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<b>Re-assessing HAM-ISI Performance Noise Budget Model for aLIGO</b>		
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# 1 Introduction

With the addition of HEPI, as well as a few extra control paths which are possible from the addition of Stage 0 L4Cs, it is necessary to re-calculate the performance of the platform for noise-budgeting techniques originally developed in eLIGO. This document describes the details of this recalculation.

## 2 eLIGO Model

*Section 2 is an excerpt from Chapter 5 of the author's thesis, P1000103, only slightly updated to follow newer nomenclature and to be more clear. We remind the reader that in eLIGO, the prototype HAM-ISIs were installed into HAM6, whose support stage was "rigidly" secured to the ground, i.e. no external, hydraulic pre-isolation.*

The model for the X (and Y) degree of freedom optical table displacement, "Stage 1 (ST1)," or  $x_{ST1}$ , of the eLIGO HAM-ISI is shown in Figure 2. Ground motion,  $x_{gnd}$ , is suppressed by the transmission of the passive isolation system, called the "plant,"  $P_x$ . Residual motion of Stage 1 is sensed by the displacement and inertial sensors. The displacement sensor signal measures the relative displacement between both Stage 1 and the support stage, "Stage 0 (ST0)," or  $x_{ST0}$ , motion which, in the case of eLIGO, we consider to be equivalent to  $x_{gnd}$ . This relative signal is corrected for motion of the support stage / ground by blending in a signal from an inertial sensor signal mounted on the ground, high-passed with a filter set  $F_{gnd}^{SC}$ , a technique known as "sensor correction (SC)." The inertial-sensor-corrected displacement sensor signal is then low-passed with  $F_x^{LP}$ . The inertial sensor signal is high-passed with a filter  $F_x^{HP}$ , complementary to  $F_x^{LP}$ . From there, the signals are added to form the super sensor. A final control filter  $K_x$  shapes the super sensor signal into a force which is fed back to the actuators, further reducing the motion of the platform.

In addition to ground motion,  $x_{gnd}$ , we include several noise sources which we measure or model. For translational degrees of freedom (X and Y), these sources are sensor noise from each of the sensors: the on-board GS13 inertial sensors,  $n_{GS13}$ , the displacement sensors  $n_{CPS}$ , and ground inertial sensor  $n_{STS}$ ; and residual tilt of the platform,  $ry_{ST1}$ . We will find that these sensor noises are those limiting the platform performance. Other, non-limiting, noise sources not considered include ground tilt coupling into the ground inertial sensor translation signal, actuator noise, ADC/DAC noise, and non-linear coupling.

The frequency response from the sensors and output from actuators have been compensated and given the appropriate gain to transfer any information in the colocated basis (e.g. sensor noise) into the cartesian basis. The raw sensor noise (Figure 1) for the on-board sensors (CPSs and GS13s) that make up a given signal in the cartesian basis (three per degree of freedom) are assumed to be independent and therefore added in quadrature. The noise for the STS is already in the cartesian basis, and is assumed to be similar for all three degrees of freedom.

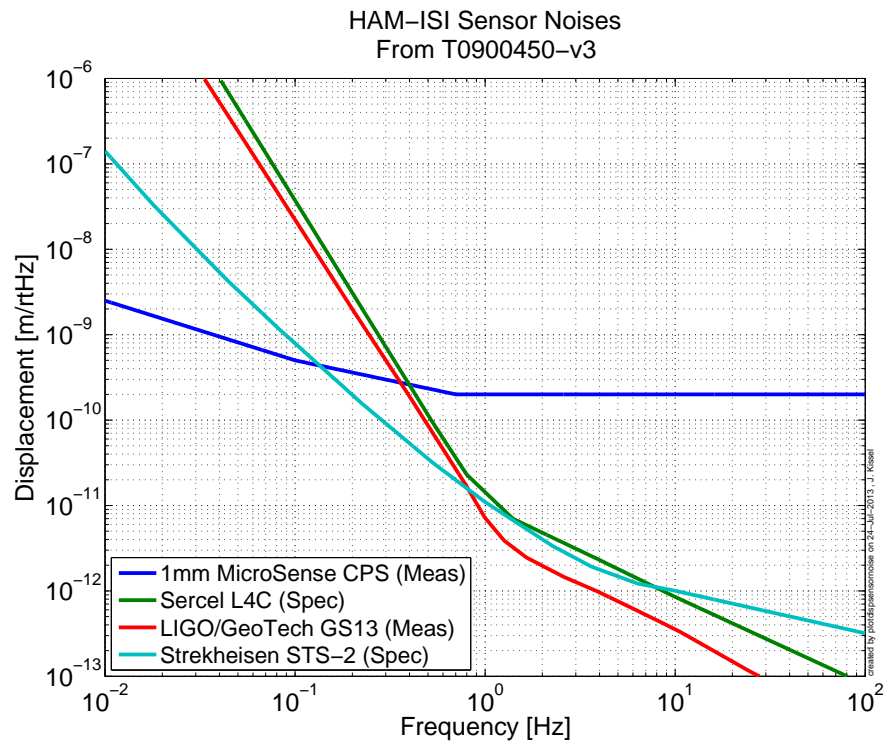


Figure 1: Displacement noise,  $n$ , for individual sensors of a given type that are used in, on, and around the HAM-ISI.

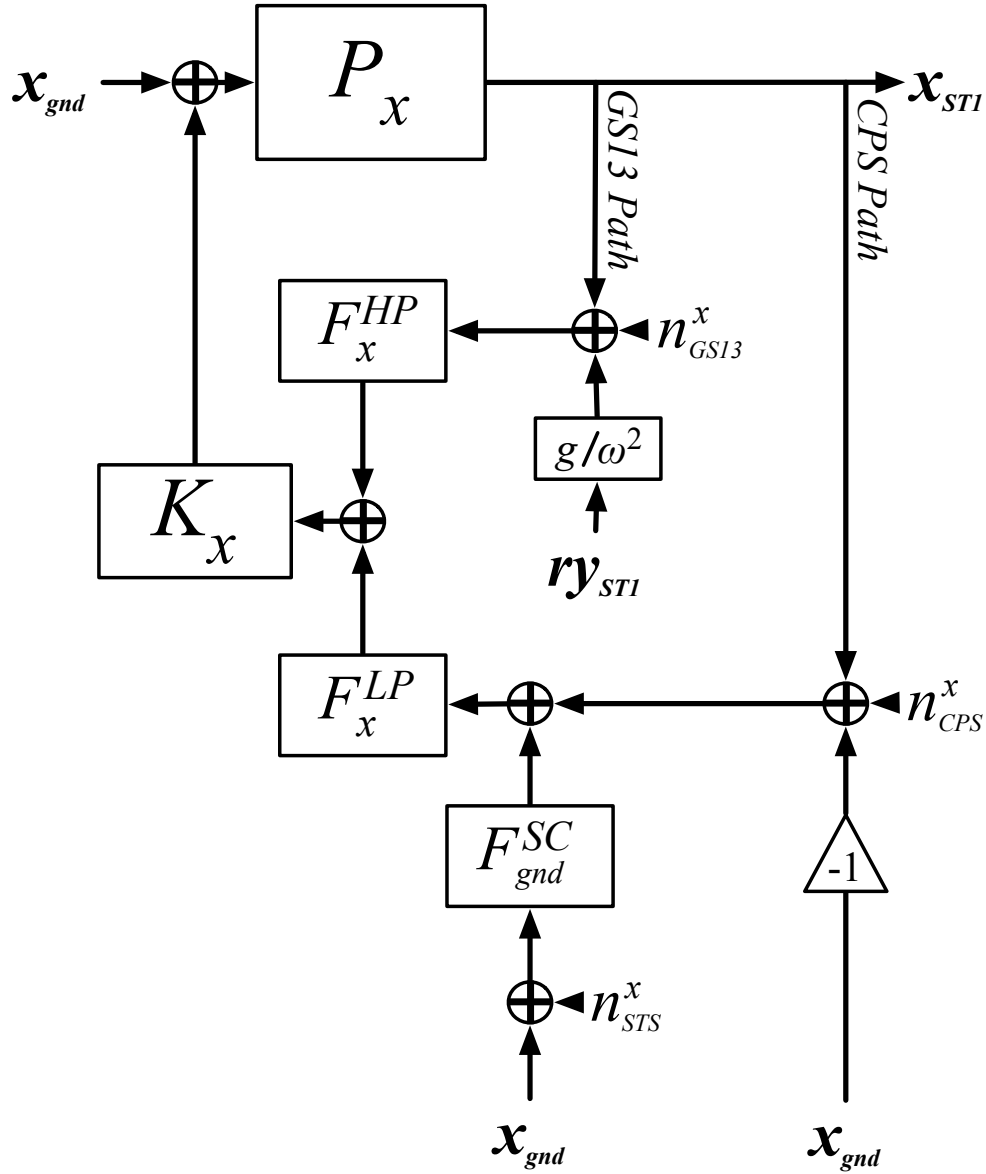


Figure 2: eLIGO Model of active control loop and noise couplings to the platform motion  $x_{ST1}$ . The loop shown is for the X translation direction, but where appropriate, noise couplings are treated to be the same for all degrees of freedom. These noise sources are input ground motion,  $x_{gnd}$ ; sensor noise from the on-board inertial sensors,  $n_{GS13}^x$ , the capacitive displacement sensors  $n_{CPS}^x$ , and ground inertial sensor  $n_{STS}^x$ ; and platform tilt noise (originating from residual ground motion or sensor noise),  $ry_{ST1}$ .

The horizontal platform motion  $x_{ST1}$  is determined by working counter-clockwise through the model

$$\begin{aligned}
 x_{ST1} = P_x x_{gnd} + P_x K_x \left[ \right. & F_x^{HP} \left\{ x_{ST1} + n_{GS13}^x + \frac{g}{\omega^2} ry_{ST1} \right\} \\
 & + F_x^{LP} \{ x_{ST1} - x_{gnd} + n_{CPS}^x \} \\
 & \left. + F_x^{LP} F_{gnd}^{SC} \{ x_{gnd} + n_{STS} \} \right].
 \end{aligned} \tag{1}$$

Defining the open loop gain transfer function,  $G_x$ , as

$$G_x = P_x K_x (F_x^{HP} + F_x^{LP}) = P_x K_x, \tag{2}$$

where we have used  $(F_x^{HP} + F_x^{LP}) = 1$  because they are designed as a complementary pair, then we may solve for the platform motion in terms of the uncorrelated noise source terms,

$$\begin{aligned}
 x_{ST1} = \left( \frac{G_x}{1 - G_x} \right) \left( \frac{P_x}{G_x} + F_x^{LP} (F_{gnd}^{SC} - 1) \right) x_{gnd} \\
 + \left( \frac{G_x}{1 - G_x} \right) F_x^{HP} \frac{g}{\omega^2} ry_{ST1} \\
 + \left( \frac{G_x}{1 - G_x} \right) F_x^{HP} n_{GS13}^x \\
 + \left( \frac{G_x}{1 - G_x} \right) F_x^{LP} n_{CPS}^x \\
 + \left( \frac{G_x}{1 - G_x} \right) F_x^{LP} F_{gnd}^{SC} n_{STS}.
 \end{aligned} \tag{3}$$

The remaining degrees of freedom are calculated in a similar fashion, but are simpler because they are insensitive to tilt of the platform. The model for vertical motion,  $z_{ST1}$ , tilt,  $t_{ST1} = rx_{ST1} = ry_{ST1}$ , and rotation about the Z axis,  $rz_{ST1}$  are,

$$\begin{aligned}
 z_{ST1} = \frac{G_z}{1 - G_z} \left( \frac{P_z}{G_z} + F_z^{LP} (F_{gnd}^{SC} - 1) \right) z_{ST0} \\
 + \frac{G_z}{(1 - G_z)} F_z^{HP} n_{GS13}^z \\
 + \frac{G_z}{(1 - G_z)} F_z^{LP} n_{CPS}^z \\
 + \frac{G_z}{(1 - G_z)} F_z^{LP} F_{gnd}^{SC} n_{STS}
 \end{aligned} \tag{4}$$

$$t_{ST1} = \frac{G_t}{(1 - G_t)} \left( \frac{P_t}{G_t} - F_t^{LP} \right) t_{gnd} + \frac{G_x}{(1 - G_x)} F_x^{HP} n_{GS13}^t + \frac{G_x}{(1 - G_x)} F_x^{LP} n_{CPS}^t \tag{5}$$

$$\begin{aligned}
 rz_{ST1} = \frac{G_{rz}}{(1 - G_{rz})} \left( \frac{P_{rz}}{G_{rz}} - F_{rz}^{LP} \right) rz_{gnd} + \frac{G_{rz}}{(1 - G_{rz})} F_{rz}^{HP} n_{GS13}^{rz} + \frac{G_{rz}}{(1 - G_{rz})} F_{rz}^{LP} n_{CPS}^{rz}
 \end{aligned} \tag{6}$$

### 3 aLIGO Model

For the aLIGO HAM-ISIs, there are a few notable differences, as indicated in red in Figure 3. We will focus the model on the SRCL HAM-ISIs (i.e. HAM4 and HAM5) as they will potentially have the most complicated loop structure. To calculate the performance of other platforms such as those in HAMS 2, 3, and 6, one can simply ignore paths which are not used.

All aLIGO HAM-ISIs are mounted on HAM-HEPIs, which create a non-negligible transfer function between the ground motion and the support stage of the HAM-ISI. Thus, we now explicitly differentiate between the two noise sources: the support stage – Stage 0 – displacement  $x_{ST0}$ , and the ground displacement  $x_{gnd}$ . An additional reason to differentiate is that  $x_{ST0}$  can be directly measured by either the HEPI L4Cs or the Stage 0 L4Cs, especially the rotation degrees of freedom. The eLIGO performance model suffered from using models the of the ground rotation.

To improve the performance around 10 [Hz], we have added an array of L4Cs on Stage 0. One can either use these to sensor correct the displacement sensors (which cannot not be rolled off sufficiently fast with  $F_x^{LP}$ ), or as a direct feed-forward to the actuators. Both paths are included below for completeness, though likely only one path will be used. When used as sensor correction, support stage motion  $x_{ST0}$  and inertial sensor noise  $n_{L4C}^x$  is filtered by the high-frequency sensor correction filter,  $F_{ST0}^{SC}$  and added into the displacement sensor path. When used in a feed-forward path,  $x_{ST0}$  and  $n_{L4C}^x$  are filtered by the feed-forward filter  $F_{ST0}^{FF}$  and added directly to the actuators.

Note that we assume here that the ground sensor correction, through  $F_{GND}^{SC}$ , will continue to be fed to the HAM-ISI’s capacitive position sensors as in eLIGO, instead of the baseline aLIGO plan to sensor correct the HAM-HEPI inductive position sensors as had been done in iLIGO.

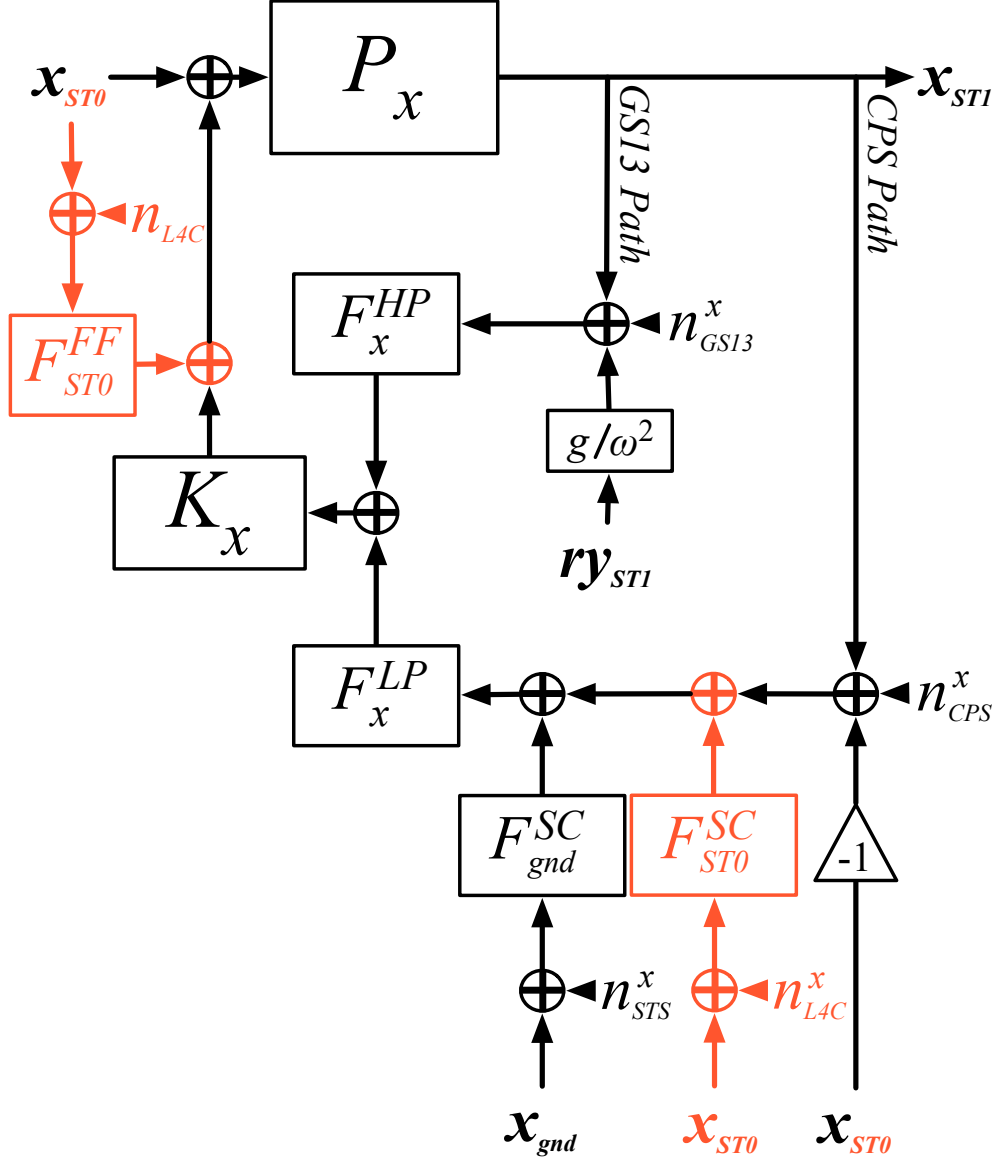


Figure 3: aLIGO Model of active control loop and noise couplings to the platform motion  $x_{ST1}$ . The differences between the eLIGO and aLIGO model are highlighted in red. These include highlighting that the input motion from the ground  $x_{gnd}$  and the support stage  $x_{ST0}$  are non-negligibly different, and the inclusion of either feed-forward or sensor correction using the new in-vacuum L4Cs on the support stage, through  $F_{ST0}^{FF}$  or  $F_{ST0}^{SC}$ , respectively.

Under the above aLIGO topology, the horizontal performance becomes

$$\begin{aligned}
x_{ST1} = P_x x_{ST0} + P_x \left[ F_{ST0}^{FF} (x_{ST0} + n_{LAC}^x) \right. \\
+ K_x \left( \begin{aligned}
& F_x^{HP} \{ x_{ST1} + n_{GS13}^x + \frac{g}{\omega^2} ry_{ST1} \} \\
& + F_x^{LP} \{ x_{ST1} - x_{ST0} + n_{CPS} \} \\
& + F_x^{LP} F_{gnd}^{SC} \{ x_{GND} + n_{STS}^x \} \\
& + F_x^{LP} F_{ST0}^{SC} \{ x_{ST0} + n_{LAC}^x \} \end{aligned} \right) \left. \right] \quad (7)
\end{aligned}$$

and after similar re-arrangement of terms, solving for the performance,

$$\begin{aligned}
x_{ST1} = \left( \frac{G_x}{1 - G_x} \right) \left[ \frac{P_x}{G_x} (1 + F_{ST0}^{FF}) + F_x^{LP} (F_{ST0}^{SC} - 1) \right] x_{ST0} \\
+ \left( \frac{G_x}{1 - G_x} \right) F_x^{LP} F_{gnd}^{SC} x_{gnd} \\
+ \left( \frac{G_x}{1 - G_x} \right) F_x^{HP} \frac{g}{\omega^2} ry_{ST1} \\
+ \left( \frac{G_x}{1 - G_x} \right) F_x^{HP} n_{GS13}^x \\
+ \left( \frac{G_x}{1 - G_x} \right) F_x^{LP} n_{CPS}^x \\
+ \left( \frac{G_x}{1 - G_x} \right) F_x^{LP} F_{gnd}^{SC} n_{STS} \\
+ \left( \frac{G_x}{1 - G_x} \right) \left( \frac{P_x}{G_x} F_{ST0}^{FF} + F_x^{LP} F_{ST0}^{SC} \right) n_{LAC}^x \quad (8)
\end{aligned}$$

which comfortably, reduces to Eq. 3 if all new terms are removed.

As before, the remaining degrees of freedom are calculated in the same fashion. Interestingly, because the aLIGO HAM-ISIs have information the rotational input motion of the support stage, the performance model can include feed-forward or sensor correction for these degrees of freedom.



$$\begin{aligned}
z_{ST1} = & \left( \frac{G_z}{1-G_z} \right) \left( \frac{P_z}{G_z} (1 + F_{ST0}^{FF}) + F_z^{LP} (F_{ST0}^{SC} - 1) \right) z_{ST0} \\
& + \left( \frac{G_z}{1-G_z} \right) F_z^{LP} F_{gnd}^{SC} z_{gnd} \\
& + \left( \frac{G_z}{1-G_z} \right) F_z^{HP} n_{GS13}^z \\
& + \left( \frac{G_z}{1-G_z} \right) F_z^{LP} n_{CPS}^z \\
& + \left( \frac{G_z}{1-G_z} \right) F_z^{LP} F_{gnd}^{SC} n_{STS} \\
& + \left( \frac{G_z}{1-G_z} \right) \left( \frac{P_z}{G_z} F_{ST0}^{FF} + F_z^{LP} F_{ST0}^{SC} \right) n_{LAC}^z
\end{aligned} \tag{9}$$

$$\begin{aligned}
t_{ST1} = & \frac{G_t}{(1-G_t)} \left( \frac{P_t}{G_t} (1 + F_{ST0}^{FF}) + F_t^{LP} (F_{ST0}^{SC} - 1) \right) t_{ST0} \\
& + \left( \frac{G_t}{1-G_t} \right) F_x^{HP} n_{GS13}^t \\
& + \left( \frac{G_t}{1-G_t} \right) F_x^{LP} n_{CPS}^t \\
& + \left( \frac{G_t}{1-G_t} \right) \left( \frac{P_t}{G_t} F_{ST0}^{FF} + F_t^{LP} F_{ST0}^{SC} \right) n_{LAC}^t
\end{aligned} \tag{10}$$

$$\begin{aligned}
rz_{ST1} = & \left( \frac{G_{rz}}{1-G_{rz}} \right) \left( \frac{P_{rz}}{G_{rz}} (1 + F_{ST0}^{FF}) + F_{rz}^{LP} (F_{ST0}^{SC} - 1) \right) rz_{ST0} \\
& + \left( \frac{G_{rz}}{1-G_{rz}} \right) F_{rz}^{HP} n_{GS13}^{rz} \\
& + \left( \frac{G_{rz}}{1-G_{rz}} \right) F_{rz}^{LP} n_{CPS}^{rz} \\
& + \left( \frac{G_{rz}}{1-G_{rz}} \right) \left( \frac{P_{rz}}{G_{rz}} F_{ST0}^{FF} + F_{rz}^{LP} F_{ST0}^{SC} \right) n_{LAC}^{rz}
\end{aligned} \tag{11}$$

## 4 Conclusion

Though the inclusion of several more loops into the HAM-ISI control topology may seem like noise budgeting the performance is daunting, the new topology results in performance terms which are not terribly different from the eLIGO topology, with new terms that appear rather obviously in comparison.

## References

- [1] J. Kissel. “Calibrating and Improving the Sensitivity of the LIGO Detectors.” LIGO-P1000103