T1300421 Ghost Beam Scatter in Signal Recycling Cavity 8/24/11

BRDF of porcelainized steel, sr^-1	$BRDF_{bd} \coloneqq 0.05$	
BRDF of chamber wall, sr^-1	$BRDF_{wall} = 0.1$	
Motion of HEPI @ 200 Hz, m/rt Hz	$x_{hepi} := 2 \cdot 10^{-10}$	
Motion of HAM table @ 100 Hz, m/rt Hz	$x_{ham} = 3.7 \cdot 10^{-14}$	
Motion of HAM flange @ 100 Hz, m/rt Hz	$x_{hamflange} := 1.7 \cdot 10^{-11}$	
laser wavelength, m	$\lambda := 1.064 10^{-6}$	
wave number, m^-1	$k := 2 \cdot \frac{\pi}{\lambda}$	$k = 5.905 \times 10^6$
IFO waist size, m	w _{ifo} := 0.012	
solid angle of IFO mode, sr	$\Delta_{ifo} \coloneqq \pi \cdot \left(\frac{\lambda}{\pi \cdot w_{ifo}}\right)^2$	$\Delta_{\rm ifo} = 2.502 \times 10^{-9}$
Transfer function @ 100 Hz, ITM AR	$\text{TF}_{\text{itmar}} := 3.16 10^{-11}$	
Transfer function @ 100 Hz, BS from SR	$\text{TF}_{\text{srbs}} \coloneqq 4.46 10^{-11}$	
Transfer function @ 100 Hz, SRM	$TF_{srm} := 4.22 \cdot 10^{-10}$	
Ref. T070247		
transmissivity of SRM HR	T _{srmhr} := 0.2	
Transmissivity of ITM HR	$T_{itmhr} := 0.014$	
Reflectivity of ITM HR	$R_{itmhr} \coloneqq 1 - T_{itmhr}$	$R_{itmhr} = 0.986$
Transmissivity of ETM HR	$T_{etm} = 5 \cdot 10^{-6}$	

ETM transmitted power, W

 $P_{\text{etmtr}} := 4.4$

input laser power, W	$P_{psl} := 125$	
arm cavity gain	$G_{ac} := 13000$	
arm cavity power, W	$\mathbf{P}_{\mathbf{a}} \coloneqq \frac{\mathbf{P}_{\mathbf{psl}}}{2} \cdot \mathbf{G}_{\mathbf{ac}}$	$P_a = 8.125 \times 10^5$
Ref. Hiro e-mail 8/29/11		
power in power recycling cavity both arms, W	$P_{rc} := \frac{2P_a \cdot T_{itmhr}}{4}$	$P_{\rm rc} = 5.688 \times 10^3$
Gaussian power parameter in recycling cavity arm	$P_{Orc} := \frac{P_{rc}}{2}$	$P_{0rc} = 2.844 \times 10^3$
Power recycling cavity gain	$G_{rc} := \frac{P_{rc}}{P_{psl}}$	G _{rc} = 45.5
refl port signal ratio	$G_{refl} \coloneqq 0.001$	
as port signal ratio	$G_{as} := 0.00108$	
output signal power, W	$P_{srm} := P_{psl} \cdot G_{as}$	$P_{\rm srm} = 0.135$
power in signal recycling cavity, W	$P_{src} \coloneqq \frac{P_{srm}}{T_{srmhr}}$	$P_{\rm src} = 0.675$
Asymmetry coefficient for common mode field rejection	$C_{assy} := \sqrt{\frac{P_{src}}{P_{rc}}}$	$C_{assy} = 0.0109$
Gaussian irradiance parameter from ITM	$P_{0itm} := 2 \cdot P_{0rc}$	$P_{0itm} = 5.688 \times 10^3$

reflectivity of BS HR

 $R_{bshr} := 0.5$

reflectivity of BS AR	$R_{bsar} \coloneqq 50.10^{-6}$	
Reflectivity of ITM HR	Ritmahar := 1 - Titmhr	$R_{itmhr} = 0.986$
Reflectivity of ITM AR	$R_{itmar} := 50 \cdot 10^{-6}$	
Reflectivity of CP AR	$R_{cpar} := 50.10^{-6}$	
reflectivity of AS septum port	$R_{sp} := 0.0025$	
reflectivity of SRM HR	$R_{srmhr} \coloneqq 1 - T_{srmhr}$	$R_{srmhr} = 0.8$
reflectivity of SRM AR	$R_{srmar} \approx 50 \cdot 10^{-6}$	
transmissivity of SRM AR	T _{srmar} := 1 - R _{srmar}	$T_{srmar} = 1$
reflectivity of PR2 HR	R _{pr2hr} := 0.9999	
transmissivity of PR2 HR	$T_{pr2hr} := 1 - R_{pr2hr}$	$T_{pr2hr} = 10 \times 10^{-5}$
reflectivity of PR2 AR	$R_{pr2ar} := 50 \cdot 10^{-6}$	
transmissivity of PR2 AR	$T_{pr2ar} \coloneqq 1 - R_{pr2ar}$	$T_{pr2ar} = 1$
reflectivity of SR2 HR	$R_{sr2hr} := R_{pr2hr}$	$R_{sr2hr} = 1$
reflectivity of SR2 AR	$R_{sr2ar} \coloneqq R_{pr2ar}$	$R_{sr2ar} = 5 \times 10^{-5}$
transmissivity of SR2 HR	$T_{sr2hr} := T_{pr2hr}$	$T_{sr2hr} = 10 \times 10^{-5}$
transmissivity of SR2 AR	$T_{sr2ar} := T_{pr2ar}$	$T_{sr2ar} = 0.99995$
reflectivity of PR3 HR	R _{pr3hr} := 0.9999	
transmissivity of PR3 HR	$T_{pr3hr} := 1 - R_{pr3hr}$	$T_{pr3hr} = 10 \times 10^{-5}$
reflectivity of PR3 AR	$R_{pr3ar} := 50.10^{-6}$	

transmissivity of PR3 AR	$T_{pr3ar} := 1 - R_{pr3ar}$	$T_{pr3ar} = 1$
reflectivity of SR3 HR	$R_{sr3hr} := R_{pr3hr}$	$R_{sr3hr} = 1$
reflectivity of SR3 AR	$R_{sr3ar} := R_{pr3ar}$	$R_{sr3ar} = 5 \times 10^{-5}$
transmissivity of SR3 HR	$T_{sr3hr} \coloneqq T_{pr3hr}$	$T_{sr3hr} = 10 \times 10^{-5}$
transmissivity of SR3 AR	$T_{sr3ar} \coloneqq T_{pr3ar}$	$T_{sr3ar} = 1$
reflectivity of FM HR	$R_{FMhr} \coloneqq R_{pr3hr}$	$R_{FMhr} = 1$
reflectivity of Hartmann dichroic bs	$R_{hartbs} := 0.0025$	
reflectivity of BS AR	$R_{bsar} = 5 \times 10^{-5}$	
Reflectivity of SR3	R _{SR3} := 1	
Reflectivity of dichroic HWSY M1	$R_{HWSYM1} := 0.01$	
Reflectivity of dichroic HWSY M2	$R_{HWSYM2} \coloneqq 0.01$	
Reflectivity of HPY-F1	$R_{HPYF1} \coloneqq 1$	
Reflectivity of HWSY M3	$R_{HWSYM3} \coloneqq 1$	
Reflectivity of HWSY M4	$R_{HWSYM4} \coloneqq 1$	
Reflectivity of HWSY M5	$R_{HWSYM5} \coloneqq 1$	
Reflectivity of viewport	R _{vp} := 0.0025	
Reflectivity of dichroic HWSX M1	$R_{HWSXM1} \coloneqq 0.01$	
Reflectivity of dichroic HWSX M2	$R_{HWSXM2} := 0.01$	
Reflectivity of HWSX M3	$R_{HWSXM3} := 1$	

$R_{HWSXM4} \coloneqq 1$
$R_{HWSXM5} \coloneqq 1$
$R_{HPXF1} \coloneqq 1$
$T_{sr2hr} = 10 \times 10^{-5}$
BRDF _{hartm} := 0.01
$BRDF_{vp} := 0.005$
$BRDF_{hartm} = 0.01$
$w_{sr30} := 0.000114$
$w_{sr20} := 0.000094$
$w_{srm0} := 0.000841$
w _{hpyf10} := 0.0000850
$w_{hpxf10} := 0.0000650$

ITM Ghost Beams

ITM_GBAR1_BD H1

Power incident on SR2 Scraper Baffle from both arms, W

 $P_{itmar1bd} := P_{rc} \cdot R_{bshr} \cdot R_{itmar}$

 $P_{itmar1bd} = 0.142$

both ITM AR1 BD scattered power into BS from SR2 Scraper baffle, W

$$P_{itmar1bds} := P_{itmar1bd} \cdot BRDF_{bd} \cdot \frac{w_{ifo}^{2}}{w_{sr30}^{2}} \cdot \Delta_{ifo} \cdot R_{bshr}^{0} \cdot R_{itmar}$$

$$P_{itmar1bds} = 9.857 \times 10^{-12}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{itmar1bd} := TF_{srbs} \cdot \left(\frac{P_{itmar1bds}}{P_{psl}}\right)^{0.5} \cdot x_{ham} \cdot 2 \cdot k$$

$$\mathrm{DN}_{\mathrm{itmar1bd}} = 5.473 \times 10^{-24}$$

ITM_GBAR3_BD H1

power incident on SR2 Scraper Baffle from both arms, W $P_{itmar3bd} \coloneqq P_{rc} \cdot R_{bshr} \cdot R_{itmhr}^{2} \cdot R_{itmar} \cdot (1 - R_{itmar})^{2}$

$$P_{itmar3bd} = 0.1382$$

power scattered from SR2 Scraper Baffle, W

$$P_{itmar3bds} \coloneqq P_{itmar3bd} \cdot BRDF_{bd} \cdot \frac{w_{ifo}^{2}}{w_{sr30}^{2}} \cdot \Delta_{ifo} \cdot R_{bshr}^{0} \cdot R_{itmhr}^{2} \cdot R_{itmar} \cdot (1 - R_{itmar})^{2}$$

$$P_{itmar3bds} = 9.314 \times 10^{-12}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{itmar3bd} \coloneqq TF_{srbs} \cdot \left(\frac{P_{itmar3bds}}{P_{psl}}\right)^{0.5} \cdot x_{ham} \cdot 2 \cdot k$$
$$DN_{itmar3bd} = 5.32 \times 10^{-24}$$

BS_GBAR3P H1

The stray light from both arms are almost anti-resonant, and the wavefronts overlap; their coherent sum is reduced by the square of the asymmetry coefficient for common mode field rejection

power incident on SR2 Scraper Baffle from both arms, W

$$P_{bsar3sr2baf} := P_{rc} \cdot \left[\left(1 - R_{bsar} \right) \cdot R_{bshr} + \left(1 - R_{bshr} \right) \cdot R_{bsar} \right] \cdot R_{bshr} \cdot \left(1 - R_{bsar} \right) \cdot C_{assy}^{2}$$
$$P_{bsar3sr2baf} = 0.169$$

power scattered from SR2 Scraper Baffle, W

$$P_{bsar3sr2bafs} := P_{bsar3sr2baf} \cdot BRDF_{bd} \cdot \frac{w_{ifo}^{2}}{w_{sr30}^{2}} \cdot \Delta_{ifo} \cdot (1 - R_{bsar}) \cdot R_{bshr} \cdot R_{bsar} \cdot \left[(1 - R_{bshr}) + R_{bshr} \cdot (1 - R_$$

 $P_{bsar3sr2bafs} = 5.848 \times 10^{-12}$

displacement noise @ 100 Hz,
$$DN_{bsar3sr2baf} := TF_{itmar} \cdot \left(\frac{P_{bsar3sr2bafs}}{P_{psl}}\right)^{0.5} \cdot x_{ham} \cdot 2 \cdot k$$

$$DN_{bsar3sr2baf} = 2.987 \times 10^{-24}$$

CP_GBAR1

power incident on SR2 Scraper Baffle from both arms, W

$$P_{cpar1sr2baf} \coloneqq P_{rc} \cdot R_{bshr} \cdot R_{cpar}$$
$$P_{cpar1sr2baf} = 0.142$$

power scattered from SR2 Scraper Baffle, W

$$P_{cpar1sr2bafs} := P_{cpar1sr2baf} \cdot BRDF_{bd} \cdot \frac{w_{ifo}^{2}}{w_{sr30}^{2}} \cdot \Delta_{ifo} \cdot R_{bshr}^{0} \cdot R_{cpar}$$

$$P_{cpar1sr2bafs} = 9.857 \times 10^{-12}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{cpar1sr2baf} := TF_{itmar} \cdot \left(\frac{P_{cpar1sr2bafs}}{P_{psl}}\right)^{0.5} \cdot x_{ham} \cdot 2 \cdot k$$

 $DN_{cpar1sr2baf} = 3.878 \times 10^{-1}$

CP_GBAR3

power incident on SR2 Scraper Baffle from both arms, W

$$P_{cpar3sr2baf} := P_{rc} \cdot R_{bshr} \cdot R_{itmhr} \cdot R_{cpar}$$

 $P_{cpar3sr2baf} = 0.14$

power scattered from SR2 Scraper Baffle, W

 $P_{cpar3sr2bafs} \coloneqq P_{cpar3sr2baf} \cdot BRDF_{bd} \cdot \frac{w_{ifo}^{2}}{w_{sr30}^{2}} \cdot \Delta_{ifo} \cdot R_{bshr}^{0} \cdot R_{itmhr} \cdot R_{cpar}$

$$P_{cpar3sr2bafs} = 9.583 \times 10^{-12}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{cpar3sr2baf} := TF_{itmar} \cdot \left(\frac{P_{cpar3sr2bafs}}{P_{psl}}\right)^{0.5} \cdot x_{ham} \cdot 2 \cdot k$$

$$DN_{cpar3sr2baf} = 3.823 \times 10^{-24}$$

SR3 GBHR3

power incident on SR3 GBHR3 (forward and backward beams), W

$$P_{sr3gbhr3} \coloneqq 2 \cdot P_{src} \cdot T_{sr3hr} \cdot R_{sr3ar} \cdot T_{sr3hr}$$

$$P_{sr3gbhr3} = 6.75 \times 10^{-13}$$

power scattered from SR3 GBHR3 toward BS, W

$$P_{sr3gbhr3bss} \coloneqq \frac{P_{sr3gbhr3}}{2} \cdot BRDF_{wall} \cdot \Delta_{ifo} \cdot T_{sr3hr} \cdot R_{sr3ar} \cdot T_{sr3hr}$$

$$P_{sr3gbhr3bss} = 4.223 \times 10^{-35}$$

power scattered from SR3 GBHR3 toward SR2 W

$$P_{sr3gbhr3sr2s} := \frac{P_{sr3gbhr3}}{2} \cdot BRDF_{wall} \cdot \Delta_{ifo} \cdot \frac{w_{ifo}^2}{w_{sr30}^2} \cdot \left(T_{sr3hr} \cdot R_{sr3ar} \cdot T_{sr3hr}\right)$$

$$P_{sr3gbhr3sr2s} = 4.679 \times 10^{-5}$$

total power scattered from SR3 GBHR3

$$P_{sr3gbhr3s} := P_{sr3gbhr3bss} + P_{sr3gbhr3sr2s}$$

$$P_{sr3gbhr3s} = 4.68 \times 10^{-31}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{sr3gbhr3} := TF_{srbs} \cdot \left(\frac{P_{sr3gbhr3s}}{P_{psl}}\right)^{0.5} \cdot x_{hamflange} \cdot 2 \cdot k$$

$$DN_{sr3gbhr3} = 5.479 \times 10^{-31}$$

SR3 GBAR3

power incident on GBAR3 AR Baffle (forward and backward beams), W

 $P_{sr3gbar3} := 2 \cdot P_{src} \cdot T_{sr3hr} \cdot R_{sr3ar} \cdot R_{sr3hr} \cdot T_{sr3ar}$

$$P_{sr3gbar3} = 6.749 \times 10^{-9}$$

power scattered from SR3 AR Baffle toward BS, W

 $P_{sr3gbar3bss} \coloneqq \frac{P_{sr3gbar3}}{2} \cdot BRDF_{bd} \cdot \Delta_{ifo} \cdot T_{sr3hr} \cdot R_{sr3ar} \cdot R_{sr3hr} \cdot T_{sr3ar}$

$$P_{sr3gbar3bss} = 2.111 \times 10^{-27}$$

power scattered from SR3 AR Baffle toward SR2, W

$$P_{sr3gbar3sr2s} := \frac{P_{sr3gbar3}}{2} \cdot BRDF_{bd} \cdot \Delta_{ifo} \cdot \frac{w_{ifo}^2}{w_{sr30}^2} \cdot T_{sr3hr} \cdot R_{sr3ar} \cdot R_{sr3hr} \cdot T_{sr3ar}$$

$$P_{sr3gbar3sr2s} = 2.339 \times 10^{-23}$$

total power scattered from SR3 AR Baffle, W

$$P_{sr3gbar3s} = 2.339 \times 10^{-23}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{sr3gbar3} \coloneqq TF_{srbs} \cdot \left(\frac{P_{sr3gbar3s}}{P_{psl}}\right)^{0.5} \cdot x_{ham} \cdot 2 \cdot k$$

$$DN_{sr3gbar3} = 8.431 \times 10^{-30}$$

SR3 AR Baffle

power incident on SR3 AR Baffle (forward and backward beams), W

 $P_{sr3arbaf} := 2P_{src} \cdot T_{sr3hr} \cdot T_{sr3ar}$ $P_{sr3arbaf} = 1.35 \times 10^{-4}$

power scattered from SR3 AR Baffle toward BS, $\ensuremath{\mathsf{W}}$

$$P_{sr3arbafbss} \coloneqq \frac{P_{sr3arbaf}}{2} \cdot BRDF_{bd} \cdot \Delta_{ifo} \cdot T_{sr3hr} \cdot T_{sr3ar}$$

$$P_{sr3arbafbss} = 8.445 \times 10^{-19}$$

power scattered from SR3 AR Baffle toward SR2 W

$$P_{sr3arbafsr2s} := \frac{P_{sr3arbaf}}{2} \cdot BRDF_{bd} \cdot \Delta_{ifo} \cdot \frac{w_{ifo}^2}{w_{sr30}^2} \cdot T_{sr3hr} \cdot T_{sr3ar}$$

$$P_{sr3arbafsr2s} = 9.357 \times 10^{-15}$$

power scattered from SR3 AR Baffle, W

 $P_{sr3arbafs} := P_{sr3arbafbss} + P_{sr3arbafsr2s}$

displacement noise @ 100 Hz, m/rtHz

 $DN_{sr3arbaf} := TF_{srbs} \cdot \left(\frac{P_{sr3arbafs}}{P_{psl}}\right)^{0.5} \cdot x_{ham} \cdot 2 \cdot k$

 $DN_{sr3arbaf} = 1.686 \times 10^{-25}$

SRM_GBHR3

power of SRM GBHR3, W

$$P_{srmhr3} := P_{srm} \cdot R_{srmar} \cdot T_{srmhr}$$

$$P_{srmhr3} := 1.35 \times 10^{-6}$$

1
 srmhr3 $^{-1.55 \times 10}$

power scattered from SRM GBHR3 Mode Cleaner Tube Baffle, W

$$P_{srmhr3bafs} \coloneqq P_{srmhr3} \cdot BRDF_{bd} \cdot \frac{\frac{w_{ifo}^{2}}{w_{srm0}^{2}}}{\frac{w_{ifo} \cdot R_{srmar} \cdot T_{srmhr}}{w_{srm0}^{2}}}$$

$$P_{\text{srmhr3bafs}} = 3.439 \times 10^{-19}$$

displacement noise @ 100 Hz,
$$DN_{srmhr3bafs} \coloneqq TF_{srm} \cdot \left(\frac{P_{srmhr3bafs}}{P_{psl}}\right)^{0.5} \cdot x_{ham} \cdot 2 \cdot k$$

$$DN_{srmhr3bafs} = 9.673 \times 10^{-27}$$

SRM_GBAR3

power incident on SRM AR Baffle, W

$$P_{srmarbaf} := P_{srm} \cdot R_{srmar} \cdot R_{srmhr} \cdot T_{srmar}$$

 $P_{srmarbaf} = 5.4 \times 10^{-6}$

power scattered from SRM AR Baffle, W

 $P_{srmarbafs} := P_{srmarbaf} \cdot BRDF_{bd} \cdot \frac{w_{ifo}^{2}}{w_{srm0}^{2}} \cdot \Delta_{ifo} \cdot R_{srmar} \cdot R_{srmhr} \cdot T_{srmar}$

$$P_{srmarbafs} = 5.502 \times 10^{-18}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{srmarbafs} := TF_{srm} \cdot \left(\frac{P_{srmarbafs}}{P_{psl}}\right)^{0.5} \cdot x_{ham} \cdot 2 \cdot k$$

$$DN_{srmarbafs} = 3.869 \times 10^{-26}$$

SR2 GBHR3

power incident on SR2 GBHR3 (forward and backward beams), W

$$P_{sr2gbhr3} \coloneqq 2 \cdot P_{src} \cdot T_{sr2hr} \cdot R_{sr2ar} \cdot T_{sr2hr}$$
$$P_{sr2gbhr3} = 6.75 \times 10^{-13}$$

power scattered from SR2 GBHR3 toward SR3, W

$$P_{sr2gbhr3sr3s} := \frac{P_{sr2gbhr3}}{2} \cdot BRDF_{wall} \cdot \frac{w_{ifo}^{2}}{w_{sr30}^{2}} \cdot \Delta_{ifo} \cdot T_{sr2hr} \cdot R_{sr2ar} \cdot T_{sr2hr}$$

$$P_{sr2gbhr3sr3s} = 4.679 \times 10^{-31}$$

power scattered from SR2 GBHR3 toward SRM, W

$$P_{sr2gbhr3srms} := \frac{P_{sr2gbhr3}}{2} \cdot BRDF_{wall} \cdot \frac{w_{ifo}^2}{w_{srm0}^2} \cdot \Delta_{ifo} \cdot T_{sr2hr} \cdot R_{sr2ar} \cdot T_{sr2hr}$$

$$P_{sr2gbhr3srms} = 8.598 \times 10^{-33}$$

total power scattered from SR2 GBHR3

 $P_{sr2gbhr3s} := P_{sr2gbhr3sr3s} + P_{sr2gbhr3srms}$

$$P_{sr2gbhr3s} = 4.765 \times 10^{-31}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{sr2gbhr3} \coloneqq TF_{srbs} \cdot \left(\frac{P_{sr2gbhr3s}}{P_{psl}}\right)^{0.5} \cdot x_{hamflange} \cdot 2 \cdot k$$

$$DN_{sr2gbhr3} = 5.529 \times 10^{-3}$$

SR2 GBAR3

 $w_{sr20} = 9.4 \times 10^{-5}$

power incident on SR2 GBAR3 AR Baffle (forward and backward beams), W

$$P_{sr2gbar3} \coloneqq 2 \cdot P_{src} \cdot T_{sr2hr} \cdot R_{sr2ar} \cdot R_{sr2hr} \cdot T_{sr2ar}$$

$$P_{sr2gbar3} = 6.749 \times 10^{-2}$$

power scattered from SR3 AR Baffle, W

$$P_{sr2gbar3s} := P_{sr2gbar3} \cdot BRDF_{bd} \cdot \frac{w_{ifo}^{2}}{w_{sr20}^{2}} \cdot \Delta_{ifo} \cdot T_{sr2hr} \cdot R_{sr2ar} \cdot R_{sr2hr} \cdot T_{sr2ar}$$

$$P_{sr2gbar3s} = 6.88 \times 10^{-23}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{sr2gbar3} := TF_{srbs} \cdot \left(\frac{P_{sr2gbar3s}}{P_{psl}}\right)^{0.5} \cdot x_{ham} \cdot 2 \cdot k$$

$$DN_{sr2gbar3} = 1.446 \times 10^{-29}$$

$$P_{a} = 6749 \times 10^{-9}$$

 R_{bsar}

r