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Violin Mode Frequency Summary

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1 Introduction

1.1 Purpose and Scope

This is a summary of estimated violin mode frequencies and Qs for all wires in all suspensions.

1.2 References

LLO alog entries [4470](#), [4472](#), [5097](#), [5280](#)

G. Cagnoli et al., Phys. Lett. A 255 (1999), p230

[T0900415](#): Upper Limit to Suspension Thermal Noise from LIGO 1 and Implications for Wire Suspensions in Advanced LIGO

T070101: [Dissipation Dilution](#)

T080096: [Wire Attachment Points and Flexure Corrections](#)

LIGO-T0900435: [HAM Small Triple Suspension \(HSTS\) Final Design Document](#)

LIGO-D020700: [HSTS Overall Assembly](#)

Cumming et al., Design and development of the advanced LIGO monolithic fused silica suspension, Class. Quantum Grav. 29 (2012) 035003.

T1200418: [LLO MC2 Violin Mode Q](#)

1.3 Version history

10/24/13: -v1.

3/6/14: -v2. Added damping theory, calculated Qs and associated parameters. Used updated models for BS, TMTS First Article and Production, HAUX and HTTS.

9/10/14: -v3 with renumbering of equations and fix to Eq. 1.11 (r should have been d).

3/9/15: -v4 with correction of fibre modes (had used neck thickness rather than centre section thickness, so modes too low by about $\sqrt{2}$), and update of model from 20120601TMproductionTM to .

2 Theory

The following theory is borrowed from T1200418 (LLO MC2 Violin Mode Q).

2.1 Mode Frequencies

Per Eq. 2.67 of Fletcher and Rossing, to second order in small quantities, the frequency of a violin mode is

$$f_n = n f_1^0 \left[1 + b + b^2 + \frac{n^2}{8} b^2 \right] \quad (1.1)$$

(Their β has been renamed b to avoid confusion with the thermodynamic material property β used below.)

Here $n = 1, 2, 3, \dots$ is the mode number, and

$$f_1^0 = \frac{1}{2L} \sqrt{\frac{T}{\rho_L}}, \quad (1.2)$$

is the frequency of a wire without bending stiffness but the same length L , tension T and mass per length ρ_L .

The dimensionless quantity b (formerly β) is

$$b = \frac{2K}{L} \sqrt{\frac{YA}{T}} \quad (1.3)$$

where K is the radius of gyration of the wire, Y is the Young's Modulus, and A is the cross-sectional area, but it is closely related to the usual flexure length, defined (T080096) as

$$a = \sqrt{\frac{YI}{T}} = \frac{bL}{2} \quad (1.4)$$

Here, I is the second moment of area of the wire in the bending direction, equal to $\pi r^4 / 4$ in any direction for a wire of circular cross-section. (The moments of area of the bottom wires in the longitudinal and transverse directions are called M31 and M32 in the model code.)

It is convenient and instructive to put the above formula in terms of a :

$$f_n = \frac{n}{2L \left(1 - \frac{2a}{L} - \frac{n^2 \pi^2 a^2}{2L^2} \right)} \sqrt{\frac{T}{\rho_L}} \quad (1.5)$$

This makes it obvious that to first order in $a/L = b/2$ (≈ 0.00248 for the HSTS) the effect is simply to shorten the wire by one flexure length a at each end for all harmonics. This is consistent with the fact that a wire of non-zero bending stiffness does not bend sharply at the clamp point but along a curve that for most purposes gives the effect of a pivot a away from the attachment point. In addition, there is also a tiny shortening $n^2 \pi^2 a^2 / 2L$ second order in both a/L and mode number n .

The plain b^2 term disappears because it turns out to be an artifact of doing the expansion in the numerator rather than the denominator, i.e.,

$$1/(1-b) = 1 + b + b^2 + O(b^3) \quad (1.6)$$

In a practical suspension with multiple wires which may not be exactly vertical, the tension is given by

$$T = \frac{mg}{n_w \cos \theta} \quad (1.7)$$

where m is the net mass supported by a set of wires, g is local gravity (taken to be 9.81 m/s^2), n_w is the number of wires sharing the load, and θ is the angle of the wires to the vertical. The cross-sectional area and moment of area are

$$A = !r^2 \quad (1.8)$$

$$I = \frac{!r^4}{4} \quad (1.9)$$

where r is the radius.

2.2 Damping

The Q of the violin mode depends on the material damping factor ϕ and the dissipation dilution factor D . The damping factor is modeled as a frequency-independent structural term $\phi_{struct} = 2 \times 10^{-4}$ (Cagnoli et al. 1999; also T0900415) plus a thermoelastic term:

$$\phi(f) = \phi_{struct} + \phi_{thermo} = \phi_{struct} + \frac{2\pi f \tau \Delta}{1 + (2\pi f \tau)^2} \quad (1.10)$$

where (e.g., Cumming et al.)

$$\tau = 0.0732 C d^2 \rho_v / \kappa \quad (1.11)$$

is a time constant for heat diffusion across the wire (C is heat capacity, κ is heat conductivity and $d = 2r$ is diameter), and

$$\Delta = \frac{Y T_w}{\rho_v C} \left(\alpha - \frac{\sigma \beta}{Y} \right)^2 \quad (1.12)$$

is twice the thermoelastic damping at the peak frequency $1/2\pi\tau$ (T_w is temperature, α is linear expansion, $\beta = \frac{1}{Y} \frac{dY}{dT_w}$, and $\sigma = T/A$ is stress). The magic number 0.0732 is a geometrical factor for wires of cylindrical shape, equal to $1/4!^2$ where $!$ is the first zero of the derivative of the first Bessel function of the first kind:

$$\left. \frac{dJ_1(x)}{dx} \right|_{x=\xi} = \frac{1}{2} (J_0(\xi) - J_2(\xi)) = 0 \quad (1.13)$$

Because the energy in a violin mode is stored in second-order stress changes of the elastic material, dissipation dilution is applicable (T070101) and the quality factor Q is not just $1/\phi$ for the material, but D/ϕ where

$$D = \frac{2a}{L \#} 1 + \frac{n^2 !^2 a \%}{2L \&} \quad (1.14)$$

Again there is a higher order term proportional to n^2 , which turns out to be significant.

3 Results

The data used to calculate the modes in the following tables were extracted from the indicated Mathematica model/case pairs (specified by location in the SUS SVN relative to

^/trunk/Common/MathematicaModels/). In each case, the notebook used to generate the numbers lives in the mark.barton calculation directory and has a name like XXXXXXXXViolinModes.nb. So for example, the Quad Monolithic notebook is

```
^/trunk/Common/MathematicaModels/QuadLite2Lateral/mark.barton/20120601TMproductionTM/mark.barton/ASUS4XLLateralModelCalcViolinModes.nb
```

Note that in the above it's not a mistake that there two levels called mark.barton. The first is a directory of related cases (the production quad model really should be moved out of that personal directory to the top level and will be the next time there's a pretext to do an update), and the second is a directory of customized calculations for a particular case.

LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

3.1 Monolithic QUAD

For main chain of standard QUAD (ITM or ETM).

QUAD fibre: QuadLite2Lateral/mark.barton/20140304TMproductionTM												
Top Mass Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
123.3	2	0.9344	7800.	2.12E11	0.55	449.	4.534	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	336.	n=2	673.	n=3	1012.	n=4	1354.				
Q	n=1	1.46E5	n=2	1.9E5	n=3	1.872E5	n=4	1.663E5				
UIM Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
101.3	4	0.891	7800.	2.12E11	0.3555	308.4	2.752	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	495.5	n=2	992.1	n=3	1491.	n=4	1994.				
Q	n=1	1.125E5	n=2	1.586E5	n=3	1.657E5	n=4	1.538E5				
PUM Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
79.26	4	0.9939	7800.	2.12E11	0.3175	339.9	2.923	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	421.3	n=2	843.6	n=3	1268.	n=4	1695.				
Q	n=1	97700.	n=2	1.458E5	n=3	1.609E5	n=4	1.565E5				
Fibres												

A more accurate calculation of the fibre f's and Q's can be found in Cumming et al., Class. Quantum Grav. 29 (2012) 035003												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
39.63	4	1.	2200.	7.27E10	0.2 (centre), 0.4 (neck)	594.6	3.859	6.191E-8	3.9E-7	0.000152	772.	1.38
f (Hz)	n=1	505.3	n=2	1011.	n=3	1518.	n=4	2028.				
Q	n=1	1.205E9	n=2	1.103E9	n=3	9.658E8	n=4	8.226E8				

3.2 Rehung QUAD

For the ITMy at LHO, rehung on wires after the fibre break incident; slightly different from pilot hang.

QUAD rehang: QuadLite2Lateral/mark.barton/20120831TMproductionTMrehang												
Top Mass Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
123.9	2	0.9344	7800.	2.12E11	0.55	449.2	4.522	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	336.7	n=2	674.5	n=3	1014.	n=4	1357.				
Q	n=1	1.458E5	n=2	1.901E5	n=3	1.875E5	n=4	1.667E5				
UIM Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
101.9	4	0.8912	7800.	2.12E11	0.3555	308.6	2.744	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	496.7	n=2	994.6	n=3	1495.	n=4	1999.				

Q	n=1	1.119E5	n=2	1.581E5	n=3	1.654E5	n=4	1.538E5				
PUM Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
79.95	4	0.9953	7800.	2.12E11	0.3175	330.8	2.917	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	434.7	n=2	870.4	n=3	1308.	n=4	1749.				
Q	n=1	97550.	n=2	1.448E5	n=3	1.59E5	n=4	1.538E5				
Wires (yes, wires)												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
39.56	4	1.	7800.	2.12E11	0.2285	604.3	2.163	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	229.5	n=2	459.1	n=3	688.9	n=4	918.9				
Q	n=1	1.03E5	n=2	1.49E5	n=3	1.874E5	n=4	2.117E5				

3.3 Pilot Hang QUAD

For the new-style metal build with a wire loop, and also various wire-hung pilot optics.

QUAD rehang: QuadLite2Lateral/20120831TMproductionTMrehang												
Top Mass Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
124.3	2	0.9331	7800.	2.12E11	0.55	445.	4.507	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	340.6	n=2	682.3	n=3	1026.	n=4	1373.				
Q	n=1	1.465E5	n=2	1.903E5	n=3	1.871E5	n=4	1.66E5				

UIM Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
102.3	4	0.8929	7800.	2.12E11	0.355	311.	2.739	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	493.9	n=2	989.	n=3	1486.	n=4	1987.				
Q	n=1	1.137E5	n=2	1.607E5	n=3	1.684E5	n=4	1.567E5				
PUM Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
79.99	4	0.9943	7800.	2.12E11	0.3175	339.	2.912	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	424.3	n=2	849.5	n=3	1277.	n=4	1707.				
Q	n=1	98330.	n=2	1.467E5	n=3	1.618E5	n=4	1.573E5				
Wires (yes, wires)												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
39.6	4	1.	7800.	2.12E11	0.2285	604.	2.162	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	229.7	n=2	459.5	n=3	689.5	n=4	919.8				
Q	n=1	3.943E8	n=2	5.718E8	n=3	6.407E8	n=4	6.479E8				

3.4 QUAD CP

For the Compensation Plate (reaction chain of ITM QUAD).

QUAD CP: QuadLite2Lateral/mark.barton/20120831TMproductionCP

Top Mass Wires

m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
122.8	2	0.9344	7800.	2.12E11	0.55	449.2	4.543	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	335.2	n=2	671.4	n=3	1010.	n=4	1351.				
Q	n=1	1.448E5	n=2	1.887E5	n=3	1.861E5	n=4	1.654E5				
UIM Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
100.8	4	0.8912	7800.	2.12E11	0.355	308.6	2.752	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	494.6	n=2	990.5	n=3	1489.	n=4	1990.				
Q	n=1	1.111E5	n=2	1.57E5	n=3	1.643E5	n=4	1.528E5				
PRM Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
79.27	4	0.994	7800.	2.12E11	0.3175	330.8	2.924	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	433.1	n=2	867.3	n=3	1304.	n=4	1743.				
Q	n=1	96980.	n=2	1.44E5	n=3	1.58E5	n=4	1.528E5				
CP Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
20.04	4	1.	7800.	2.12E11	0.2285	604.1	3.039	0.0001	1.2E-5	-0.00025	460.	49.

f (Hz)	n=1	163.9	n=2	327.9	n=3	492.2	n=4	656.8				
Q	n=1	69620.	n=2	86140.	n=3	1.025E5	n=4	1.117E5				

3.5 ERM

For the End Reaction Mass (reaction chain of ETM QUAD).

QUAD ERM: QuadLite2Lateral/mark.barton/20120831TMproductionERM												
Top Mass Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
122.7	2	0.9344	7800.	2.12E11	0.55	449.2	4.545	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	335.	n=2	671.1	n=3	1009.	n=4	1351.				
Q	n=1	1.446E5	n=2	1.886E5	n=3	1.859E5	n=4	1.653E5				
UIM Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
100.7	4	0.8912	7800.	2.12E11	0.355	308.6	2.753	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	494.3	n=2	989.8	n=3	1488.	n=4	1989.				
Q	n=1	1.11E5	n=2	1.568E5	n=3	1.641E5	n=4	1.526E5				
PRM Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
79.13	4	0.994	7800.	2.12E11	0.3175	330.8	2.926	0.0001	1.2E-	-0.00025	460.	49.

									5			
f (Hz)	n=1	432.8	n=2	866.6	n=3	1302.	n=4	1741.				
Q	n=1	96840.	n=2	1.437E5	n=3	1.577E5	n=4	1.526E5				
ERM Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
25.99	4	1.	7800.	2.12E11	0.2285	604.1	2.668	0.0001	1.2E-5	-0.00025	460.	49.
f (Hz)	n=1	186.4	n=2	373.	n=3	559.7	n=4	746.8				
Q	n=1	80280.	n=2	1.062E5	n=3	1.295E5	n=4	1.434E5				

3.6 BS

Note the BS has a horizontal wedge, so in reality the left and right wires will have slightly different tensions and thus frequencies.

BSFM: TripleLite2/mark.barton/20120120bsNW												
Upper Mass Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
40.42	2	0.9962	7800.	2.119E11	0.3125	612.	2.814	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	237.8	n=2	475.8	n=3	714.	n=4	952.7				
Q	n=1	1.142E5	n=2	1.689E5	n=3	1.948E5	n=4	2.001E5				
Intermediate Mass Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
27.79	4	0.9747	7800.	2.119E11	0.2	596.	1.902	0.0002	1.2E-5	-0.00025	486.	49.

f (Hz)	n=1	225.5	n=2	451.1	n=3	676.8	n=4	902.7				
Q	n=1	1.032E5	n=2	1.336E5	n=3	1.611E5	n=4	1.775E5				
Optic Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
14.21	4	1.	7800.	2.119E11	0.125	500.	1.08	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	303.	n=2	606.1	n=3	909.2	n=4	1212.				
Q	n=1	1.589E5	n=2	1.505E5	n=3	1.681E5	n=4	1.855E5				

3.7 HLTS

HLTS: TripleLite2/mark.barton/20120120hlts												
Upper Mass Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
36.46	2	0.9651	7800.	2.119E11	0.3048	202.5	2.687	0.0002	1.2E-5	-0.00025	486.	\
f (Hz)	n=1	724.2	n=2	1452.	n=3	2188.	n=4	2936.				
Q	n=1	72420.	n=2	85170.	n=3	77780.	n=4	65220.				
Intermediate Mass Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
24.37	4	0.9433	7800.	2.119E11	0.1702	203.6	1.4	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	744.2	n=2	1489.	n=3	2237.	n=4	2987.				
Q	n=1	67510.	n=2	96720.	n=3	1.071E5	n=4	1.053E5				

Optic Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
12.14	4	1.	7800.	2.119E11	0.1346	255.	1.355	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	513.3	n=2	1027.	n=3	1542.	n=4	2057.				
Q	n=1	63750.	n=2	81280.	n=3	94420.	n=4	99490.				

3.8 HSTS

Note that some HSTS have a horizontal wedge, so in reality the left and right wires will have slightly different tensions and thus frequencies.

HSTS: TripleLite2/mark.barton/20120120hsts												
Upper Mass Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
8.987	2	0.997	7800.	2.119E11	0.178	295.	1.938	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	409.9	n=2	820.4	n=3	1232.	n=4	1645.				
Q	n=1	58340.	n=2	80280.	n=3	91680.	n=4	93430.				
Intermediate Mass Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
5.872	4	0.9745	7800.	2.119E11	0.1	167.	1.034	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	744.6	n=2	1490.	n=3	2237.	n=4	2987.				
Q	n=1	51730.	n=2	59200.	n=3	66270.	n=4	68430.				
Optic Wires												

m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
2.889	4	1.	7800.	2.119E11	0.0597	220.	0.5462	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	650.6	n=2	1301.	n=3	1952.	n=4	2604.				
Q	n=1	2.012E5	n=2	1.357E5	n=3	1.196E5	n=4	1.149E5				

3.9 OMCS

OMCS: DualLite2/20130612OMCSmetal4904												
Upper Mass Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
10.02	2	1.	7800.	2.12E11	0.178	249.6	1.843	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	511.8	n=2	1024.	n=3	1539.	n=4	2056.				
Q	n=1	57710.	n=2	81240.	n=3	91050.	n=4	90640.				
OMC Bench Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
7.124	4	1.	7800.	2.12E11	0.1005	220.	0.986	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	609.4	n=2	1219.	n=3	1830.	n=4	2441.				
Q	n=1	73380.	n=2	78070.	n=3	88000.	n=4	93680.				

3.10 TMTS First Article

TMTS: DualLite2DBLateral/20131224TMTS_FirstArticle												
Upper Mass Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
123.	2	0.933	7800.	2.119E11	0.55	456.2	4.529	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	330.4	n=2	661.7	n=3	995.1	n=4	1331.				
Q	n=1	1.157E5	n=2	1.325E5	n=3	1.218E5	n=4	1.037E5				
TransMon Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
78.6	4	0.9891	7800.	2.119E11	0.55	801.9	8.743	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	103.4	n=2	207.2	n=3	311.8	n=4	417.5				
Q	n=1	59790.	n=2	79430.	n=3	80120.	n=4	72440.				

3.11 TMTS Production

TMTS: DualLite2DBLateral/20131224TMTS_Production												
Upper Mass Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
123.9	2	0.933	7800.	2.119E11	0.55	456.2	4.511	0.0002	1.2E-5	-0.00025	486.	49.

f (Hz)	n=1	331.7	n=2	664.3	n=3	998.9	n=4	1336.				
Q	n=1	1.164E5	n=2	1.332E5	n=3	1.225E5	n=4	1.043E5				
TransMon Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
79.86	4	0.9889	7800.	2.119E11	0.55	801.9	8.671	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	104.2	n=2	208.9	n=3	314.2	n=4	420.7				
Q	n=1	60580.	n=2	80520.	n=3	81260.	n=4	73500.				

3.12 HAUX

Note this model has not had much validation against installed suspensions.

HAUX: TwoWireSimpleBlades/20131231HAUXdamp												
Optic Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
0.3738	2	0.986	7800.	1.65E11	0.0762	253.8	1.705	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	228.3	n=2	456.9	n=3	686.1	n=4	916.3				
Q	n=1	1.316E5	n=2	79260.	n=3	56340.	n=4	43490.				

3.13 HTTS (Tip-Tilt)

Note: to match observed pendulum frequencies, this model has fitted values of the MOIs which were very different (an order of magnitude) from those in the source document P1100090. Very likely the mass is off as well, in which case the tension and VM frequencies could be wrong too.

HTTS: TwoWireSimpleBlades/20140123HTTSdamp												
Optic Wires												
m (kg)	cos	nw	ρ_L (kg/m ³)	Y (Pa)	r (mm)	l (mm)	a (mm)	ϕ	α (K ⁻¹)	β (K ⁻¹)	C (J/(kg.K))	κ (W/(m.K))
0.0885	2	0.9867	7800.	1.65E11	0.0635	141.9	2.447	0.0002	1.2E-5	-0.00025	486.	49.
f (Hz)	n=1	243.9	n=2	490.1	n=3	740.7	n=4	998.4				
Q	n=1	58300.	n=2	31380.	n=3	18720.	n=4	12130.				