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Scattering on aLIGO Transmission Monitor (TransMon) Optics

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1 References

1. J. Fishner and S. J. Waldman, "Advanced LIGO Transmon beam dump", `https://dcc.ligo.org/T1000300`

2. J. Kissel, "aLIGO TMTS "Level 2" Damping Loop Design", `https://dcc.ligo.org/g1300621`

2 Summary

Scattering caused by Transmission Monitor (TransMon) optics is studied in this document. The prompt back scattering is not a problem because of the optical attenuation provided by the ETM HR coating and the seismic isolation provided by ISI and TransMon Suspension (TMS). If the TMS optic shows a wider angle scattering, however, that is a potential problem as the light might hit the chamber that is not isolated seismically nor acoustically, and be reintroduced to the arm.

3 Prompt back scattering

Here we consider the case where a part of the scattered light couples directly to the back propagating arm mode. This is represented by the power coupling constant S of the scattering optic on the TransMon Suspension (TMS). The underlying calculation is quite similar to the one presented in TransMon beam dump requirement [1].

TMS longitudinal motion performance [2] indicates that there's no fringe wrapping above 0.1 Hz or so, therefore we only consider the linear coupling. Also, assuming that S is not large (i.e. not on the order of 1), we ignore the cavity formed by the ETM and the scattering optic.

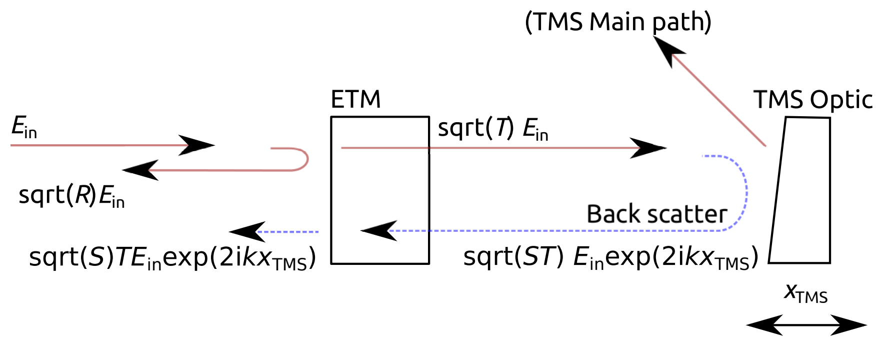

Figure 1: Prompt back scattering.

Under these assumptions, the effective amplitude reflectivity of the ETM becomes

$$\begin{aligned} r_{\text{eff}} &= \sqrt{R} + T\sqrt{S}\exp(2ikx_{\text{tms}} + i\phi) \\ &\sim 1 + 2ikT\sqrt{S}x_{\text{tms}}\cos\phi \end{aligned} \tag{1}$$

where k is the wave number of the light, x_{tms} is the small motion of TMS, ϕ is the phase term that represents the average distance between the TMS and the ETM (Figure 1). The power reflectivity and transmissivity of the ETM are represented by $R \sim 1$ and $T \sim 5$ ppm.

In the worst case ($\phi = n\pi$), x_{tms} causes the phase change in the reflection from the ETM that is equivalent of $T\sqrt{S}x_{\text{tms}}$ in ETM motion:

$$\begin{aligned} x_{\text{equiv}} &\sim T\sqrt{S}x_{\text{tms}} \\ &= 5 \times 10^{-6}\sqrt{S}x_{\text{tms}}. \end{aligned}$$

If you look at the performance estimate of TMS [1] again, the bulk motion of TMS is about $2\times10^{-16}\text{m}/\sqrt{\text{Hz}}$ at 10 Hz and rolls off with roughly f^{-4} for $f > 10$ Hz, so the effective ETM displacement caused by the TMS is

$$x_{\text{equiv}} \sim 1 \times 10^{-21}\sqrt{S}\ \text{m}/\sqrt{Hz}\ (\text{at 10Hz, rolls off with } f^{-4}). \tag{2}$$

On the other hand, the requirement for the QUAD suspension is $10^{-19}\mathrm{m}/\sqrt{Hz}$ at 10 Hz with f^{-4} or faster roll off. This means that the prompt back scattering effect is orders of magnitude smaller than the QUAD suspension requirement for all practical values of S.

Even if we are somehow overestimating the TMS performance by two orders of magnitude, back scattering as large as $S = 1$ % will give us a factor of 10 head room.

4 Wide angle scattering

The scattered light can also hit the chamber, somehow comes back to one of the optics in the main beam line and then scattered back into the arm mode.

4.1 TMS-chamber-TMS-arm

In this scenario the TMS scatters the light, the light hits the chamber, a part of the light comes back from the chamber to the TMS, and the TMS scatters it back to the arm mode (Figure 2). Since there's no baffling to the side of the TMS, this path is wide open.

Since we're only interested in a rough estimate, the entire process is represented by only three parameters: The coupling constant of the TMS scattering (S_{tms}), that of the chamber (S_{c}) and the chamber displacement (x_{c}). The effective reflectivity of the ETM is written by

$$r_{\mathrm{eff}} = 1 + \sqrt{S_{\mathrm{c}}} S_{\mathrm{tms}} T \exp(2ikx_{\mathrm{c}}).$$

For the linear coupling, the equivalent ETM displacement would be

$$\begin{aligned} x_{\mathrm{equiv}} &\sim \sqrt{S_{\mathrm{c}}} S_{\mathrm{tms}} T x_{\mathrm{c}} \\ &= 10^{-20} \times \sqrt{\frac{S_{\mathrm{c}}}{0.1\%}} \frac{S_{\mathrm{tms}}}{10 \text{ ppm}} \frac{x_{\mathrm{c}}}{5 \times 10^{-9}}. \end{aligned} \quad (3)$$

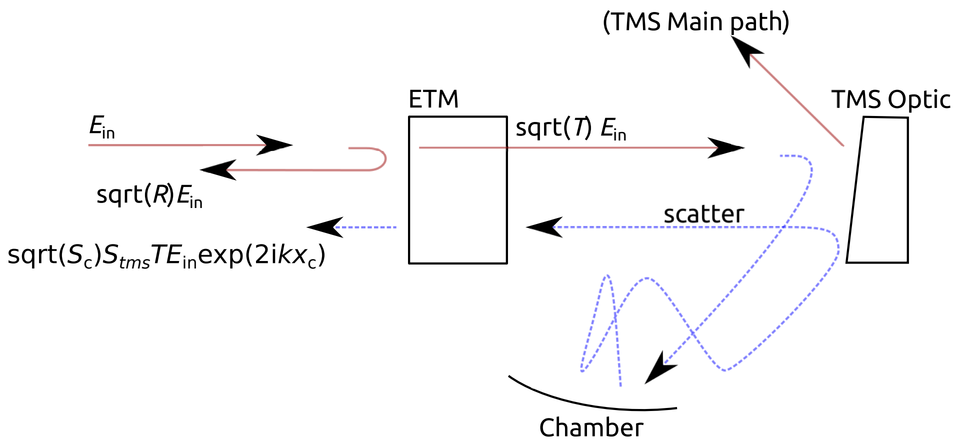

Figure 2: TMS-chamber-TMS-arm process.

This means that, if for example the TMS coupling is 10 ppm and the chamber scattering/reflection coupling is 0.1 %, and if we require that $x_{\rm equiv}$ is a factor of 10 smaller than the QUAD requirement, i.e. $10^{-20}{\rm m}/\sqrt{Hz}$ at 10 Hz and rolls off with f^{-4} or faster, $x_{\rm c}$ should not exceed $5\times10^{-9}{\rm m}/\sqrt{Hz}$ at 10Hz and should roll off with f^{-4}.

As for the fringe wrapping effect, the peak amplitude is

$$\begin{aligned} x_{\rm wrap} &= \frac{\sqrt{S_{\rm c}}S_{\rm tms}T}{2k} \\ &= \frac{\sqrt{S_{\rm c}}S_{\rm tms}T}{4\pi}\lambda \\ &\sim 10^{-19}\times\sqrt{\frac{S_{\rm c}}{0.1\ \%}\frac{S_{\rm tms}}{10\ {\rm ppm}}}[{\rm m}]. \end{aligned}$$

4.2 TMS-chamber-ETMHR-arm

In this scenario the scattering from the TMS hits the chamber, somehow goes to the front surface of the ETM and is scattered back to the arm mode (Figure 3).

The linear coupling is represented by

$$x_{\rm equiv} \sim \sqrt{S_{\rm etm}S_{\rm c}S_{\rm tms}T}x_{\rm c}$$

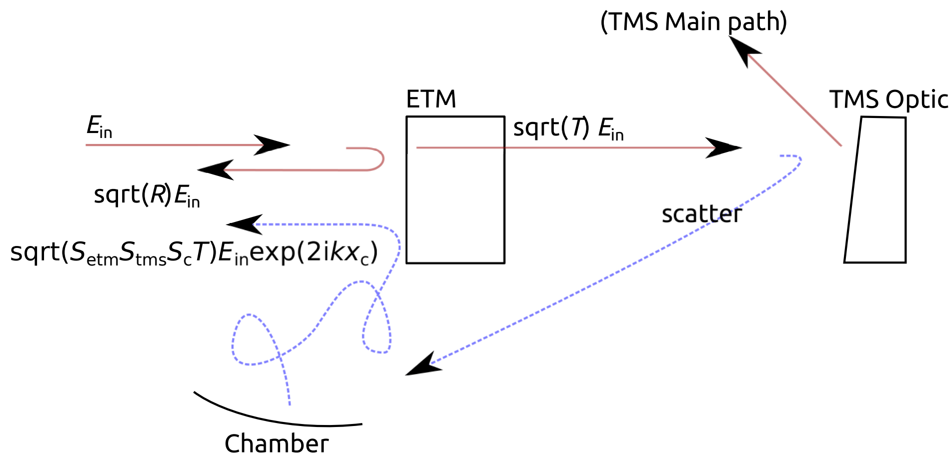

Figure 3: TMS-chamber-ETMHR-arm process.

$$\sim \quad 10^{-20} \times \sqrt{\frac{S_{\rm etm}}{1 \text{ ppm}} \frac{S_{\rm c}}{0.1 \text{ \%}} \frac{S_{\rm tms}}{10 \text{ ppm}}} \frac{x_{\rm c}}{5 \times 10^{-11}} \tag{4}$$

where $S_{\rm etm}$ is the coupling constant of the ETM. This seems to be about two orders of magnitude larger than before assuming that $S_{\rm etm} \sim 1$ ppm is a reasonable assumption (note that the total scattering of the ETM HR is about 10ppm or smaller).

For example if the factors in the square root are all one, $x_{\rm c}$ should be $5 \times 10^{-11}{\rm m}/\sqrt{Hz}$ at 10Hz (instead of 5×10^{-9}) or smaller and should roll off with f^{-4} or faster to satisfy the same requirement as before.

The fringe wrapping peak amplitude will be

$$\begin{aligned} x_{\rm wrap} &= \frac{\sqrt{S_{\rm etm} S_{\rm c} S_{\rm tms} T}}{4\pi}\lambda \\ &\sim 2 \times 10^{-17} \sqrt{\frac{S_{\rm etm}}{1 \text{ ppm}} \frac{S_{\rm c}}{0.1 \text{ \%}} \frac{S_{\rm tms}}{10 \text{ ppm}}} \text{ [m]}. \end{aligned}$$