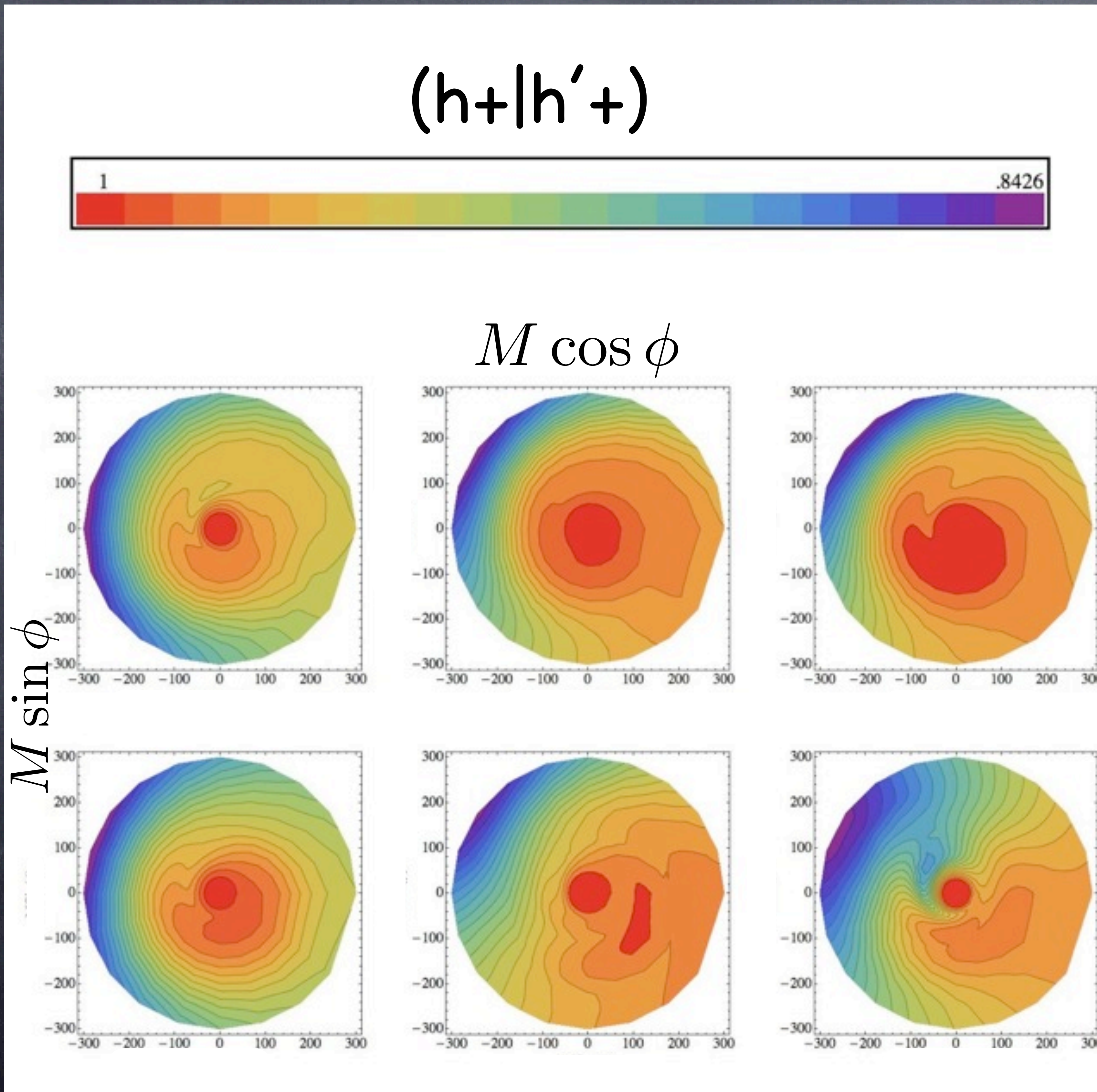
$$< h|h >= \int_{fm}^{fM} \frac{\tilde{h}(f)\tilde{h}^*(f)}{S_n(f)} df$$

Sn(f)=2015 Advanced LIGO

fm=30Hz

fM=2000Hz

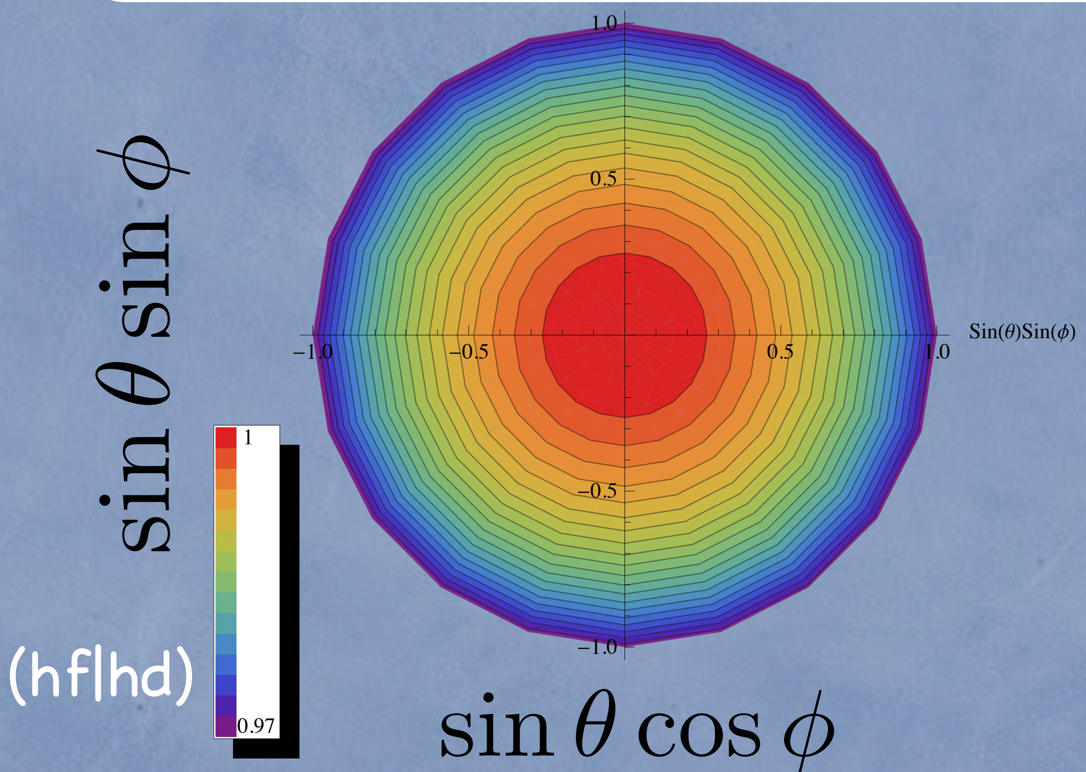
Effect of Phase Errors: q=6



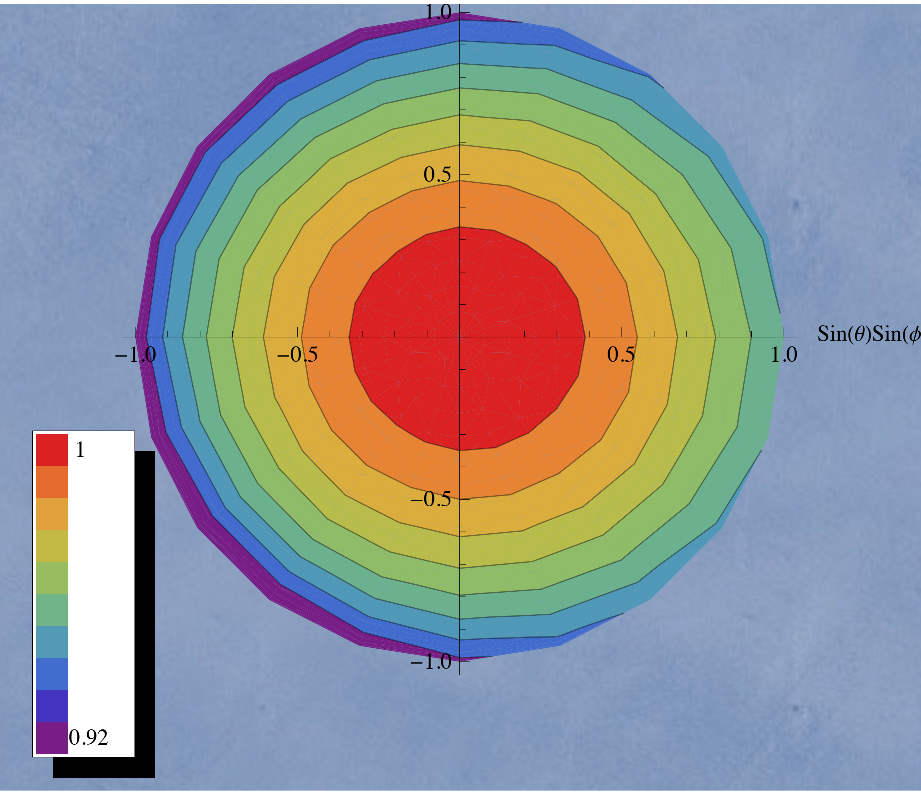
$\epsilon_{l,m} \sim 0, \forall h_{l,m}$

$_{,m} \in (-\pi/6, \pi/6), \forall h'_{l,m}$

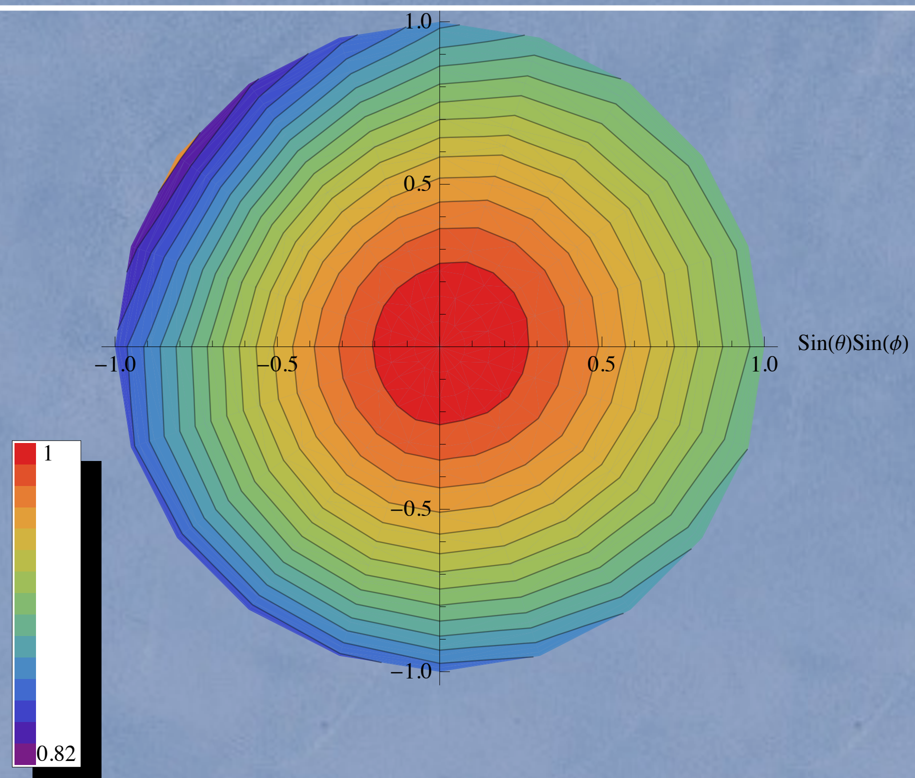
M=20Mo Min=0.97



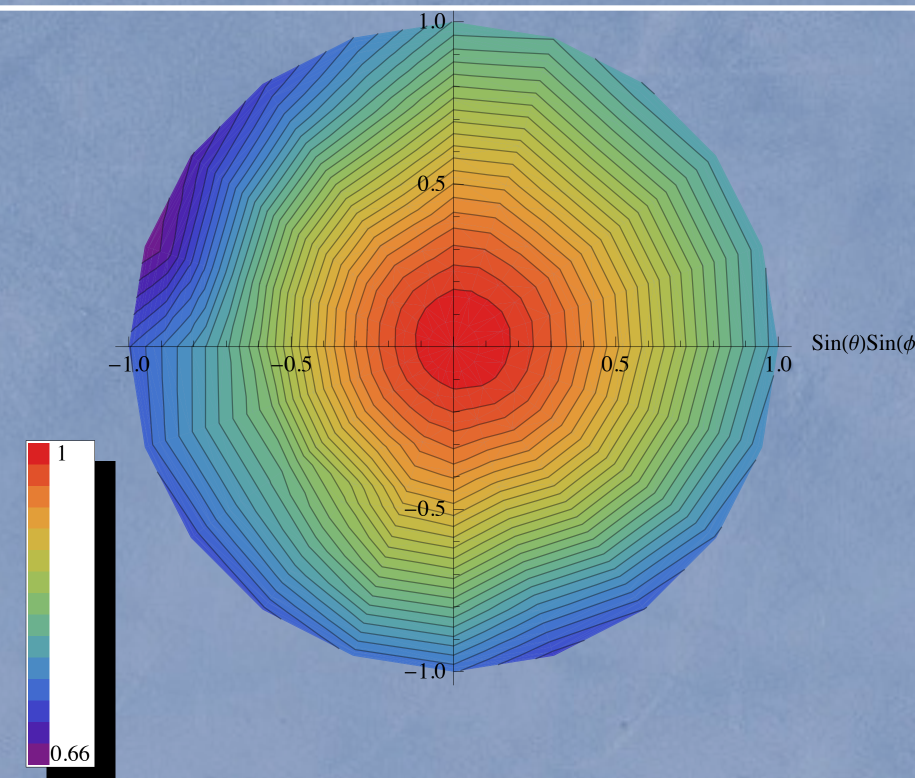
M=100Mo Min=0.92



M=160Mo Min=0.82



M=300Mo Min=0.66



Potential Volume Gain II

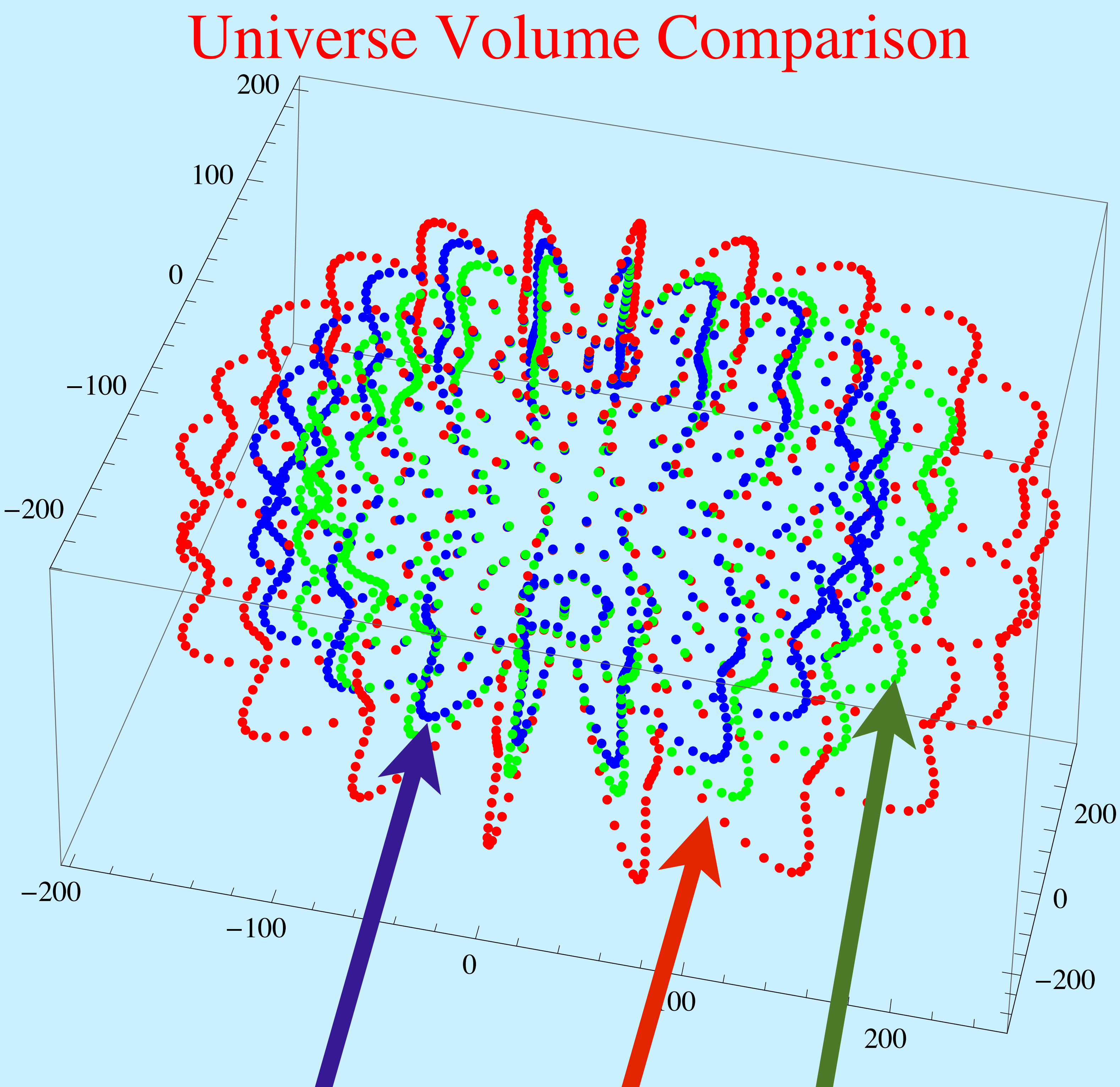
Horizon Distance: Maximum Distance at which a source produces a threshold SNR (8)

$$\boxed{\frac{d_h^{22}}{d_h^F} = \frac{d_{\rho=8}^{22}}{d_{\rho=8}^F} = \frac{\rho_{d_0}^{22}}{\rho_{d_0}^F} = \frac{\rho_{d_0}^F (h^{22}|h^F)}{\rho_{d_0}^F} = (h_{22}|h^F) \equiv O}$$

$$V_h = \sum_i \sin\theta_i \rho^3(\theta_i, \phi_i)$$

$$V_d = \sum_i \sin\theta_i \rho^3 \times O^3(\theta_i, \phi_i)$$

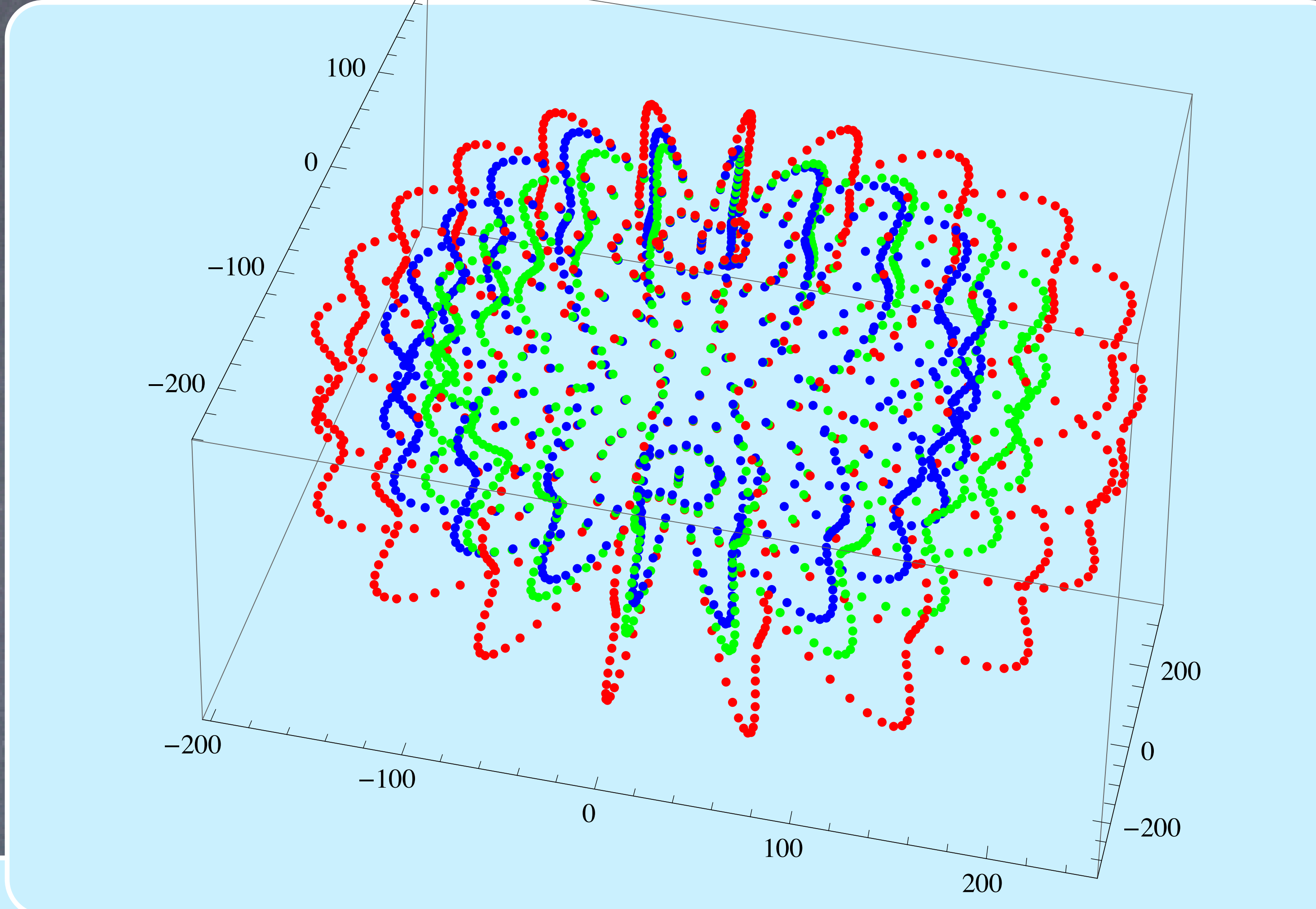
$$\Delta V = 100(Vh/Vd - 1)\%$$



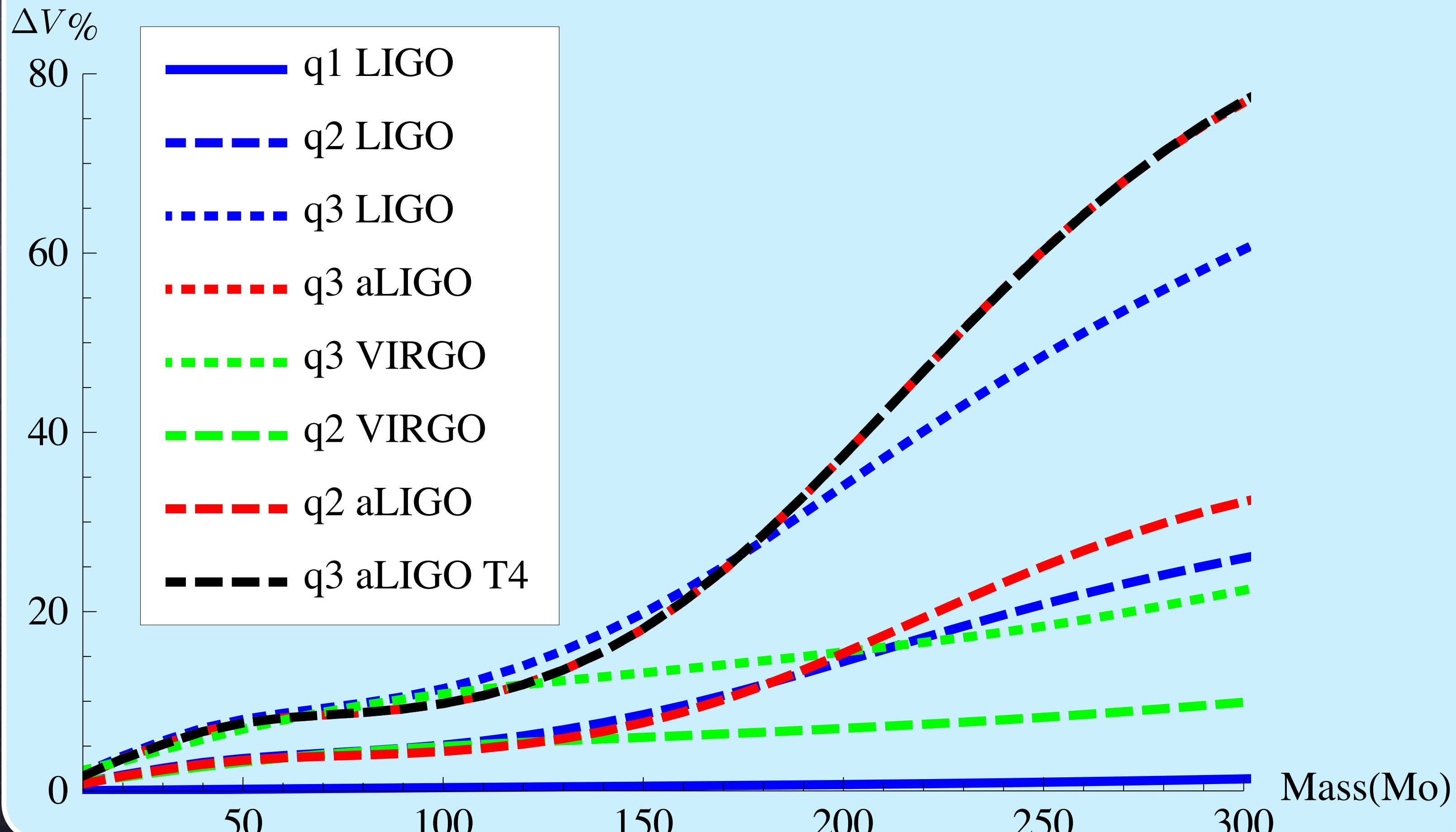
Blue: (D vs D) Green:(Full vs D)

Red:(Full vs Full) q=3 M=300Mo

Blue: (D vs D)
Green:(F vs D)
Red:(F vs F)
q=3 M=300Mo



Volume Gain



At a zero parameter bias cost

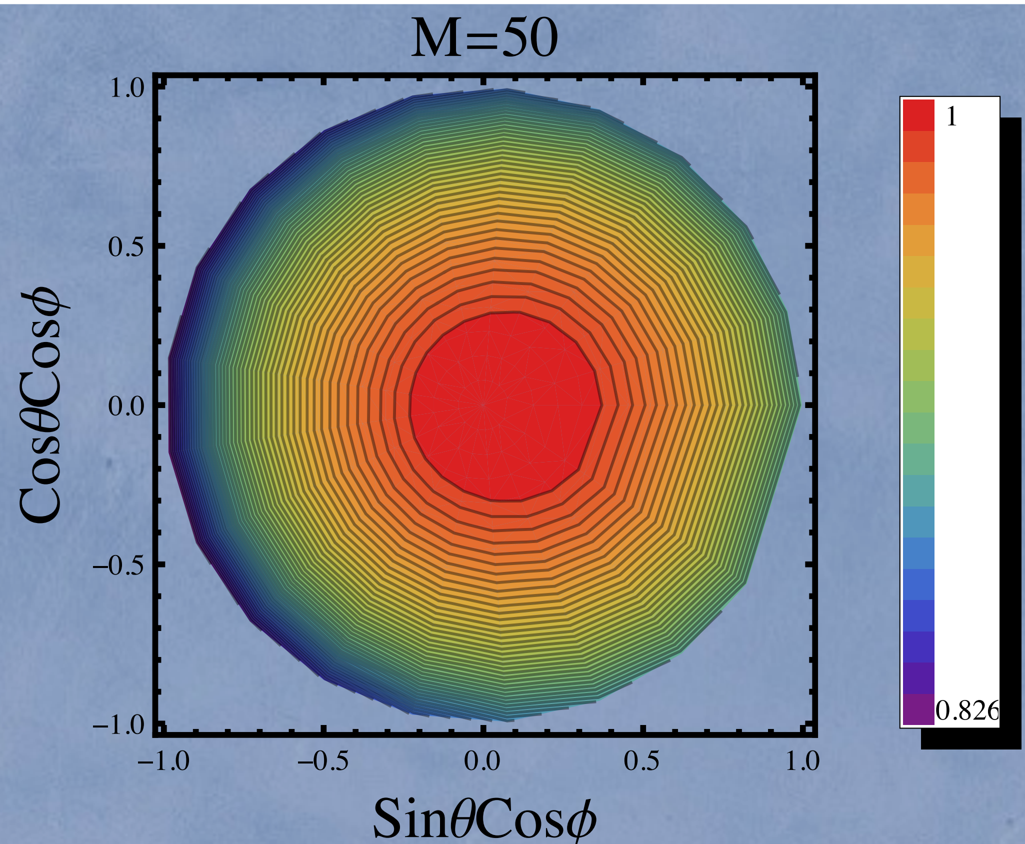
Fitting Factors

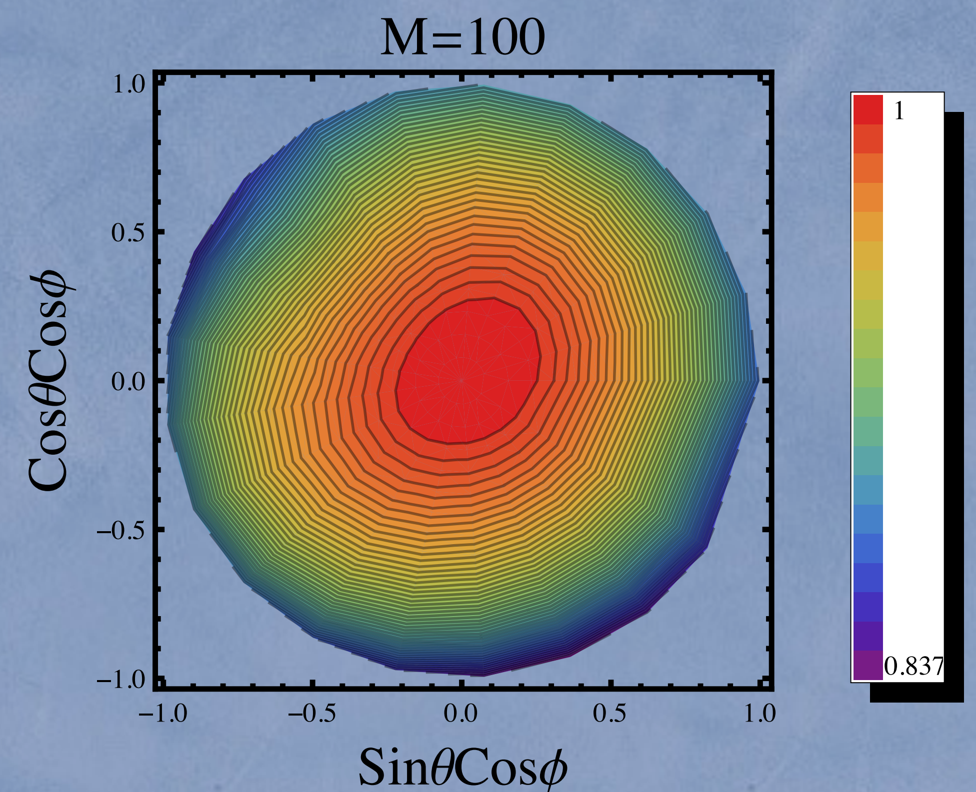
$$H = \{h_i(q, M, \theta, \phi, \nu) / h \in PhenomC\}$$

$$g = g(\theta_g, \phi_g, \nu_g)$$

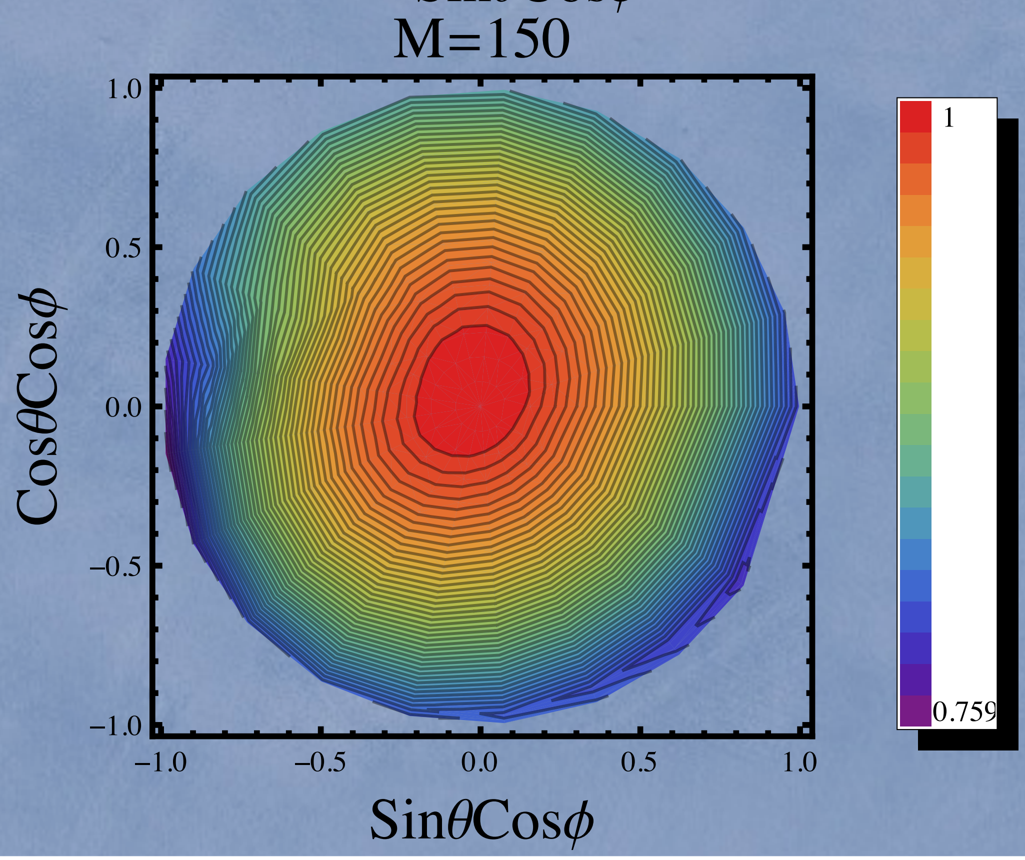
$$FF(\theta_g, \phi_g, \nu_g) = \max_H (g(\theta_g, \phi_g, \nu_g) | h_i)$$

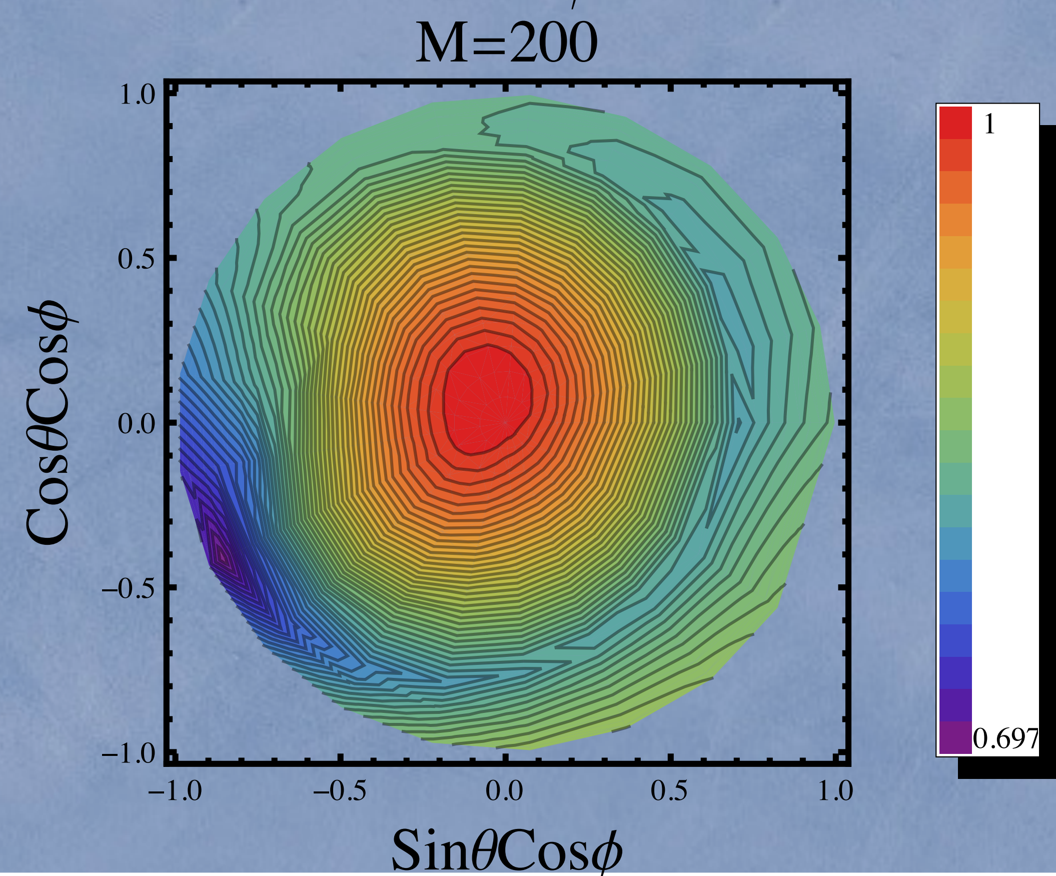
Fitting Factors



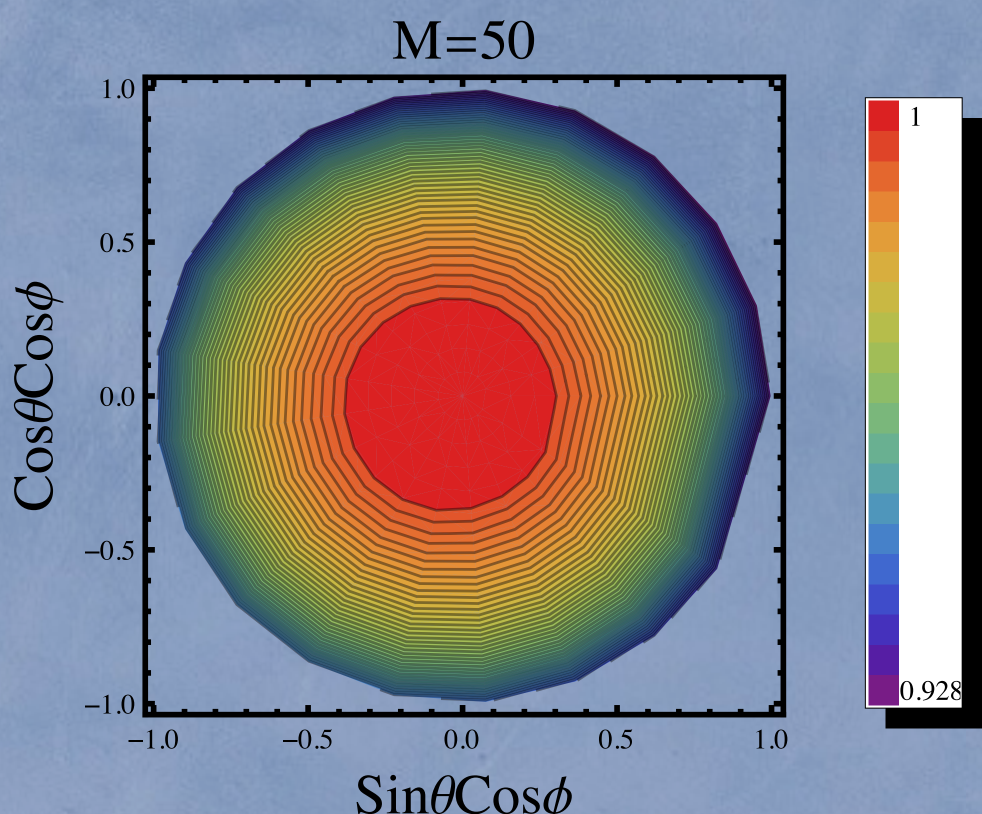


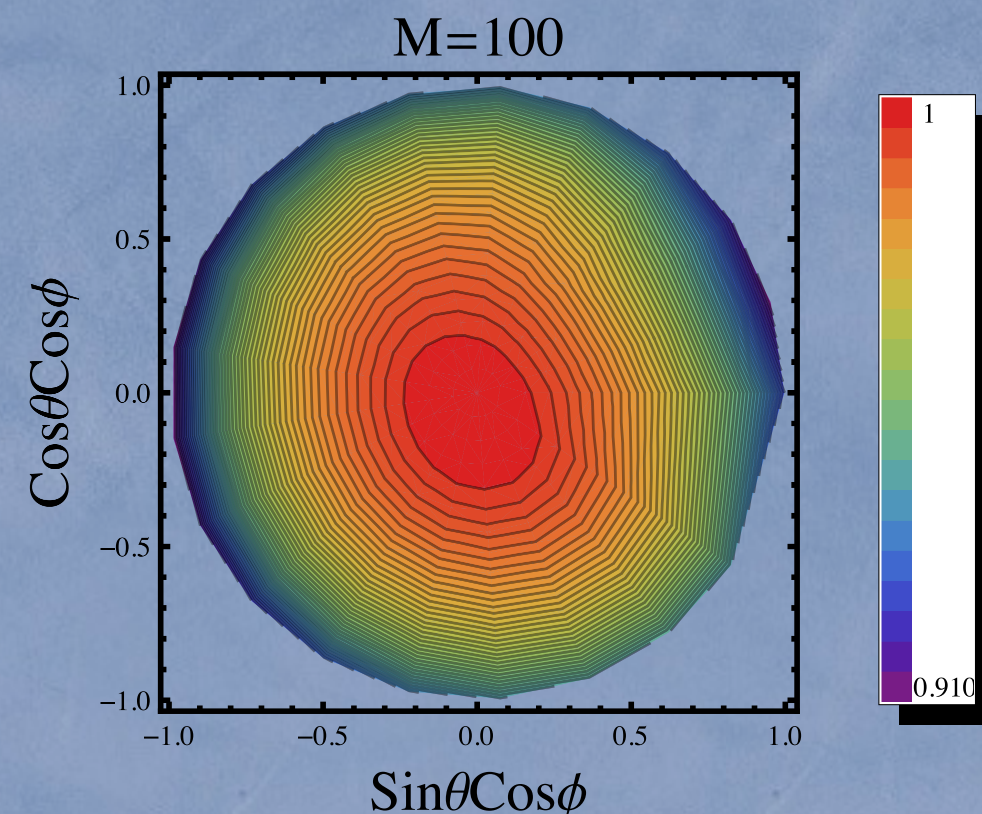
q=6
s=0





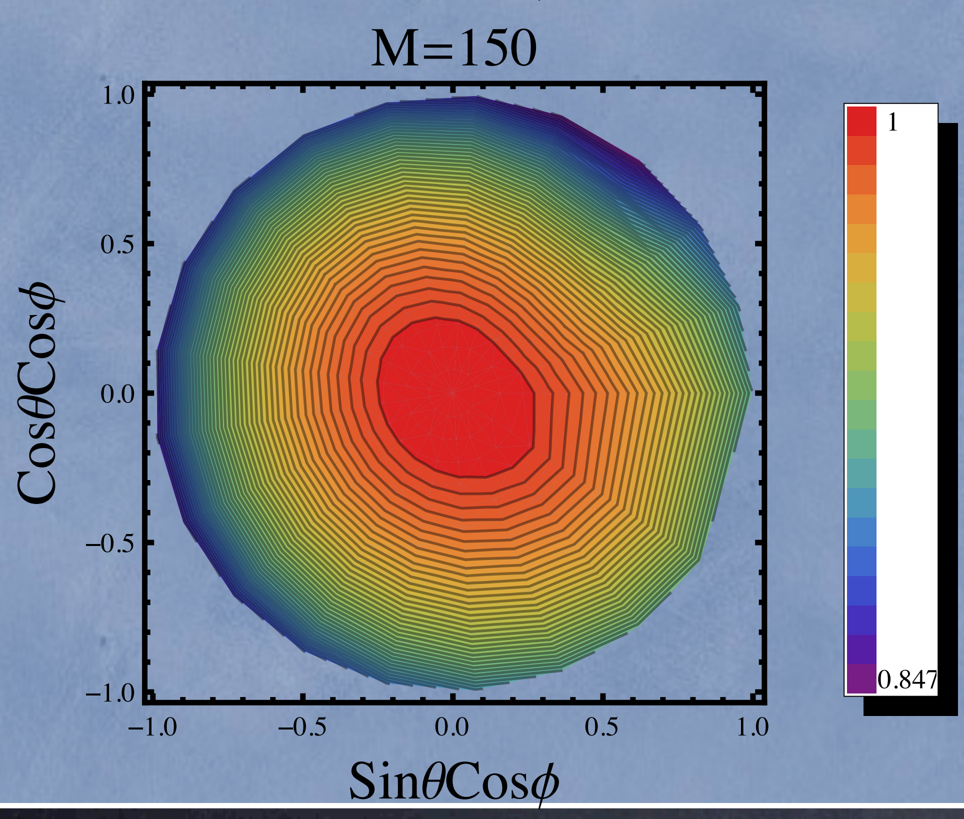
Fitting Factors

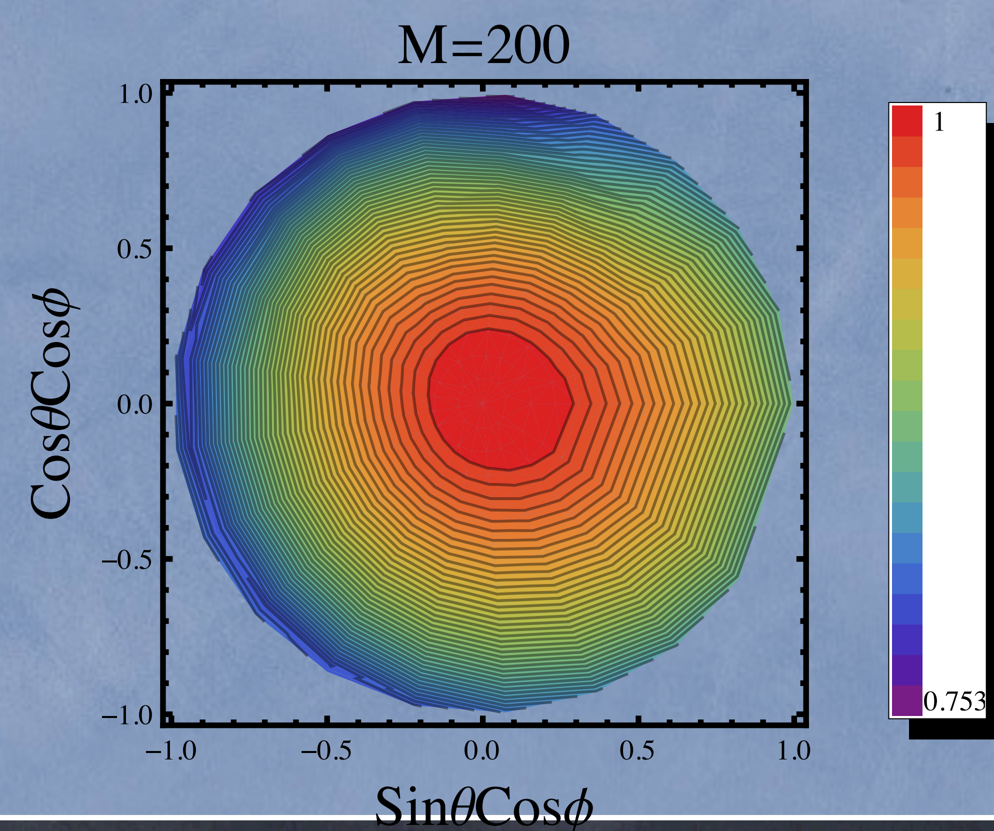




q=3

s=0





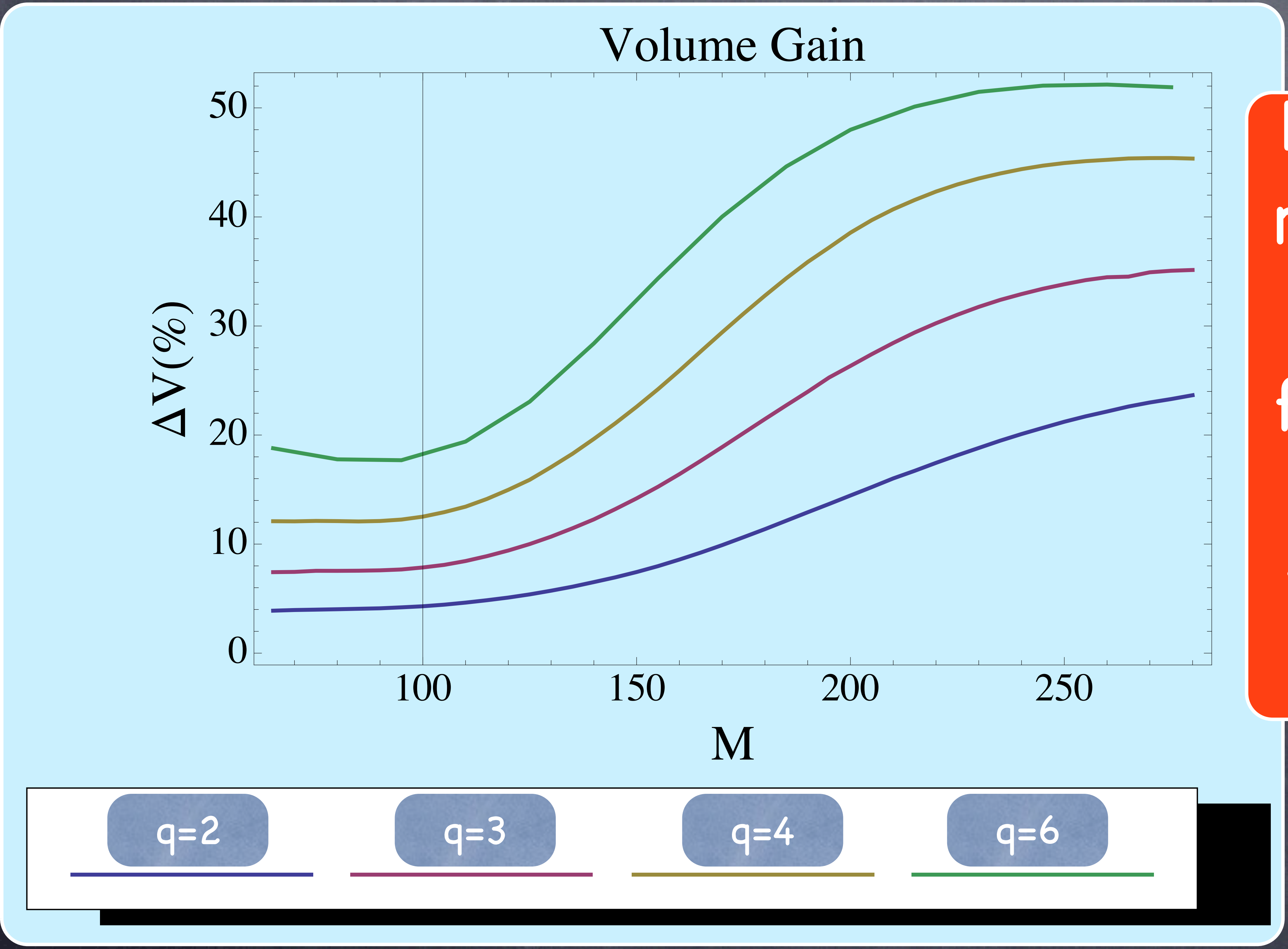
Potential Volume Gain II

$$V_h = \sum_i \sin\theta_i \rho^3(\theta_i, \phi_i)$$

$$V_d = \sum_i \sin\theta_i \rho^3 \times FF^3(\theta_i, \phi_i)$$

$$\Delta V = 100(Vh/Vd - 1)\%$$

Potential Volume Gain II

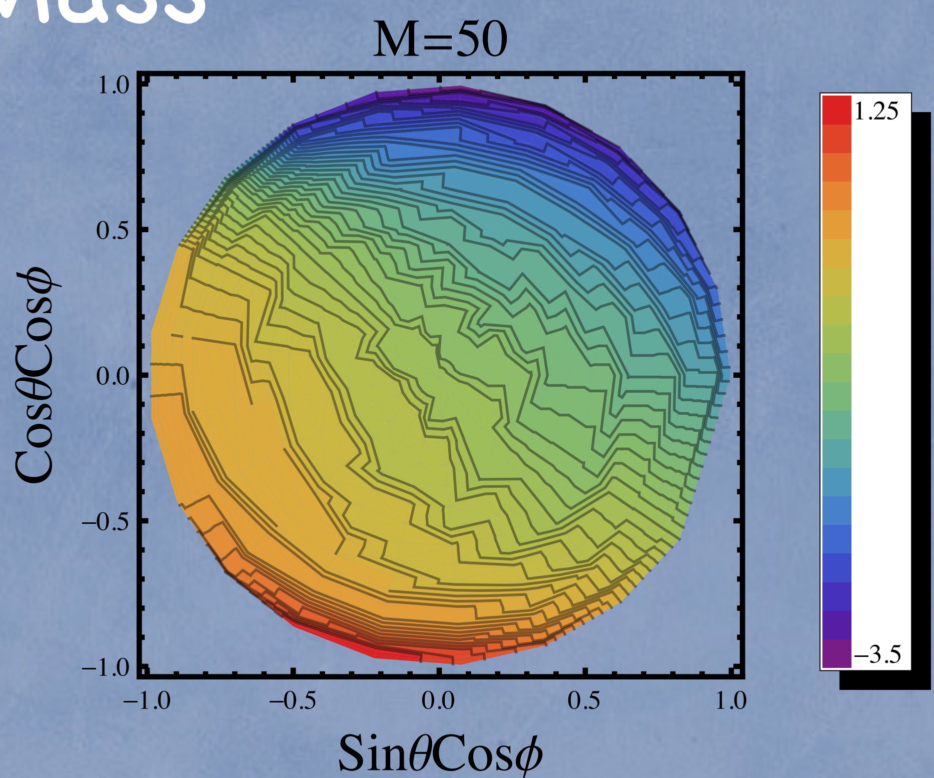


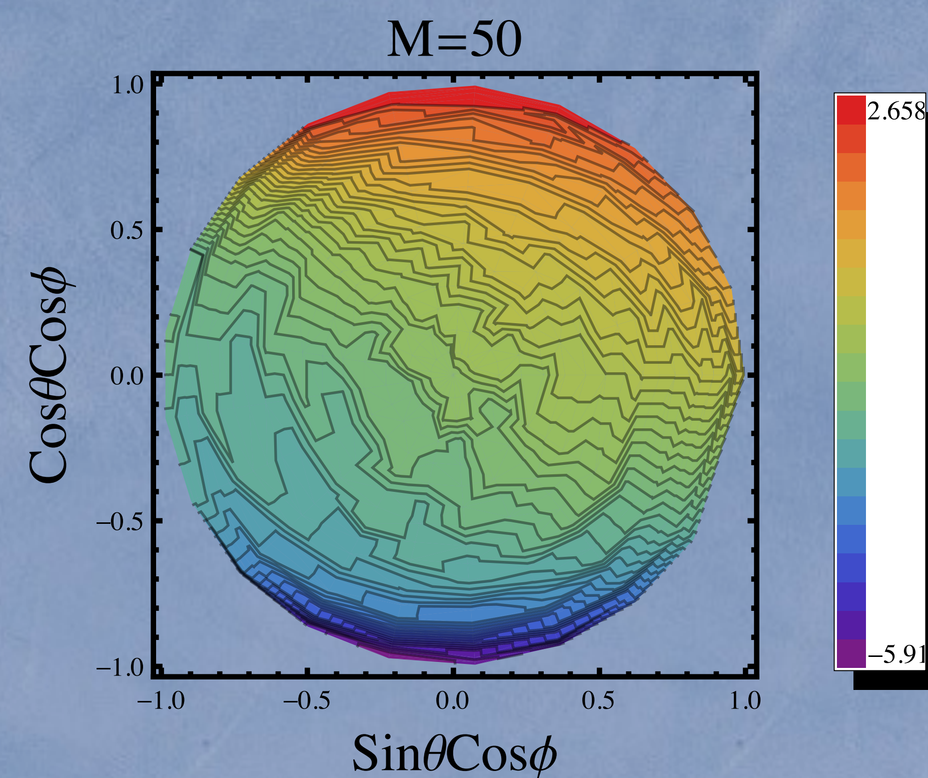
However, the minimum SNR would raise from 8 to 8.3
->
Reduction of 10%

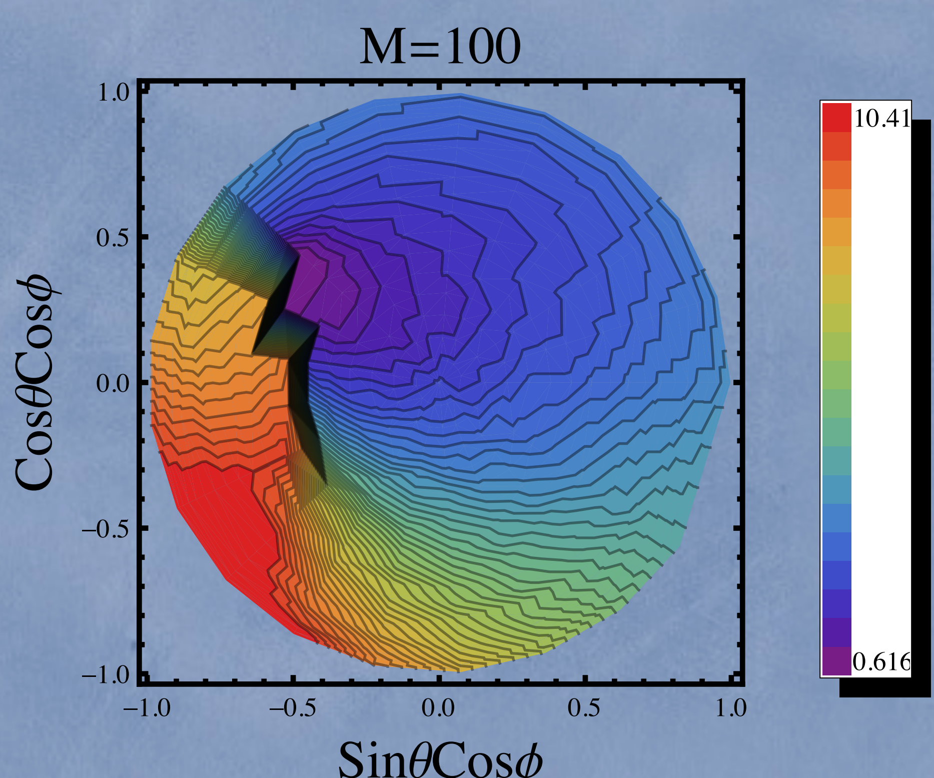
Parameter Bias q6

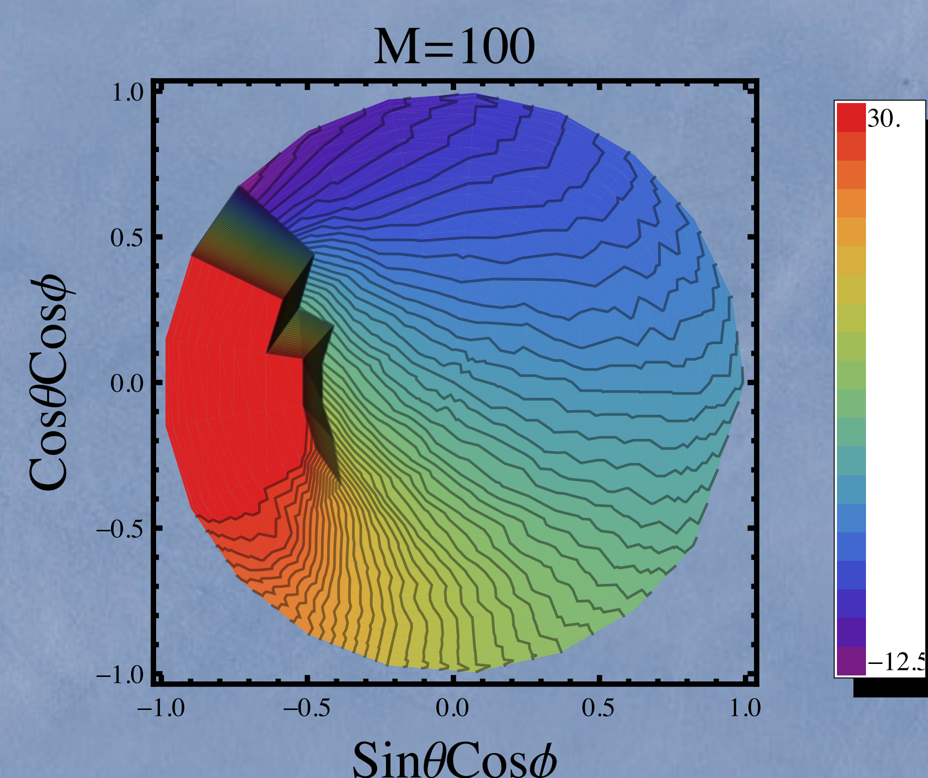
Mass

Mass Ratio



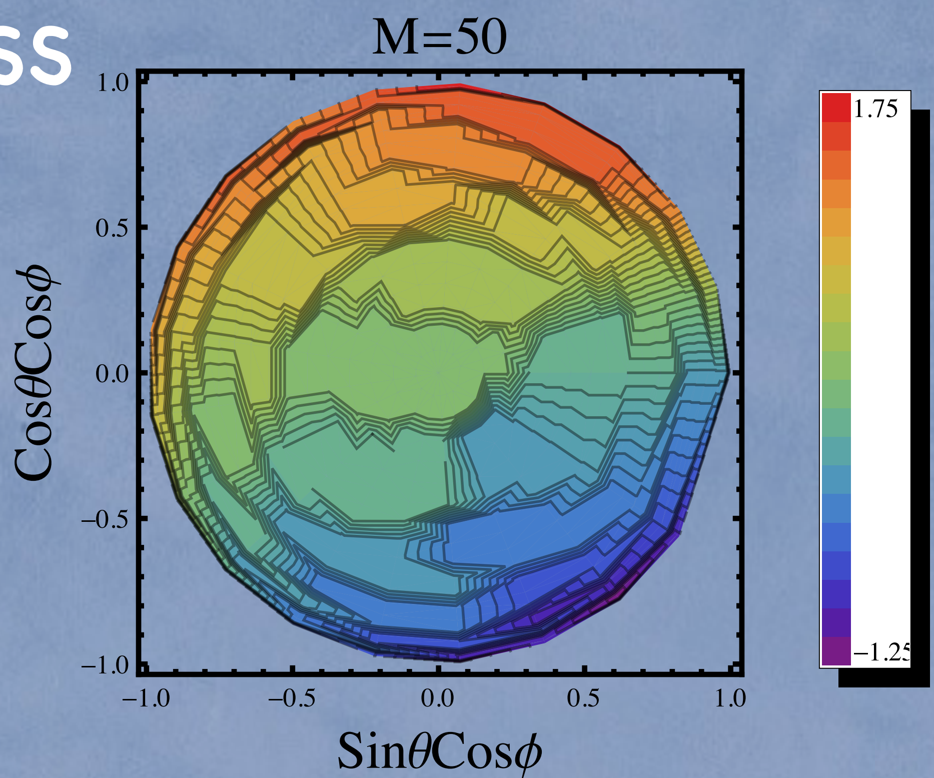




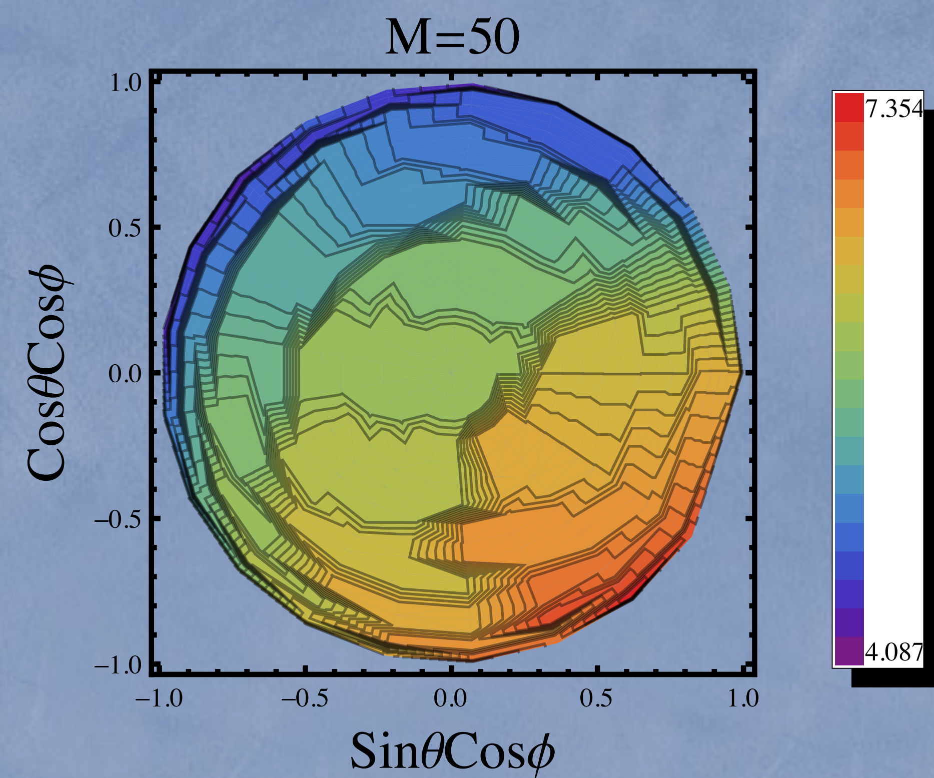


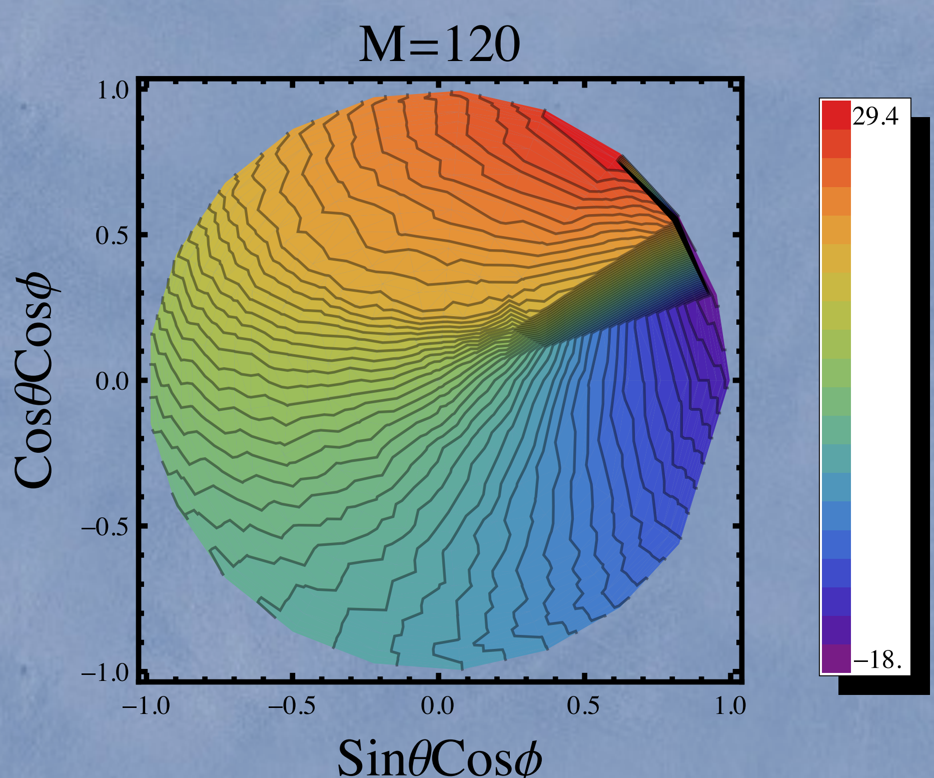
Parameter Bias: q3

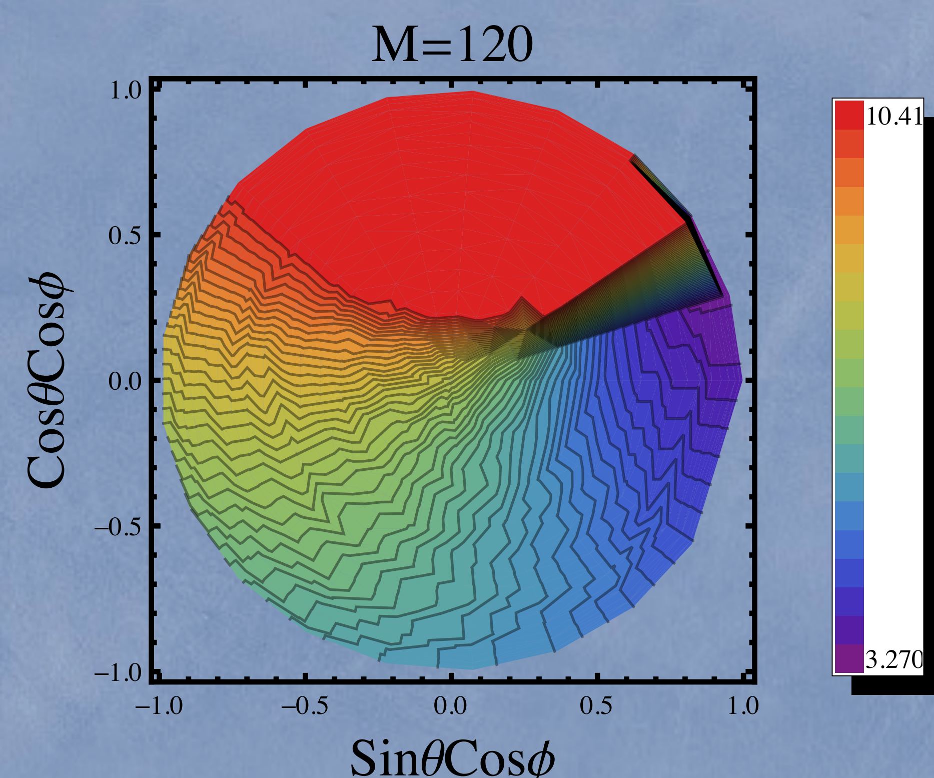
Mass



Mass Ratio





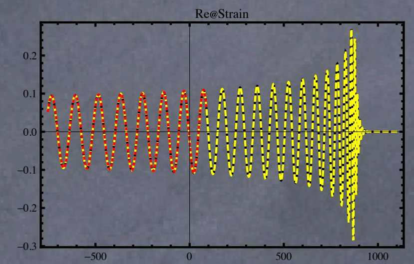


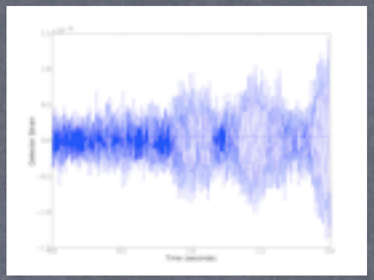
Preliminar Real Analysis

- Injection of Full Waveforms in Real Recolored Noise Simulating 2015-adv. LIGO.
- SpEC+TaylorT1 q=3 s=0 Hybrid Waveforms.
- EOBNR Template Bank with 1Mo<M<100Mo [Buonanno et al.2007Phys.Rev.D76104049].

Detection Results

1.Injection of a full waveform in real data



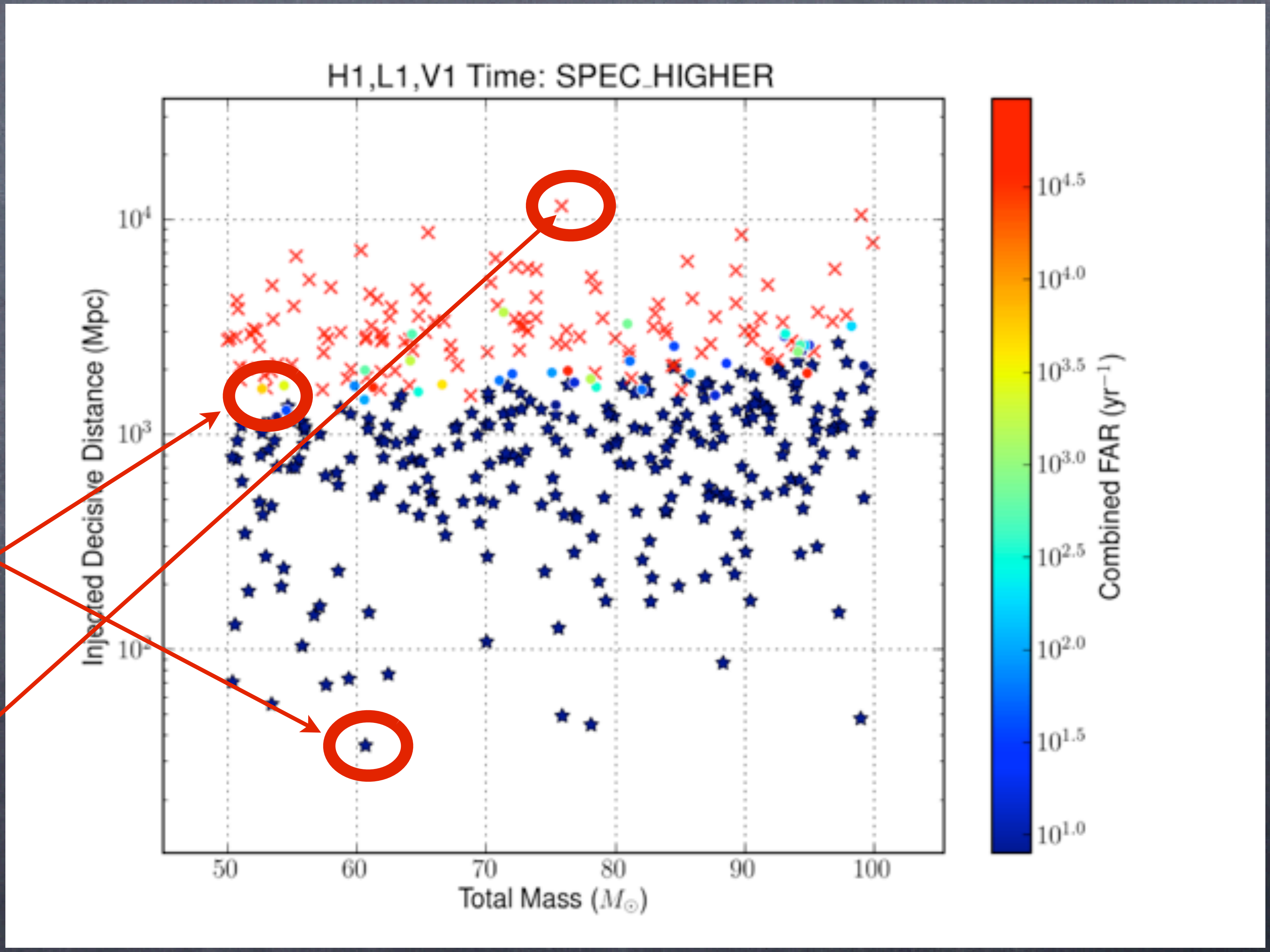


2. Filter the resulting data with a only 22 mode Template Bank

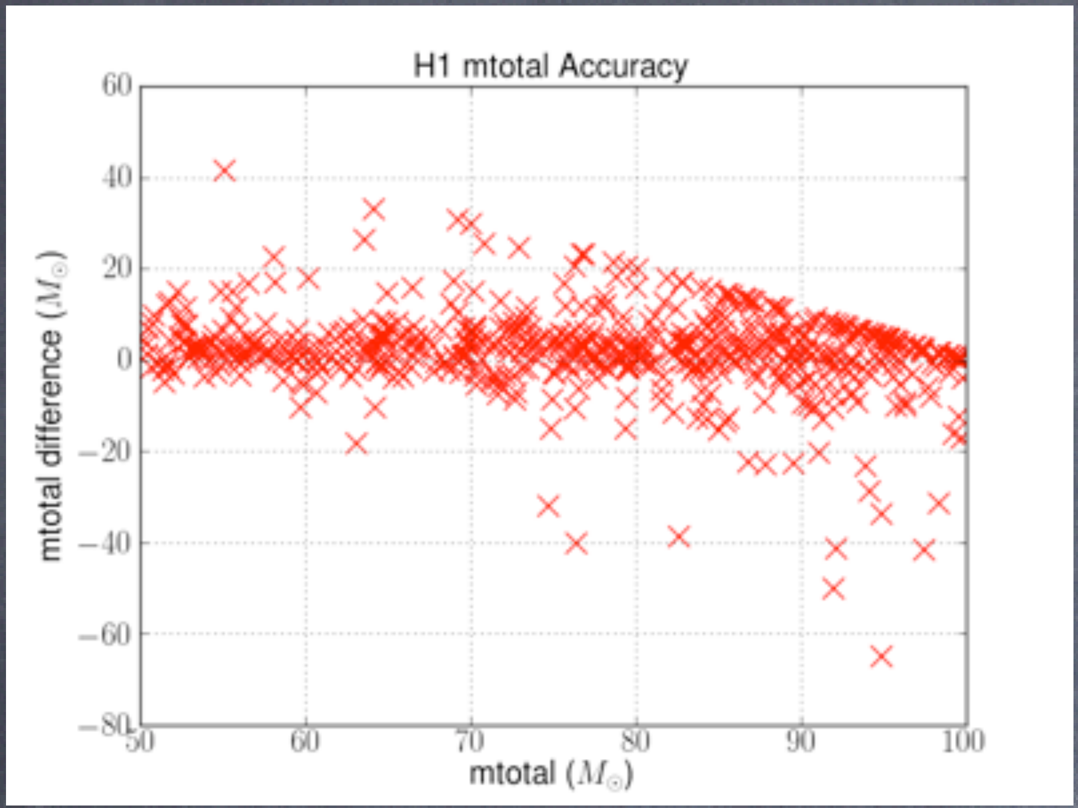
A. Signal is Detected with zero FAR: blue star

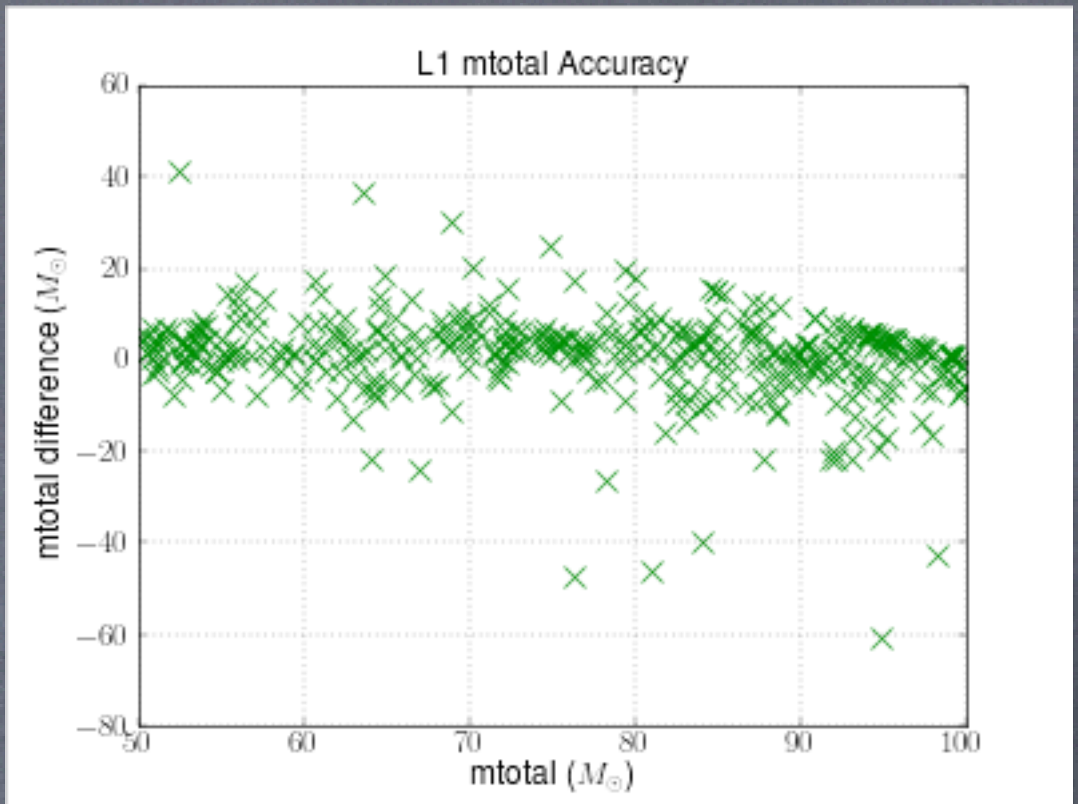
B. Signal is detected with non zero FAR: colored dot

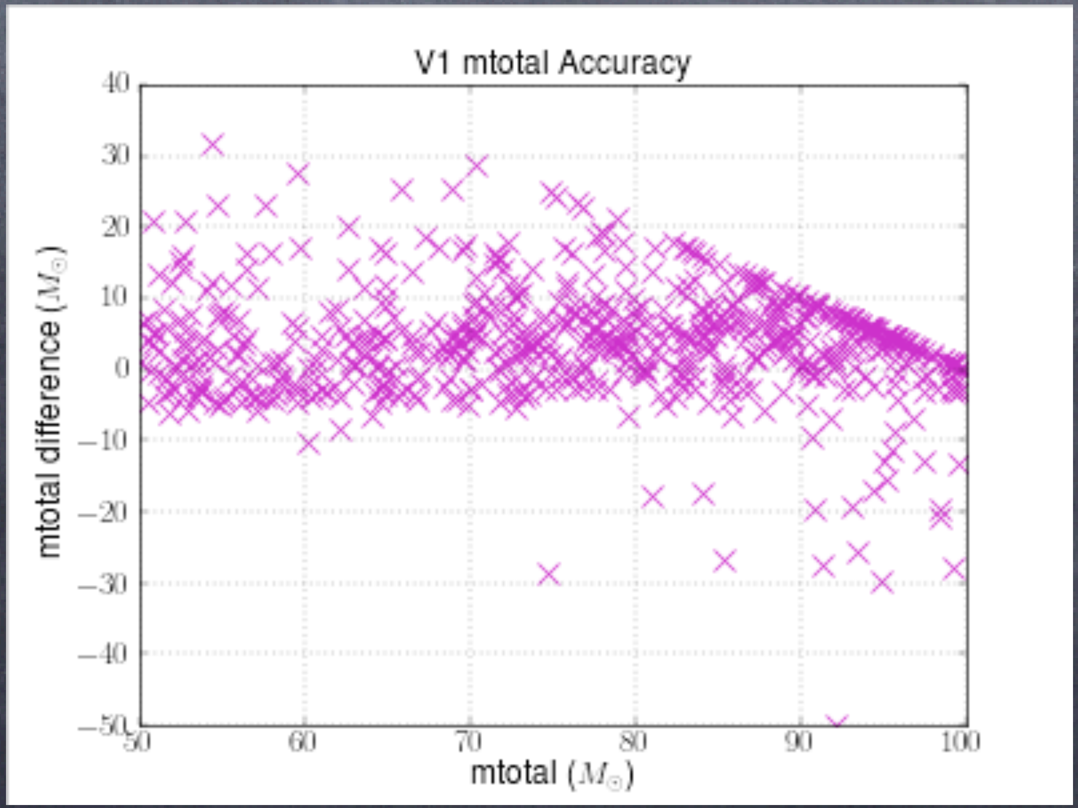
C. SNR is lower than 8, signal not detected: red cross



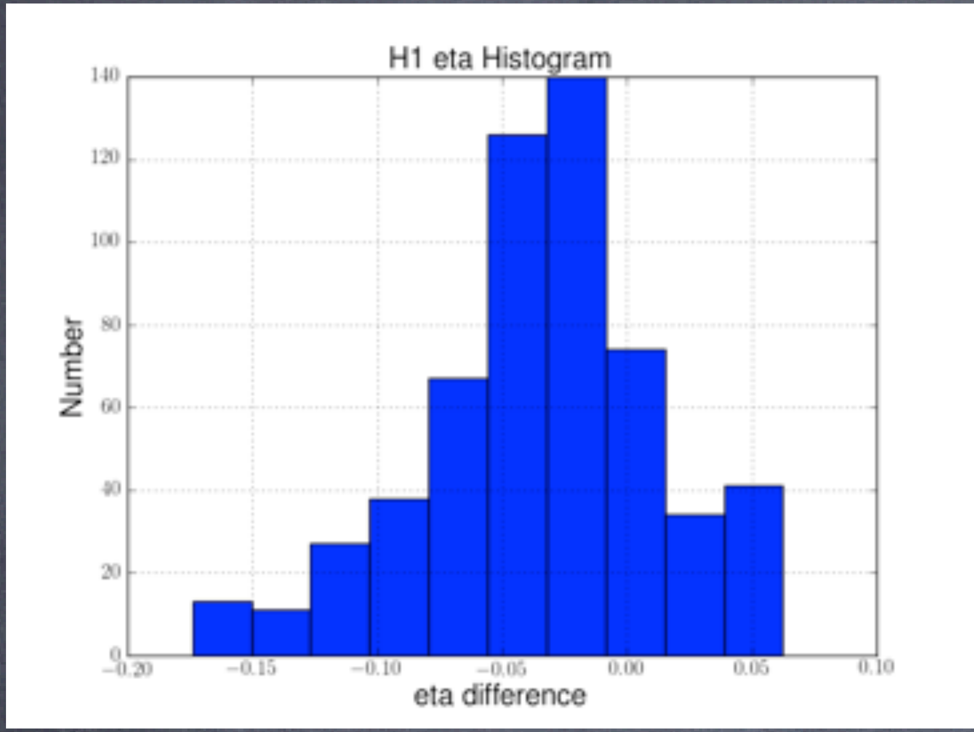
Total Mass Recovery

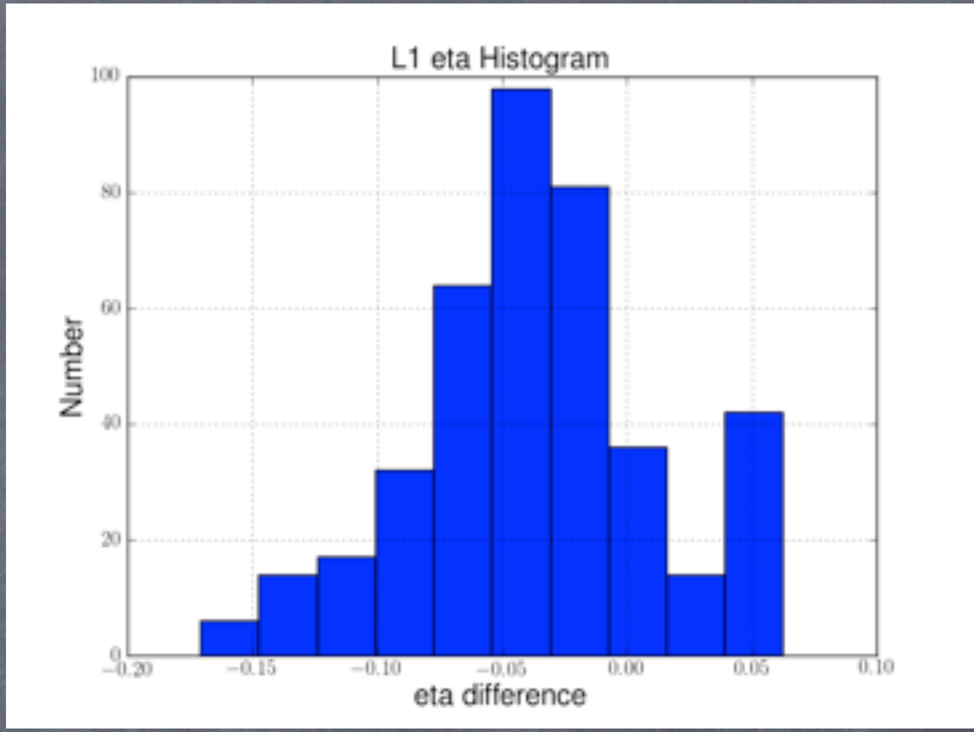


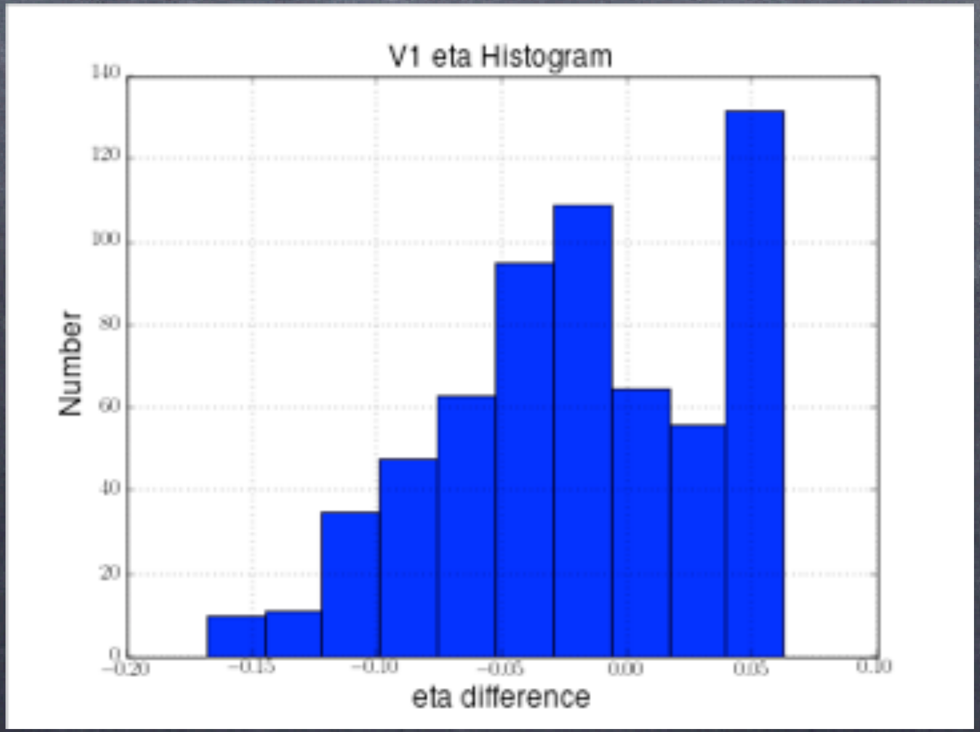




Mass Ratio Recovery







Summary

- A method for building hybrid waveforms including higher modes has been developed.
- Higher modes importance increases with mass, mass ratio and polar angle.
- Significant increment of events-rate may be achieved in the early advanced detector era due to inclusion of higher modes.
- Non-inclusion of higher modes can generate a huge bias in parameter estimation.

Thanks for your attention