The Search for Gravitational Waves

John T. Whelan
john.whelan@astro.rit.edu

Center for Computational Relativity & Gravitation
& School of Mathematical Sciences
Rochester Institute of Technology

ICTS Bangalore Seminar
2014 January 13
LIGO-G1400023-v2

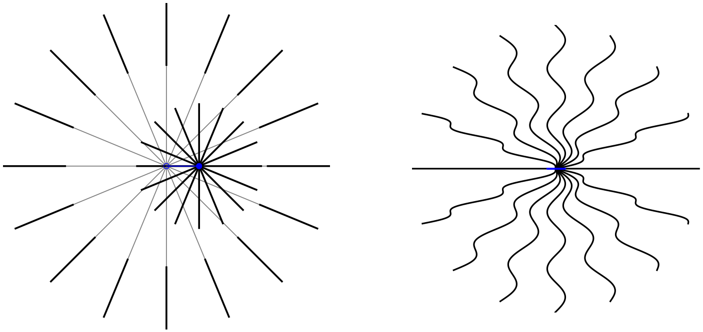
Outline

Outline

1. Gravitational Waves
   - Crash Course in Gravitational Wave Physics
   - Gravitational-Wave Sources & Signals
2. Initial and Advanced Gravitational-Wave Observations
   - Upper Limit Results from Initial Detectors
   - Prospects for Detections with Advanced Detectors
3. Periodic Gravitational Waves from Low-Mass X-Ray Binaries
   - Gravitational Wave Signal and Detection Problem
   - Cross-Correlation Search

Outline

Gravity + Causality = Gravitational Waves



- In Newtonian gravity, force dep on distance btwn objects
- If massive object suddenly moved, grav field at a distance would change instantaneously
- In relativity, no signal can travel faster than light
  $\longrightarrow$ time-dep grav fields must propagate like light waves

Gravity as Geometry

- Minkowski Spacetime (Special Relativity):
  Invariant spacetime interval (all inertial observers agree):

$$ds^2 = -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2$$

$$= \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}^{\rm tr} \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} \eta_{\mu\nu} dx^\mu dx^\nu$$

- General Spacetime:

$$ds^2 = \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\rm tr} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} g_{\mu\nu} dx^\mu dx^\nu$$

Metric tensor $\{g_{\mu\nu}(\{x^\lambda\})\}$ determined by masses via Einstein's equations. (10 non-linear PDEs!)

Gravitational Wave as Metric Perturbation

- For GW propagation & detection, work to 1st order in $h_{\mu\nu} \equiv$ difference btwn actual metric $g_{\mu\nu}$ & flat metric $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

($h_{\mu\nu}$ "small" in weak-field regime, e.g. for GW detection)

- Small coord changes induce "gauge transformation" on $h_{\mu\nu}$ Convenient choice of gauge is transverse-traceless:

$$h_{0\mu} = h_{\mu 0} = 0 \qquad \eta^{\nu\lambda} \frac{\partial h_{\mu\nu}}{\partial x^\lambda} = 0 \qquad \eta^{\mu\nu} h_{\mu\nu} = \delta^{ij} h_{ij} = 0$$

In this gauge:
  - Test particles w/constant coords are freely falling
  - Vacuum Einstein eqns $\Longrightarrow$ wave equation for $\{h_{ij}\}$:

$$\left(-\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \nabla^2 \right) h_{ij} = 0$$

Gravitational Wave Polarization States

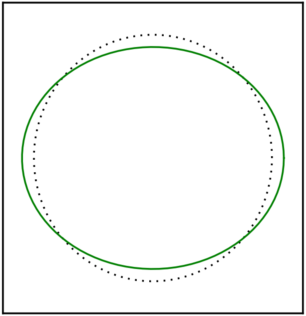
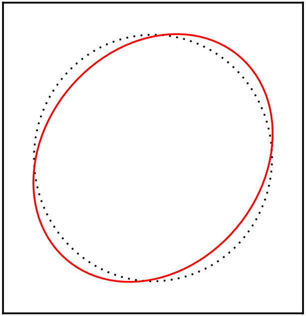
Far from source, GW looks like plane wave prop along $\vec{k}$
TT conditions mean, in convenient basis,

$$\{k_i\} \equiv \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \{h_{ij}\} \equiv \mathbf{h} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where $h_+\left(t - \frac{x^3}{c}\right)$ and $h_\times\left(t - \frac{x^3}{c}\right)$ are components in "plus" and "cross" polarization states

Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

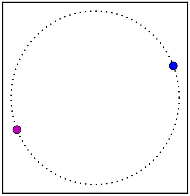
Plus (+) Polarization	Cross (×) Polarization
	

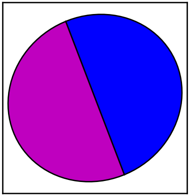
Outline

1. Gravitational Waves
   - Crash Course in Gravitational Wave Physics
   - Gravitational-Wave Sources & Signals
2. Initial and Advanced Gravitational-Wave Observations
   - Upper Limit Results from Initial Detectors
   - Prospects for Detections with Advanced Detectors
3. Periodic Gravitational Waves from Low-Mass X-Ray Binaries
   - Gravitational Wave Signal and Detection Problem
   - Cross-Correlation Search

Generation of Gravitational Waves

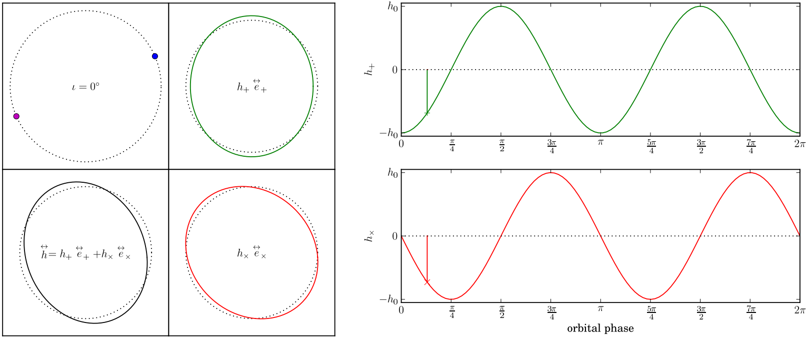
- EM waves generated by moving/oscillating charges
- GW generated by moving/oscillating masses
- Lowest multipole is quadrupole
- Different types of signals:
  - Burst (transient, unmodelled)
  - Stochastic (long-lived, unmodelled)
  - Binary coalescence (transient, modelled)
  - Periodic (long-lived, modelled)





Gravitational Waves from Binary Orbit

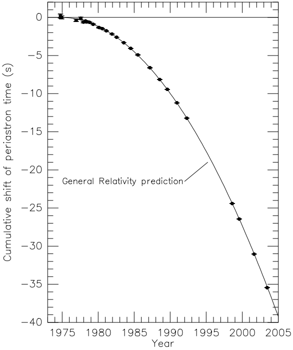
- Orbital motion → oscillating quadrupole moment → GWs



Gravitational Waves from Binary Orbit

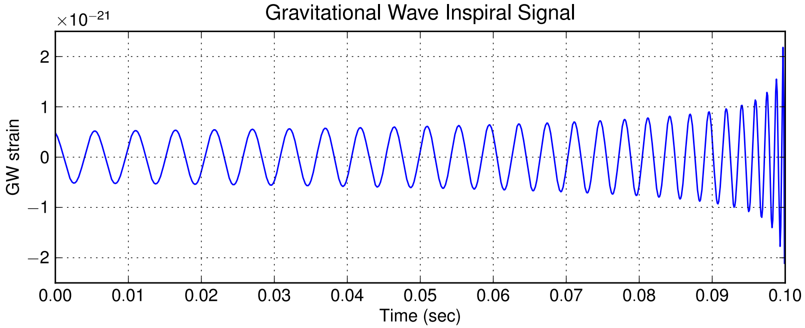
- Orbital motion → oscillating quadrupole moment → GWs
- GW emission removes energy → orbit gets tighter → amplitude & freq increase in "chirp"
- Hulse & Taylor saw this evolution in binary pulsar 1913+16
  1993 Nobel Prize

Weisberg, Nice & Taylor
ApJ **722**, 1030 (2010)



Gravitational Waves from Binary Orbit

- Orbital motion → oscillating quadrupole moment → GWs
- GW emission removes energy → orbit gets tighter → amplitude & freq increase in "chirp"

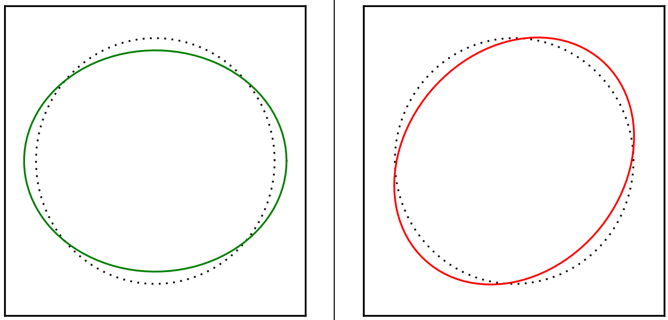


Outline

Effects of Gravitational Wave

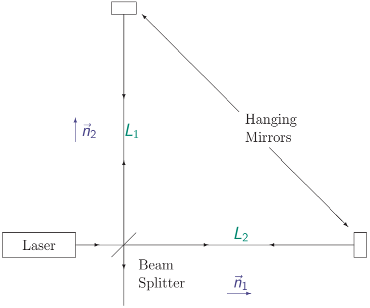
Fluctuating geom changes distances btwn particles in free-fall:

Plus (+) Polarization	Cross (×) Polarization



Measuring GWs w/Laser Interferometry

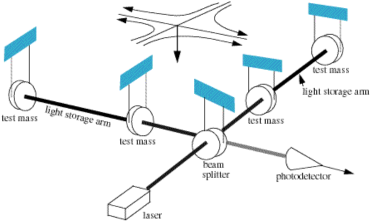
Interferometry: Measure GW-induced distance changes



- Measure small change in
  $$L_1 - L_2 \approx L_0 \frac{h_{11} - h_{22}}{2} \sim L_0 h_+$$
- Plausible signals: $h \lesssim 10^{-20}$
  $\rightarrow$ need L_0 very big!
- For LIGO, $L_0 = 4\,\mathrm{km}$

Measuring GWs w/Laser Interferometry

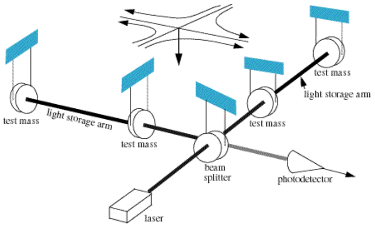
Interferometry: Measure GW-induced distance changes



- Measure small change in $L_1 - L_2 \approx L_0 \frac{h_{11} - h_{22}}{2} \sim L_0 h_+$
- Plausible signals: $h \lesssim 10^{-20}$ $\rightarrow$ need L_0 very big!
- For LIGO, $L_0 = 4\,\text{km}$

Measuring GWs w/Laser Interferometry

Interferometry: Measure GW-induced distance changes



- Measure small change in $L_1 - L_2 \approx L_0 \frac{h_{11} - h_{22}}{2} \sim L_0 h_+$
- Plausible signals: $h \lesssim 10^{-20}$ $\rightarrow$ need L_0 very big!
- For LIGO, $L_0 = 4\,\mathrm{km}$

Note: other detection methods include resonant bars, pulsar timing arrays & planned space-based interferometers

Rogues' Gallery of Ground-Based Interferometers

LIGO Hanford (Wash.)

LIGO Livingston (La.)

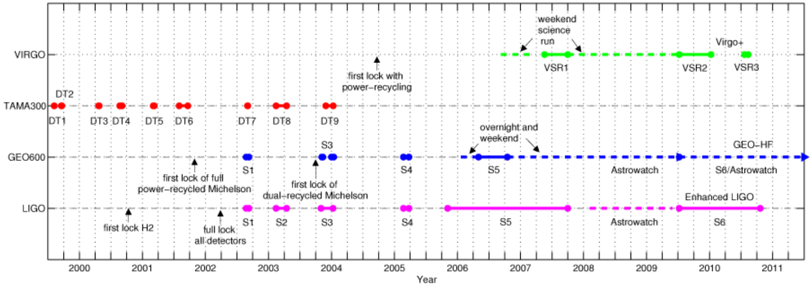
GEO-600 (Germany)

Virgo (Italy)

Initial Gravitational Wave Detector Network

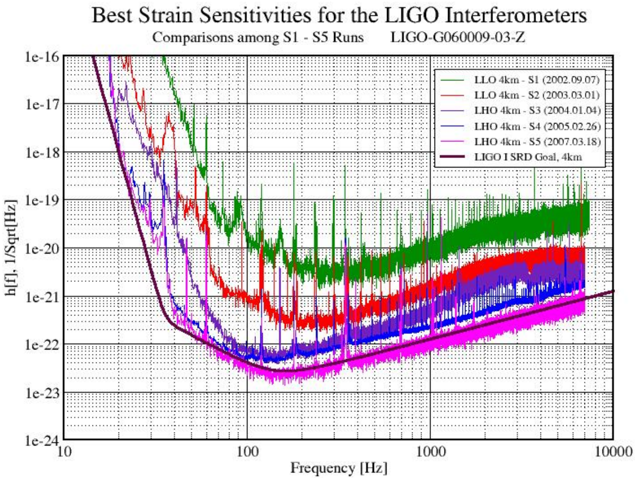
- "1st generation" ground-based interferometric GW detectors (kilometer scale):
  - TAMA 300 (Tokyo, Japan) first online, late 90s; now offline
  - LSC (LIGO Scientific Collaboration) detectors conducting science runs since 2002
    - LIGO Hanford (4km H1 & 2km H2)
    - LIGO Livingston (4km L1)
    - GEO-600 (600m G1)
  - Virgo (3km V1) started science runs in 2007
  - LSC-Virgo long joint runs @ design sensitivity 2005-2010
- LIGO and Virgo being upgraded to 2nd generation "advanced" detectors ($10\times$ improvement in sensitivity)
- GEO-600 remains operational in "astrowatch" mode in case there's a nearby supernova

Initial Gravitational Wave Detector Network



Living Reviews in Relativity **14**, 5 (2011)

Evolution of LIGO Sensitivity S1-S5



Advanced Gravitational Wave Detector Network

Aasi et al (LSC & Virgo) arXiv:1304.0670

- “2nd generation” ground-based interferometric GW detectors:
  - Adv LIGO expected to take science data from 2015
    4km detectors in Livingston, La. & Hanford, Wa.
  - Advanced Virgo should be on comparable timescale
  - KAGRA (cryogenic detector in Kamioka mine, Japan)
    uses 2.5-generation technology
  - Third advanced LIGO detector (4km)
    to be installed in India, taking data 2019+
    Big payoff for sky localization via triangulation
- Planning for 3rd generation already underway:
  - Einstein Telescope in Europe
  - USA 3G plans still under development

A Few Words About Collaborations

- LIGO Scientific Collaboration LSC: hundreds of researchers at dozens of institutions world-wide working on instrument science, data analysis, astrophysics, etc.
  - LSC scientists operate LIGO & GEO detectors
  - and indigo consortium are LSC members
- VIRGO Collaboration operates Virgo (Italy) and includes institutions in Italy, France, Netherlands & Hungary
  - LIGO & Virgo conduct data analysis jointly
- KAGRA: Japanese collaboration constructing detector in Kamioka mine

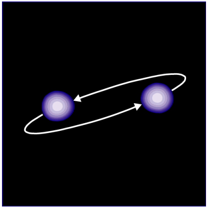
Outline

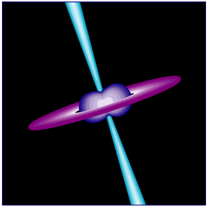
1. Gravitational Waves
   - Crash Course in Gravitational Wave Physics
   - Gravitational-Wave Sources & Signals
2. Initial and Advanced Gravitational-Wave Observations
   - Upper Limit Results from Initial Detectors
   - Prospects for Detections with Advanced Detectors
3. Periodic Gravitational Waves from Low-Mass X-Ray Binaries
   - Gravitational Wave Signal and Detection Problem
   - Cross-Correlation Search

Results of Initial Detector Observations

- 80+ Observational papers from initial LIGO/Virgo/GEO: https://www.lsc-group.phys.uwm.edu/ppcomm/Papers.html
- No detections (although some analyses still trickling out)
- Assortment of null results and upper limits
- As sensitivity improved, some of these results gave new information to complement astronomical observations: "Multi-Messenger Astronomy"
- Some highlights:
  - GW associated w/γ-ray bursts (rule out nearby NS merger)
  - GW from known pulsars (beat spindown limit)
  - Stochastic background of GWs (beat nucleosynthesis limit)

Gravitational Waves from Gamma-Ray Burst Events

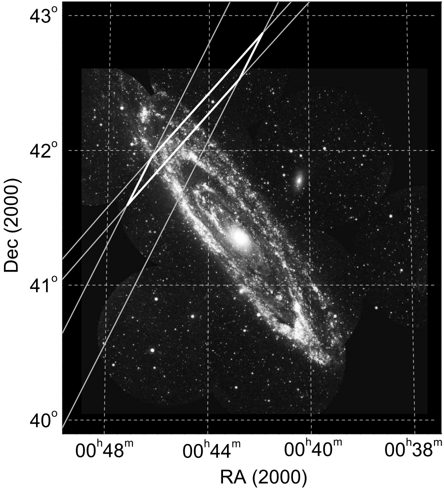




- GRBs are bursts of high energy photons observed by orbiting satellites like Swift and Fermi
- One possible source is the merger of a neutron star w/another neutron star or a black hole
- Search for the GWs emitted by the neutron star as it inspirals; search is “triggered” by the GRB, so can compare data at GRB time to data at other times

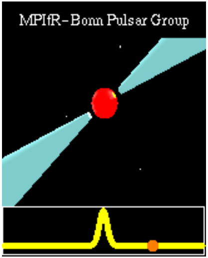
GRB070201

- 2007 Feb 1: short GRB whose error box overlapped spiral arm of M31 (770 kpc away)
- LHO 4 km & 2 km detectors operating & sensitive to inspiral out to 35.7 & 15.3 Mpc
- No GW seen; rule out binary progenitor in M31 w/ $>$ 99% conf
- *ApJ* **681**, 1419 (2008)



Similar result for GRB051103 & M81; *ApJ* **755**, 2 (2012)

Searching for Known Pulsars



- Pulsar=rapidly rotating neutron star emitting radio or X-ray "pulses" as it spins (pulse comes when magnetic pole points at Earth)
- Pulsars spin down mostly due to drag of magnetic field through nebula
- If pulsar has small bump, will emit GWs
- Can search for periodic GW signal modulated by Doppler effect as Earth rotates & orbits Sun
- Parameters like freq, sky position, etc known from pulsar
- Spindown produces indirect upper limit
  - GW emission above limit $\longrightarrow$ more spindown than seen
  - Pulsars w/rapid spindown have "more room" for GW
  - LIGO/Virgo have surpassed spindown limit for Crab & Vela

Crab Pulsar Upper Limit

- Pulsar in Crab Nebula
- Created by SN 1054
- $\sim 2\,\mathrm{kpc}$ away
- $f_{\mathrm{rot}} = 29.7\,\mathrm{Hz}$
- $f_{\mathrm{gw}} = 59.4\,\mathrm{Hz}$

Image credit: Hubble/Chandra

- Initial LIGO (S5 & S6) upper limits beat spindown limit
- Abbott et al (LSC) *ApJL* **683**, L45 (2008)
- Abbott et al (LSC & Virgo) + Bégin et al *ApJ* **713**, 671 (2010)
- No more than 2% of spindown energy loss can be in GW
- Similar limit set on Vela using Virgo data (lower freq)
  Abadie et al (LSC & Virgo) + Buchner et al *ApJ* **737**, 93 (2011)

Searching for a Stochastic Background

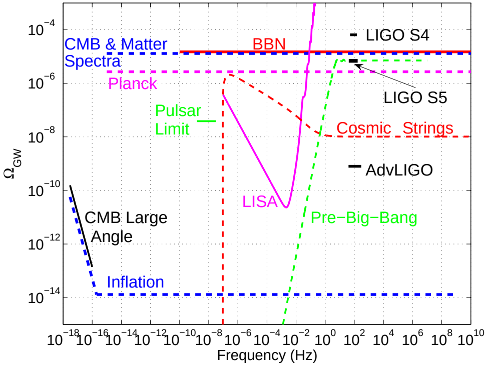
- Expect a random (stochastic) background of GWs left over from Big Bang (like the cosmic microwave background radiation) or from confusion of many faint sources
- Need to find a random signal in random noise!
- Noisy data from GW Detector:
  $x(t) = n(t) + h(t) = n(t) + \overleftrightarrow{h}(t) : \overleftrightarrow{d}$
- Look for correlations between detectors

$$\langle x_1 x_2 \rangle = \overbrace{\cancel{\langle n_1 n_2 \rangle}}^{\text{avgto0}} + \overbrace{\cancel{\langle n_1 h_2 \rangle}}^{\text{avgto0}} + \overbrace{\cancel{\langle h_1 n_2 \rangle}}^{\text{avgto0}} + \langle h_1 h_2 \rangle$$

- Details of expected correlation will depend on sky distribution of background

Allen & Romano *PRD* **59**, 102001 (1999)

Isotropic Stochastic Background Limit



S5 limit $\Omega_{gw}(f) < 6.9 \times 10^{-6} \left(\frac{72\,\mathrm{km/s/Mpc}}{H_0} \right)^2$

[Abbott et al (LSC & Virgo) *Nature* **460**, 990 (2009)]

surpasses indirect limit from Big-Bang Nucleosynthesis

Outline

Improved Sensitivity w/Advanced Detectors

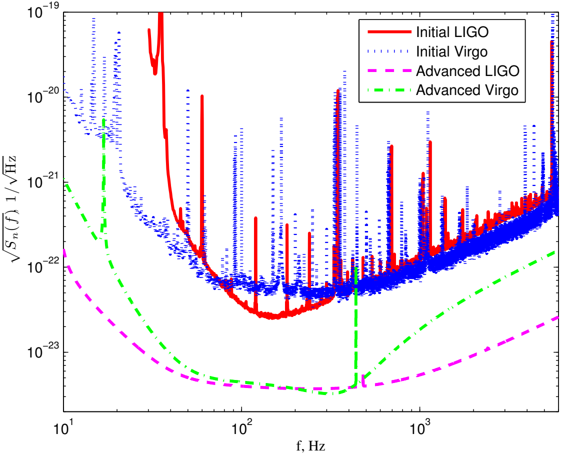
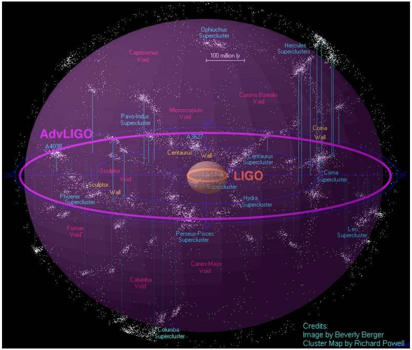


figure from *CQG* **27**, 173001 (2010)

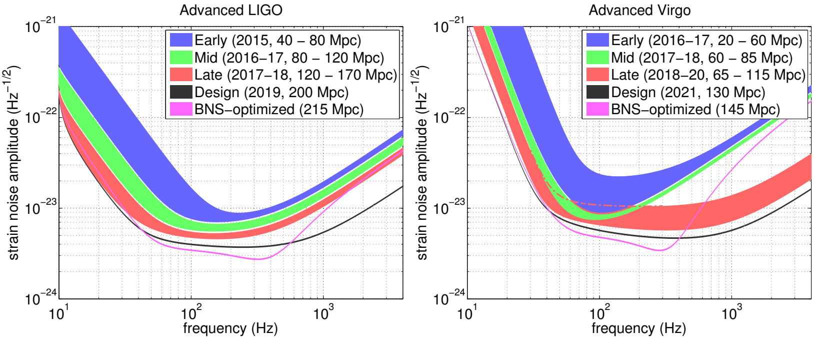
Expected Event Rates w/Advanced Detectors

CQG **27**, 173001 (2010)

- Advanced detectors should see NS binary inspiral up to 400 Mpc & BH binary coalescence up to 2 Gpc away
- $\Longrightarrow$ Expect between a few and hundreds of events/year



Anticipated Evolution of Advanced Detector Sensitivity



Figures from arXiv:1304.0670

Average BNS ranges are $\frac{\text{optimal range}}{2.26}$

Expansion of the GW Detector Network

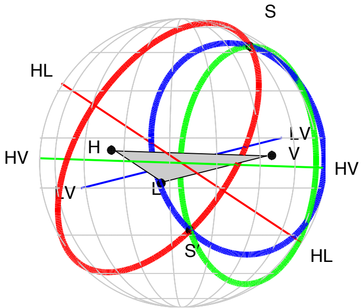
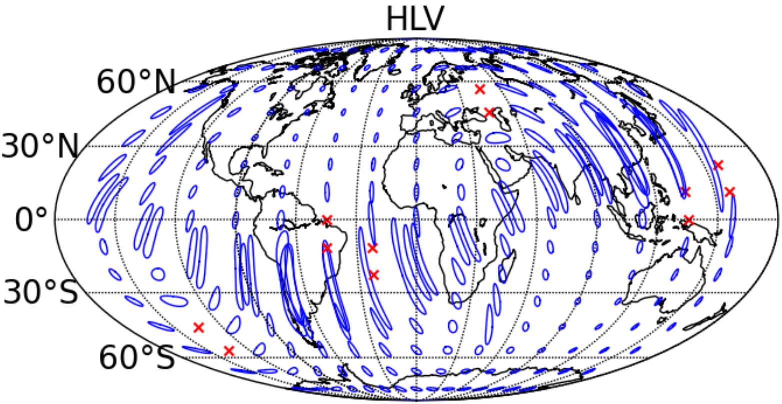


Figure from arXiv:1304.0670

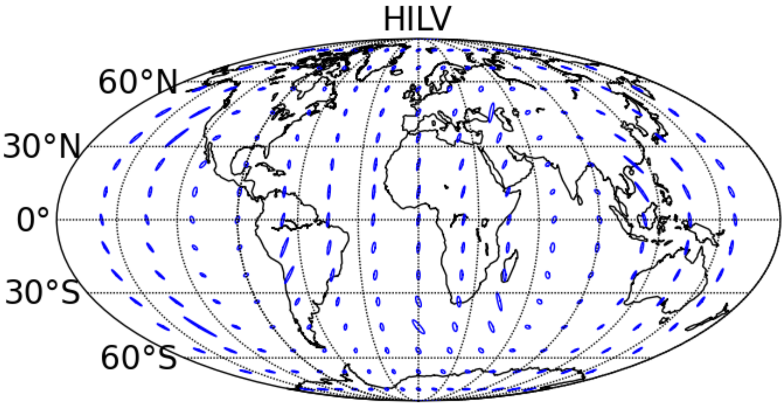
- Sky loc for GW transients can be found by triangulation
- Spread detectors around globe to make this more accurate
- Put 3rd LIGO detector in India to improve sky localization and aid in identification of electromagnetic counterparts

Improvement in Triangulation with LIGO-India



Figures from arXiv:1304.0670

Improvement in Triangulation with LIGO-India



Figures from arXiv:1304.0670

Outline

1. Gravitational Waves
   - Crash Course in Gravitational Wave Physics
   - Gravitational-Wave Sources & Signals
2. Initial and Advanced Gravitational-Wave Observations
   - Upper Limit Results from Initial Detectors
   - Prospects for Detections with Advanced Detectors
3. Periodic Gravitational Waves from Low-Mass X-Ray Binaries
   - Gravitational Wave Signal and Detection Problem
   - Cross-Correlation Search

Outline

Gravitational Waves from Low-Mass X-Ray Binaries

- LMXB: compact object (neutron star or black hole) in binary orbit w/companion star
- If NS, accretion from companion provides "hot spot"; rotating non-axisymmetric NS emits gravitational waves
  Note: waves from NS rotation, NOT binary orbit
- Bildsten *ApJL* **501**, L89 (1998)
  suggested GW spindown may balance accretion spinup
- Torque balance would give ≈ constant GW freq
- Signal at solar system modulated by binary orbit

GW Signal from Periodic Source

GW signal arriving time τ at Solar System Barycenter

$$\overleftrightarrow{h}(\tau) = h_0 \left[\frac{1+\cos^2\iota}{2} \cos\Phi(\tau) \overleftrightarrow{e}_+ + \cos\iota \sin\Phi(\tau) \overleftrightarrow{e}_\times \right]$$

- Amplitude h_0 depends on distance, frequency, ellipticity
- Pol basis $\{\overleftrightarrow{e}_+, \overleftrightarrow{e}_\times\}$ depends on sky position $\{\alpha, \delta\}$ and polarization angle ψ
- Phase evolution e.g., $\Phi(\tau) = \phi_0 + 2\pi \left(f_0\tau + \frac{f_1\tau^2}{2} + \cdots \right)$ (+Doppler mod if NS in binary; note constant Doppler shift OK)
- Signal $h(t) = \overleftrightarrow{h}(\tau(t)) : \overleftrightarrow{d}$ received in detector has $\{\alpha, \delta\}$-dep Doppler shift $\tau(t)$ due to daily & yearly motion of detector
- Divide signal parameters into
    - amplitude params: $\{h_0, \iota, \psi, \phi_0\}$
    - phase params: $\{\alpha, \delta, f_0, f_1, \ldots\}$ + orbital params for LMXB

Coherent Maximum-Likelihood Search ($\mathcal{F}$-statistic)

- Divide signal parameters into
  - amplitude params: $\{h_0, \iota, \psi, \phi_0\}$
  - phase params: $\lambda \equiv \{\alpha, \delta, f_0, f_1, \ldots\}$ + orb params for LMXB
- Jaranowski, Królak, Schutz *PRD* **58**, 063001 (1998) showed signal linear in $\{\mathcal{A}^\mu\}$, fcns of amplitude params

$$h(t) = \mathcal{A}^\mu h_\mu(t) \qquad \text{(assume } \textstyle\sum_{\mu=1}^{4}\text{)}$$

  template waveforms $h_\mu(t)$ depend on phase params λ
- Mismatch of obs data w/signal model quadratic in $\{\mathcal{A}^\mu\}$:

$$\chi^2(\mathcal{A}, \lambda) = \mathcal{A}^\mu \mathcal{M}_{\mu\nu}(\lambda)\mathcal{A}^\nu - 2\mathcal{A}^\mu x_\mu(\lambda) + \chi^2(0, \lambda)$$

- $\mathcal{F}$-stat method uses best-fit amp params $\widehat{\mathcal{A}}^\mu = \mathcal{M}^{\mu\nu}(\lambda)x_\nu(\lambda)$ ($\mathcal{M}^{\mu\nu}$ is inv of $\mathcal{M}_{\mu\nu}$); detection statistic is max log-likelihood

$$\mathcal{F} = -\frac{\chi^2(\widehat{\mathcal{A}}, \lambda) - \chi^2(0, \lambda)}{2} = \frac{1}{2} x_\mu(\lambda)\mathcal{M}^{\mu\nu}(\lambda)x_\nu(\lambda)$$

Bayesian Interpretation ($\mathcal{B}$-statistic)

- Assume λ known; likelihood $P(x|\mathcal{A}) \propto e^{-\chi^2(\mathcal{A})/2}$
- Bayes's theorem says $P(\mathcal{H}|x) = \frac{P(x|\mathcal{H})P(\mathcal{H})}{P(x)}$
- Odds ratio $\frac{P(\mathcal{H}_1|x)}{P(\mathcal{H}_0|x)} = \frac{P(x|\mathcal{H}_1)}{P(x|\mathcal{H}_0)} \frac{P(\mathcal{H}_1)}{P(\mathcal{H}_0)}$; Bayes Factor $\mathcal{B}_{10} = \frac{P(x|\mathcal{H}_1)}{P(x|\mathcal{H}_0)}$
- $\mathcal{H}_1 \equiv$ noise + signal w/some $\mathcal{A}$; $\mathcal{H}_0 \equiv$ noise only
- $\mathcal{F}$-stat is maximized log-likelihood: $\max_{\mathcal{A}} \frac{P(x|\mathcal{A})}{P(x|0)} = e^{\mathcal{F}}$
- But $\mathcal{H}_1$ is composite hypoth. $P(x|\mathcal{H}_1) = \int P(x|\mathcal{A})P(\mathcal{A}|\mathcal{H}_1)d^4\mathcal{A}$
- Don't maximize; marginalize! $\mathcal{B}$-statistic (Prix): $\mathcal{B} = \int \frac{P(x|\mathcal{A})}{P(x|0)} P(\mathcal{A}|\mathcal{H}_1)d^4\mathcal{A} = \int e^{-\frac{1}{2}\mathcal{A}^\mu \mathcal{M}_{\mu\nu}\mathcal{A}^\nu + \mathcal{A}^\mu x_\mu} P(\mathcal{A}|\mathcal{H}_1)d^4\mathcal{A}$
- Prix & Krishnan *CQG* **26**, 204013 (2009): If $P(\mathcal{A}|\mathcal{H}_1)$ uniform in $\{\mathcal{A}^\mu\}$, $\mathcal{B} = e^{\mathcal{F}}$
  Unphysical; implies $P(h_0, \cos\iota, \psi, \phi_0|\mathcal{H}_1) \propto h_0^3(1-\cos^2\iota)^3$
- JTW, Prix, Cutler & Willis arXiv:1311.0065: choice of coords $\{\mathcal{A}^{\breve{\mu}}\}$ aids in approximate eval of $\mathcal{B}$-stat integral w/physical priors

Computational Costs & Phase Parameter Resolution

- If $\lambda \equiv$ {freq, sky pos etc} known, can do most sensitive fully coherent search (correlate all data)
- If some params unknown, have to search over them
- Long coherent observation $\rightarrow$ fine resolution in freq etc $\rightarrow$ need too many templates $\rightarrow$ computationally impossible

$$\text{e.g.} \qquad N_{\text{tmplts}} \sim \frac{1}{\Delta f} \frac{1}{\Delta \dot{f}} \frac{1}{\Delta \text{sky}} \sim T \cdot T^2 \cdot (fT)^2$$

- Most CW searches semi-coherent: deliberately limit coherent integration time & param space resolution to keep number of templates manageable

Brightest LMXB: Scorpius X-1

- Scorpius X-1
  - $1.4 M_\odot$ NS w/ $0.4 M_\odot$ companion
  - unknown params are f_0, $a \sin i$, orbital phase
  - Parameters from Steeghs & Casares *ApJ* **568**, 273 (2002)
    Update by Galloway et al *ApJ* **781**, 14 (2014)
- Promising source for Advanced Detectors
- Initial LSC/Virgo searches for Sco X-1:
  - Coherent $\mathcal{F}$-stat search w/ 6 hr of S2 data
    Abbott et al (LSC) *PRD* **76**, 082001 (2007)
  - Directed stochastic ("radiometer") search (unmodelled)
    Abbott et al (LSC) *PRD* **76**, 082003 (2007)
    Abbott et al (LSC) *PRL* **107**, 271102 (2011)
- Mock data challenge to compare Sco X-1 search methods
  Poster by Messenger et al at GWPAW 2013
- One method: Cross-corr specialized to periodic signal
  Dhurandhar et al *PRD* **77**, 082001 (2008)

Outline

1. Gravitational Waves
   - Crash Course in Gravitational Wave Physics
   - Gravitational-Wave Sources & Signals
2. Initial and Advanced Gravitational-Wave Observations
   - Upper Limit Results from Initial Detectors
   - Prospects for Detections with Advanced Detectors
3. Periodic Gravitational Waves from Low-Mass X-Ray Binaries
   - Gravitational Wave Signal and Detection Problem
   - Cross-Correlation Search

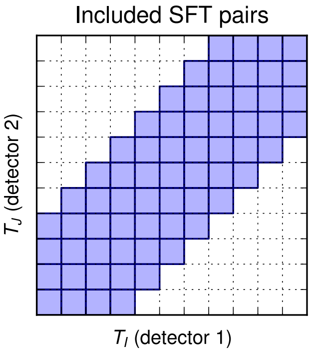
Basics of Cross-Correlation Method

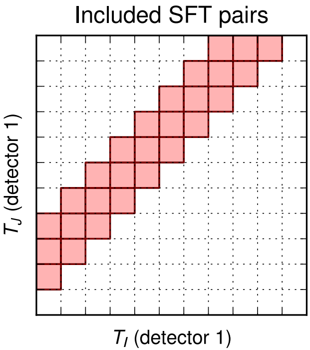
Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

- [BTW, other targets include SN1987A supernova remnant; see Chung, Melatos, Krishnan & JTW *MNRAS* **414**, 2650 (2011)]
- Divide data into segments of length T_{sft} & take "short Fourier transform" (SFT) $\tilde{x}_I(f)$
- Label SFTs by I, J, … and pairs by α, β, …
  - ☞ I & J can be same or different times or detectors
- Construct cross-correlation $\mathcal{Y}_{IJ} = \frac{\tilde{x}_I^*(f_{\tilde{k}_I})\tilde{x}_J(f_{\tilde{k}_J})}{(T_{\text{sft}})^2}$
  - ☞ $f_{\tilde{k}_I} \approx$ signal freq @ time T_I Doppler shifted for detector I
- Use CW signal model to determine expected cross-correlation btwn SFTs & combine pairs into optimal statistic $\rho = \sum_\alpha (u_\alpha \mathcal{Y}_\alpha + u_\alpha^* \mathcal{Y}_\alpha^*)$

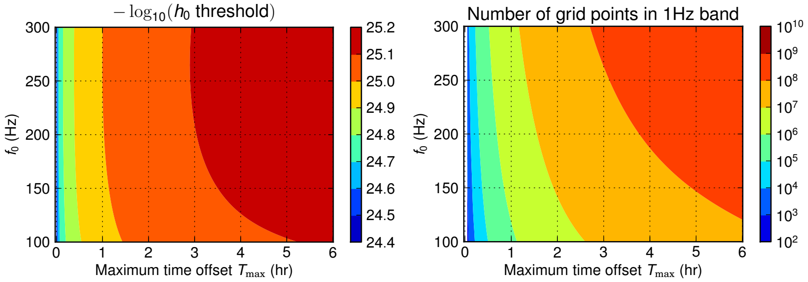
Tuning the Cross-Correlation Search

- Computational considerations limit coherent integration time
- Can make tunable semi-coherent search by restricting which SFT pairs α are included in $\rho = \sum_\alpha (u_\alpha \mathcal{Y}_\alpha + u_\alpha^* \mathcal{Y}_\alpha^*)$
- E.g., only include pairs where $|T_I - T_J| \equiv |T_\alpha| \leq T_{\text{max}}$





Sensitivity and Computational Cost



- Can tune sensitivity vs # of param space points
- Methods paper detailing search projections plus assorted technical issues (param space metric,
  windowing & leakage, marginalization on ι & ψ, etc)
  JTW, Sundaresan, Zhang & Peiris in preparation

Summary

- Gravitational waves: predicted by Einstein
  confirmed indirectly (Hulse-Taylor binary pulsar)
- Advanced GW detectors in USA/Italy/Japan/India
  preparing to make first direct detections
  and initiate gravitational-wave astronomy
- Periodic GWs from rotating neutron stars
  require coherent or semi-coherent search techniques
  depending on knowledge of parameters
- Cross-correlation: one promising method to search for
  GW from Low-Mass X-Ray Binaries like Sco X-1