T1300324-v3 ITM Elliptical Baffle Scatter
8/12/13

BRDF of ellip baf, sr^-1 $BRDF_{ellbaf} := 0.030$

BRDF of edge, sr^-1 $BRDF_{edge} := 0.1$

BRDF of chamber wall, sr^-1 $BRDF_{wall} := 0.1$

Motion of suspended baffle @ 100 Hz, m/rt Hz $x_{baf} := 3 \cdot 10^{-14}$

laser wavelength, m $\lambda := 1.064 \cdot 10^{-6}$

wave number, m^-1 $k := 2 \cdot \frac{\pi}{\lambda}$ $k = 5.9052 \times 10^{6}$

Transfer function @ 100 Hz, ITM AR $TF_{itmar} := 3.16 \cdot 10^{-11}$

ITM beam radius, m $w_{itm} := 0.053168$

virtual beam waist looking toward ITM AR
(see H1 Signal Recycling Cavity beam
size_8-12-13) $w_{itmar0} := 0.008342$

distance from ITM AR to virtual beam waist, m $l_{itmar0} := 1.293 \times 10^{3}$

solid angle of ITM AR virtual beam
waist, sr $\Delta_{itmar} := \frac{\lambda^2}{\pi \cdot w_{itmar0}^{\,2}}$

$\Delta_{itmar} = 5.1784 \times 10^{-9}$

Ref: T1000090-v5, aLIGO Baffle Design using SIS

elliptical baffle horizontal
semi-axis, m $a := \frac{0.21 + 0.014}{2}$ $a = 0.112$

elliptical baffle vertical semi-axis, m	$b := \frac{0.260 + 0.014}{2}$	$b = 0.137$
baffle incidence angle, rad	$\theta_b := 33 \cdot \frac{\pi}{180}$	
Reflectivity of Elliptical Baffle	$R_{ellbaf} := 2.4e\text{-}5$	
Transmissivity of ITM HR	$T_{itmhr} := 0.0140$	
Reflectivity of ITM HR	$R_{itmhr} := 0.9951 - T_{itmhr}$	$R_{itmhr} = 0.9811$
Reflectivity of ITM AR	$R_{itmar} := 50 \cdot 10^{-6}$	
Transmissivity of ETM HR	$T_{etm} := 5 \cdot 10^{-6}$	
Transmissivity of FM HR	$T_{FMhr} := 10 \times 10^{-5}$	

Ref. T070247

input laser power, W	$P_{psl} := 125$	
arm cavity gain	$G_{ac} := 13000$	
arm cavity power, W	$P_a := \frac{P_{psl}}{2} \cdot G_{ac}$	$P_a = 8.125 \times 10^5$

Ref. Hiro e-mail 8/29/11

power in power recycling cavity arm, W	$P_{rca} := \frac{P_a \cdot T_{itmhr}}{4}$	$P_{rca} = 2.8438 \times 10^3$
Gaussian power parameter in recycling cavity	$P_{0rc} := P_{rca}$	
Power recycling cavity gain	$G_{rc} := \frac{2 \cdot P_{rca}}{P_{psl}}$	$G_{rc} = 45.5$

Gaussian beam equation in recycling cavity

$$I_{rc}(x,y) := 2\cdot\frac{P_{0rc}}{\pi\cdot w_{itm}^{2}}\cdot e^{-2\cdot\left(\frac{x^2+y^2}{w_{itm}^{2}}\right)}$$

total integrated power in recycling cavity arm, W

$$P_{rca} := 4\cdot\int_{0}^{b}\int_{0}^{a\cdot\sqrt{1-\frac{y^2}{b^2}}} I_{rc}(x,y)\,dx\,dy$$

$$P_{rca} = 2.8436\times10^{3}$$

radius of ITM, m

$$r_{itm} := 0.170$$

horizontal displacement of ITM Elliptical Baffle, m

$$\delta x := 0$$

vertical displacement of ITM Elliptical Baffle, m

$$\delta y := 0$$

exitance function from ITM

$$I_{itm}(x,y) := 2\cdot\frac{4\cdot P_{0rc}}{\pi\cdot w_{itm}^{2}}\cdot e^{-2\cdot\left(\frac{x^2+y^2}{w_{itm}^{2}}\right)}$$

power exiting from ITM toward elliptical baffle, W

$$P_{itm} := 4\cdot\int_{0}^{r_{itm}}\int_{0}^{r_{itm}\cdot\sqrt{1-\frac{y^2}{r_{itm}^{2}}}} I_{itm}(x,y)\,dx\,dy$$

$$P_{itm} = 1.1375\times10^{4}$$

also check

$$P_{itm} := P_a\cdot T_{itmhr} \qquad P_{itm} = 1.1375\times10^{4}$$

ITM ELLIPTICAL BAFFLE

Power hitting ITM Ellip Baf, direct integration

$$P_{baf} := \int_{-r_{itm}}^{-a}\int_{0}^{\sqrt{r_{itm}^2-x^2}} 2\cdot I_{itm}(x,y)\,dy\,dx + \int_{a}^{r_{itm}}\int_{0}^{\sqrt{r_{itm}^2-x^2}} 2\cdot I_{itm}(x,y)\,dy\,dx + \int_{-a}^{a}\int_{\sqrt{\left(1-\frac{x^2}{a^2}\right)\cdot b^2}}^{\sqrt{r_{itm}^2-x^2}}$$

$$P_{baf} = 0.5342$$

intercepton efficiency for ITM power

$$\eta_{itmellbaf} := \frac{P_{baf}}{P_{itm}}$$

$$\eta_{itmellbaf} = 4.6958 \times 10^{-5}$$

SCATTERED LIGHT PHASE INTERFERENCE

distance from scattering point to virtual ITM AR beam waist

$$d(x,y,\theta_b) := \sqrt{x^2 + y^2 + \left(\frac{x}{\tan(\theta_b)} - l_{itmar0}\right)^2}$$

Power scattered from ITM Ellip Baf surface

$$P_{Tbafs} = \int I_{itm}(x,y)\cdot BRDF_{ellbaf}\cdot\Delta_{itmar}\,dA$$

$$P_{Tbafs} = P1_{bafs} + P2_{bafs} + P3_{bafs}$$

Partial power integrals

$$P1_{bafs} := \int_{-r_{itm}}^{-a}\int_{0}^{\sqrt{r_{itm}^2-x^2}} 2\cdot I_{itm}(x,y)\cdot BRDF_{ellbaf}\cdot\Delta_{itmar}\,dy\,dx$$

$$P2_{bafs} := \int_{a}^{r_{itm}}\int_{0}^{\sqrt{r_{itm}^2-x^2}} 2\cdot I_{itm}(x,y)\cdot BRDF_{ellbaf}\cdot\Delta_{itmar}\,dy\,dx$$

$$P3_{bafs} := \int_{-a}^{a}\int_{\sqrt{\left(1-\frac{x^2}{a^2}\right)\cdot b^2}}^{\sqrt{r_{itm}^2-x^2}} 2\cdot I_{itm}(x,y)\cdot BRDF_{ellbaf}\cdot\Delta_{itmar}\,dy\,dx$$

total scattered power from two baffles

$$P_{bafs} := \sqrt{2}\cdot\left(P1_{bafs}+P2_{bafs}+P3_{bafs}\right)$$

$$P_{bafs} = 1.1732\times 10^{-10}$$

Scattered electric field into IFO with round trip phase difference

$$dE_{bafs} = \sqrt{I_{itm}(x,y)\cdot dA\cdot BRDF_{ellbaf}\cdot\Delta_{itmar}}\cdot e^{i\cdot 2\cdot k\cdot d\left(x,y,\theta_b\right)}$$

$$dE_{bafs} = \sqrt{I_{itm}(x,y)\cdot dA\cdot BRDF_{ellbaf}\cdot\Delta_{itmar}\cdot e^{i\cdot 4\cdot k\cdot d\left(x,y,\theta_b\right)}}$$

$$E_{bafs} = \sqrt{\iint I_{itm}(x,y)\cdot BRDF_{ellbaf}\cdot\Delta_{itmar}\cdot e^{i\cdot 4\cdot k\cdot d\left(x,y,\theta_b\right)}\,dA}$$

$$E_{bafs} = \sqrt{P_{s1}\left(\theta_b\right)+P_{s2}\left(\theta_b\right)+P_{s3}\left(\theta_b\right)}$$

scattered power integral 1

$$P_{s1}\left(\theta_b\right) := \int_{-r_{itm}}^{-a}\int_{0}^{\sqrt{r_{itm}^2-x^2}} 2\cdot I_{itm}(x,y)\cdot BRDF_{ellbaf}\cdot\Delta_{itmar}\cdot e^{i\cdot 4\cdot k\cdot d\left(x,y,\theta_b\right)}\,dy\,dx$$

scattered power integral 2

$$P_{s2}(\theta_b) := \int_a^{r_{itm}} \int_0^{\sqrt{r_{itm}^2 - x^2}} 2 \cdot I_{itm}(x,y) \cdot BRDF_{ellbaf} \cdot \Delta_{itmar} \cdot e^{i \cdot 4 \cdot k \cdot d(x,y,\theta_b)} \, dy \, dx$$

scattered power integral 3

$$P_{s3}(\theta_b) := \int_{-a}^{a} \int_{\sqrt{\left(1 - \frac{x^2}{a^2}\right) \cdot b^2}}^{\sqrt{r_{itm}^2 - x^2}} 2 \cdot I_{itm}(x,y) \cdot BRDF_{ellbaf} \cdot \Delta_{itmar} \cdot e^{i \cdot 4 \cdot k \cdot d(x,y,\theta_b)} \, dy \, dx$$

Total scattered field per baffle

$$E_{bafs}(\theta_b) := \sqrt{P_{s1}(\theta_b) + P_{s2}(\theta_b) + P_{s3}(\theta_b)}$$

$$E_{bafs}(\theta_b) = 1.1705 \times 10^{-6} + 1.0958i \times 10^{-6}$$

total scattered power from two baffles

$$P_{bafs}(\theta_b) := \sqrt{2} \cdot \left(\left|E_{bafs}(\theta_b)\right|\right)^2$$

$$P_{bafs}(\theta_b) = 3.6355 \times 10^{-12}$$

displacement noise @ 100 Hz,
m/rtHz

$$DN_{itmbaf}(\theta_b) := TF_{itmar} \cdot \left(\frac{P_{bafs}(\theta_b)}{P_{psl}}\right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmbaf}(\theta_b) = 1.3502 \times 10^{-24}$$

Scattered electric field into IFO with no phase difference

$$d(x,y,\theta_b) := 0$$

scattered power integral 1

$$P_{s1}(\theta_b) := \int_{-r_{itm}}^{-a} \int_{0}^{\sqrt{r_{itm}^2 - x^2}} 2 \cdot I_{itm}(x, y) \cdot BRDF_{ellbaf} \cdot \Delta_{itmar} \cdot e^{i \cdot 4 \cdot k \cdot d(x, y, \theta_b)} \, dy \, dx$$

scattered power integral 2

$$P_{s2}(\theta_b) := \int_{a}^{r_{itm}} \int_{0}^{\sqrt{r_{itm}^2 - x^2}} 2 \cdot I_{itm}(x, y) \cdot BRDF_{ellbaf} \cdot \Delta_{itmar} \cdot e^{i \cdot 4 \cdot k \cdot d(x, y, \theta_b)} \, dy \, dx$$

scattered power integral 3

$$P_{s3}(\theta_b) := \int_{-a}^{a} \int_{\sqrt{\left(1 - \frac{x^2}{a^2}\right) \cdot b^2}}^{\sqrt{r_{itm}^2 - x^2}} 2 \cdot I_{itm}(x, y) \cdot BRDF_{ellbaf} \cdot \Delta_{itmar} \cdot e^{i \cdot 4 \cdot k \cdot d(x, y, \theta_b)} \, dy \, dx$$

Total scattered field

$$E_{bafs}(\theta_b) := \sqrt{P_{s1}(\theta_b) + P_{s2}(\theta_b) + P_{s3}(\theta_b)}$$

$$E_{bafs}(\theta_b) = 9.1081 \times 10^{-6}$$

total scattered power from two baffles

$$P_{bafs\_0phase}(\theta_b) := \sqrt{2} \cdot \left(\left| E_{bafs}(\theta_b) \right| \right)^2$$

$$P_{bafs\_0phase}(\theta_b) = 1.1732 \times 10^{-10}$$

displacement noise @ 100 Hz,
m/rtHz

$$DN_{itmbaf\_0phase}(\theta_b) := TF_{itmar} \cdot \left(\frac{P_{bafs\_0phase}(\theta_b)}{P_{psl}} \right)^{0.5} \cdot x_{baf} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{itmbaf\_0phase}(\theta_b) = 7.67 \times 10^{-24}$$

compare with phase calculation

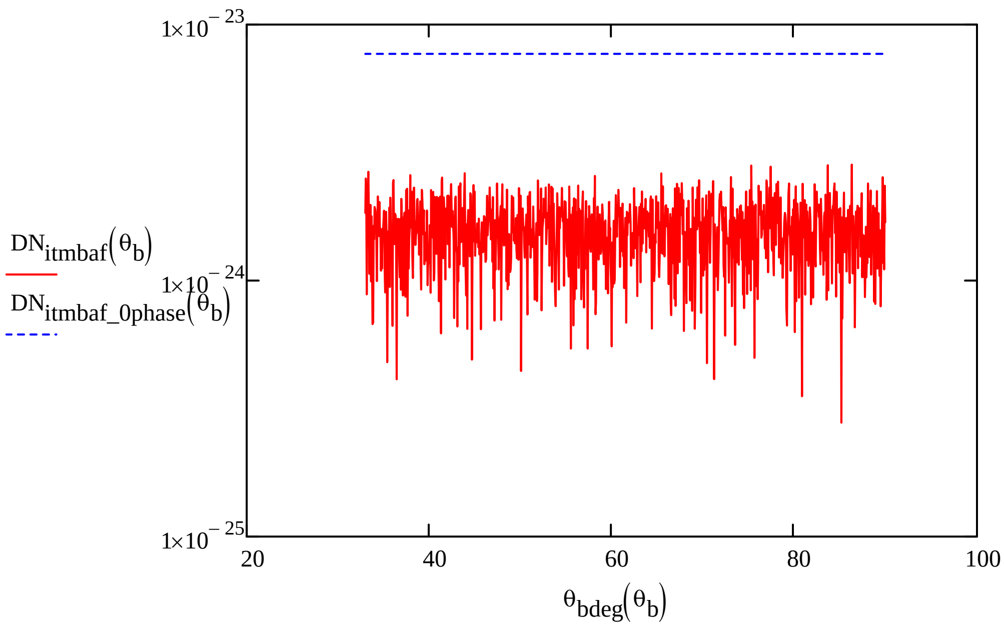
$DN_{itmbaf}(\theta_b) = 1.3502 \times 10^{-24}$

$P_{bafs}(\theta_b) = 3.6355 \times 10^{-12}$

$\theta_b := 33 \cdot \frac{\pi}{180}$ $\qquad$ $\theta_b = 0.5759587$

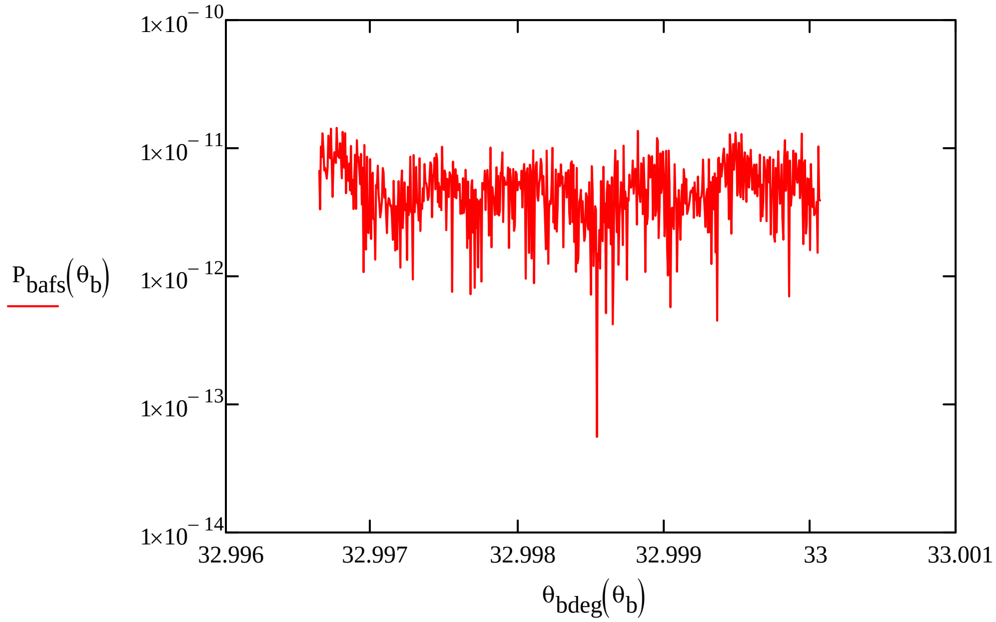
$\theta_b := 0.576, 0.577 .. 1.57$

$\theta_{bdeg}(\theta_b) := \theta_b \cdot \frac{180}{\pi}$



$\theta_b := 0.5759, 0.5759001 .. 0.57596$

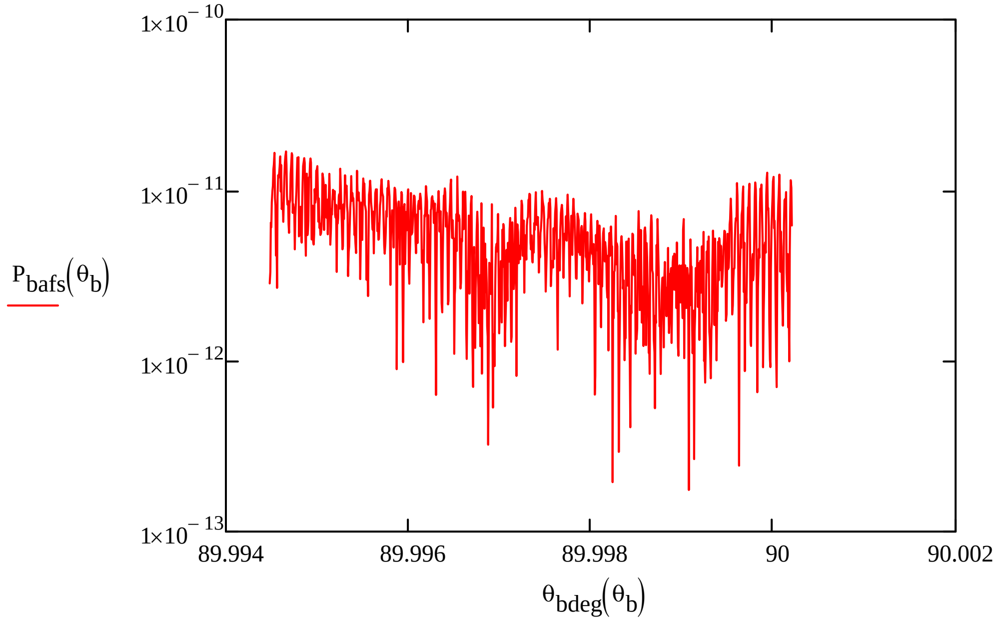
$\theta_{bdeg}(\theta_b) := \theta_b \cdot \frac{180}{\pi}$



$\theta_b := 1.5707, 1.5707001 .. 1.5708$

$\frac{\pi}{2} = 1.5708$

$\theta_{bdeg}(\theta_b) := \theta_b \cdot \frac{180}{\pi}$


1×10^− 10
1×10^− 11
1×10^− 12
1×10^− 13
Pbafs(θb)
89.994
89.996
89.998
90
90.002
θbdeg(θb)

$2 \cdot I_{itm}(x, y)\ dy\ dx$