A Model-Based Cross-Correlation Search for Gravitational Waves from Scorpius X-1

John T. Whelan[1], Santosh Sundaresan[2] and Prabath Peiris[1]

[1]Center for Computational Relativity & Gravitation Rochester Institute of Technology, Rochester, NY, USA;
[2]Indian Institute for Science Education and Research, Kolkata, India

john.whelan@astro.rit.edu

Gravitational Waves from LMXBs

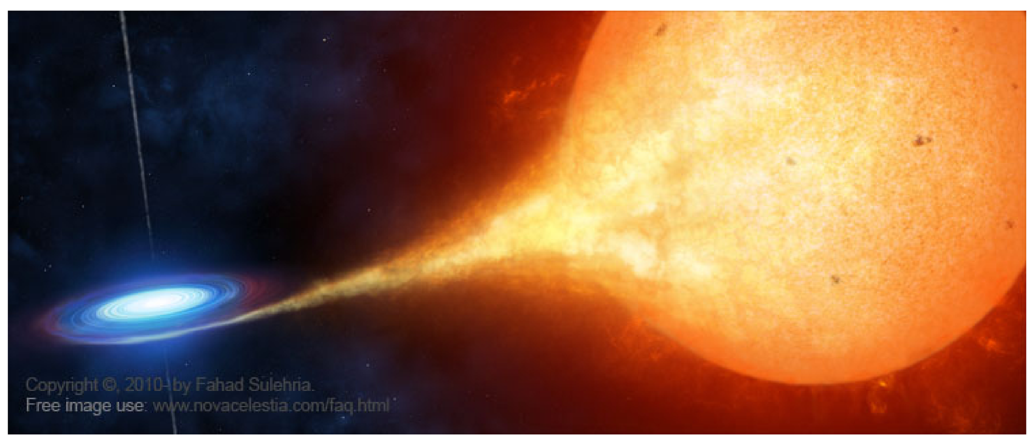

Figure 1: *Artist's impression of a low-mass X-ray binary. From Astronomical Illustrations and Space Art, by Fahad Sulehria, http://www.novacelestia.com/*

A low-mass X-ray binary is a binary of a compact object (neutron star or black hole) & a companion star. If the CO is a NS, accretion from the companion can produce a hot spot & power GW emission from the non-axisymmetric NS. If GW spindown balances accretion spinup, GW strength can be estimated from X-ray flux, and GW freq $\approx$ constant [1]. Sco X-1, the brightest LMXB, is thought to be a $1.4M_\odot$ NS + $0.42M_\odot$ companion[2]. Proposed & applied search methods include a fully coherent search over a small amount of data [3], an unmodelled search for a monochromatic stochastic signal [4], a search for a pattern of sidebands arising from the Doppler modulation of the signal by the binary orbit [5], and the modelled cross-correlation search described here[6]. These methods are currently being compared in a Mock Data Challenge; see poster C2.34.

Cross-Correlation Method

- Divide data into segments of length T_{sft} & take "short Fourier transform" (SFT) $\tilde{x}_I(f)$.
- Label segments by I, J, ... (I & J can be same or different times or detectors) & pairs by $\alpha, \beta, \ldots$.
- Use CW signal model ($\mathcal{A}_+ = \frac{1+\cos^2\iota}{2}$; $\mathcal{A}_\times = \cos\iota$)

$$h(t) = h_0\left[\mathcal{A}_+\cos\Phi(\tau(t))F_+ + \mathcal{A}_\times\sin\Phi(\tau(t))F_\times\right]$$

- expected cross-correlation between SFTs I & J

$$E\left[\tilde{x}_I^*(f_{k_I})\,\tilde{x}_J(f_{k_J})\right] = \tilde{h}_I^*(f_{k_I})\,\tilde{h}_J(f_{k_J}) = h_0^2\,\tilde{\mathcal{G}}_{IJ}\,\delta_{T_{\text{sft}}}(f_{k_I}-f_I)\,\delta_{T_{\text{sft}}}(f_{k_J}-f_J)$$

☞ f_I is signal freq @ time T_I Doppler shifted for detector I

☞ $\delta_{T_{\text{sft}}}(f-f') = \int_{-T_{\text{sft}}/2}^{T_{\text{sft}}/2} e^{i2\pi(f-f')t}\,dt$ so $\delta_{T_{\text{sft}}}(0) = T_{\text{sft}}$.

- Construct $\mathcal{Y}_{IJ} = \frac{\tilde{x}_I^*(f_{k_I})\tilde{x}_J(f_{k_J})}{(T_{\text{sft}})^2}$ (where $f_{k_I} \approx f_I$) s.t.

$$E[\mathcal{Y}_\alpha] \approx h_0^2\tilde{\mathcal{G}}_\alpha \qquad \text{Var}[\mathcal{Y}_{IJ}] \approx \sigma_{IJ}^2 = \frac{S_I(f_0)S_J(f_0)}{4(T_{\text{sft}})^2}$$

- Optimally combine into $\rho = \sum_\alpha(u_\alpha\mathcal{Y}_\alpha + u_\alpha^*\mathcal{Y}_\alpha^*)$ w/$u_\alpha \propto \frac{\tilde{\mathcal{G}}_\alpha^*}{\sigma_\alpha^2}$ so $E[\rho] = h_0^2\sqrt{2\sum_\alpha|\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}$ & $\text{Var}[\rho] = 1$

Computational considerations limit coherent integration time. Can make tunable semi-coherent search by restricting which SFT pairs α are included in $\rho = \sum_\alpha(u_\alpha\mathcal{Y}_\alpha + u_\alpha^*\mathcal{Y}_\alpha^*)$ E.g., only include pairs where $|T_I - T_J| \equiv |T_\alpha| \le T_{\max}$

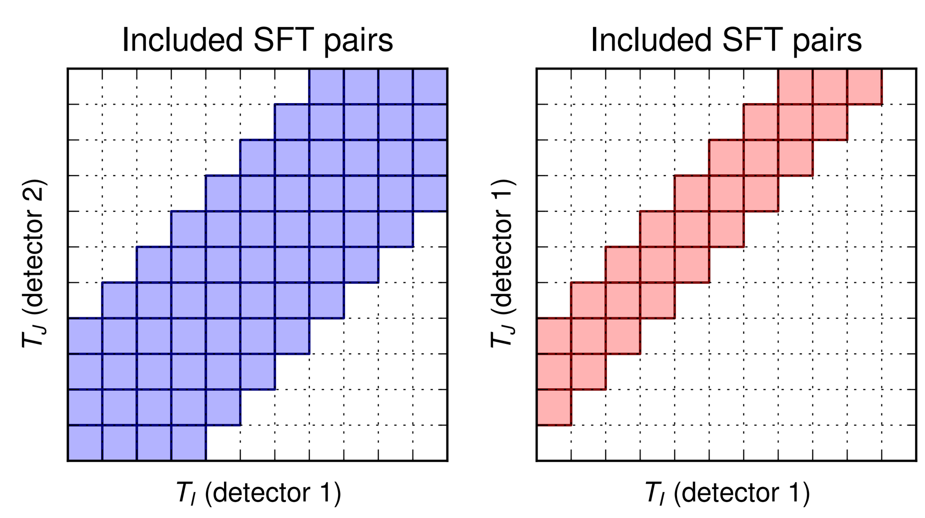

Figure 2: *SFT pairs for inclusion in sliding cross-correlation search. Left: data from different detectors at same or different times. Right: data from same detector at different times. In this illustrative example, $T_{max} = 3T_{sft}$.*

Parameter Space Metric

Consider dependence of ρ on parameters $\boldsymbol{\lambda} \equiv \{\lambda_i\}$. Can define Parameter space metric via

$$\frac{E[\rho]-E[\rho^{\text{true}}]}{E[\rho^{\text{true}}]} = -g_{ij}(\Delta\lambda^i)(\Delta\lambda^j) + \mathcal{O}([\Delta\lambda]^3)$$

$$g_{ij} = -\frac{1}{2}\frac{E[\rho_{,ij}]\big|_{\lambda=\lambda_{\text{true}}}}{E[\rho^{\text{true}}]}$$

Assume dominant contribution to $E[\rho_{,ij}]$ is from variation of $\Delta\Phi_{IJ} = \Phi_I - \Phi_J$; get phase metric

$$g_{ij} = \frac{1}{2}\frac{\sum_\alpha \Delta\Phi_{\alpha,i}\Delta\Phi_{\alpha,j}|\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}{\sum_\beta|\tilde{\mathcal{G}}_\beta|^2/\sigma_\beta^2} \equiv \frac{1}{2}\left\langle\Delta\Phi_{\alpha,i}\Delta\Phi_{\alpha,j}\right\rangle_\alpha$$

Note $\langle\ \rangle_\alpha$ is average over pairs weighted by $|\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2$; ignoring weighting factor would give usual metric [7]

$$\langle\Phi_{I,i}\Phi_{I,j}\rangle_I - \langle\Phi_{I,i}\rangle_I\langle\Phi_{J,j}\rangle_J$$

Define $T_{IJ} = T_I - T_J \equiv T_\alpha$ as time offset between SFTs; T_α^{av} is average time. For each detector pair, avg over pairs is avg over T_α & T_α^{av}. If we assume the avg over T_α^{av} evenly samples orbital phase, the metric in parameters f_0 (signal frequency), a_p (orbit radius projected along line of sight), $\tilde{T}$ (time orbit crosses reference point) & P_{orb} (orbit period) is approximately diagonal, with

$$g_{f_0f_0} \approx 2\pi^2\left\langle T_\alpha^2\right\rangle_\alpha \qquad g_{P_{\text{orb}}P_{\text{orb}}} = \frac{4\pi^4 f_0^2 a_p^2\sigma_T^2}{P_{\text{orb}}^4}\left\langle\sin^2\frac{\pi T_\alpha}{P_{\text{orb}}}\right\rangle_\alpha$$

$$g_{a_pa_p} = \pi^2 f_0^2\left\langle\sin^2\frac{\pi T_\alpha}{P_{\text{orb}}}\right\rangle_\alpha \qquad g_{\tilde{T}\tilde{T}} = \frac{4\pi^4 f_0^2 a_p^2}{P_{\text{orb}}^2}\left\langle\sin^2\frac{\pi T_\alpha}{P_{\text{orb}}}\right\rangle_\alpha$$

(For reasonable values of σ_T ~observation time, $g_{P_{\text{orb}}P_{\text{orb}}}$ is small enough that we don't need to search over P_{orb}.)

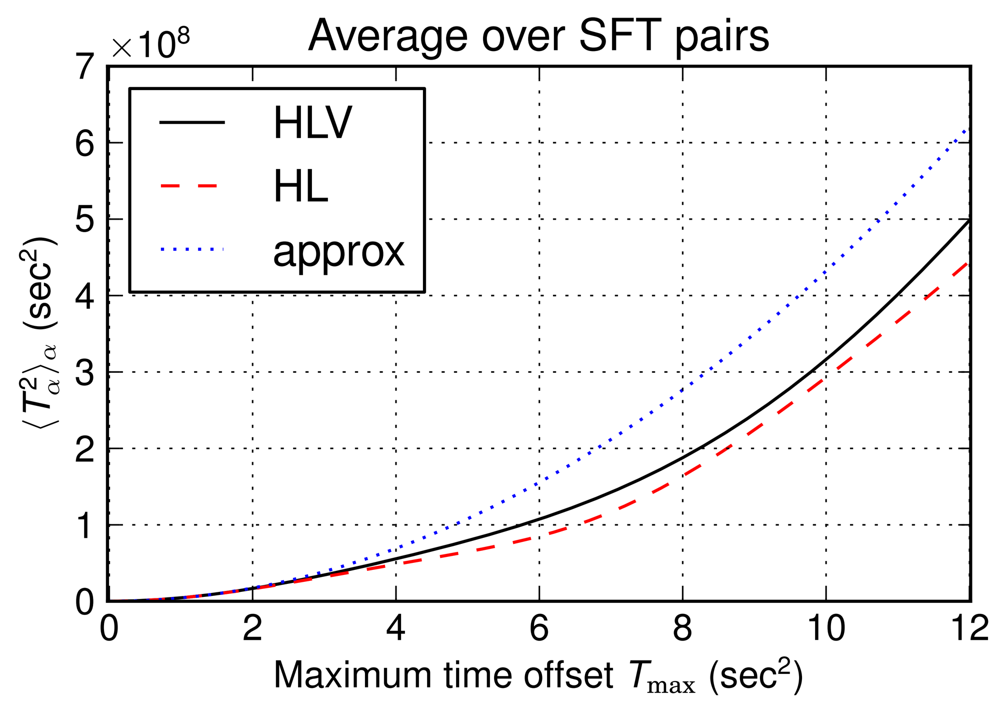

Figure 3: *Factor $\left\langle T_\alpha^2\right\rangle_\alpha$ appearing in metric element $g_{f_0f_0}$. The metric is slightly over-estimated if geometry factors are ignored, relative to actual calculations for Hanford-Livingston (HL) and Hanford-Livingston-Virgo (HLV) network. Behavior of factor $\left\langle\sin^2\frac{\pi T_\alpha}{P_{orb}}\right\rangle_\alpha$ appearing in other metric components is similar.*

The metric determines how large a lag time $T_{\max}$ can be allowed while keeping computing cost manageable.

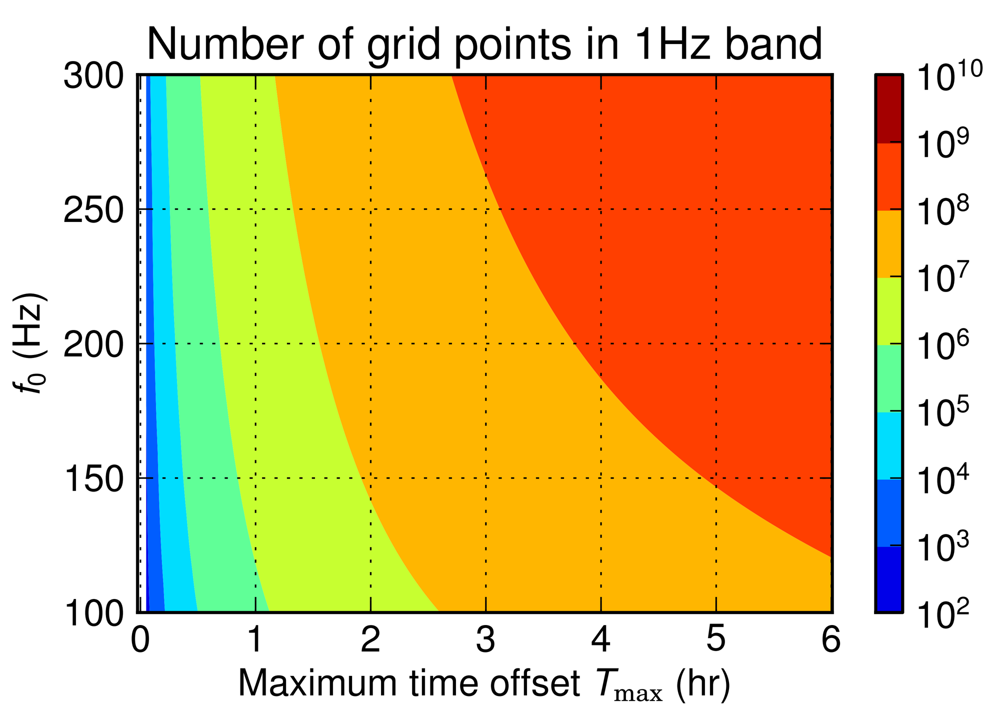

Figure 4: *Number of templates needed for a search for GW from Sco X-1 at each frequency as a function of lag time. We allow a 20% mismatch and cover the one-sigma uncertainties in the parameters f_0, a_p and $\tilde{T}$ from [8].*

Sensitivity Estimates

Search is sensitive to signal of amplitude

$$h_0 = \left(\frac{\mathcal{S}^2}{\sum_\alpha|\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}\right)^2$$

where $\mathcal{S}$ is a statistical factor. $\tilde{\mathcal{G}}_\alpha$ depends on (unknown) spin orientation angles ι & ψ; standard approach is to average value of $\tilde{\mathcal{G}}_\alpha$ over $\cos\iota$ & ψ. The ψ effect is small after average over sidereal time. The ι effect means actually

$$E[\rho] \approx h_0^2\frac{\mathcal{A}_+^2+\mathcal{A}_\times^2}{2}\sqrt{2\sum_\alpha|\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}$$

Net effect is to change statistical factor $\mathcal{S}$, which reduces the sensitivity of the search to h_0 by a factor of $\sqrt{\mathcal{S}^{\text{eff}}/\mathcal{S}}$.

	$\mathcal{S}$			$\mathcal{S}^{\text{eff}}$			$\sqrt{\mathcal{S}^{\text{eff}}/\mathcal{S}}$		
	FD			FD			FD		
FA	0.10	0.05	0.01	0.10	0.05	0.01	0.10	0.05	0.01
0.10	1.81	2.07	2.55	3.49	4.45	6.27	1.39	1.47	1.57
0.05	2.07	2.33	2.81	4.15	5.16	7.03	1.42	1.49	1.58
0.01	2.55	2.81	3.29	5.42	6.52	8.47	1.46	1.52	1.60

Table 1: *Approximate modification of search sensitivity, as a function of desired false alarm probability and false dismissal probability, resulting from filtering with a template averaged over the signal parameters $\cos\iota$ and ψ. The detectable signal amplitude h_0 is proportional to $\sqrt{\mathcal{S}^{\text{eff}}}$, so the sensitivity is reduced by the factor in the third group of columns. Note that the worst-case value for this is* $\sqrt{16/5} \approx 1.79$

We illustrate the sensitivity of the search, and its dependence on the maximum allowed time lag $T_{\max}$, using the advanced LIGO and Virgo design noise spectra from [9], and assuming a one-year observation. We plot the h_0 level that could be detected with 10% false-alarm and false-dismissal.

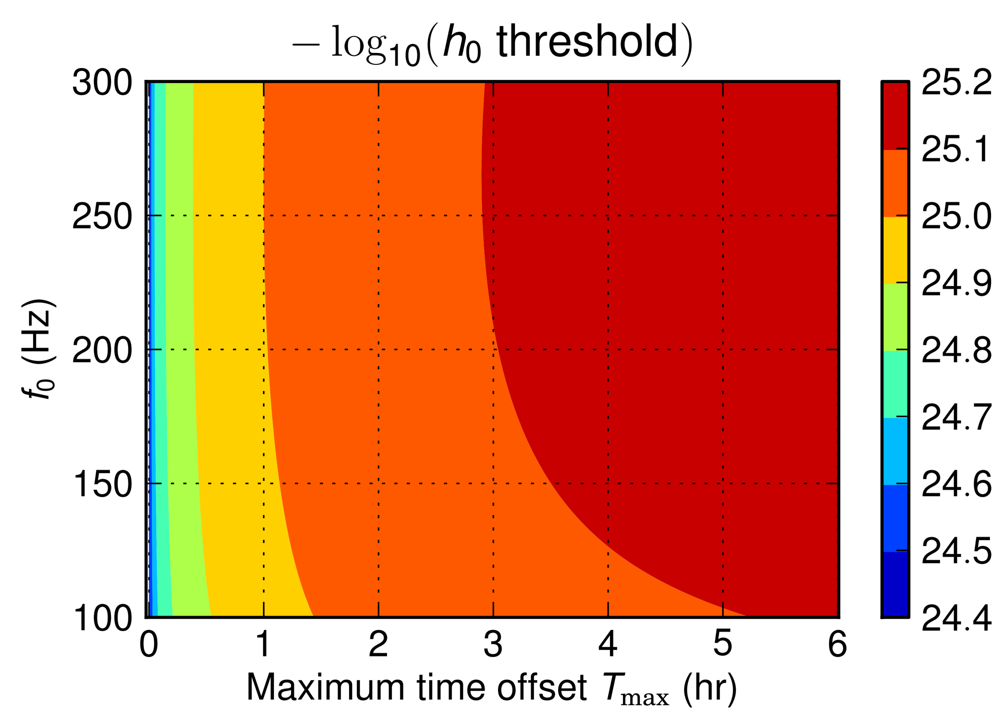

Figure 5: *Sensitivity of a cross-correlation search for Sco X-1 with one year of advanced LIGO and Virgo design-sensitivity data, assuming 10% false-alarm &-dismissal. Note that the $T_{max} = 0$ measurement is effectively the directed stochastic "radiometer" search.*

References

[1] Bildsten, *ApJL* **501**, L89 (1998)

[2] Steeghs & Casares *ApJ* **568**, 273 (2002)

[3] LSC, *PRD* **76**, 082001 (2007)

[4] Ballmer, *CQG* **23**, S179 (2006)
LSC, *PRD* **76**, 082003 (2007)

[5] Messenger & Woan, *CQG* **24**, S469 (2007)

[6] Dhurandhar et al, *PRD* **77**, 082001 (2008)

[7] Pletsch, *PRD* **82**, 042002 (2010)

[8] Galloway et al in preparation

[9] LSC & Virgo, arXiv:1304.0670
https://dcc.ligo.org/LIGO-P1200087-v18/public